# 20th June 2022-shift 2-16-30

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## Section A

1) The mean and variance of the data $4, 5, 6, 6, 7, 8, x, y$ where $x < y$ are 6 and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to [20th June shift 2, 2020]

a) 162
b) 320
c) 674
d) 420

2) If a point $A(x, y)$ lies in the region bounded by the y-axis, straight lines $2y + x = 6$ and $5x - 6y = 30$, then the probablity that $y < 1$ is: [20th June shift 2, 2020]

a) $\frac{1}{6}$
b) $\frac{5}{6}$
c) $\frac{2}{3}$
d) $\frac{6}{7}$

3) The value of $\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is [20th June shift 2, 2020]

a) $\frac{26}{25}$
b) $\frac{25}{26}$
c) $\frac{50}{51}$
d) $\frac{52}{51}$

4) $\alpha = \sin 36°$ is a root of which of the following equation [20th June shift 2, 2020]

a) $10x^4 - 10x^2 - 5 = 0$
b) $16x^4 + 20x^2 - 5 = 0$
c) $16x^4 - 20x^2 + 5 = 0$
d) $16x^4 - 10x^2 + 5 = 0$

5) Which of the following statement is a tautology? [20th June shift 2, 2020]

a) $((\sim q \cap p) \cap q)$
b) $((\sim q) \cap p) \cap (p \cap (\sim p))$
c) $((\sim q) \cap p) \cup (\cup(\sim p)))$
d) $((p\hat{q})\hat{(} \sim (p \cap q))$

6) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Define the function $f : S \to S$ as follows: [20th June shift 2, 2020]

$$f(n) = \begin{cases} 2n, & \text{if } n = 1, 2, 3, 4, 5, \\ 2(11 - n), & \text{if } n = 6, 7, 8, 9, 10. \end{cases}$$

Let $g : S \to S$ be a function such that:

$$g(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd}, \\ 11 - \frac{n}{2}, & \text{if } n \text{ is even}. \end{cases}$$

Find the value of $g(10)(g(1) + g(2) + g(3) + g(4) + g(5))$. [20th June shift 2, 2020]

7) Let $\alpha, \beta$ be the roots of the equation:

$$x^2 - 4x + 5 = 0,$$

and let $\alpha\gamma, \beta\gamma$ be the roots of the equation:

$$x^2 - \left(3\sqrt{2} + 7\sqrt{3}\right)x + \left(7 + 3\sqrt{5}\right) = 0.$$

If $\beta + \gamma = 3\sqrt{2}$, then find $(\alpha + 2\beta + \gamma)^2$. [20th June shift 2, 2020]

8) Let $A$ be a matrix of order $2 \times 2$, whose entries are from the set $\{0, 1, 2, 3, 4, 5\}$. If the sum of all the entries of $A$ is a prime number $p$, $2 \leq p < 8$, find the number of such matrices $A$. [20th June shift 2, 2020]

9) If the sum of the coefficients of all the positive powers of $x$ in the binomial expansion of

$$\left(x + \frac{2}{x}\right)^n$$

is 939, find the sum of all the possible integral values of $n$. [20th June shift 2, 2020]

10) Let $\lfloor t \rfloor$ denote the greatest integer $\leq t$ and $\{t\}$ denote the fractional part of $t$. Then the integral value of $\alpha$ for which the left-hand limit of the function:

$$f(x) = \lfloor 1 + x \rfloor + \frac{\alpha x^{3/2}\{x\} - 1}{2\lfloor x \rfloor + \{x\}}$$

at $x = 0$ is equal to $\alpha - \frac{4}{3}$. [20th June shift 2, 2020]

11) If $y(x) = x^x$, $x > 0$, then find the value of: [20th June shift 2, 2020]

$$\frac{d^2x}{dy^2} \text{ at } x = 1.$$

12) If the area of the region $\{(x, y) : x^3 + y^3 \leq 1, x + y \geq 0, y \geq 0\}$ is $A$, find the value of $\frac{256A}{\pi}$. [20th June shift 2, 2020]
13) Let $v$ be the solution of the differential equation:

$$\left(1 - x^2\right) \frac{dy}{dx} = \left(xy + \left(x^3 + 2\right) \sqrt{1 - x^2}\right), -1 < x < 1,$$

and $y(0) = 0$. If:

$$\int_{-1/2}^{1/2} \sqrt{1 - x^2} y(0)\, dx = k,$$

then $k^{-1}$ is equal to: [20th June shift 2, 2020]
14) Let a circle $C$ of radius 5 lie below the x-axis. The line $L_1 : 4x + 3y - 2 = 0$ passes through the center $P$ of the circle $C$ and intersects the line $L_2 : 3x - 4y - 11 = 0$ at $Q$. The line $L_1$ touches $C$ at the point $Q$. Then the distance of $P$ from the line $5x - 12y + 51 = 0$ is: [20th June shift 2, 2020]
15) Let $S = \{E_1, E_2, \ldots, E_8\}$ be a sample space of random experiments such that $P(E_n) = \frac{n}{36}$ for every $n = 1, 2, \ldots, 8$. Then the number of elements in the set

$$\{A \subset S : P(A) \geq \frac{4}{5}\}$$

is: [20th June shift 2, 2020]