LONDON'S GLOBAL UNIVERSITY

UCL

UCL COMP0105 Past Paper Answers

Not Official - Student Made

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Abstract

GitHub repo link: https://github.com/ignRyann/COMP0105_Answers.
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1 2021/22 Paper

Q1b

- Contact Value: $500 \times \$80 = \$40,000$
- Initial Margin: $0.15 \times \$40,000 = \$6,000$
- Maintenance Margin: $0.75 \times \$6,000 = \$4,500$

On Day 1, our margin account is at a balance of $\$6,000 - \$2 \times 500 = \$5,000$.

On Day 2, our margin account is at a balance of $\$5,000 - \$3 \times 500 = \$3,500$. This is less than the maintenance margin, thus a margin call is initiated. We require \$2,500 to meet the margin call.

So in total, we need $\$6,000 + \$2,500 = \$8,500$ liquid cash *before* we enter into the contract so that we can make our first margin call.

Q2a

Refer to 2023 Q2a.

Q2b (i)

The general formula is given by

$$(1 + r_n)^{n+1} = (1 + r)(1 + f_1)...(1 + f_n) \tag{1}$$

Using this, we can calculate both f_1 as follows:

$$(1 + r_1)^2 = (1 + r)(1 + f_1) \tag{2}$$

$$(1.025)^2 = (1.02)(1 + f_1) \tag{3}$$

Thus, we get a value of $f_1 = 0.03002\dot{4}5... \approx 0.03$ or 3%. Similarly for f_2, we have:

$$(1 + r_2)^3 = (1 + r)(1 + f_1)(1 + f_2) \tag{4}$$

$$(1.0267)^3 = (1.02)(1.03)(1 + f_2) \tag{5}$$

Thus, we get a value of $f_2 = 0.030132976 \approx 0.0301$ or 3.01%.

Q2c

Refer to 2023 Q2c.

Q3c (i)

We know that

$$\sigma_p^2 = (w_A\sigma_A)^2 + (w_B\sigma_B)^2 + 2\rho w_A\sigma_A w_B\sigma_B \tag{6}$$

Since $\omega_A + \omega_B = 1$, we can let $\omega_B = 1 - \omega_A$:

$$\sigma_p^2 = w_A^2\sigma_A^2 + \sigma_B^2 - 2w_A\sigma_B^2 + w_A^2\sigma_B^2 + 2\rho w_A\sigma_A\sigma_B - 2\rho w_A^2\sigma_A\sigma_B \tag{7}$$

We then differentiate in respect to w_A to get

$$\frac{d\sigma_p^2}{dw_A} = w_A\sigma_A^2 - \sigma_B^2 + w_A\sigma_B^2 + \rho\sigma_A\sigma_B - 2\rho w_A\sigma_A\sigma_B = 0 \tag{8}$$

Plugging in our values and rearranging, we have that $w_A = 0.7$ and consequently, $w_B = 0.3$.

Q3c (ii)

The expected return is calculated as

$$0.7(0.2) + 0.3(0.3) = 0.23 = 23\% \tag{9}$$

We then also plug in our values for our standard deviation

$$\sigma_p = (0.7 \times 0.2)^2 + (0.3 \times 0.3)^2 + 2(0.2)(0.7)(0.2)(0.3)(0.3) \approx 0.15 = 15\% \tag{10}$$

2 2022/23 Paper

Q1a

$$100 = \frac{\$15,500,000,000}{\text{D}} \tag{11}$$

Re-arranging gives us a divisor value of D $= 155,000,000$. If the index currently has a value of 92 then the total market capitalisation of the companies included is

$$92 = \frac{x}{155,000,000} \tag{12}$$

Therefore, $x = \$14,260,000,000$ is the current total market capitalisation of the companies included in the index.

Q1b

Given that it is based on the total market capitalisation of the companies in the index, it would have no effect as the total value of the stocks after a stock split would be the same.

For example, if a companies stock were to face a 3:1 split, the overall value would be the same.

$$\$40 \times 3 = \$120 \times 1 \tag{13}$$

Q1c (i)

- Contact Value: $2000 \times \$55 = \$110,000$
- Initial Margin: $0.12 \times \$110,000 = \$13,200$
- Maintenance Margin: $0.75 \times \$13,200 = \$9,900$

Q1c (ii)

To get our first margin call, we'd have to experience a loss of \$3,300. This is equivalent to a \$1.65 price drop for each share. Thus, we'd experience a margin call when the price is of the share is less than \$53.35. We'll have our first margin call on Day 8.

Our margin account will have a balance of $\$13,200 - (\$55 - \$53.11) \times 2000 = \$9,420$. We'll have to pay \$3,780.

Q2a

So in total, we have 19 payments. Our first payment is compounded for 16 time periods (of 6 months at 1.015% per period). Thus, we have that

$$\text{FV} = (\$200 \times (1.015)^{18}) + (\$200 \times (1.015)^{17}) + .. + (\$200 \times (1.015)^{0}) \tag{14}$$

$$\text{FV} = \$200 \times \sum_{i=0}^{18} 1.015^{i} \tag{15}$$

So we have that $a = \$200$, $r = 1.015$ and $n = 19$. This means that we have

$$FV = \$200 \times \left(\frac{1 - 1.015^{19}}{1 - 1.015}\right) = \$4359.34 \tag{16}$$

Q2b (i)

A nominal interest rate of 3% per annum with bi-annual compounding is 1.5% per period (6 months). The bond pays out \$75 every 6 months which is 1.5% of \$5,000. Thus, the bond is at par immediately after a coupon payment and is worth \$5,000.

Q2b (ii)

Every 6 months, the bond pays 1.5% of its face value. Thus, the dirty price is

$$\$5,000 \times 1.015^{\frac{60}{180}} = \$5,024.88 \tag{17}$$

Q2b (iii)

When calculating the clean price from the dirty price, we subtract the proportional amount of the coupon payment.

$$\$5,000 - \frac{60}{180}(\$75) = \$4,999.88 \tag{18}$$

Q2c

$$\triangle V = \frac{\$9,000 \times 4.8 \times 0.01}{1.015} = \$425.62 \tag{19}$$

Q3a

If we buy 200 shares at \$100 per share, our profit/loss is calculated as:

$$\text{P/L} = 200V - \$20,000, \tag{20}$$

where V is the price of the share.

If we were to use leverage, we'd have 1000 shares at \$100 per share. However, \$80,000 was borrowed at 2% per annum so we'd have to pay back \$81,600. Thus, our profit/loss is calculated as:

$$\text{P/L} = 1000V - (\$81,600 + \$20,000) \tag{21}$$

Thus, equating both equations will give us the share price such that they both make the same amount.

$$200V - 20,000 = 1000V - 101,600 \tag{22}$$

$$800V = \$81,600 \tag{23}$$

We have that the price of our shares must be $V = \$102$, which is a \$2 increase in share price. This is equivalent to a 2% increase in share price value.

Q4c

If CRACRB = 0.8520/30 and CRACRC = 1.5500/20, then

$$\text{CRCCRB} = (1.5510)^{-1} \times 0.8525 = 0.5496 \tag{24}$$

Q4d (i)

- **EUR:** £100 − $\left[(100 \times 1.1111) \div 1.2736\right]$ = £12.76
- **USD:** £12.75
- **CAD:** £14.58
- **AUD:** £13.26
- **BBD:** £18.27
- **BGN:** £18.06
- **CNY:** £18.00

Q4e (i)

If we have x CRX, it'll be worth $1.02x$ in 1 year.
If we convert it to CRY, we'll have $1.5x$ and it'll be worth $1.545x$ in 1 year.
Thus, the 1 year forward exchange rate, to avoid the possibility of arbitrage, should be

$$\text{CRXCRY} = 1.5147 \tag{25}$$

Q5d

We create a butterfly spread payoff using the the following:

- We buy a call option at K_1.
- We sell 2 call options at K_2.
- We buy a call options at K_3

We define the relationship between the different strike prices as $K_1 < K_2 < K_3$.

Q5e

$$100 \times \frac{0.015}{0.055} = 27.\dot{2}\dot{7} \approx 27 \tag{26}$$

$$(100 \times 0.627) - (27 \times 0.537) = 48.201 \approx 48 \tag{27}$$

Thus, our portfolio is structured as

$$\Pi = 27C_b - 100C_w + 48S \tag{28}$$

Perfect hedging is not possible as we had to round our values and Γ changes as the price moves.

Q6b (i)

$$\begin{bmatrix} 0.85 & 0.1 & 0.05 & 0.0 \\ 0.05 & 0.8 & 0.1 & 0.05 \\ 0.0 & 0.05 & 0.8 & 0.15 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Q6b (ii)

$$\begin{bmatrix} 0.7275 & 0.1675 & 0.0925 & 0.0125 \\ 0.0825 & 0.65 & 0.1625 & 0.105 \\ 0.0025 & 0.08 & 0.6450 & 0.2725 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$