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Beyond the Circle: Deforming Contours in Inverse Z-Transform

Final Year Project Report

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[1]**Disclaimer:** This report is submitted as part requirement for the MEng degree in Mathematical Computation at UCL. It is substantially the result of my own work except where explicitly indicated in the text. The report may be freely copied and distributed provided the source is explicitly acknowledged.

Abstract

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Chapter 1

Introduction

1.1 Motivation

High-Level overview for pricing options.

How this problem relates to the inverse Z-transform.

1.2 Aims and Objectives

Implementing widely used numerical methods for the inverse Z-transform.

Testing out Levendorskii's method

Benchmarking the performance of these methods.

1.3 Overview

2-3 sentences of each chapter

Chapter 2

Background

"By definition, a complex number z is an ordered pair (x, y) of real numbers x and y, written $z = (x, y)$" [Kreyszig, 2010]. In practice, complex numbers are written in the form $z = x + iy$, where x and y are real numbers and i is the imaginary unit. The set of complex numbers is denoted by $\mathbb{C}$.

2.1 $\mathcal{Z}$-Transform

Come up with a more catchy Z-Transform title

The z-transform is a transformation of a real or complex continuous time function $x(t)$, often used for discrete time signals and is commonly described as the discrete time Fourier transform (DTFT). Taking the Fourier transform of a sampled function results in

$$\mathcal{F}\left[x(t)\sum_{n=-\infty}^{\infty}\delta(t-n\triangle t)\right] = \int_{-\infty}^{\infty} x(t)\sum_{n=-\infty}^{\infty}\delta(t-n\triangle t)e^{-j\omega t}dt \tag{2.1}$$

We can then make use of the sifting property of the delta function.

$$= \sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} x(t)\delta(t-n\triangle t)e^{-j\omega t}dt \tag{2.2}$$

$$= \sum_{n=-\infty}^{\infty} x(n\triangle t)e^{-j\omega nt} \tag{2.3}$$

If we normalise the sampling interval to 1, we get

$$\sum_{n=-\infty}^{\infty} x(n)e^{-jn\omega} \tag{2.4}$$

The sequence $x(n)$ is sampled at discrete time intervals $t_n = n\triangle t$, where the sampling interval $\triangle t$ is the time between consecutive samples and the time index n numbers the samples [Loveless and Germano, 2021].

2.2 Discrete Pricing Options

2.3 Optimisation Techniques

Chapter 3

Experiment

Finding different parameters to use for the experiment making use of Machine Learning techniques.

Chapter 4

Results

Chapter 5

Conclusion

5.1 Summary

5.2 Future Work

5.3 Acknowledgements

Bibliography

[Kreyszig, 2010] Kreyszig, E. (2010). *Advanced Engineering Mathematics*. John Wiley & Sons.

[Loveless and Germano, 2021] Loveless, B. and Germano, G. (2021). Review of numerical inversion techniques of the z-transform. Working Paper.

Appendices

Appendix A

Initial Project Plan

Numerical Benchmarking on Inverse Z-Transform and Its Uses in Discrete Pricing Options

Project Plan

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Chapter 1

Aims and Objectives

1.1 Aims

We aim to understand a new efficient method for numerical evaluation of the inverse Z-transform, which states to be faster and more accurate than the standard trapezoid rule. A specific area of applying this method would be to the pricing of discretely monitored exotic options, such as lookback and barrier options, and see how it compares to other methods; Abate and Whitt's approach, C. Cavers' method with Euler, Shanks and epsilon accelerations, etc.

1.2 Objectives

- Understanding Levendorskii's inverse Z-transform and the common numerical evaluation methods
- Implementing the function as a code
- Numerical benchmarking; average error, maximum error and CPU time
- Exploring its uses in discrete pricing options

1.3 Deliverables

- numerical benchmarking results to add to '*Review of numerical inversion techniques of the z-transform'* by Loveless and Germano
- results and implementation in regards to discrete pricing options (*Accurate numerical inverse z-transform and it's use in the Fourier-z pricing of discretely monitored path-dependent options* by Loveless, Phelan and Germano)

Chapter 2

Work Plan

2.1 Project Start $\rightarrow$ 30$^{\text{th}}$ November '23

- background reading on complex numbers & contour integration based methods, fourier transform, z-transform and its inverse, numerical approaches to inverse z-transform and pricing options (barrier and lookback options)
- coding implementation of Levendorskii's inverse z-transform

2.2 1$^{\text{st}}$ December '23 $\rightarrow$ 24$^{\text{th}}$ January '24

- preliminary research on Loveless' and Germano's *'Review of numerical inversion techniques of the z-transform'*
- understanding the other methods; AW, C, CEuler, CShanks and CEpsilon
- going over the different functions; Heaviside Step, Polynomial, Decaying Exp, Sinusodial
- reviewing the code for numerical benchmarking
- implementing it for Levendorskii's method
- begin work on interim report

2.3 24$^{\text{th}}$ January '24 $\rightarrow$ 15$^{\text{th}}$ March '24

- preliminary recap on discrete pricing options (barrier and lookback options) and the need for z-transform
- use-case in discrete pricing options
- start work on project report; however, to be worked on throughout the year/stages

2.4 5$^{\text{th}}$ March '24 $\rightarrow$ 26$^{\text{th}}$ April '24

- extra time to deal with any unexpected problems or delays
- final touches