2022-EE-”40-52”

ai24btech11028 - Ronit Ranjan

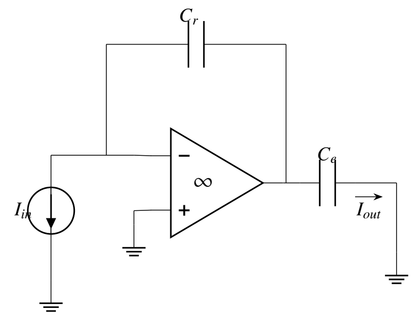
1) e^A denotes the exponential of a square matrix A. Suppose λ is an eigenvalue and v is the corresponding eigen-vector of matrix A.
Consider the following two statements:
Statement 1: e^λ is an eigenvalue of e^A.
Statement 2: v is an eigenvector of e^A.
Which one of the following options is correct?
   a) Statement 1 is true and statement 2 is false
   b) Statement 1 is false and statement 2 is true
   c) Both the statements are correct.
   d) Both the statements are false.
2) Let $f(x) = \int_0^x e^t (t-1)(t-2)\, dt.$. Then $f(x)$ decreases in the interval
   a) $x \in (1,2)$
   b) $x \in (2,3)$
   c) $x \in (0,1)$
   d) $x \in (0.5,1)$
3) Consider a matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix. Then $(c+d)$ is equal to
   a) 5
   b) 17
   c) -6
   d) 11
4) The fuel cost functions in rupees/hour for two 600 MW thermal power plants are given by

$$Plant1 : C_1 = 350 + 6P_1 + 0.004P_1^2 Plant2 : C_2 = 450 + aP_2 + 0.003P_2^2$$

where P_1 and P_2 are power generated by plant 1 and plant 2, respectively, in MW and a is constant. The incremental cost of power (λ) is 8 rupees per MWh. The two thermal power plants together meet a total power demand of 550 MW. The optimal generation of plant 1 and plant 2 in MW, respectively, are
   a) 200, 350
   b) 250, 300
   c) 325, 225
   d) 350, 200
5) The current gain (I_{out}/I_{in})in the circuit with an ideal current amplifier given below is
   a) $\frac{C_f}{C_c}$
   b) $\frac{C_f}{C_c}$
   c) $\frac{C_f}{C_c}$



d) $\frac{C_f}{C_c}$

6) If the magnetic field intensity (H) in a conducting region is given by the expression $H = x^2\hat{i} + x^2y^2\hat{j} + x^2y^2z^2\hat{k}A/m$. The magnitude of the current density, in A/m^2, at x = 1m, y = 2m, and z = 1m is
   a) 8
   b) 12
   c) 16
   d) 20

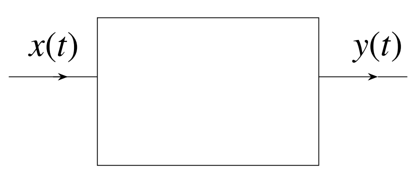
7) Let a causal LTI system be governed by the following differential equation $y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$, where $x(t)$ and $y(t)$ are the input and output respectively. Its impulse response is
   a) $2e^{-\frac{1}{4}u(t)}$
   b) $2e^{-4u(t)}$
   c) $8e^{-\frac{1}{4}u(t)}$
   d) $2e^{-4u(t)}$

8) Let an input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$ is passed through an LTI system having an impulse response,

$$h(t) = 2\left(\frac{2\sin(10\pi t)}{\pi t}\right)\cos(40\pi t)$$

   The output of the system is
   a) $2\sin(10\pi t) + 5\cos(15\pi t)$
   b) $5\cos(15\pi t) + 7\sin(42\pi t)$
   c) $7\sin(42\pi t) + 4\cos(45\pi t)$
   d) $2\sin(10\pi t) + 4\cos(45\pi t)$

9) Consider the system as shown below where $y(t) = x(e^t)$. The system is



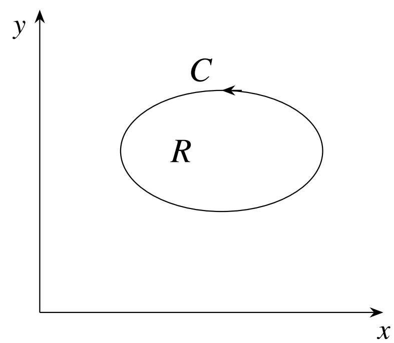
   a) linear and causal.
   b) linear and non-causal.
   c) non-linear and causal.
   d) non-linear and non-causal.

10) The discrete time Fourier series representation of a signal $x[n]$ with period N is written as $x[n] = \sum_{k=0}^{N-1} a_k e^{j(2\pi kn/N)}$. A discrete time periodic signal with period $N = 3$, has the non-zero Fourier series coefficients: $a_{-3} = 2$ and $a_4 = 1$. The signal is:
   a) $2 + 2e^{-j\frac{2\pi}{6}n}\cos\left(\frac{2\pi}{6}n\right)$

b) $1 + 2e^{-j\frac{2\pi}{6}n} \cos\left(\frac{2\pi}{6}n\right)$
c) $1 + 2e^{j\frac{2\pi}{6}n} \cos\left(\frac{2\pi}{6}n\right)$
d) $2 + 2e^{j\frac{2\pi}{6}n} \cos\left(\frac{2\pi}{6}n\right)$

11) Let $f(x, y, z) = 4x^2 + 7xy + 3xz^2$. The direction in which the function $f(x, y, z)$ increases most rapidly at point $P = (1, 0, 2)$ is:
a) $20\hat{i} + 7\hat{j}$
b) $20\hat{i} + 7\hat{j} + 12\hat{k}$
c) $20\hat{i} + 12\hat{k}$
d) $20\hat{i}$

12) Let R be a region in the first quadrant of the xy plane enclosed by a closed curve C considered in counter-clockwise direction. Which of the following expressions does not represent the area of the region R?



a) $\iint_R dx\,dy$
b) $\oint_C x\,dy$
c) $\oint_C y\,dx$
d) $\frac{1}{2}\oint_C (x\,dy - y\,dx)$

13) Let $\hat{E}(x, y, z) = 2xz^2\,\hat{i} + 5y\,\hat{j} + 3z\,\hat{k}$. The value of $\iiint_V (\nabla \cdot \overrightarrow{E}), dV$, where V is the volume enclosed by the unit cube defined by $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$, is:
a) 3
b) 8
c) 10
d) 12