

# JEE Chapter 3 A,B

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## SECTION A

- 1) The mean and variance of the data 4, 5, 6, 6, 7, 8,  $x$ ,  $y$  where  $x < y$  are 6 and  $\frac{9}{4}$  respectively. Then  $x^4 + y^2$  is equal to

- a) 162                      c) 674  
b) 320                      d) 420

- 2) If a point  $A(x, y)$  lies in the region bounded by the  $y$ -axis, straight lines  $2y + x = 6$  and  $5x - 6y = 30$ , then the probability that  $y < 1$  is:

- a)  $\frac{1}{6}$                       c)  $\frac{2}{3}$   
b)  $\frac{5}{6}$                       d)  $\frac{3}{7}$

- 3) The value of  $\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$  is

- a)  $\frac{26}{25}$                       c)  $\frac{50}{51}$   
b)  $\frac{25}{26}$                       d)  $\frac{51}{50}$

- 4)  $\alpha = \sin 36^\circ$  is a root of which of the following equation

- a)  $10x^4 - 10x^2 - 5 = 0$     c)  $16x^4 - 20x^2 + 5 = 0$   
b)  $16x^4 + 20x^2 - 5 = 0$     d)  $16x^4 - 10x^2 + 5 = 0$

- 5) Which of the following statement is a tautology?

- a)  $((\sim q \cap p) \cap q)$                       c)  $((\sim q) \cap p) \cup (p \cup (\sim p))$   
b)  $((\sim q) \cap p) \cap (p \cap (\sim p))$   
d)  $((p \hat{q}) \hat{(\sim (p \cap q))})$

- 6) Let  $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ . Define the function  $f : S \rightarrow S$  as follows:

$$f(n) = \begin{cases} 2n, & \text{if } n = 1, 2, 3, 4, 5, \\ 2(11 - n), & \text{if } n = 6, 7, 8, 9, 10. \end{cases}$$

Let  $g : S \rightarrow S$  be a function such that:

$$g(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd,} \\ 11 - \frac{n}{2}, & \text{if } n \text{ is even.} \end{cases}$$

Find the value of  $g(10)(g(1) + g(2) + g(3) + g(4) + g(5))$ .

- 7) Let  $\alpha, \beta$  be the roots of the equation:

$$x^2 - 4x + 5 = 0,$$

and let  $\alpha\gamma, \beta\gamma$  be the roots of the equation:

$$x^2 - (3\sqrt{2} + 7\sqrt{3})x + (7 + 3\sqrt{5}) = 0.$$

If  $\beta + \gamma = 3\sqrt{2}$ , then find  $(\alpha + 2\beta + \gamma)^2$ .

- 8) Let  $A$  be a matrix of order  $2 \times 2$ , whose entries are from the set  $\{0, 1, 2, 3, 4, 5\}$ . If the sum of all the entries of  $A$  is a prime number  $p$ ,  $2 \leq p < 8$ , find the number of such matrices  $A$ .
- 9) If the sum of the coefficients of all the positive powers of  $x$  in the binomial expansion of

$$\left(x + \frac{2}{x}\right)^n$$

is 939, find the sum of all the possible integral values of  $n$ .

- 10) Let  $[t]$  denote the greatest integer  $\leq t$  and  $\{t\}$  denote the fractional part of  $t$ . Then the integral value of  $\alpha$  for which the left-hand limit of the function:

$$f(x) = [1 + x] + \frac{\alpha x^{3/2}\{x\} - 1}{2[x] + \{x\}}$$

at  $x = 0$  is equal to  $\alpha - \frac{4}{3}$ .

- 11) If  $y(x) = x^x$ ,  $x > 0$ , then find the value of:

$$\frac{d^2x}{dy^2} \text{ at } x = 1.$$

- 12) If the area of the region  $\{(x, y) : x^3 + y^3 \leq 1, x + y \geq 0, y \geq 0\}$  is  $A$ , find the value of  $\frac{256A}{\pi}$ .

- 13) Let  $v$  be the solution of the differential equation:

$$(1 - x^2) \frac{dy}{dx} = (xy + (x^3 + 2)\sqrt{1 - x^2}), -1 < x < 1,$$

and  $y(0) = 0$ . If:

$$\int_{-1/2}^{1/2} \sqrt{1 - x^2} y(x) dx = k,$$

then  $k^{-1}$  is equal to:

- 14) Let a circle  $C$  of radius 5 lie below the x-axis. The line  $L_1 : 4x + 3y - 2 = 0$  passes through the center  $P$  of the circle  $C$  and intersects the line  $L_2 : 3x - 4y - 11 = 0$  at  $Q$ . The line  $L_1$  touches  $C$  at the point  $Q$ . Then the distance of  $P$  from the line  $5x - 12y + 51 = 0$  is:
- 15) Let  $S = \{E_1, E_2, \dots, E_8\}$  be a sample space of random experiments such that  $P(E_n) = \frac{n}{36}$  for every  $n = 1, 2, \dots, 8$ . Then the number of elements in the set

$$\left\{ A \subset S : P(A) \geq \frac{4}{5} \right\}$$

is: