Problem Set 2

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Experiments

The following field experiment studies the causal effect of providing information about corruption on voting for the incumbent. Voters were randomly assigned to receive a document with evidence that the incumbent has used public resources for personal benefit (t_ind). The experiment uses a randomized block design. The blocks are constructed using a binary indicator for having voted for the incumbent in the previous elections (block). The outcome is a binary indicator for voting for the incumbent in the next election (outcome). There is one pretreatment covariate available for the analysis (woman). Also, we have a battery of other outcomes about political attitudes that we can also be used to learn about the political effects of the treatment (outcome2 and outcome3).

```
#load packages
library(lmtest)
## Loading required package: zoo
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##
     as.Date, as.Date.numeric
library(foreign)
library(ggplot2)
library(sandwich)
library(Matrix)
library(ri)
library(ri2)
## Loading required package: randomizr
## Loading required package: estimatr
library(dplyr)
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##
     filter, lag
## The following objects are masked from 'package:base':
##
##
     intersect, setdiff, setequal, union
```

Compute the average treatment effect, traditional standard errors, and p-values using a regression. Interpret the results. Remember this is a blocked design.

```
the results. Remember this is a blocked design.
# name the data frame
df <- data.frame(t_ind, block, outcome, woman, outcome2, outcome3)</pre>
# regress to grab p-value, BLOCK DESIGN
regressQ1 <- lm(outcome ~ t_ind + block, data = df)
summary(regressQ1)
##
## Call:
## lm(formula = outcome ~ t_ind + block, data = df)
## Residuals:
##
     Min
              1Q Median
                            3Q
                                  Max
## -0.500 -0.250 0.125 0.125 0.750
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 0.5000
                            0.1025
                                    4.878 3.56e-05 ***
## t_ind
                -0.6250
                            0.1184 -5.281 1.16e-05 ***
                 0.3750
                            0.1184 3.168 0.0036 **
## block
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.3348 on 29 degrees of freedom
## Multiple R-squared: 0.5667, Adjusted R-squared: 0.5368
## F-statistic: 18.96 on 2 and 29 DF, p-value: 5.419e-06
# standard error of t ind from the regression
se_t_ind <- summary(regressQ1)$coefficients["t_ind", "Std. Error"]</pre>
se_t_ind
## [1] 0.118358
# compute confidence interval using regression standard error
z_value <- qnorm(0.975) # Z-score for 95% CI
lower_bound <- coef(regressQ1)["t_ind"] - z_value * se_t_ind</pre>
upper_bound <- coef(regressQ1)["t_ind"] + z_value * se_t_ind</pre>
# Print results
cat("Regression ATE Estimate:", coef(regressQ1)["t_ind"], "\n")
## Regression ATE Estimate: -0.625
cat("Regression SE:", se_t_ind, "\n")
## Regression SE: 0.118358
```

```
cat("95% Confidence Interval: [", lower_bound, ",", upper_bound, "]\n")
```

```
## 95% Confidence Interval: [ -0.8569775 , -0.3930225 ]
```

ANSWER: The ATE is -0.625, p = 1.16e-05, which means that receiving the corruption information reduces the chance of voting for the incumbent and is statistically significant.

The traditional standard errors are:

```
Intercept: 0.1025, p-value = 3.56e-05
t_ind: 0.1184, p = 1.16e-05
Block: 0.1184, p = 0.0036
```

I created a 95% CI for t_ind to double check in results and its (-0.8569775 , -0.3930225), which doesn't include zero making it statistically significant.

Question 2

Use a regression to show that the block is a prognostic covariate. Explain the results.

```
regressQ2 <- lm(outcome ~ block, data = df)</pre>
summary(regressQ2)
##
## Call:
## lm(formula = outcome ~ block, data = df)
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
## -0.5625 -0.1875 -0.1875 0.4375
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                 0.1875
                            0.1152
                                     1.627
                                             0.1142
## (Intercept)
## block
                 0.3750
                            0.1630
                                     2.301
                                             0.0285 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.461 on 30 degrees of freedom
## Multiple R-squared:
                        0.15, Adjusted R-squared: 0.1217
## F-statistic: 5.294 on 1 and 30 DF, p-value: 0.02852
```

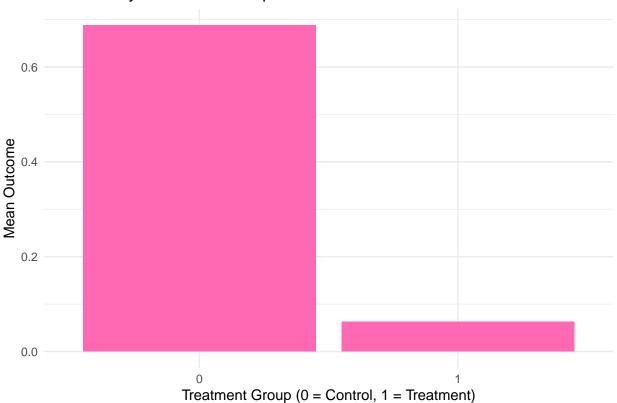
ANSWER: The intercept coefficient 0.1875 represents the baseline probability of voting for the incumbent when in a block. The block coefficient is 0.3750 which means that voters who supported the incumbent in the previous election are 37.5 **percentage points** more likely to vote for them again. The block design is statistically significant (p < 0.05), it is a prognostic covariate, a significant predictor of future support.

Question 3

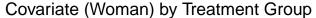
Provide a plot that compares the outcome for the treatment and control group. Do the same for the covariate. Feel free to use any plot you want.

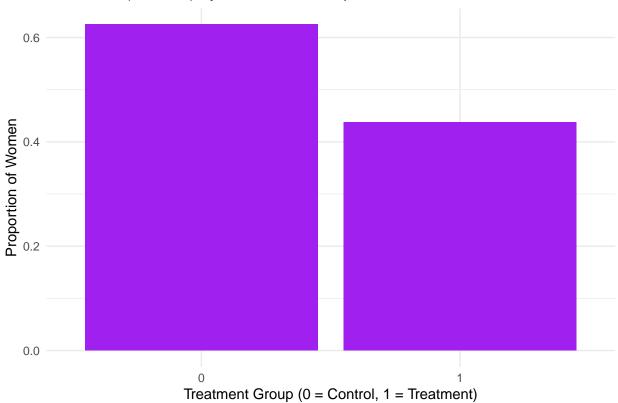
```
# outcome for the treatment and control group
ggplot(df, aes(x = factor(t_ind), y = outcome)) +
  geom_bar(stat = "summary", fun = "mean", fill = "hotpink") +
  labs(x = "Treatment Group (0 = Control, 1 = Treatment)", y = "Mean Outcome", title = "Outcome by Treatment minimal()
```

Outcome by Treatment Group



```
ggplot(df, aes(x = factor(t_ind), y = woman)) +
  geom_bar(stat = "summary", fun = "mean", fill = "purple") +
  labs(x = "Treatment Group (0 = Control, 1 = Treatment)", y = "Proportion of Women", title = "Covariat theme_minimal()
```





Compute the Bell-McCaffrey standard errors and confidence intervals. Compare the results with question 1.

```
# define function to compute Bell-McCaffrey SEs
BMlmSE <- function(model, clustervar=NULL, ell=NULL, IK=TRUE) {</pre>
  X <- model.matrix(model) # extract the design matrix</pre>
  sum.model <- summary.lm(model) # store model summary</pre>
  n <- sum(sum.model$df[1:2]) # observations</pre>
  K <- model$rank # predictors</pre>
  XXinv <- sum.model$cov.unscaled # XX^{-1}
  u <- residuals(model) # model residuals</pre>
# compute variance matrix using HC3 - more conservative
Vhat <- vcovHC(model, type="HC3")</pre>
# compute standard errors
se <- sqrt(diag(Vhat))</pre>
  return(list(vcov=Vhat, se=se))
}
# compute BMse
bm <- BMlmSE(regressQ1)</pre>
# extract robust standard errors
bm$se
```

```
## (Intercept)
                      t_{ind}
                                   block
     0.1512925
                 0.1243294
                              0.1243294
# compute 95% confidence intervals using BMse
point.estimate <- coef(regressQ1)['t_ind']</pre>
critical.value <- qt(0.975, df = regressQ1$df.residual) # t-distribution critical value
margin.of.error <- critical.value * bm$se["t_ind"]</pre>
ci <- c(point.estimate - margin.of.error, point.estimate + margin.of.error)</pre>
names(ci) <- c('lower', 'upper')</pre>
# print results
cat("Bell-McCaffrey SE:", bm$se["t_ind"], "\n")
## Bell-McCaffrey SE: 0.1243294
cat("95% Confidence Interval:", ci, "\n")
```

95% Confidence Interval: -0.8792821 -0.3707179

Defaulting to approximate method.

ANSWER: The BMse adjusted SE is 0.1243294, and Q1 traditional SE was 0.1184. The BMse is slightly larger than the traditional regression, this shows that the BMse adjustment accounts for small sample bias by being more conservative. The adjustment also gives a slight shift in the 95% CI. The BMse CI is (-0.8792821 -0.3707179), while the traditional one was (-0.8569775, -0.3930225), the BMse is slightly higher range. BMse inflates uncertainty in a smaller sample. Both methods confirm that the treatment effect (t_ind) remains statistically significant.

Question 5

Use randomization inference to compute p-values. Remember that this is a blocked design.

```
# set seed for reproducibility
set.seed(123)
# define treatment, outcome, and blocking variable
Z <- df$t ind
                         # treatment indicator
Y <- df$outcome
                           # outcome variable
block <- df$block
                           # blocking variable
# compute the exact probability of treatment assignment within each block
probs <- genprobexact(Z, blockvar = block)</pre>
table(probs)
## probs
## 0.5
# estimate the Observed Average Treatment Effect (ATE)
ate <- estate(Y, Z, prob = probs)</pre>
cat("Observed ATE:", ate, "\n") # Print ATE
## Observed ATE: -0.625
# generate simulated random treatment assignments while maintaining the block structure
# I increased from 10000 to 50000 for precision
perms <- genperms(Z, maxiter = 50000, blockvar = block)</pre>
## Too many permutations to use exact method.
```

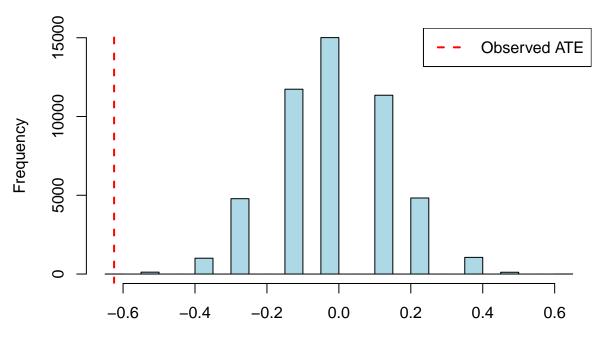
```
## Increase maxiter to at least 165636900 to perform exact estimation.
# create potential outcomes under the sharp null hypothesis
# no treatment effect
Ys <- genouts(Y, Z, ate = 0)

# generate the null distribution of the ATE under the assumption of no treatment effect
distout <- gendist(Ys, perms, prob = probs)

# compute the two-tailed p-value for the randomization test
p_value <- mean(abs(distout) >= abs(ate))
cat("Randomization Inference p-value:", p_value, "\n")
```

Randomization Inference p-value: 1e-04

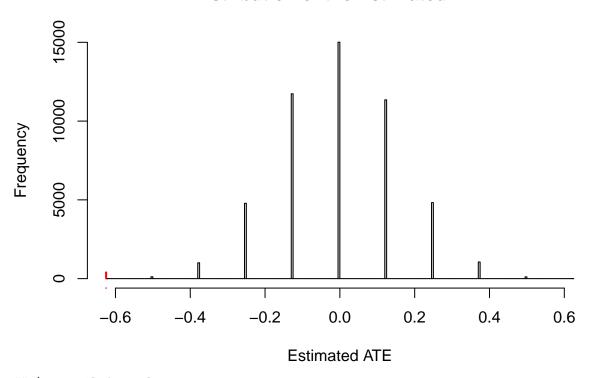
Null Distribution of ATE



Simulated ATE under the Null

display results of the null distribution
dispdist(distout, ate)

Distribution of the Estimated ATE



```
## $two.tailed.p.value
## [1] 8e-05
##
## $two.tailed.p.value.abs
## [1] 1e-04
##
## $greater.p.value
##
## $lesser.p.value
## [1] 4e-05
##
## $quantile
    2.5% 97.5%
##
   -0.25 0.25
##
## $sd
## [1] 0.161938
##
## $exp.val
## [1] -0.0003925
```

ANSWER: The randomization based p-value = 1e-04, below 0.05 confidence level. This means I reject the null hypothesis that the treatment had no effect. The effect of receiving corruption information on voting behavior is real and is not due to random chance.

Question 6

What is the proportion of woman voters in the treated and control group? Check if there is covariate balance for gender using both a regression and randomization inference.

```
women_treated <- mean(df$woman[df$t_ind == 1])</pre>
women_control <- mean(df$woman[df$t_ind == 0])</pre>
# Print results
cat("Proportion of Women in Treatment Group:", women_treated, "\n")
## Proportion of Women in Treatment Group: 0.4375
cat("Proportion of Women in Control Group:", women_control, "\n")
## Proportion of Women in Control Group: 0.625
# Run a regression to check gender balance
# woman, female or not female voter
# t_ind treatment or no treatment
regressQ6 <- lm(woman ~ t_ind, data = df)
se_robust <- vcovHC(regressQ6, type = "HCO")</pre>
coeftest(regressQ6, vcov = se_robust)
## t test of coefficients:
##
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.62500
                         0.12103    5.164    1.466e-05 ***
                           0.17329 -1.082
## t_ind
               -0.18750
                                                0.2879
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
# randomization
set.seed(123)
sims <- 10000
W_sims <- numeric(sims)</pre>
for (i in 1:sims) {
 Z_sim <- sample(df$t_ind)</pre>
 fit_sim <- lm(df$woman ~ Z_sim)</pre>
 Rbeta.hat <- coef(fit_sim)["Z_sim"]</pre>
 RVR <- vcovHC(fit_sim, type = "HCO")["Z_sim", "Z_sim"]</pre>
  W_sims[i] <- Rbeta.hat^2 / RVR</pre>
W_obs <- coef(regressQ6)["t_ind"]^2 / vcovHC(regressQ6, type = "HCO")["t_ind", "t_ind"]
p_value <- mean(W_sims >= W_obs)
cat("Randomization-Based P-Value for Gender Balance:", p_value, "\n")
```

Randomization-Based P-Value for Gender Balance: 0.4316

ANSWER:

Proportion of Women in Treatment Group: 0.4375

Proportion of Women in Control Group: 0.625

The voters were randomly assigned to receive a document with evidence that the incumbent has used public resources for personal benefit (t_{ind}) p-value = 0.2879, which shows us that gender is balanced.

Additionally, the randomization p-value is greater than 0.05, p = 0.4316 indicating that there is effective randomization.

Question 7

Compute the heterogeneous treatment effect for women voters using a regression and traditional SE. Interpret the constant, the coefficient for the treatment, and the coefficient for the interaction between the treatment and woman.

```
regressQ7 <- lm(outcome ~ t_ind * woman + block, data = df)
summary(regressQ7)</pre>
```

```
##
## Call:
## lm(formula = outcome ~ t_ind * woman + block, data = df)
##
## Residuals:
##
       Min
                  1Q
                      Median
                                    3Q
                                            Max
##
  -0.54553 -0.23800 0.07305 0.17988
                                       0.69164
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
               0.40922
                           0.16179
                                     2.529 0.01758 *
## t_ind
               -0.58085
                           0.18200
                                    -3.192 0.00357 **
## woman
                0.13631
                           0.17933
                                     0.760
                                           0.45376
## block
                0.38618
                           0.12327
                                     3.133
                                           0.00414 **
## t ind:woman -0.04249
                           0.25106
                                    -0.169
                                           0.86685
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3414 on 27 degrees of freedom
## Multiple R-squared: 0.5804, Adjusted R-squared: 0.5183
## F-statistic: 9.338 on 4 and 27 DF, p-value: 7.143e-05
```

ANSWER: The regression estimates whether the effect of the corruption information on voting for the incumbent differs between women and non-women.

Constant: The constant (0.40922, p = 0.01758) represents the probability of voting for the incumbent for non-women (women = 0) in the control group (no treatment, t_i = 0), and in the block. Their predicted probability = 0.409 or 40.9%

Coefficient for the treatment: The treatment effect (-0.58085, p=0.00357) measures the effect of receiving the information on the voting behavior among non-women (women = 0). The coefficient is negative, meaning that exposure to the corruption information reduces support for the incumbent among non-women.

The coefficient for the interaction between the treatment and woman: The interaction $t_{ind:woman}$ (-0.04249, p = 0.86685) treatment effect for women is 4.25 percentages points weaker than it is for non-women. However, the p-value is high indicating that its not statistically significant. This indicates that there is no evidence that gender moderates the treatment effect, women and non-women respond similarly to the corruption information.

Estimate the average treatment effect for each block using both regressions and randomization inference. Can we obtain valid causal inference when computing the ATE within each block? Why?

```
regressQ8_b0 <- lm(outcome ~ t_ind, data = subset(df, block == 0))
regressQ8_b1 <- lm(outcome ~ t_ind, data = subset(df, block == 1))</pre>
summary(regressQ8_b0)
##
## Call:
## lm(formula = outcome ~ t_ind, data = subset(df, block == 0))
## Residuals:
##
     Min
             1Q Median
                           3Q
                                 Max
## -0.375 -0.375 0.000 0.000 0.625
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.3750
                           0.1294
                                  2.898 0.0117 *
                           0.1830 -2.049
               -0.3750
                                            0.0596 .
## t ind
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.366 on 14 degrees of freedom
## Multiple R-squared: 0.2308, Adjusted R-squared: 0.1758
## F-statistic:
                 4.2 on 1 and 14 DF, p-value: 0.05965
summary(regressQ8 b1)
##
## Call:
## lm(formula = outcome ~ t_ind, data = subset(df, block == 1))
## Residuals:
     Min
             1Q Median
                           3Q
                                 Max
## -0.125 -0.125 0.000 0.000 0.875
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 1.00000
                          0.08839
                                   11.31 1.98e-08 ***
## t_ind
              -0.87500
                          0.12500
                                    -7.00 6.25e-06 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.25 on 14 degrees of freedom
## Multiple R-squared: 0.7778, Adjusted R-squared: 0.7619
                 49 on 1 and 14 DF, p-value: 6.248e-06
## F-statistic:
# Define treatment, outcome, and blocking variable
Z <- df$t_ind
Y <- df$outcome
block <- df$block
# Generate probability of treatment assignment within blocks
```

```
probs <- genprobexact(Z, blockvar = block)</pre>
table(probs)
## probs
## 0.5
## 32
# Generate simulated random assignments
perms <- genperms(Z, maxiter = 100000, blockvar = block)
## Too many permutations to use exact method.
## Defaulting to approximate method.
## Increase maxiter to at least 165636900 to perform exact estimation.
# Subset probabilities and permutations
probs_b0 <- probs[df$block == 0]</pre>
probs_b1 <- probs[df$block == 1]</pre>
perms_b0 <- perms[, df$block == 0]</pre>
perms_b1 <- perms[, df$block == 1]</pre>
# ATE Estimation Using RI for Block = 0
df b0 <- subset(df, block == 0)</pre>
ate_b0 <- estate(df_b0$outcome, df_b0$t_ind, prob = probs_b0)</pre>
# ATE Estimation Using RI for Block = 1
df_b1 <- subset(df, block == 1)</pre>
ate_b1 <- estate(df_b1$outcome, df_b1$t_ind, prob = probs_b1)</pre>
cat("ATE for Block 0 (Regression):", coef(regressQ8_b0)["t_ind"], "\n")
## ATE for Block 0 (Regression): -0.375
cat("ATE for Block 1 (Regression):", coef(regressQ8_b1)["t_ind"], "\n")
## ATE for Block 1 (Regression): -0.875
cat("ATE for Block 0 (Randomization Inference):", ate_b0, "\n")
## ATE for Block 0 (Randomization Inference): -0.375
cat("ATE for Block 1 (Randomization Inference):", ate_b1, "\n")
## ATE for Block 1 (Randomization Inference): -0.875
Estimate the average treatment effect for each block using both regressions and randomization
inference:
ATE for Block 0 (Regression): -0.375
ATE for Block 1 (Regression): -0.875
ATE for Block 0 (Randomization Inference): -0.375
ATE for Block 1 (Randomization Inference): -0.875
```

Can we obtain valid causal inference when computing the ATE within each block? Why?: Yes, because the treatment was randomized in each block, making sure there are unbiased estimates. Since the comparisons are made within groups that were randomized separately, the estimated effects are not confounded by differences across the blocks. The difference in ATE between blocks suggests potential treatment effect heterogeneity.

Estimate the effect of the treatment on outcomes 2 and 3 using a regression and traditional SE. Correct for multiple comparisons.

```
# set seed for reproducibility
set.seed(123)
# Regressions for outcomes 2 and 3
regressQ9_outcome2 <- lm(outcome2 ~ t_ind + block, data = df)</pre>
regressQ9_outcome3 <- lm(outcome3 ~ t_ind + block, data = df)
# Traditional SEs
se_outcome2 <- coeftest(regressQ9_outcome2, vcov = vcovHC(regressQ9_outcome2, type = "HCO"))</pre>
se_outcome3 <- coeftest(regressQ9_outcome3, vcov = vcovHC(regressQ9_outcome3, type = "HCO"))</pre>
# p-values for multiple comparison
p_values <- c(se_outcome2["t_ind", 4], se_outcome3["t_ind", 4])</pre>
# Alpha Level
alpha \leftarrow 0.05
# Multiple comparison
p_bonferroni <- p.adjust(p_values, "bonferroni") < alpha</pre>
p_holm <- p.adjust(p_values, "holm") < alpha</pre>
p_bh <- p.adjust(p_values, "BH") < alpha</pre>
# print out results
cat("Regression Results for Outcome 2:\n")
## Regression Results for Outcome 2:
print(se_outcome2)
##
## t test of coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 5.31250 0.78755 6.7456 2.108e-07 ***
## t_ind
               0.37500
                           0.84288 0.4449 0.65969
## block
               -1.75000
                           0.84288 -2.0762
                                              0.04684 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
cat("\nRegression Results for Outcome 3:\n")
##
## Regression Results for Outcome 3:
print(se_outcome3)
## t test of coefficients:
##
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.93750
                           0.72627 5.4215 7.864e-06 ***
                           0.96597 0.5176
                0.50000
## t_ind
                                               0.6087
```

```
## block
              -0.37500
                          0.96597 -0.3882
                                             0.7007
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
cat("\nMultiple Comparisons Correction:\n")
##
## Multiple Comparisons Correction:
cat("Original p-values:", p_values, "\n")
## Original p-values: 0.6596934 0.608655
cat("Bonferroni-adjusted p-values:", p_bonferroni, "\n")
## Bonferroni-adjusted p-values: FALSE FALSE
cat("Holm-adjusted p-values:", p_holm, "\n")
## Holm-adjusted p-values: FALSE FALSE
cat("Benjamini-Hochberg-adjusted p-values:", p_bh, "\n")
```

Benjamini-Hochberg-adjusted p-values: FALSE FALSE

ANSWER:

Outcome 2:

The intercept (5.3125, p = 2.108e-07) represents the expected value of Outcome 2 for individuals in the control group $(t_ind = 0)$ and the reference block. It is highly significant, indicating confidence in this baseline estimate.

The treatment effect (0.3750, p = 0.6597) represents the change in Outcome 2 due to treatment (t_ind = 1). Since the p-value is well above 0.05, the treatment does not have a statistically significant effect on Outcome 2.

The block effect (-1.7500, p = 0.0468) is statistically significant at the 5% level, suggesting that block assignment has a meaningful impact on Outcome 2, independent of treatment.

Outcome 3:

The intercept (3.9375, p = 7.864e-06) represents the expected value of Outcome 3 for individuals in the control group (t_ind = 0) and the reference block. It is highly significant, indicating confidence in this baseline estimate.

The treatment effect (0.5000, p = 0.6087) represents the change in Outcome 3 due to treatment (t_ind = 1). Since the p-value is not statistically significant (p > 0.05), the treatment does not have a meaningful impact on Outcome 3.

The block effect (-0.3750, p = 0.7007) is not statistically significant, suggesting that block assignment does not have a meaningful impact on Outcome 3.

Multiple Comparison Correction:

All corrections returned back as FALSE, indicating that neither result is significant after adjusting for multiple tests. This means that there is **no statistical evidence that the treatment affects either outcome.**

Question 10

Let's suppose that some people moved to a new house and never got the mail with the treatment. Can you still estimate the ATE? Why?

ANSWER: Yes, but there are limitations. If some individuals moved and don't receive the mail with the treatment, there is a non-compliance issue, someone who was assigned the treatment did not receive/undergo the treatment. This can affect how we estimate the ATE. But the assignment to treatment was still randomized, I can estimate the ITT effect, which measures the effect of being assigned to treatment, regardless of whether it was actually received. However, the ATE may be biased if non-receivers systematically differ from receivers.