Práctica Unidad 10 –
Resolución de Ecuaciones Diferenciales por Método Numérico

1-

Condición de Existencia y Unicidad de Solución de Problemas del Valor Inicial

$$\begin{cases} f(t,y)\ continua\ en\ R=\{a<t<b\wedge c<y<d\}/(t_0,y_0)\in R \\ \exists|f'_y(t,y)|\leq L\ CONDICIÓN\ LIPSCHITZ \end{cases}$$

a)

$$yCos(x) es\ continua\ en\ R \Rightarrow Está\ bien\ Planteada$$
$$Max|f'_y(t,y)| = Max|Cos(x)| = 1 \leq L \Rightarrow L=1 \Rightarrow Cumple\ Lipschitz$$

b)

$$\frac{2}{x}y + e^x x^2\ es\ continua\ en\ R-\{x\in\{0\}\} \Rightarrow f(t,y) es\ Continua\ [1,2]\ Está\ bien\ Planteada$$
$$Max|f'_y(t,y)| = Max\left|\frac{2}{x}\right| = 3 \leq L \Rightarrow L=2 \Longrightarrow Cumple\ Lipschitz$$

c)

$$-xy+\frac{4x}{y}\ \ es\ continua\ en\ R \Rightarrow\ Está\ bien\ Planteada$$
$$Max|1-y| = Max|-1| \Rightarrow L=1 \Rightarrow Cumple\ Lipschitz$$

e)

$$x^2y+1\ \ es\ continua\ en\ R \Rightarrow\ Está\ bien\ Planteada$$
$$Max|x^2| = 1\leq L \Rightarrow L=1 \Rightarrow Cumple\ Lipschitz$$

2-

$$-ty\ es\ continua\ en\ R \Rightarrow Está\ bien\ Planteada$$
$$Max|f'_y(t,y)| = Max|-t| = 3 \leq L \Rightarrow L=3 \Rightarrow b)$$

4-

Método de Euler

$$w_{i+1} = w_i + hf(t_i,w_i) \quad siendo \begin{cases} f(t,y)=y' \\ w_0=y_0 \end{cases}$$

a)

$$w_{i+1} = w_i + 0.02(w_i\ +1)$$
$$w_1 = w_0 + 0.02(w_0+1) = 0.02$$
$$w_2 = w_1 + 0.02(w_1+1) = 0.0404$$
$$w_3 = w_2 + 0.02(w_2+1) = 0.061208$$
$$w_4 = w_3 + 0.02(w_3+1) = 0.08243216$$
$$w_5 = w_4 + 0.02(w_4+1) = 0.1040808 = y(0.1)$$

b)

$$w_{i+1} = w_i + 0.1\left(\frac{2(w_i-1)}{t_i}\right)$$
$$w_1 = w_0 + 0.1\left(2*\frac{(w_0-1)}{1}\right) = 2.2$$
$$w_2 = w_1 + 0.1\left(2*\frac{(w_1-1)}{1.1}\right) = \frac{133}{55} = 2.4181818181$$
$$w_3 = w_2 + 0.1\left(2*\frac{(w_2-1)}{1.2}\right) = \frac{146}{55} = 2.6545454545$$
$$w_4 = w_3 + 0.1\left(2*\frac{(w_3-1)}{1.3}\right) = \frac{32}{11} = 2.909090909$$
$$w_5 = w_4 + 0.1\left(2*\frac{(w_4-1)}{1.4}\right) = \frac{35}{11} = 3.1818181818 = y(1.5)$$

c)
Para Resolverlo por Calculadora:

$$1 =$$

$$ans_1 = ans + 0.2(2x0 - 0\char`^2 + ans) = 1.2$$
$$ans_2 = ans + 0.2(2x0.2 - 0.2\char`^2 + ans) = 1.512$$
$$ans_3 = ans + 0.2(2x0.4 - 0.2\char`^4 + ans) = 1.9424$$
$$ans_4 = ans + 0.2(2x0.6 - 0.6\char`^2 + ans) = 2.49888$$
$$ans_5 = ans + 0.2(2x0.8 - 0.8\char`^2 + ans) = 3.190656$$
$$w_5 = w_4 + 0.1\left(2*\frac{(w_4-1)}{1.4}\right) = \frac{35}{11} = 3.1818181818 = y(1)$$

5-

Error de Euler

$$\frac{h^2}{2}Max|f''(\phi)| < e\ en\ [t_0,t_n]$$

$$Como\ Max|y''(\phi)| = 0 \Rightarrow Euler\ da\ Exacto \Rightarrow c)$$

7-

Método de Heun

$$w_{i+1} = w_i + \frac{h}{2}\left[f(t_i,w_i) + f\big(t_i+h; w_i + hf(t_i,w_i)\big)\right]$$

$$w_{i+1} = w_i + \frac{h}{2}\left[\frac{x_i}{y_i} + \left(\frac{x_{i+1}}{w_i + h\left(\frac{x_i}{w_i}\right)}\right)\right]$$
$$w_1 = 1 + \frac{0.1}{2}\left[\frac{0}{1} + \left(\frac{0.1}{1+0.1\left(\frac{0}{1}\right)}\right)\right] = 1.005$$
$$w_2 = 1.005 + \frac{0.1}{2}\left[\frac{0.1}{1.005} + \left(\frac{0.2}{1.0005+0.1\left(\frac{0.1}{1.0005}\right)}\right)\right] = 1.019827824066843$$

8-

a)

$$w_{i+1} = w_i + h\left[\frac{w_i}{t_i^2+t_i}\right]$$
$$w_1 = 0.5 + 0.1\left[\frac{0.5}{1^2+1}\right] = 0.525$$
$$w_2 = 0.525 + 0.1\left[\frac{0.525}{1.1^2+1.1}\right] = 0.5477272$$
$$w_3 = 0.5477272 + 0.1\left[\frac{0.5477272}{1.2^2+1.2}\right] = 0.568474517$$
$$w_4 = 0.568474517 + 0.1\left[\frac{0.568474517}{1.3^2+1.3}\right] = 0.587487043$$
$$w_5 = 0.587487043 + 0.1\left[\frac{0.587487043}{1.4^2+1.4}\right] = 0.604971777$$
$$w_6 = 0.604971777 + 0.1\left[\frac{0.604971777}{1.5^2+1.5}\right] = 0.621104357$$

$$w_7 = 0.621104357 + 0.1\left[\frac{0.621104357}{1.6^2 + 1.6}\right] = 0.636034751$$
$$w_8 = 0.636034751 + 0.1\left[\frac{0.636034751}{1.7^2 + 1.7}\right] = 0.649891717$$
$$w_9 = 0.649891717 + 0.1\left[\frac{0.649891717}{1.8^2 + 1.8}\right] = 0.662786394$$
$$w_{10} = 0.662786394 + 0.1\left[\frac{0.662786394}{1.9^2 + 1.9}\right] = 0.674815185$$

b)

Runge-Kutta de 4to Orden

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + +2k_2 + 2k_3 + k_4)$$

$$\begin{cases} k_{1;i} = hf(t_i; w_i) \\ k_{2;i} = hf\left(t_i + \frac{h}{2}; w_i + \frac{k_1}{2}\right) \\ k_{3;i} = hf\left(t_i + \frac{h}{2}; w_i + \frac{k_2}{2}\right) \\ k_{4;i} = hf(t_i + h, w_i + k_3) \end{cases}$$

$$\begin{cases} k_{1;0} = 0.5\frac{0.5}{1^2 + 1} = 0.125 \\ k_{2;0} = 0.5\frac{0.5 + \frac{0.125}{2}}{1.25^2 + 1.25} = 0.1 \\ k_{3;0} = 0.5\frac{0.5 + \frac{0.1}{2}}{1.25^2 + 1.25} = 0.0977777777 \\ k_{4;0} = 0.5\frac{0.5 + 0.089086419}{1.5^2 + 1.5} = 0.079703703 \end{cases}$$

$$w_1 = 0.5 + \frac{1}{6}(0.125 + 0.2 + 0.195555555555 + 0.078544855) = 0.6$$

$$\begin{cases} k_{1;1} = 05.\frac{0.6}{1.5^2 + 1.5} = 0.08 \\ k_{2;1} = 0.5\frac{0.6 + \frac{0.08}{2}}{1.75^2 + 1.75} = 0.066493506 \\ k_{3;1} = 0.5\frac{0.6 + \frac{0.066493506}{2}}{1.75^2 + 1.75} = 0.06579187 \\ k_{4;1} = 0.5\frac{0.6 + 0.06579187}{2^2 + 2} = 0.055482655 \end{cases}$$

$w_2 =$ 0.666675568298945 (VERIFICAR NO ME DA COMO LA GUÍA)

9-

a) por Calculadora

a)

$$w_{i+1} = ans + h(t_i\ x\ ans)$$
$$1 =$$
$$w_1 = ans + 0.1(-0\ x\ ans) = 1$$
$$pad(\leftarrow)$$
$$Busco\ t_i\ DEL$$
$$shift + Del = ins$$
$$t_i\ por\ t_i + h$$
$$shift + Del$$
$$=$$
$$w_2 = ans + 0.1(-0.1\ x\ ans) = 0.99$$
Repetir despues de cada iteración
$$w_3 = ans + 0.1(-0.2\ x\ ans) = 0.9702$$
$$w_4 = ans + 0.1(-0.3\ x\ ans) = 0.941094$$
$$w_5 = ans + 0.1(-0.4\ x\ ans) = 0.90345024$$
$$w_6 = ans + 0.1(-0.5\ x\ ans) = 0.858277728$$
$$w_7 = ans + 0.1(-0.6\ x\ ans) = 0.80678106432$$
$$w_8 = ans + 0.1(-0.7\ x\ ans) = 0.7503063898176$$
$$w_9 = ans + 0.1(-0.8\ x\ ans) = 0.690281878632192$$
$$w_{10} = ans + 0.1(-0.9\ x\ ans) = 0.628156509555295$$

b)

$$w_{i+1} = ans + \frac{h}{2}[-t_i ans - (t_i + h) * (ans + h(-t_i ans))]$$
$$w_1 = ans + \frac{0.2}{2}[-0 * ans - (0 + 0.2) * (ans + 0.2(-0 * ans))] = 0.98$$
$$w_2 = 0.922768$$
$$w_3 = 0.8349204864$$
$$w_4 = 0.726046854973440$$
$$w_5 = 0.606975170757796 (MEJOR\ APROX)$$

c)

Polinomio de Taylor de Orden n

$$w_{i+1} = w_i + hf(t_i, w_i) + \frac{h^2}{2}f''(t_i, w_i) + \cdots + \frac{h^n}{n!}f^{(n)}(t_i, w_i)$$

$$w_1 = w_0 + 0.5 * \left(-t_0 e^{-\frac{t_0^2}{2}}\right) + \frac{0.5^2}{2}\left(e^{-\frac{t_0^2}{2}}(t_0^2 - 1)\right) = 0.875$$

$$w_2 = 0.571641689736545$$

d)

$$\begin{cases} k_{1;0} = -t_0 y_0 = 0 \\ k_{2;0} = -(t_0 + 0.5)(y_0) = -0.5 \\ k_{3;0} = -(t_0 + 0.5)\left(y_0 - \frac{0.5}{2}\right) = -0.375 \\ k_{4;0} = -(t_0 + 1)(y_0 - 0.375) = -0.6255 \end{cases}$$

$$w_1 = 1 + \frac{1}{6}(0 - 1 - 0.75 - 0.6255) = 0.604083333$$