Numerical Benchmarking on Inverse Z-Transform and Its Uses in Discrete Pricing Options

Final Year Project Report
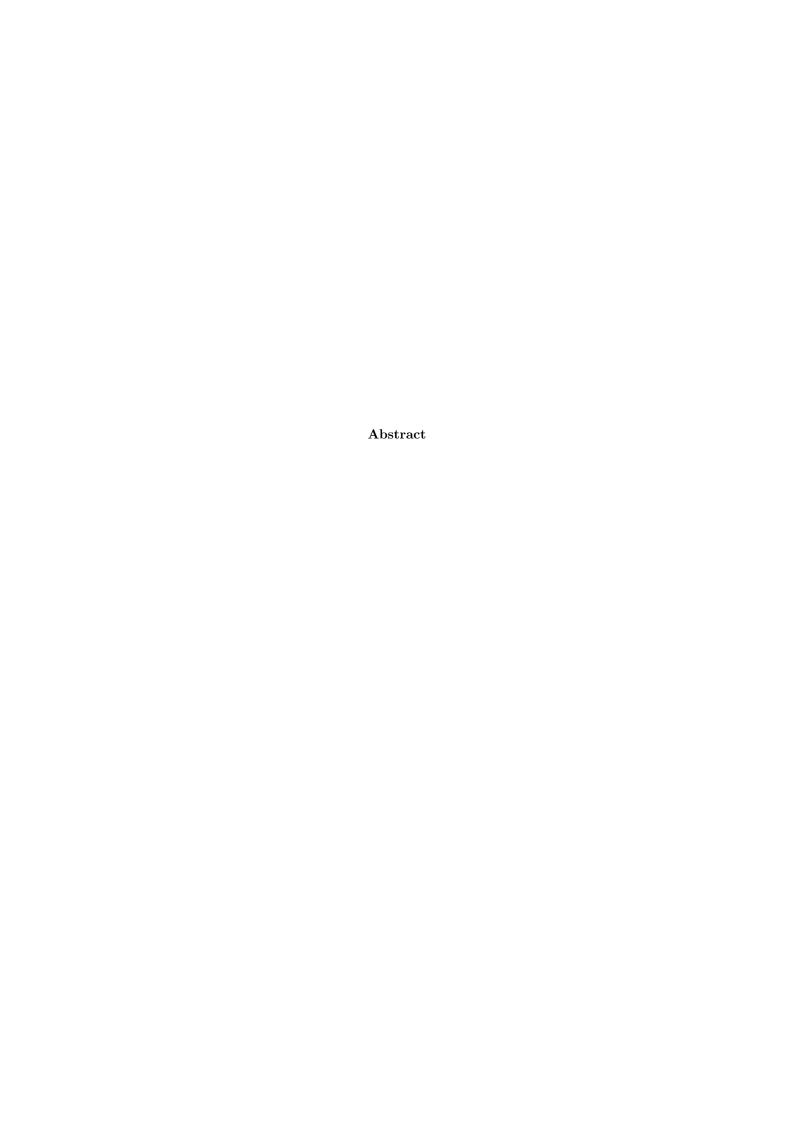
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Submission date: March 3, 2024

Abstract

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High-Level overview for pricing options.

How this problem relates to the inverse Z-transform.

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Benchmarking the performance of these methods.

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Initial Project Plan

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Project Plan

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Submission date: 16 November 2023

Chapter 1

Aims and Objectives

1.1   Aims

We aim to understand a new efficient method for numerical evaluation of the inverse Z-transform, which states to be faster and more accurate than the standard trapezoid rule. A specific area of applying this method would be to the pricing of discretely monitored exotic options, such as lookback and barrier options, and see how it compares to other methods; Abate and Whitt's approach, C. Cavers' method with Euler, Shanks and epsilon accelerations, etc.

1.2   Objectives

- Understanding Levendorskii's inverse Z-transform and the common numerical evaluation methods

- Implementing the function as a code

- Numerical benchmarking; average error, maximum error and CPU time

- Exploring its uses in discrete pricing options

1.3   Deliverables

- numerical benchmarking results to add to '*Review of numerical inversion techniques of the z-transform*' by Loveless and Germano

- results and implementation in regards to discrete pricing options (*Accurate numerical inverse z-transform and it's use in the Fourier-z pricing of discretely monitored path-dependent options* by Loveless, Phelan and Germano)

Chapter 2

Work Plan

2.1 Project Start → 30$^{\text{th}}$ November '23

- background reading on complex numbers & contour integration based methods, fourier transform, z-transform and its inverse, numerical approaches to inverse z-transform and pricing options (barrier and lookback options)
- coding implementation of Levendorskii's inverse z-transform

2.2 1$^{\text{st}}$ December '23 → 24$^{\text{th}}$ January '24

- preliminary research on Loveless' and Germano's *'Review of numerical inversion techniques of the z-transform'*
- understanding the other methods; AW, C, CEuler, CShanks and CEpsilon
- going over the different functions; Heaviside Step, Polynomial, Decaying Exp, Sinusodial
- reviewing the code for numerical benchmarking
- implementing it for Levendorskii's method
- begin work on interim report

2.3 24$^{\text{th}}$ January '24 → 15$^{\text{th}}$ March '24

- preliminary recap on discrete pricing options (barrier and lookback options) and the need for z-transform
- use-case in discrete pricing options
- start work on project report; however, to be worked on throughout the year/stages

2.4 5$^{\text{th}}$ March '24 → 26$^{\text{th}}$ April '24

- extra time to deal with any unexpected problems or delays
- final touches