Problem Set 2

Experiments

The following field experiment studies the causal effect of providing information about corruption on voting for the incumbent. Voters were randomly assigned to receive a document with evidence that the incumbent has used public resources for personal benefit (t_ind). The experiment uses a randomized block design. The blocks are constructed using a binary indicator for having voted for the incumbent in the previous elections (block). The outcome is a binary indicator for voting for the incumbent in the next election (outcome). There is one pretreatment covariate available for the analysis (woman). Also, we have a battery of other outcomes about political attitudes that we can also be used to learn about the political effects of the treatment (outcome2 and outcome3).

```r
#load packages
library(lmtest)
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```r
library(foreign)
library(ggplot2)
library(sandwich)
library(Matrix)
library(ri)
library(ri2)
```

```
## Loading required package: randomizr
```

```
## Loading required package: estimatr
```

```r
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
t_ind = c(0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1)
block = c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)
outcome = c(0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,1,0,1,1,1,0,1,0,1,0,1,0,1,0,1,0,1,0)
woman = c(0,0,1,0,1,0,1,0,1,1,1,1,0,0,1,1,0,1,0,1,1,1,1,0,0,0,1,0,1,0,0,1)
```

```r
outcome2 = c(7,5,3,1,1,5,4,4,8,9,9,4,5,6,8,9,3,4,5,6,7,8,1,5,0,5,3,3,2,2,5,1)
outcome3 = c(2,5,4,3,9,5,5,6,6,6,3,1,5,2,3,2,1,7,8,9,7,3,0,0,0,2,4,7,1,1,2,9)
```

Question 1

Compute the average treatment effect, traditional standard errors, and p-values using a regression. Interpret the results. Remember this is a blocked design.

```r
# name the data frame
df <- data.frame(t_ind, block, outcome, woman, outcome2, outcome3)

# regress to grab p-value, BLOCK DESIGN
regressQ1 <- lm(outcome ~ t_ind + block, data = df)
summary(regressQ1)
```

```
##
## Call:
## lm(formula = outcome ~ t_ind + block, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -0.500 -0.250  0.125  0.125  0.750
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.5000     0.1025   4.878 3.56e-05 ***
## t_ind        -0.6250     0.1184  -5.281 1.16e-05 ***
## block         0.3750     0.1184   3.168   0.0036 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3348 on 29 degrees of freedom
## Multiple R-squared:  0.5667, Adjusted R-squared:  0.5368
## F-statistic: 18.96 on 2 and 29 DF,  p-value: 5.419e-06
```

```r
# standard error of t_ind from the regression
se_t_ind <- summary(regressQ1)$coefficients["t_ind", "Std. Error"]
se_t_ind
```

```
## [1] 0.118358
```

```r
# compute confidence interval using regression standard error
z_value <- qnorm(0.975)  # Z-score for 95% CI
lower_bound <- coef(regressQ1)["t_ind"] - z_value * se_t_ind
upper_bound <- coef(regressQ1)["t_ind"] + z_value * se_t_ind

# Print results
cat("Regression ATE Estimate:", coef(regressQ1)["t_ind"], "\n")
```

```
## Regression ATE Estimate: -0.625
```

```r
cat("Regression SE:", se_t_ind, "\n")
```

```
## Regression SE: 0.118358
```

```r
cat("95% Confidence Interval: [", lower_bound, ",", upper_bound, "]\n")
```

```
## 95% Confidence Interval: [ -0.8569775 , -0.3930225 ]
```

ANSWER: The ATE is -0.625, $p > 3.56e-05$ which means that receiving the corruption information reduces the chance of voting for the incumbent and is statistically significant.

The traditional standard errors are:

- Intercept: 0.1025, p-value = 3.56e-05
- t_ind: 0.1184, p = 1.16e-05
- Block: 0.1184, p = 0.0036

I created a 95% CI for t_ind to double check in results and its (-0.8569775 , -0.3930225), which doesn't include zero making it statistically significant.

Question 2

Use a regression to show that the block is a prognostic covariate. Explain the results.

```r
regressQ2 <- lm(outcome ~ block, data = df)
summary(regressQ2)
```

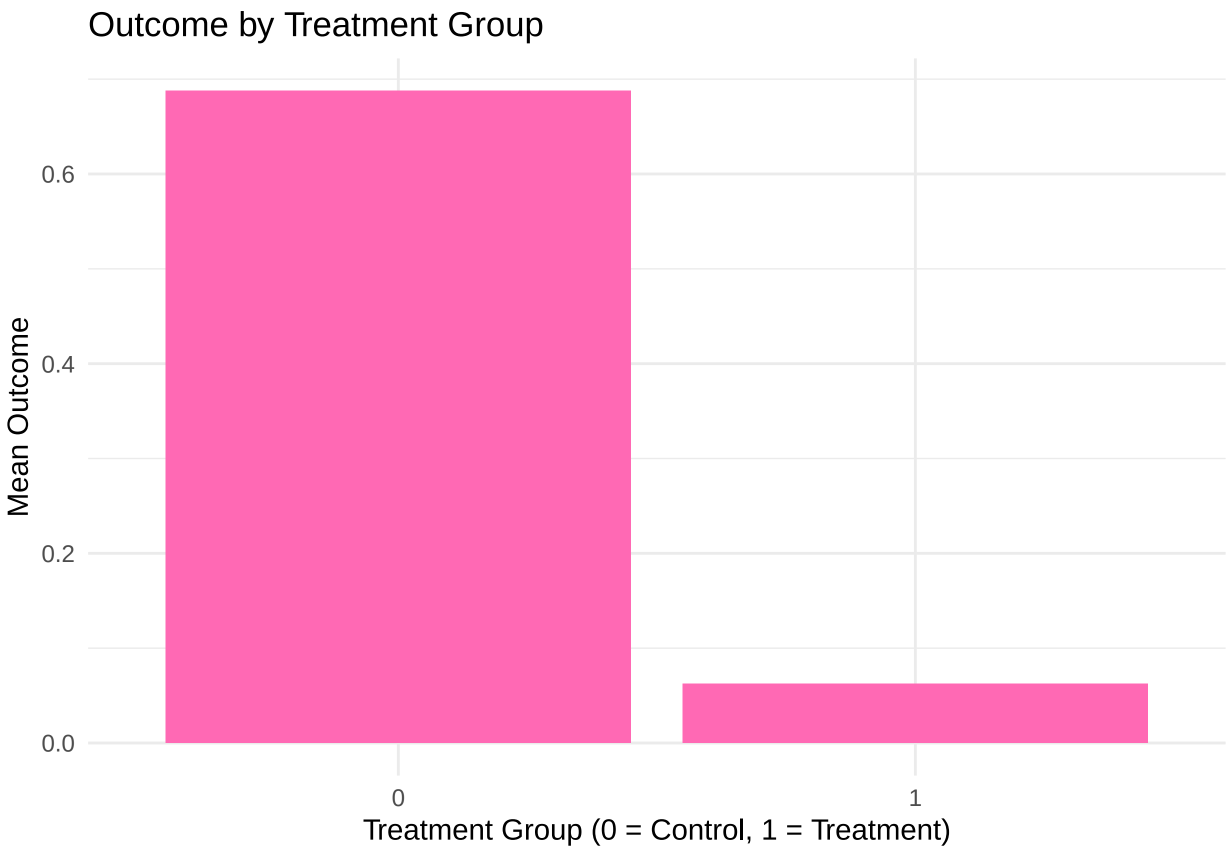
```
##
## Call:
## lm(formula = outcome ~ block, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.5625 -0.1875 -0.1875  0.4375  0.8125
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.1875     0.1152   1.627   0.1142
## block         0.3750     0.1630   2.301   0.0285 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.461 on 30 degrees of freedom
## Multiple R-squared:   0.15,  Adjusted R-squared:  0.1217
## F-statistic: 5.294 on 1 and 30 DF,  p-value: 0.02852
```

ANSWER: The intercept coefficient 0.1875 represents the baseline probability of voting for the incumbent when in a block. The block coefficient is 0.3750 which means that voters who supported the incumbent in the previous election are 37.5 **percentage points** more likely to vote for them again. The block design is statistically significant ($p < 0.05$), it is a prognostic covariate, a significant predictor of future support.

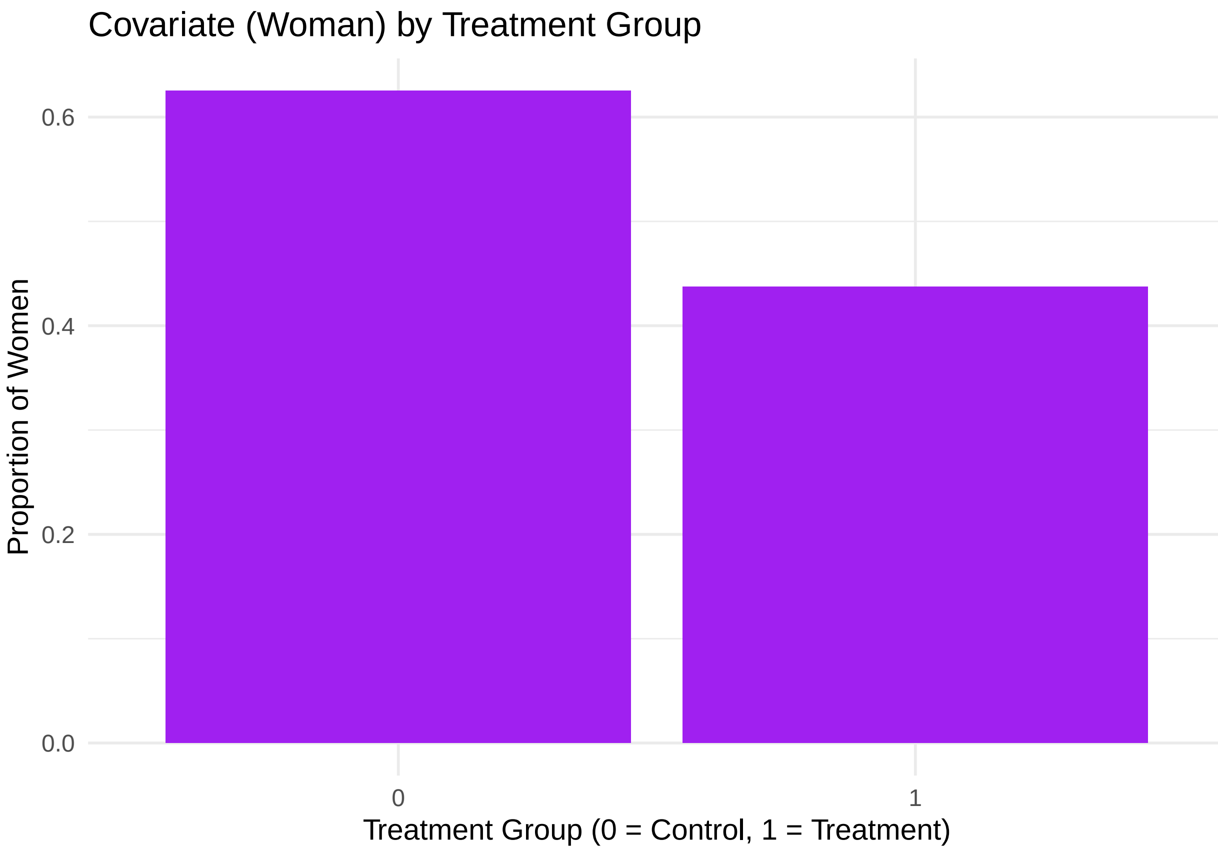
Question 3

Provide a plot that compares the outcome for the treatment and control group. Do the same for the covariate. Feel free to use any plot you want.

```r
# outcome for the treatment and control group
ggplot(df, aes(x = factor(t_ind), y = outcome)) +
  geom_bar(stat = "summary", fun = "mean", fill = "hotpink") +
  labs(x = "Treatment Group (0 = Control, 1 = Treatment)", y = "Mean Outcome", title = "Outcome by Trea
  theme_minimal()
```




```r
ggplot(df, aes(x = factor(t_ind), y = woman)) +
  geom_bar(stat = "summary", fun = "mean", fill = "purple") +
  labs(x = "Treatment Group (0 = Control, 1 = Treatment)", y = "Proportion of Women", title = "Covariat
  theme_minimal()
```



Question 4

Compute the Bell-McCaffrey standard errors and confidence intervals. Compare the results with question 1.

```
# define function to compute Bell-McCaffrey SEs
BMlmSE <- function(model, clustervar=NULL, ell=NULL, IK=TRUE) {
  X <- model.matrix(model) # extract the design matrix
  sum.model <- summary.lm(model) # store model summary
  n <- sum(sum.model$df[1:2]) # observations
  K <- model$rank # predictors
  XXinv <- sum.model$cov.unscaled # XX^{-1}
  u <- residuals(model) # model residuals

# compute variance matrix using HC3 - more conservative
Vhat <- vcovHC(model, type="HC3")

# compute standard errors
se <- sqrt(diag(Vhat))

  return(list(vcov=Vhat, se=se))
}

# compute BMse
bm <- BMlmSE(regressQ1)

# extract robust standard errors
bm$se
```

```
## (Intercept)       t_ind       block
##   0.1512925   0.1243294   0.1243294
```

```
# compute 95% confidence intervals using BMse
point.estimate <- coef(regressQ1)['t_ind']
critical.value <- qt(0.975, df = regressQ1$df.residual)  # t-distribution critical value
margin.of.error <- critical.value * bm$se["t_ind"]

ci <- c(point.estimate - margin.of.error, point.estimate + margin.of.error)
names(ci) <- c('lower', 'upper')

# print results
cat("Bell-McCaffrey SE:", bm$se["t_ind"], "\n")
```

```
## Bell-McCaffrey SE: 0.1243294
```

```
cat("95% Confidence Interval:", ci, "\n")
```

```
## 95% Confidence Interval: -0.8792821 -0.3707179
```

ANSWER: The BMse adjusted SE is 0.1243294, and Q1 traditional SE was 0.1184. The BMse is slightly larger than the traditional regression, this shows that the BMse adjustment accounts for small sample bias by being more conservative. The adjustment also gives a slight shift in the 95% CI. The BMse CI is (-0.8792821 -0.3707179), while the traditional one was (-0.8569775 , -0.3930225), the BMse is slightly higher range. BMse inflates uncertainty in a smaller sample. Both methods confirm that the treatment effect (t_ind) remains statistically significant.

Question 5

Use randomization inference to compute p-values. Remember that this is a blocked design.

```
# set seed for reproducibility
set.seed(123)

# define treatment, outcome, and blocking variable
Z <- df$t_ind              # treatment indicator
Y <- df$outcome            # outcome variable
block <- df$block          # blocking variable

# compute the exact probability of treatment assignment within each block
probs <- genprobexact(Z, blockvar = block)
table(probs)
```

```
## probs
## 0.5
##  32
```

```
# estimate the Observed Average Treatment Effect (ATE)
ate <- estate(Y, Z, prob = probs)
cat("Observed ATE:", ate, "\n")  # Print ATE
```

```
## Observed ATE: -0.625
```

```
# generate simulated random treatment assignments while maintaining the block structure
# I increased from 10000 to 50000 for precision
perms <- genperms(Z, maxiter = 50000, blockvar = block)
```

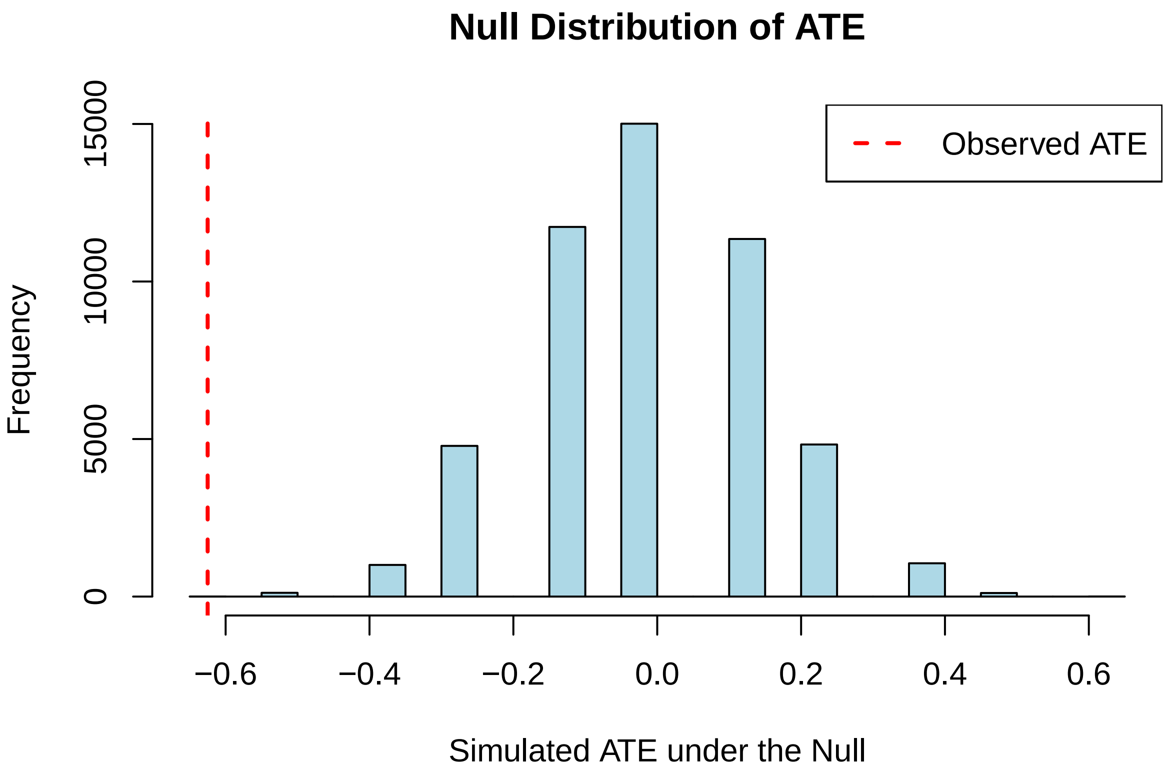
```
## Too many permutations to use exact method.
## Defaulting to approximate method.
```

```
## Increase maxiter to at least 165636900 to perform exact estimation.
```

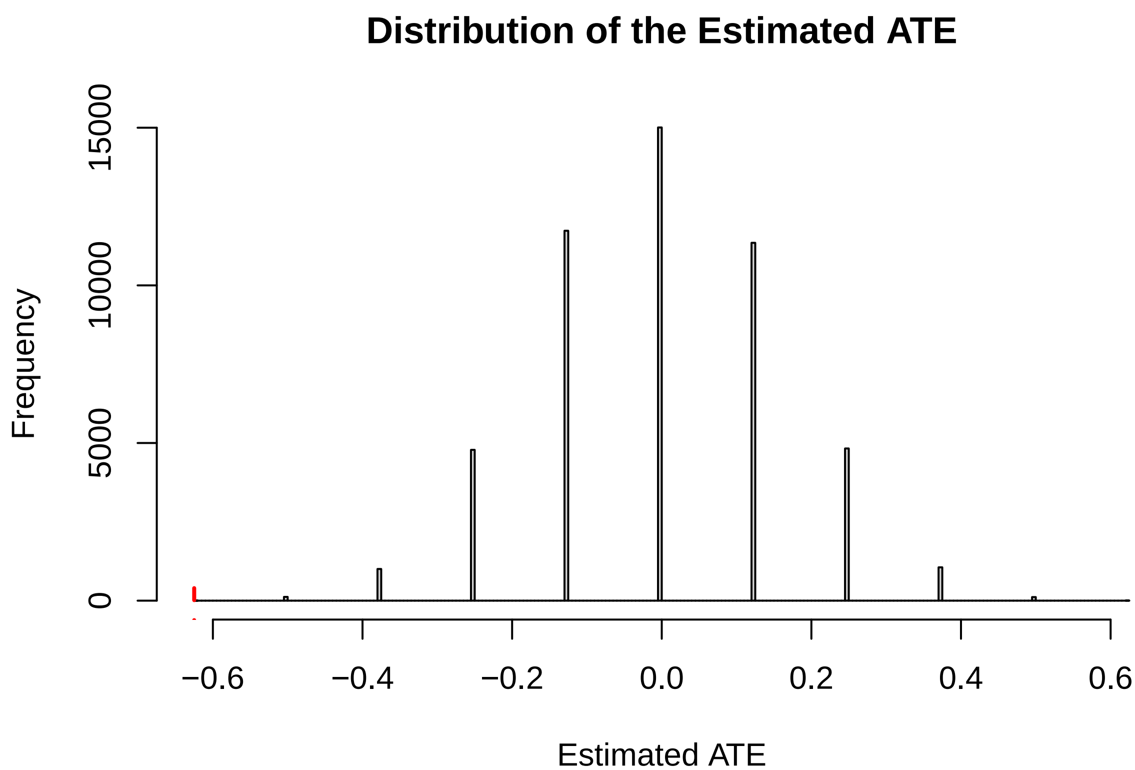
```r
# create potential outcomes under the sharp null hypothesis
# no treatment effect
Ys <- genouts(Y, Z, ate = 0)

# generate the null distribution of the ATE under the assumption of no treatment effect
distout <- gendist(Ys, perms, prob = probs)

# compute the two-tailed p-value for the randomization test
p_value <- mean(abs(distout) >= abs(ate))
cat("Randomization Inference p-value:", p_value, "\n")
```

```
## Randomization Inference p-value: 1e-04
```

```r
# visualize the null distribution of the ATE
hist(distout, breaks = 30, col = "lightblue", main = "Null Distribution of ATE",
     xlab = "Simulated ATE under the Null", border = "black")
abline(v = ate, col = "red", lwd = 2, lty = 2)  # Observed ATE
legend("topright", legend = c("Observed ATE"), col = c("red"), lwd = 2, lty = 2)
```




```r
# display  results of the null distribution
dispdist(distout, ate)
```




```
## $two.tailed.p.value
## [1] 8e-05
##
## $two.tailed.p.value.abs
## [1] 1e-04
##
## $greater.p.value
## [1] 1
##
## $lesser.p.value
## [1] 4e-05
##
## $quantile
##  2.5% 97.5%
## -0.25  0.25
##
## $sd
## [1] 0.161938
##
## $exp.val
## [1] -0.0003925
```

ANSWER: The randomization based p-value = 1e-04, below 0.05 confidence level. This means I reject the null hypothesis that the treatment had no effect. The effect of receiving corruption information on voting behavior is real and is not due to random chance.

Question 6

What is the proportion of woman voters in the treated and control group? Check if there is covariate balance for gender using both a regression and randomization inference.

```r
women_treated <- mean(df$woman[df$t_ind == 1])
women_control <- mean(df$woman[df$t_ind == 0])

# Print results
cat("Proportion of Women in Treatment Group:", women_treated, "\n")
```

```
## Proportion of Women in Treatment Group: 0.4375
```

```r
cat("Proportion of Women in Control Group:", women_control, "\n")
```

```
## Proportion of Women in Control Group: 0.625
```

```r
# Run a regression to check gender balance
# woman, female or not female voter
# t_ind treatment or no treatment
regressQ6 <- lm(woman ~ t_ind, data = df)

se_robust <- vcovHC(regressQ6, type = "HC0")

coeftest(regressQ6, vcov = se_robust)
```

```
##
## t test of coefficients:
##
##              Estimate Std. Error t value  Pr(>|t|)
## (Intercept)  0.62500    0.12103   5.164 1.466e-05 ***
## t_ind       -0.18750    0.17329  -1.082    0.2879
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# randomization
set.seed(123)
sims <- 10000
W_sims <- numeric(sims)

for (i in 1:sims) {
  Z_sim <- sample(df$t_ind)
  fit_sim <- lm(df$woman ~ Z_sim)

  Rbeta.hat <- coef(fit_sim)["Z_sim"]
  RVR <- vcovHC(fit_sim, type = "HC0")["Z_sim", "Z_sim"]
  W_sims[i] <- Rbeta.hat^2 / RVR
}

W_obs <- coef(regressQ6)["t_ind"]^2 / vcovHC(regressQ6, type = "HC0")["t_ind", "t_ind"]

p_value <- mean(W_sims >= W_obs)

cat("Randomization-Based P-Value for Gender Balance:", p_value, "\n")
```

```
## Randomization-Based P-Value for Gender Balance: 0.4316
```

ANSWER:

Proportion of Women in Treatment Group: 0.4375

Proportion of Women in Control Group: 0.625

The voters were randomly assigned to receive a document with evidence that the incumbent has used public resources for personal benefit (t_ind) p-value = 0.2879, which shows us that gender is balanced.

Additionally, the randomization p-value is greater than 0.05, p = 0.4316 indicating that there is effective randomization

Question 7

Compute the heterogeneous treatment effect for women voters using a regression and traditional SE. Interpret the constant, the coefficient for the treatment, and the coefficient for the interaction between the treatment and woman.

ANSWER:

Question 8

Estimate the average treatment effect for each block using both regressions and randomization inference. Can we obtain valid causal inference when computing the ATE within each block? Why?

Question 9

Estimate the effect of the treatment on outcomes 2 and 3 using a regression and traditional SE. Correct for multiple comparisons.

Question 10

Let's suppose that some people moved to a new house and never got the mail with the treatment. Can you still estimate the ATE? Why?