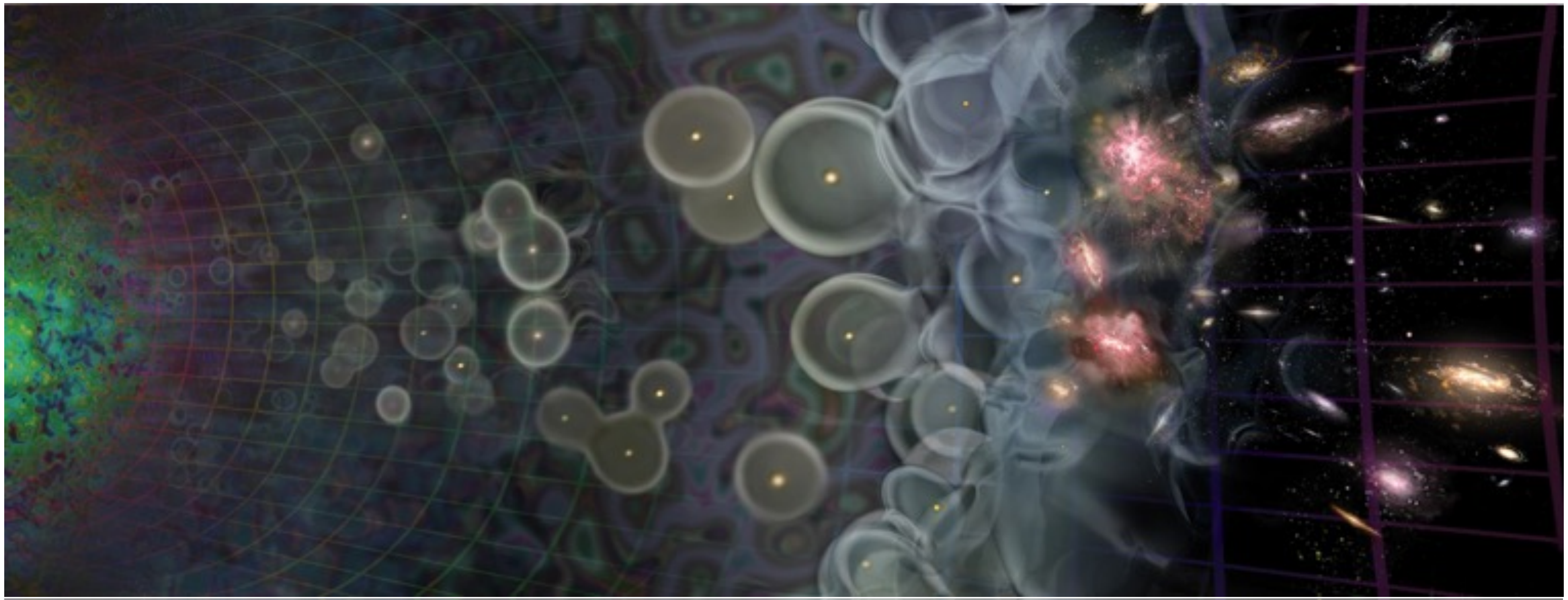


Statistical Description of Cosmological Density Perturbations

Mariana Vargas Magana

Instituto de Física

Observational Cosmology for MACSS 2017 (as part of MexiCopas), León Guanajuato.

Preliminaries

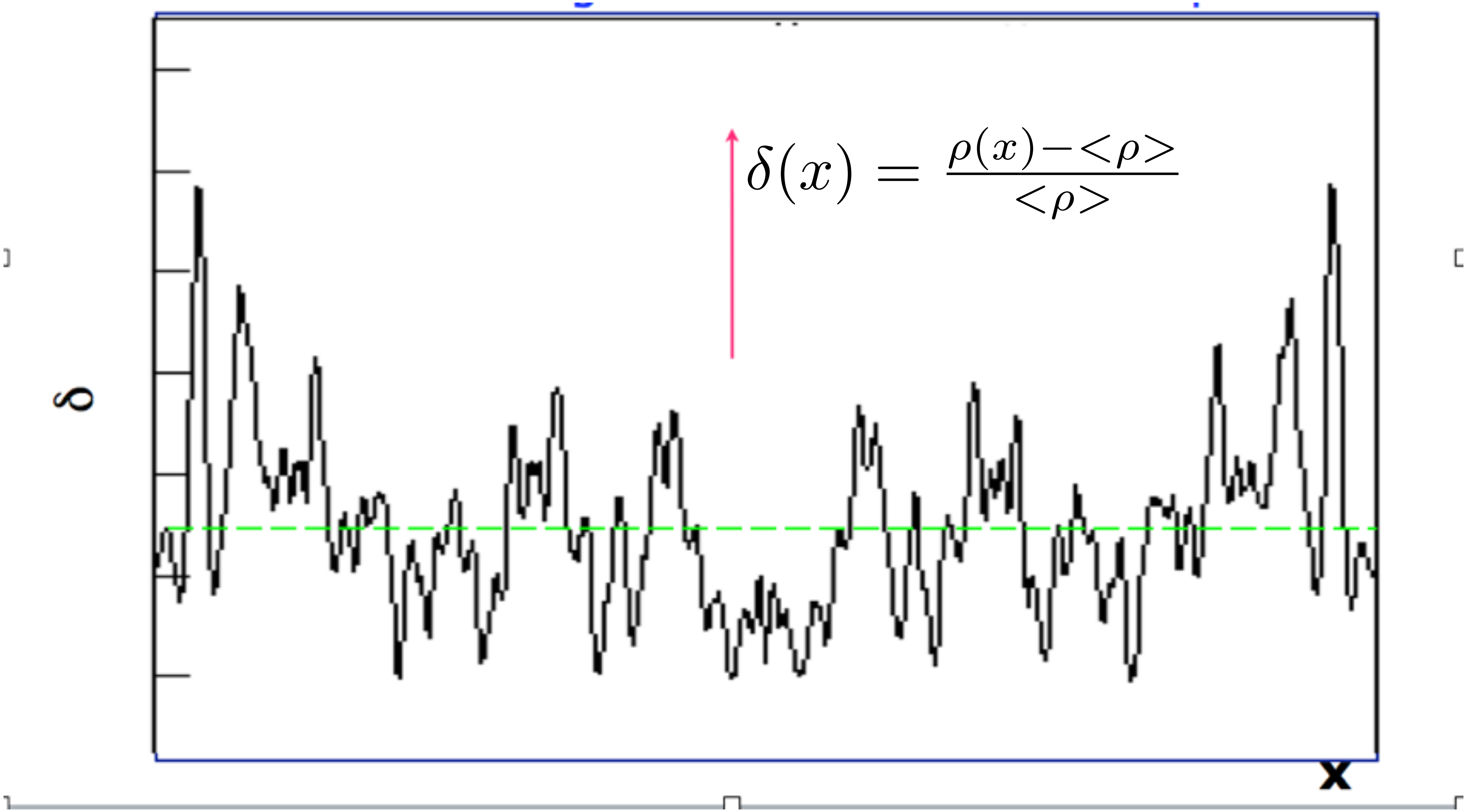
- Let $\varrho(x)$ be the density distribution of matter at a location x

$$\delta(x) = \frac{\rho(x) - <\rho>}{<\rho>}$$

- It is useful to define the corresponding over density field

- is believed to be the outcome of some random process in the early universe (i.e. Quantum fluctuations in inflation)

Density Fluctuations



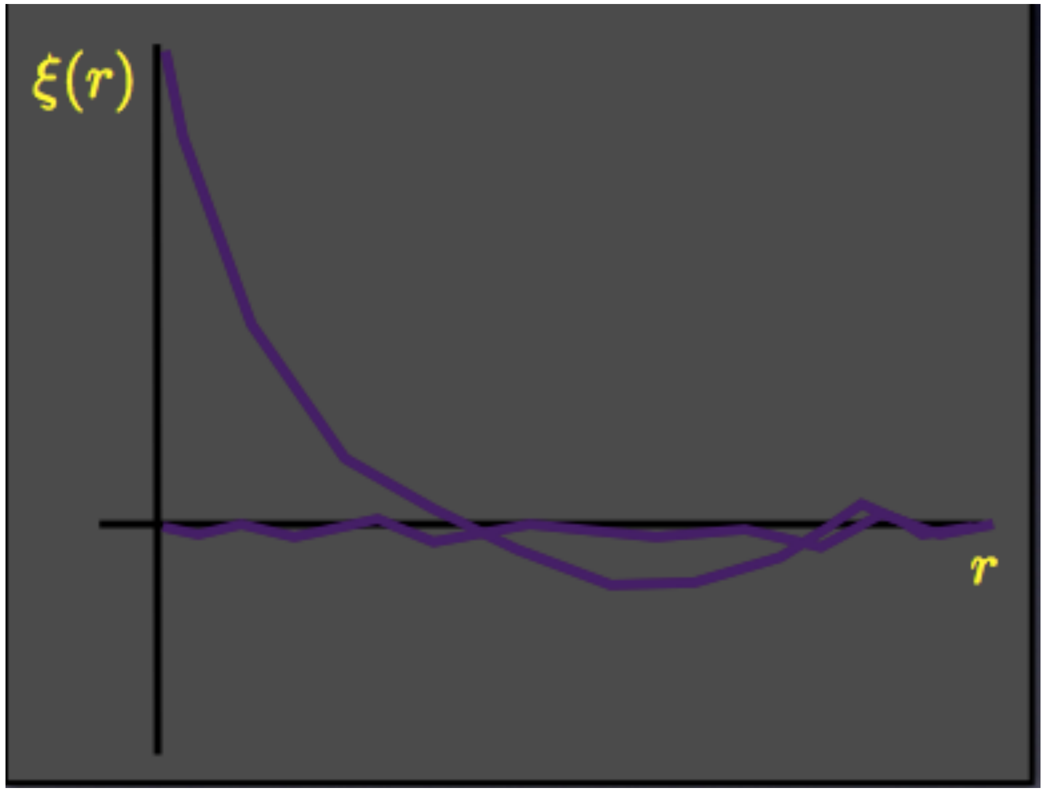
- NOTE: <*>denotes an ensemble average. For instance, means the average overdensity at for many realizations of the random process

2-point Correlation Function

Second Moment

$$\langle \delta_1 \, \delta_2 \rangle \equiv \xi(r_{12}) \qquad r_{12} = |\vec{x}_1 - \vec{x}_2|$$

- ξ(r) is called the two-point correlation function
- Note that this two-point correlation function is defined for a continuous field, . However, one can also define it for a point distribution:



2-point estimators in Fourier space

Instituto de Física

Reminder

- Overdensity δ(t,x) contains all information about the LSS in the universe at any time.

- to compare observations of δ with theory, we interpret δ as a realization of a stochastic process

- δ as a realization of a homogeneous and isotropic random field with zero mean.

- For describing a random field our goal is to describe the probability distribution

$$\langle \delta_1^{l_1} \delta_2^{l_2} \ldots \delta_N^{l_N} \rangle = \int \delta_1^{l_1} \delta_2^{l_2} \ldots \delta_N^{l_N} \mathcal{P}(\delta_1, \delta_2, \ldots, \delta_N)\, \mathrm{d}\delta_1\, \mathrm{d}\delta_2 \ldots \mathrm{d}\delta_N$$

- This probability distribution is completely specified by the moments.

Reminder

- < * > denotes an ensemble average.
- For instance, $$\langle\delta\rangle = \int \delta\,\mathcal{P}(\delta)\,\mathrm{d}\delta = \frac{1}{V}\int_V \delta(\vec{x})\,\mathrm{d}^3\vec{x} = 0$$ **First Moment**
- means the average overdensity at for many realizations of the random process.
- Ergodic Hypothesis:Ensemble average is equal to spatial average taken over one realization of the random field.
- ergodic hypothesis requires spatial correlations to decay sufficiently rapidly with increasing separation so that there exists many statistically independent volumes in one realization

Reminder

Moments

$$\boxed{\hat{\mu}_m = \langle x^m \rangle}$$

Mean

$$\langle \delta \rangle = \int \delta \, \mathcal{P}(\delta) \, \mathrm{d}\delta = \frac{1}{V} \int_V \delta(\vec{x}) \, \mathrm{d}^3\vec{x} = 0$$

First Moment

2PCF

$$\langle \delta_1 \, \delta_2 \rangle \equiv \xi(r_{12}) \qquad r_{12} = |\vec{x}_1 - \vec{x}_2|$$

Second Moment

$$\langle \delta_1^{l_1} \delta_2^{l_2} \ldots \delta_N^{l_N} \rangle = \int \delta_1^{l_1} \delta_2^{l_2} \ldots \delta_N^{l_N} \mathcal{P}(\delta_1, \delta_2, \ldots, \delta_N) \, \mathrm{d}\delta_1 \, \mathrm{d}\delta_2 \ldots \mathrm{d}\delta_N$$

- For a Gaussian distribution all moments of order higher than 2 are specified by μ1 and μ2. Or, in other words, the mean and the variance completely specify a Gaussian distribution.

Power Spectrum

- Often it is very useful to describe the matter field in Fourier space :

$$\delta(\vec{x}) = \sum_k \delta_{\vec{k}}\, e^{+i\vec{k}\cdot\vec{x}} \qquad \delta_{\vec{k}} = \frac{1}{V}\int \delta(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x}$$

- The **perturbed density field can be written as a sum of plane waves of different wave numbers k(called `modes').**

- The Fourier transform (FT) of the two-point correlation function is called the power spectrum and is given by

$$\begin{aligned} P(\vec{k}) &\equiv V\langle|\delta_{\vec{k}}|^2\rangle \\ &= \int \xi(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x} \\ &= 4\pi \int \xi(r)\, \frac{\sin kr}{kr}\, r^2\, \mathrm{d}r \end{aligned}$$

- **A Gaussian random field is completely specified by either the two-point correlation function** , or, equivalently, the power spectrum

Power spectrum (and Bispectrum) from N-body simulation

- Estimating power spectrum (and bispectrum) from the N-body simulation data is less complicated as N-body simulations have 1) the cubic box, 2) the constant mean number density. We divide the general procedure of measuring power spectrum from N-body simulation by following five steps:

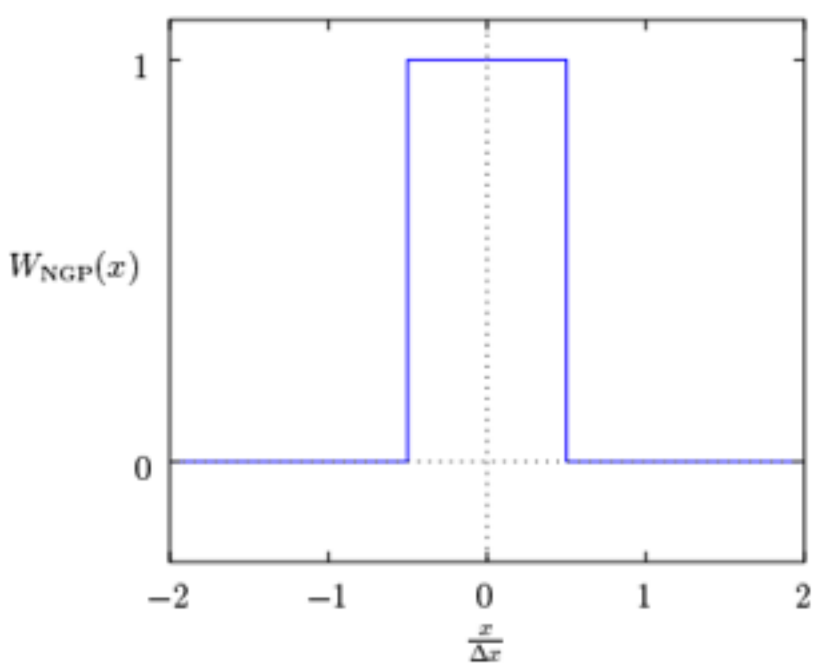
    1. **Distributing particles onto the regular grid**
    2. Fourier transformation
    3. Estimating power spectrum
    4. Deconvolving window function
    5. Subtracting shot noise

Density estimation and effect on P(k)

- The way we distribute a particle to the nearby grid points is called a 'particle distribution scheme.'
- For a given distribution scheme, we can define an associated 'shape function', which quantifies how a quantity (number) of particle is distributed. After this process, the sampling we made from the particle distribution is not a mere sampling of the underlying density field, but a sampling convolved with the 'window function' of particle distribution scheme.
- .There are different ways of placing galaxies (or particle in your simulation) on a grid:
  - **Nearest grid point, NGP.**
  - Cloud in cell, CIC.
  - triangular shaped cloud, TSC.
- For each of these we need to deconvolve the resulting P(k) for their effect. **For our course we consider NGP.**

Particle Distribution Scheme

- Nearest Grid Point (NGP) scheme assigns particles to their nearest grid points. Therefore, the number density changes discontinuously when particles cross cell boundaries. The one dimensional window function for NGP is proportional to the top-hat function.



$$W_{NGP}(x) \equiv \frac{1}{H}\mathcal{T}\left(\frac{x}{H}\right) = \begin{cases} 1/H & \text{if } \; |x| < H/2 \\ 1/(2H) & \text{if } \; |x| = H/2 \\ 0 & \text{if } \; \text{otherwise} \end{cases},$$

kN = π/H is the Nyquist frequency

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  1. Distributing particles onto the regular grid
  2. **Fourier transformation**
  3. Estimating power spectrum
  4. Deconvolving window function
  5. Subtracting shot noise

P(k) from a realistic galaxy catalog

- In the real world when you go and take the FT of your survey or even of your simulation box you will be using something like a **Fast Fourier transform code (FFT) which is a discrete Fourier transform**.

- If your box has side of size L, even if δ(r) in the box is continuous, δk will be discrete. The k-modes sampled will be given by

$$\boxed{\vec{k} = \left(\frac{2\pi}{L}\right)(i, j, k) \quad \text{where} \quad \Delta_k = \frac{2\pi}{L}}$$

- The discrete Fourier transform is obtained by placing the δ(x) on a lattice of N3 grid points with spacing L/N. Then:

$$\begin{aligned} \delta_k^{DFT} &= \frac{1}{N^3} \sum_r \exp[-i\vec{k} \cdot \vec{r}]\delta(\vec{r}) \\ \delta^{DFT}(\vec{r}) &= \sum_k \exp[i\vec{k} \cdot \vec{r}]\delta_k^{DFT} \end{aligned}$$

P(k) using FFTW

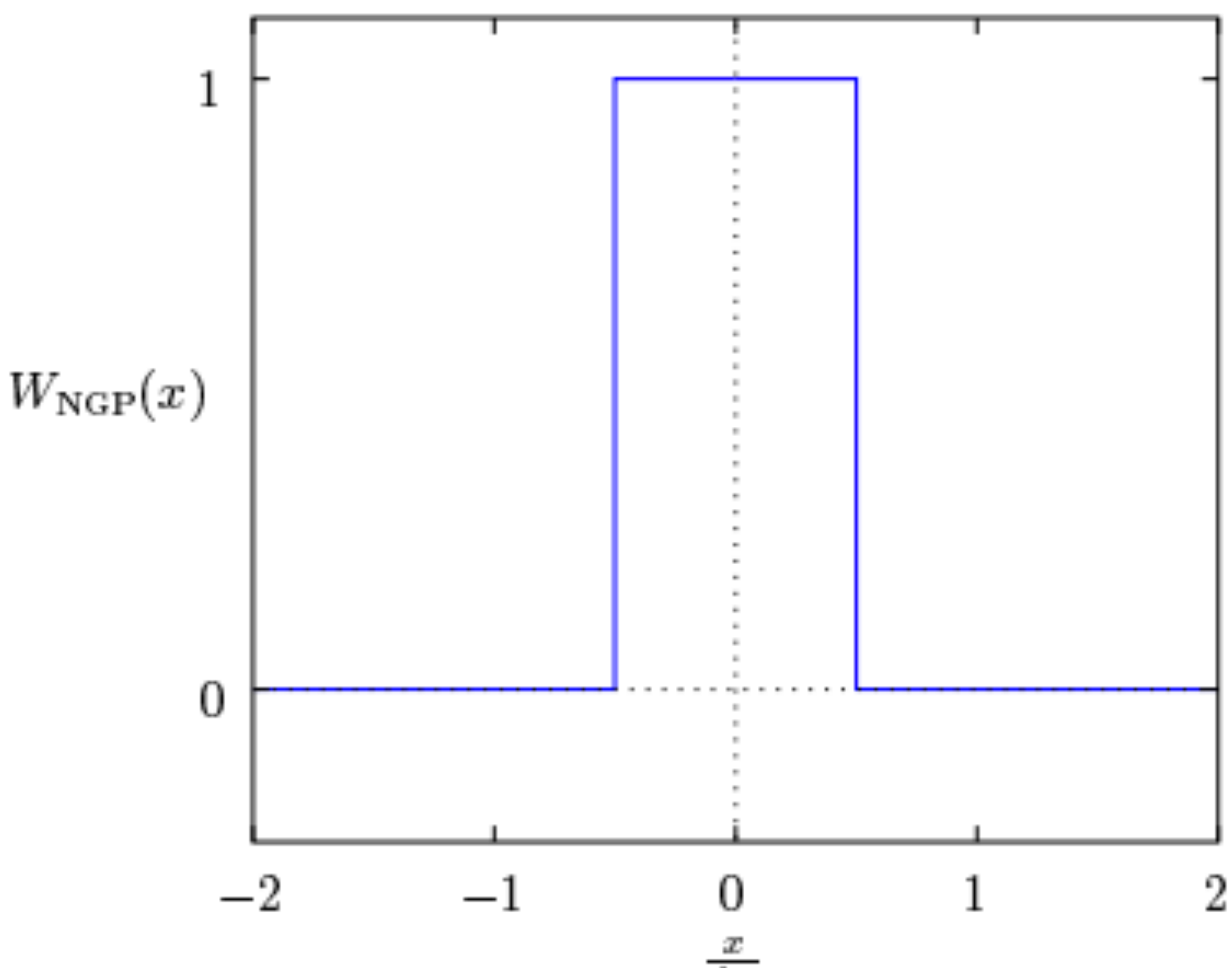
- We shall find the proper normalization to the power spectrum estimators which use the unnormalized **Fast Fourier Transformation (FFT) such as FFTW**. For denote the unnormalized discrete Fourier transform result by superscript ‘FFTW’.

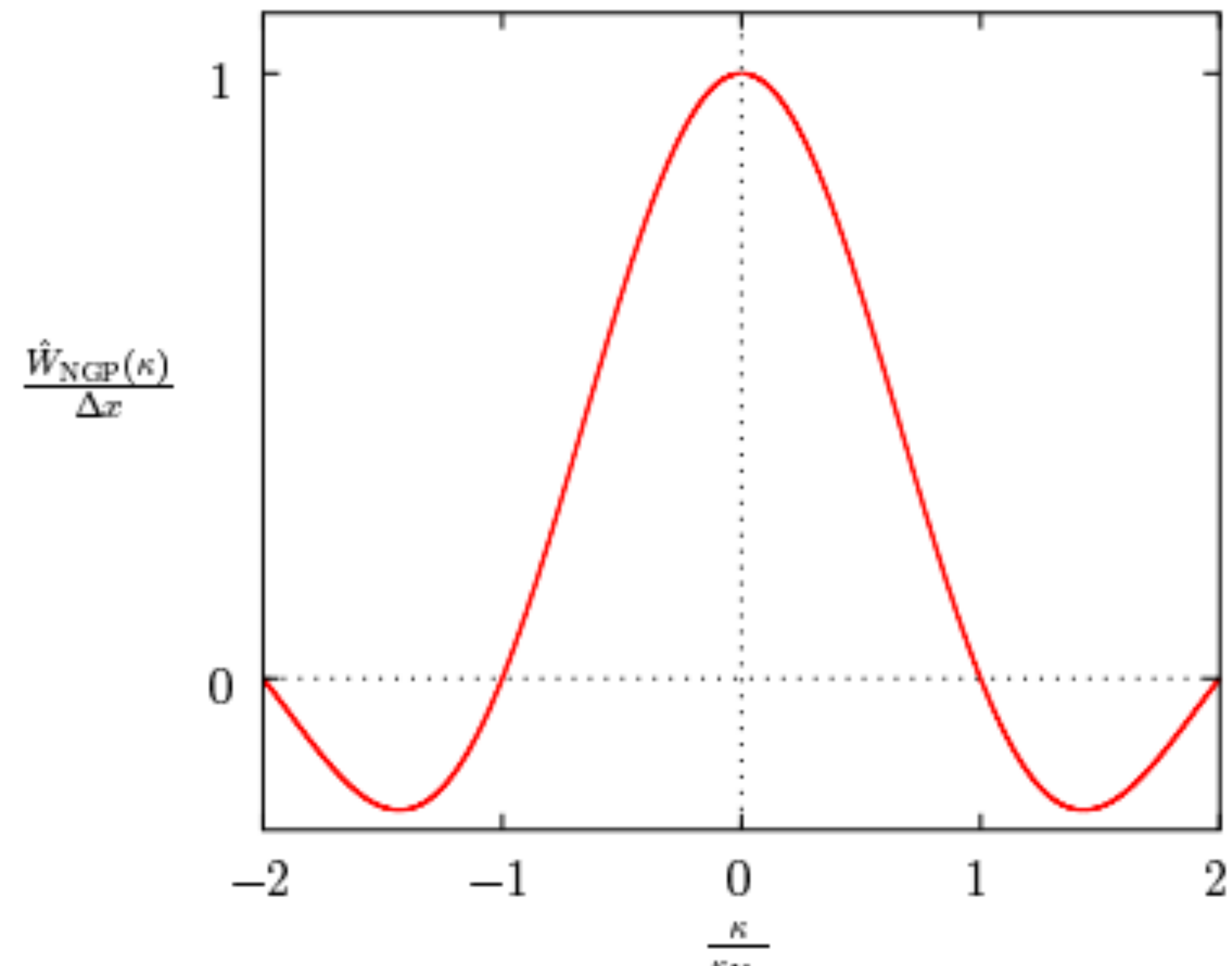
$$\boxed{P(k_F n_1) = \frac{V}{N^6}\left\langle \left|\delta^{FFTW}(\boldsymbol{n}_1)\right|^2 \right\rangle = \frac{V}{N^6}\left(\frac{1}{N_k}\sum_{|\boldsymbol{n}_k - \boldsymbol{n}_1| \le \frac{1}{2}} \left|\delta^{FFTW}(\boldsymbol{n}_k)\right|^2\right),}$$

- where V is the volume of survey, N is number of one-dimensional grid, $H^3 = V/N^3$ and $k_F{}^3 = (2\pi)^3/V$ .where we sum over all Fourier modes within $k_1 - k_F/2 < |k| < k_1 + k_F/2$ to estimate the power spectrum at $k = k_1 = k_F\, n1$.

Density estimation

the density assignment wider in configuration space.





$$\boxed{W_{NGP}(k) = \mathrm{FT}[\mathfrak{I}](Hk) = \mathrm{sinc}\left(\frac{Hk}{2}\right) = \mathrm{sinc}\left(\frac{\pi k}{2k_N}\right)}$$

3D window function

- As we use the regular cubic grid, the three dimensional window function is simply given as the multiplication of three one dimensional window functions.

$$\boxed{W(\boldsymbol{x}) = W(x_1)W(x_2)W(x_3)}$$

- Therefore, its Fourier transformation is

$$\boxed{W(\boldsymbol{k}) = \left[\mathrm{sinc}\left(\frac{\pi k_1}{2k_N}\right)\mathrm{sinc}\left(\frac{\pi k_2}{2k_N}\right)\mathrm{sinc}\left(\frac{\pi k_3}{2k_N}\right)\right]^p,}$$

- where p = 1, 2, 3 for NGP, CIC and TSC, respectively.

Deconvolution of Window Function

- We have the estimator for the power spectrum. However, as we have employed the distribution scheme, the power spectrum we would measure with those estimators are not the same as the power of the 'real' density contrast, but the **power of density contrast convolved with the window function**.

- Therefore, the power spectrum we estimate will show the **artificial power suppression on small scales**. Therefore, we have to **deconvolve the window function due to the particle distribution scheme** in order to estimate the power spectrum of the true density contrast.

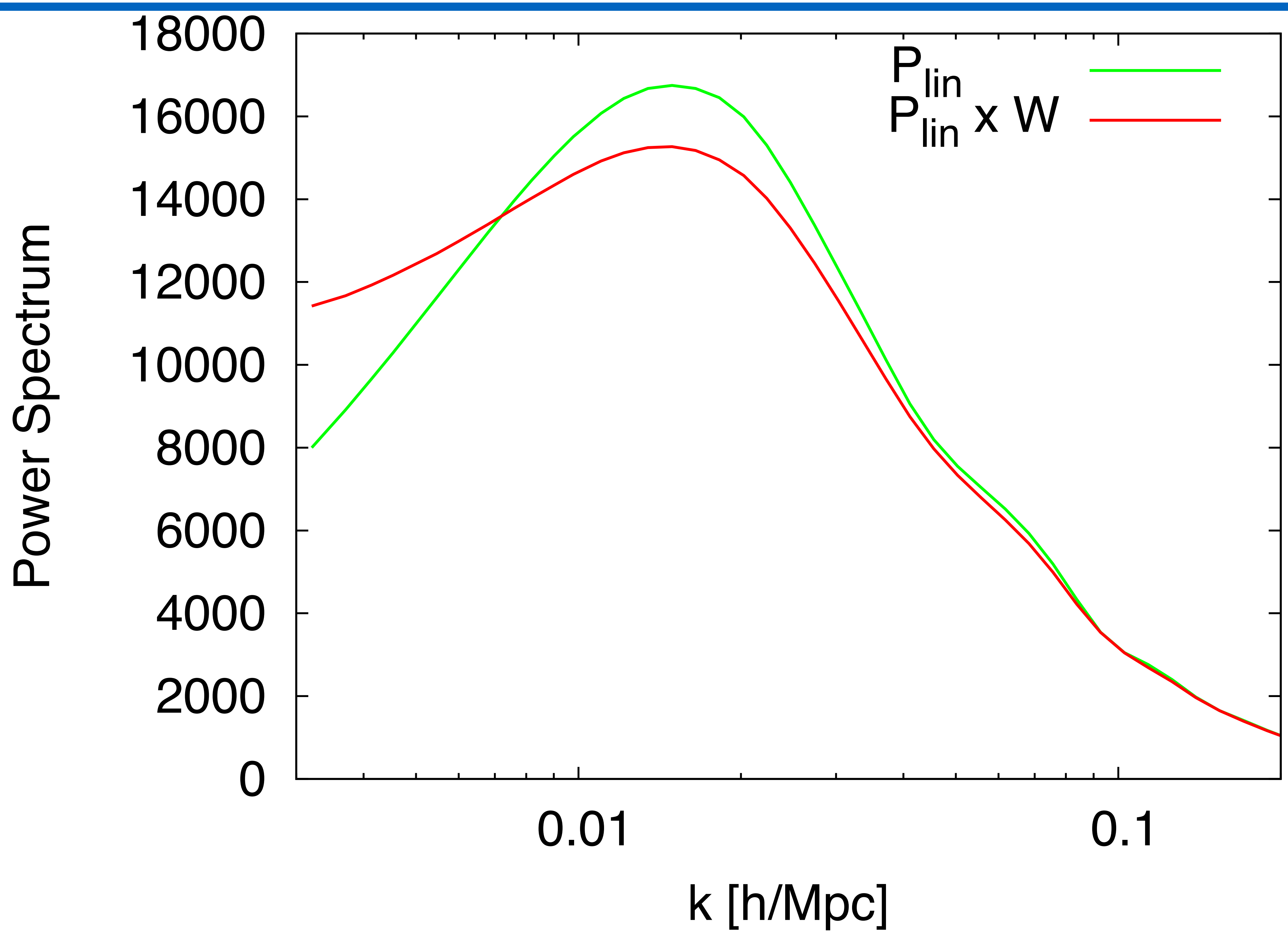
- As we know the exact shape of the window function in Fourier space, we can simply divide the resulting density contrast in Fourier space by the window function. That is, we deconvolve each k mode of density contrast as

$$\boxed{\delta(\boldsymbol{k}) = \frac{\delta^m(\boldsymbol{k})}{W(\boldsymbol{k})},}$$

- or, deconvolve the estimated power spectrum by

- for k < kN . Again, p = 1, 2, 3 for NGP, CIC and TSC scheme, respectively. Here, superscript m denote the measured quantity.

$$\boxed{P(\boldsymbol{k}) = \left|\frac{\delta^m(\boldsymbol{k})}{W(\boldsymbol{k})}\right|^2 = P^m(k_1,k_2,k_3)\left[\mathrm{sinc}\left(\frac{\pi k_1}{2k_N}\right)\mathrm{sinc}\left(\frac{\pi k_2}{2k_N}\right)\mathrm{sinc}\left(\frac{\pi k_3}{2k_N}\right)\right]^{-2p},}$$



18000
16000
14000
12000
10000
8000
6000
4000
2000
0
Power Spectrum
0.01
0.1
k [h/Mpc]
P_{lin}
P_{lin} x W

Shot NOISE

- As long as a galaxy number density is high enough (which will need to be quantified and checked for any practical application) and we have enough modes, we say that we will **have a superposition of our random field (say the dark matter one characterized by its P(k)) plus a white noise contribution coming from the discreteness which amplitude depends on the average number density of galaxies** (and should go to zero as this go to infinity), and we treat this additional contribution as if it has the same statistical properties as the underlying density field (which is an approximation).

$$\boxed{< \delta_{k_1} \delta_{k_2} >^d = (2\pi)^3 \left(P(k) + \frac{1}{\bar{n}} \right) \delta^d(\vec{k_1} + \vec{k_2})}$$

kmax and kmin

- The Nyquist frequency (**maximal k**):

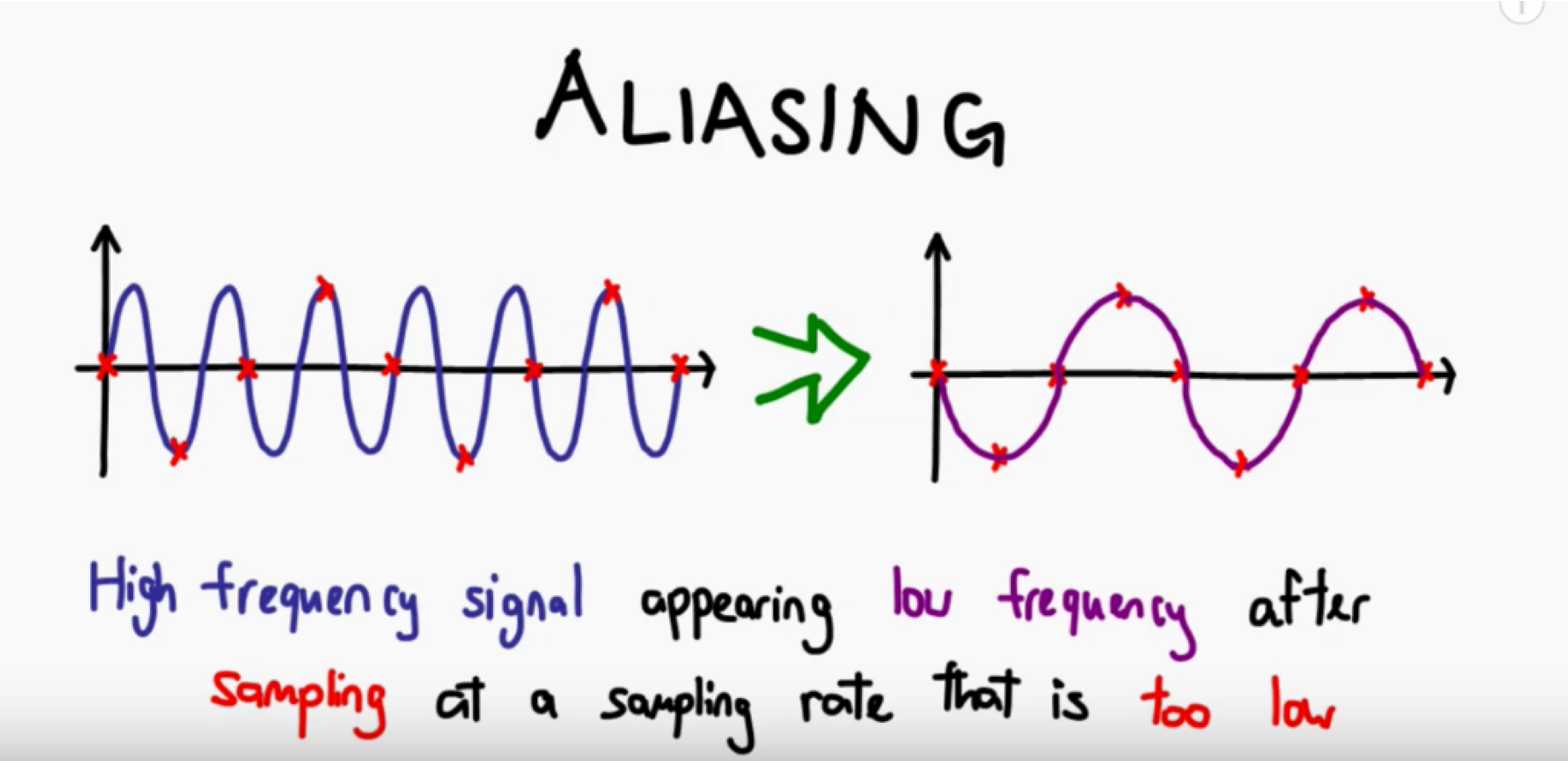
$$\boxed{\text{knquist} = 2\pi N/2L}$$

- is that of a mode which is sampled by 2 grid points. **Higher frequencies cannot be properly sampled and give aliasing (spurious transfer of power)** effects. You should always work at **k < kNy.**

- There is also a **minimum k** (largest possible scale) that you finite box can test :

$$\boxed{\text{kmin} > 2\pi/L.}$$

- **In addition DFT assume periodic boundary conditions, if you do not have periodic boundary conditions then this also introduces aliasing.**


ALIASING
High frequency signal appearing low frequency after sampling at a sampling rate that is too low

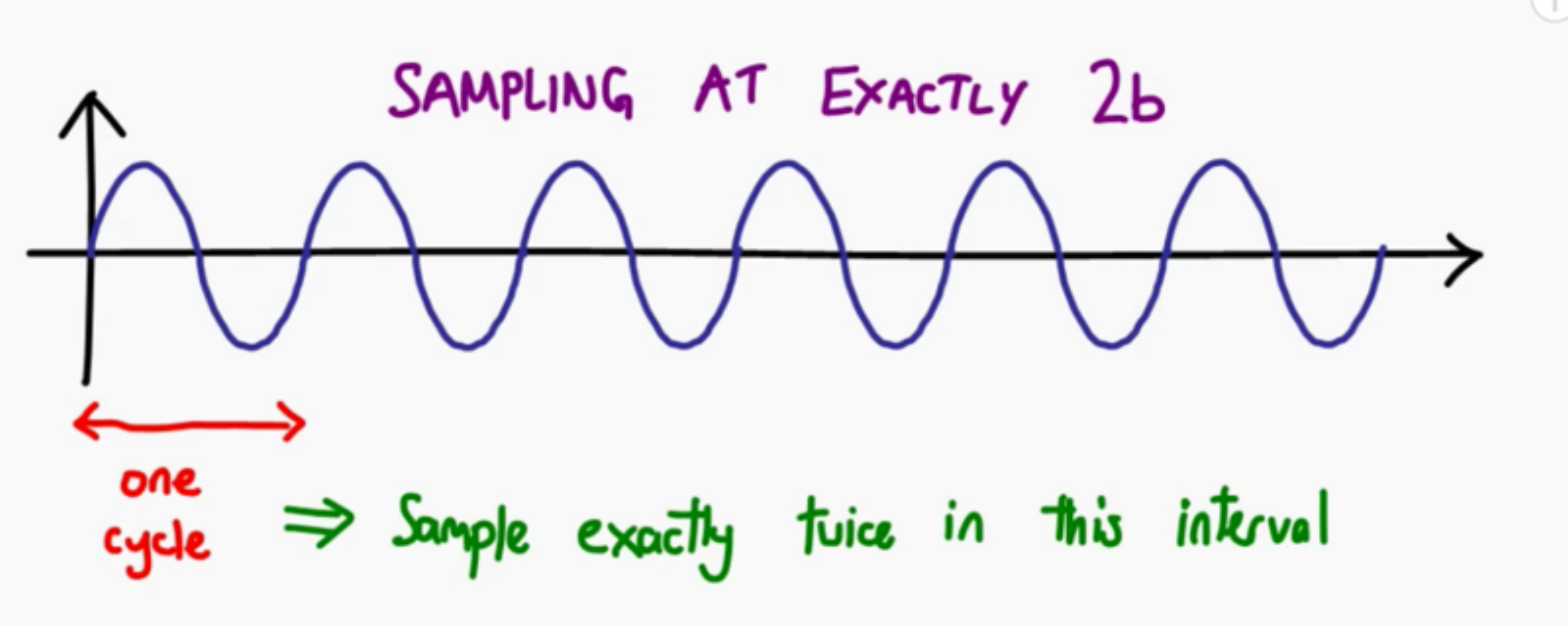
Nyquist-Shannon & aliasing

NYQUIST SAMPLING THEOREM

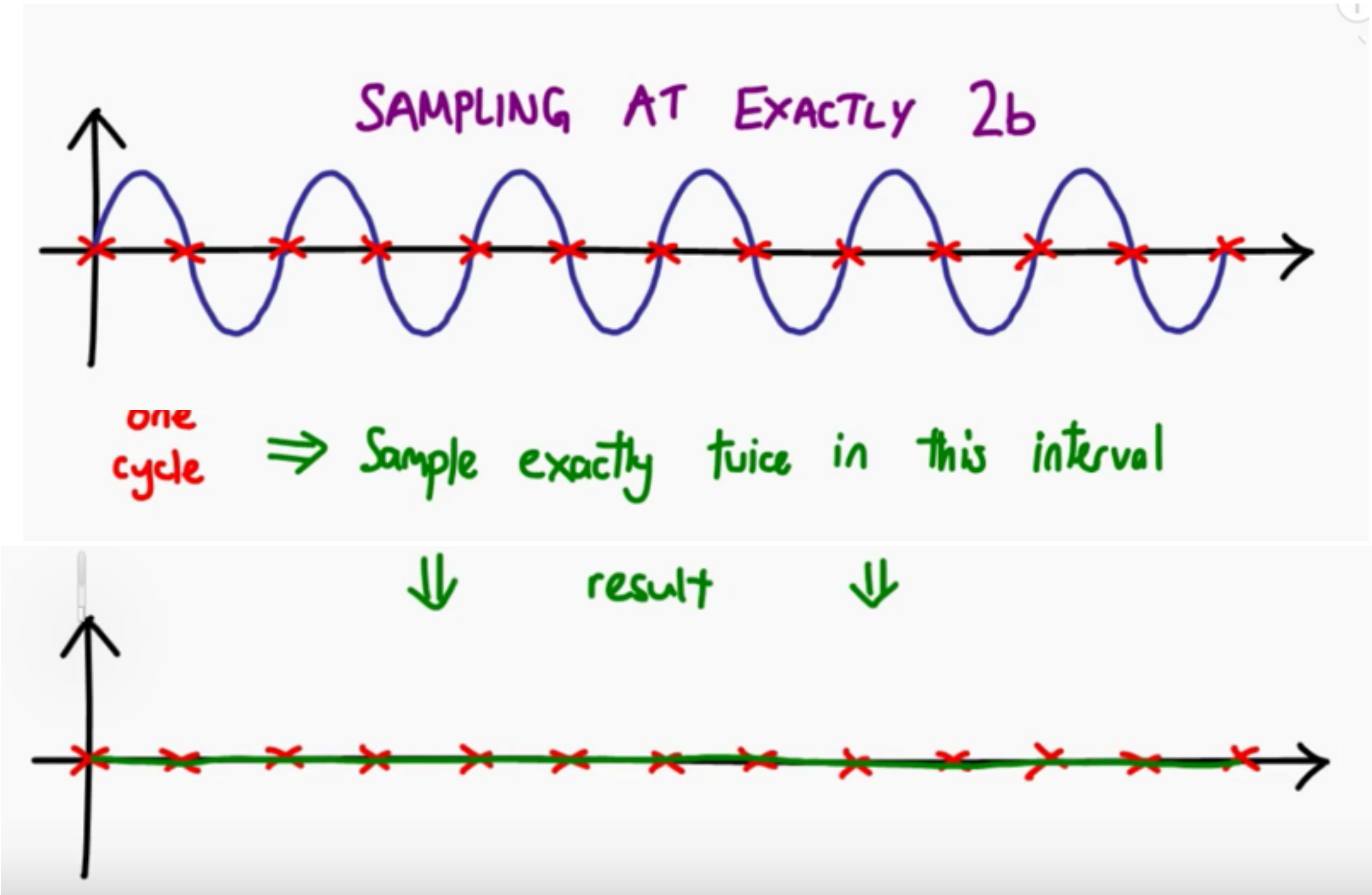
Sampling rate should be double the max. frequency

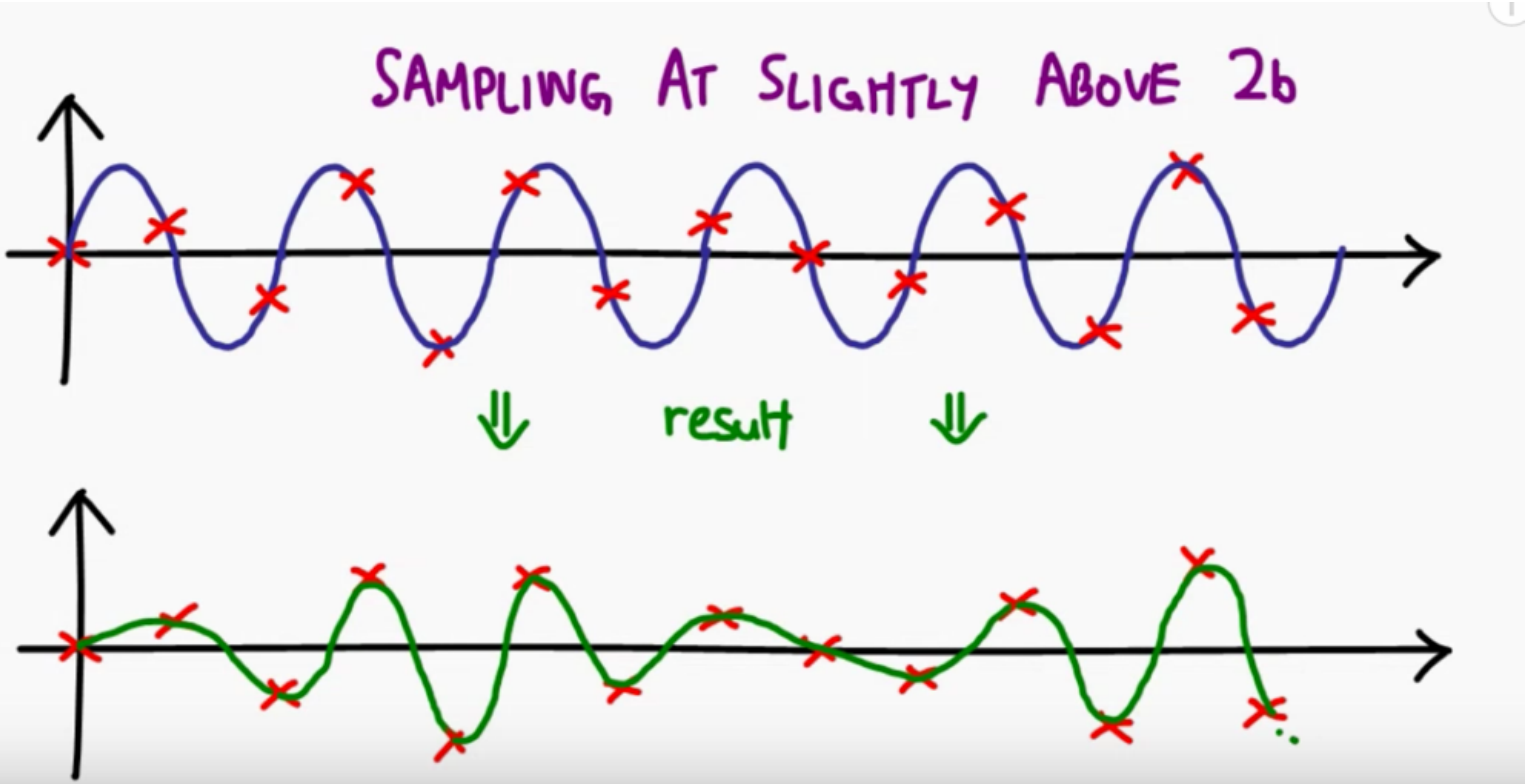
To be precise...

Sampling rate = 2b $\Rightarrow$ All frequencies LESS THAN b will not alias

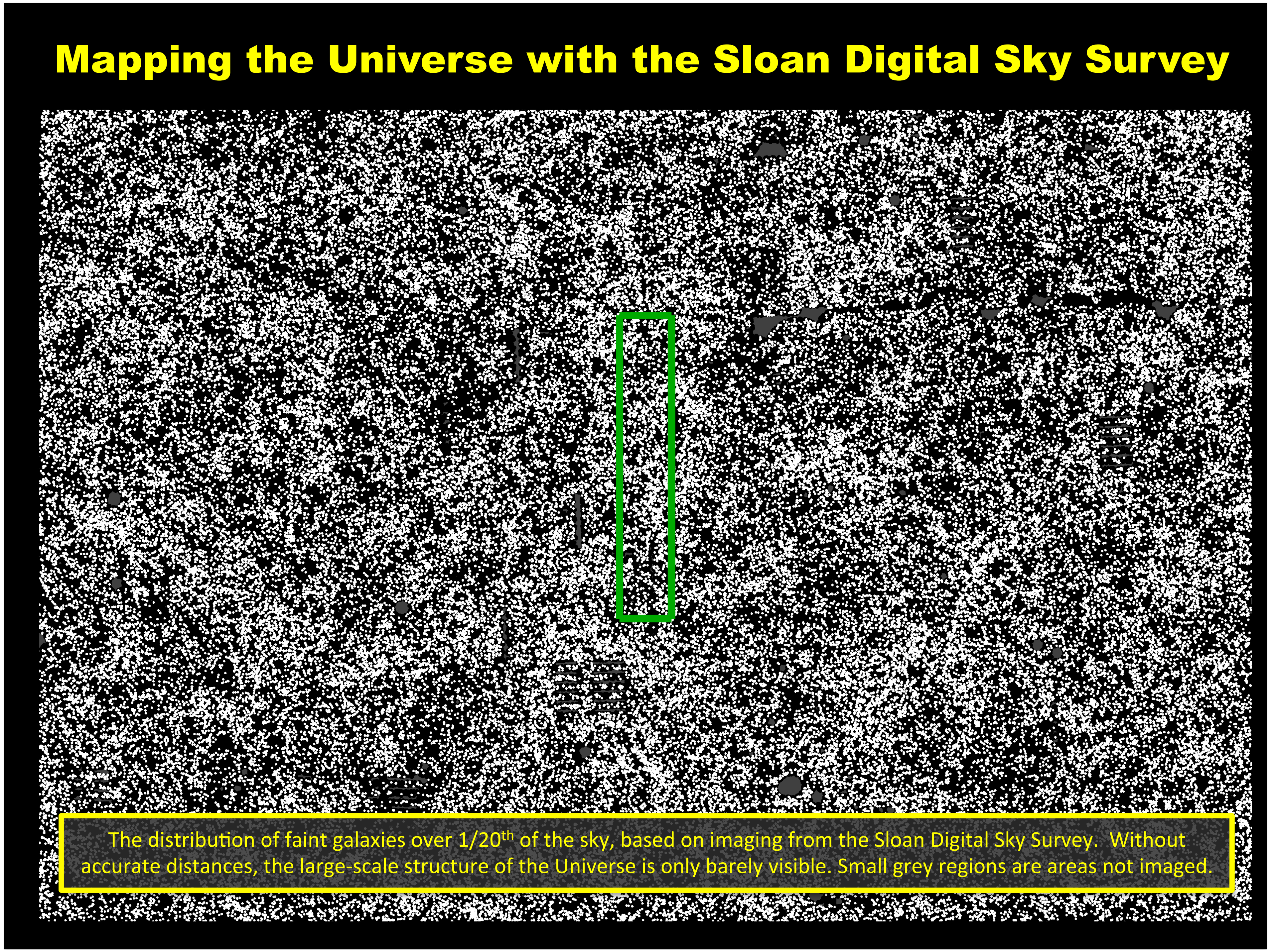

SAMPLING AT EXACTLY 2b
one cycle
⇒ Sample exactly twice in this interval


SAMPLING AT EXACTLY 2b


SAMPLING AT EXACTLY 2b
one cycle
⇒ Sample exactly twice in this interval
⇓ result ⇓


SAMPLING AT SLIGHTLY ABOVE 2b
result

P(k) from a realistic galaxy catalog



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P(k) from a realistic galaxy catalog

arXiv:astro-ph/9304022v1

POWER SPECTRUM ANALYSIS
OF
THREE-DIMENSIONAL REDSHIFT SURVEYS

Hume A. Feldman[1,a], **Nick Kaiser**[2,4,b] **and John A. Peacock**[3,4,c]

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2) CITA, University of Toronto, Ontario M5S 1A1, Canada
3) Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ, UK
4) CIAR Cosmology Program

Abstract

We develop a general method for power spectrum analysis of three dimensional redshift surveys. We present rigorous analytical estimates for the statistical uncertainty in the power and we are able to derive a rigorous optimal weighting scheme under the reasonable (and largely empirically verified) assumption that the long wavelength Fourier components are

Window function and P(k)

- Following the familiar **(FKP) method** developed by Feldman, Kaiser and Peacock. we obtain the convolved power spectrum including the window effect.

- power spectrum analysis developed by Feldman, Kaiser and Peacock ([27], hereafter FKP).

- We define the fluctuation field as:

$$\boxed{F(\mathbf{s}) = n_{\rm g}(\mathbf{s}) - \alpha n_{\rm s}(\mathbf{s}),}$$

- ng(s) is the density in the data catalog, ns(s) is the density of a random catalog that has a mean number density $1/\alpha$ times that of the galaxy catalog. Random points without any correlation, which can be constructed through a random process by mimicking the selection function of the galaxy catalog.

https://arxiv.org/abs/1308.3551

We introduce the Fourier coefficient of $F(\mathbf{s})$ by

$$\mathcal{F}_0(\mathbf{k}) = \frac{\int d^3s\psi(\mathbf{s},\mathbf{k})F(\mathbf{s})e^{i\mathbf{k}\cdot\mathbf{s}}}{[\int d^3s\bar{n}^2(\mathbf{s})\psi^2(\mathbf{s},\mathbf{k})]^{1/2}}, \tag{6}$$

where $\psi(\mathbf{s},\mathbf{k})$ is the weight function (Throughout this paper, we assume $\psi = 1$). The expectation value of $|\mathcal{F}_0(\mathbf{k})|^2$ is

$$\langle|\mathcal{F}_0(\mathbf{k})|^2\rangle = \frac{1}{(2\pi)^3}\int d^3k'P(\mathbf{k}')W(\mathbf{k}-\mathbf{k}') + (1+\alpha)S_0(\mathbf{k}) \tag{7}$$

with

$$W(\mathbf{k}-\mathbf{k}') = \frac{\left|\int d^3s\bar{n}(\mathbf{s})\psi(\mathbf{s},\mathbf{k})e^{i\mathbf{s}\cdot(\mathbf{k}-\mathbf{k}')}\right|^2}{\int d^3s\bar{n}^2(\mathbf{s})\psi^2(\mathbf{s},\mathbf{k})} \tag{8}$$

Window Function

and

$$S_0(\mathbf{k}) = \frac{\int d^3s\bar{n}(\mathbf{s})\psi^2(\mathbf{s},\mathbf{k})}{\int d^3s\bar{n}^2(\mathbf{s})\psi^2(\mathbf{s},\mathbf{k})}, \tag{9}$$

Shot Noise

where we used

$$\xi(\mathbf{s}_1,\mathbf{s}_2) = \frac{1}{(2\pi)^3}\int d^3kP(\mathbf{k})e^{-i\mathbf{k}\cdot(\mathbf{s}_1-\mathbf{s}_2)}. \tag{10}$$

Power spectrum estimator

- The estimator of the convolved power spectrum is:

$$\boxed{P^{\mathrm{conv}}(\mathbf{k}) = |\mathcal{F}_0(\mathbf{k})|^2 - (1+\alpha)S_0(\mathbf{k}),}$$

https://arxiv.org/abs/1308.3551

Window Function

$$W(\mathbf{k}) = \frac{\left|\int d^3s\alpha n_s(\mathbf{s})\psi(\mathbf{s},\mathbf{k})e^{i\mathbf{s}\cdot\mathbf{k}}\right|^2}{\int d^3s\bar{n}^2(\mathbf{s})\psi^2(\mathbf{s},\mathbf{k})} - \alpha S_0(\mathbf{k}).$$

- We can measure the multipole moments of the window function.
- The window function can be evaluated using the random catalog in a similar way of evaluating the power spectrum.
- Similar to the case of the power spectrum, we need to subtract the shot noise contribution.

https://arxiv.org/abs/1308.3551

Optimal Weights

- We estimate the variance of the power spectrum by

$$\boxed{\sigma_p^2 \equiv \left\langle \left[\hat{P}(k) - P(k)\right]^2 \right\rangle = \frac{1}{V_k^2} \int_{V_k} d^3q \int_{V_k} d^3q' \left\langle \delta\hat{P}(\boldsymbol{q})\delta\hat{P}(\boldsymbol{q}') \right\rangle}$$

- with δP^(q) = P^(q) – P (q). If F (k) obeys Gaussian statistics, then

$$\boxed{\left\langle \delta\hat{P}(\boldsymbol{q})\delta\hat{P}(\boldsymbol{q}') \right\rangle = |\langle F(\boldsymbol{q})F^*(\boldsymbol{q}')\rangle|^2}$$

- the fractional variance of the power is

$$\boxed{\frac{\sigma_p^2(k)}{P^2(k)} = \frac{(2\pi)^3}{V_k W^2} \int d^3r w^4(\boldsymbol{r})\bar{n}^4(\boldsymbol{r}) \left(1 + \frac{1+\alpha}{\bar{n}(\boldsymbol{r})P(k)}\right)^2 .}$$

- we choose the weighting function w(r) which minimizes the variance in Equation

$$\boxed{w(\boldsymbol{r}) = \frac{1}{1 + P(k)\bar{n}(\boldsymbol{r})}.}$$

Ejercicio

- Calcular el Power Spectrum from the final 2dFGRS catalogue
- http://magnum.anu.edu.au/~TDFgg/
- http://www.2dfgrs.net/Public/Release/PowSpec/

The Power Spectrum from the final 2dFGRS catalogue

The power spectrum of the final galaxy catalogue is now available. For details of the method, see **Cole et al. 2005**, to be published in MNRAS

The power spectrum data, window functions and covariance matrix are available as text files from the links below. We also provide two simple "C" functions designed to demonstrate the use of these data to determine the likelihood of a given cosmological model.

- **Refereed version of Cole et al. 2005**
- **The power spectrum data** (see header for details)
- **The power spectrum** covariance matri
- **The power spectrum window function**
- **Recovered C code to demonstrate likelihood**
  This C code demonstrates the steps necessary to compute the likelihood of a given model power spectrum.
  - reading the window function and convolving the model power spectrum, that is to be tested, by the window function
  - inverting the covariance matrix and scaling it according to the convolved model power spectrum
  - differencing the 2dFGRS and model spectra and computing the likelihood

More information about the 2dFGRS is available from a number of websites including **Mount Stromlo Observatory, Australia** and **Edinburgh**.

skip

The Power Spectrum from the final 2dFGRS catalogue

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Power Spectrum

- σ8 is

$$\boxed{\sigma_8^2 = \int \frac{d^3k}{(2\pi)^3} P(k)|W(kR)|^2 = \int \frac{dk}{k} \Delta^2(k)|W(kR)|^2}$$

- with R = 8 Mpc/h, and Δ2(k) = P(k)k3/2π2 is the dimensionless power spectrum. One can check the normalization of the power spectrum by calculating σ8.

Anisotropic Estimators of the 2P statistics

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2-point estimators in 2D

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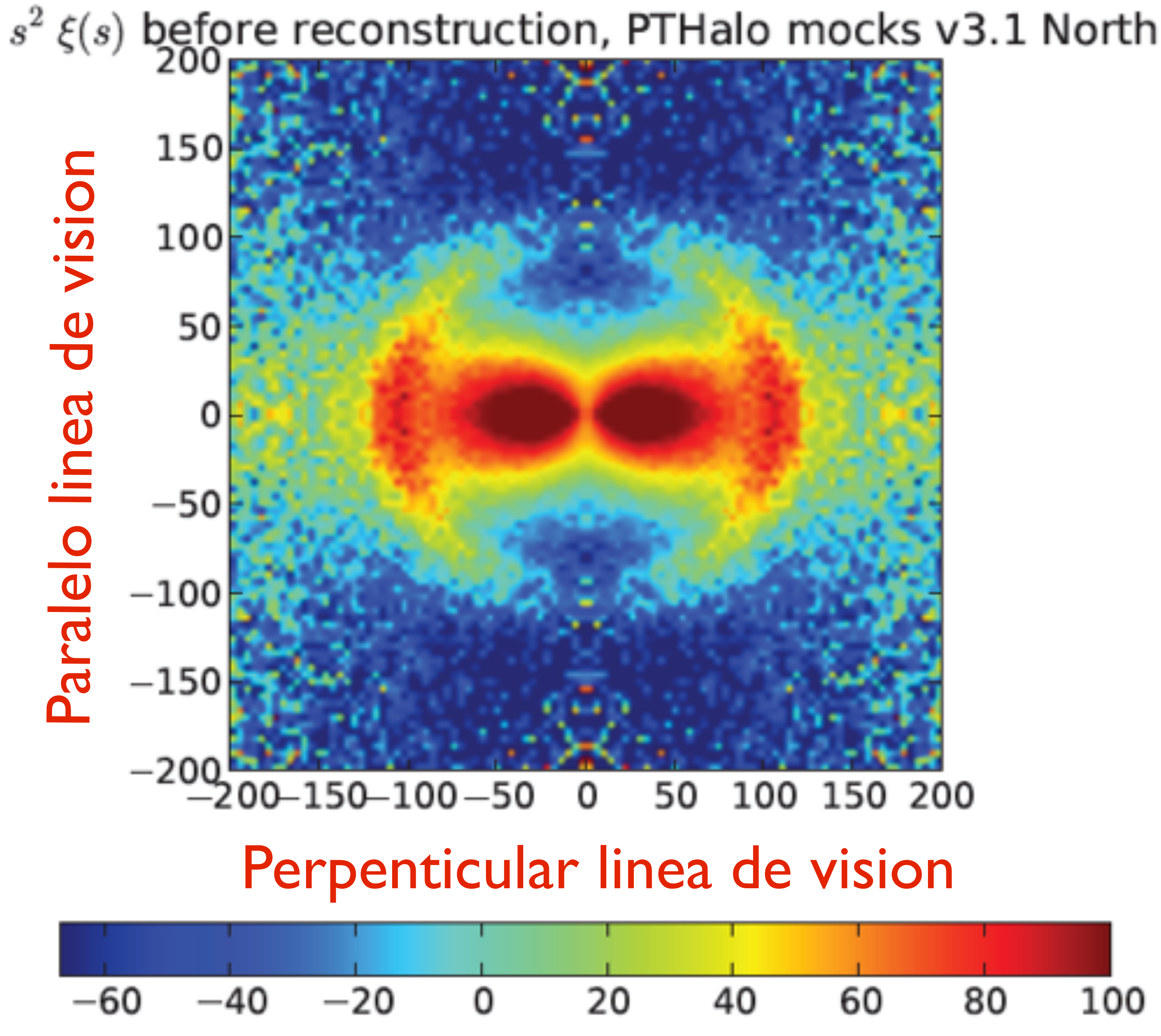
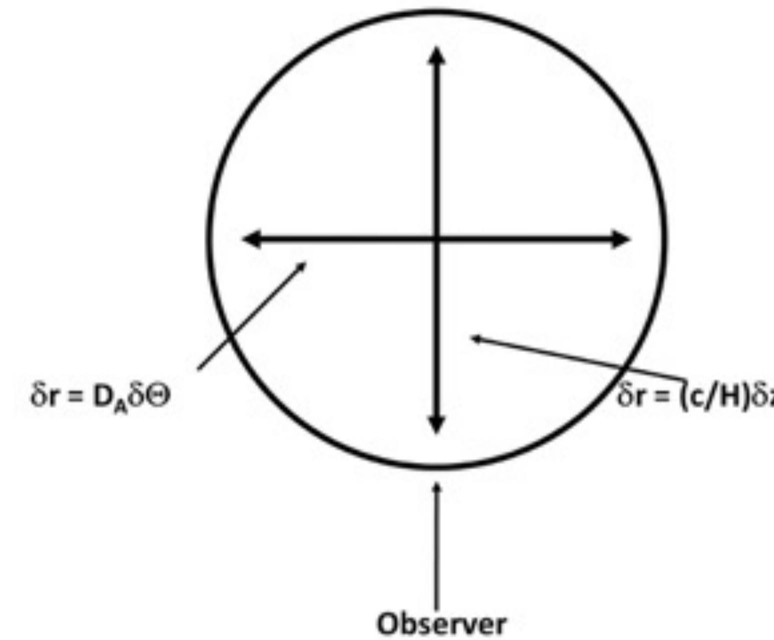
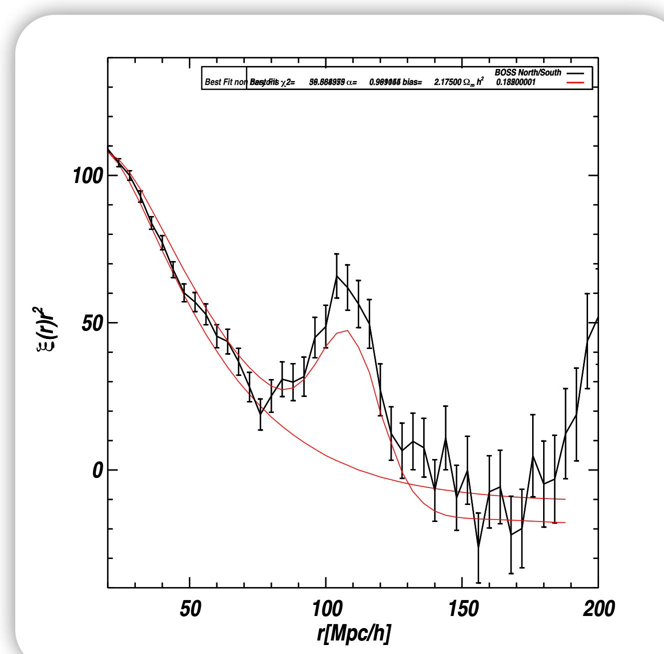
II. BAO isotropic & anisotropic

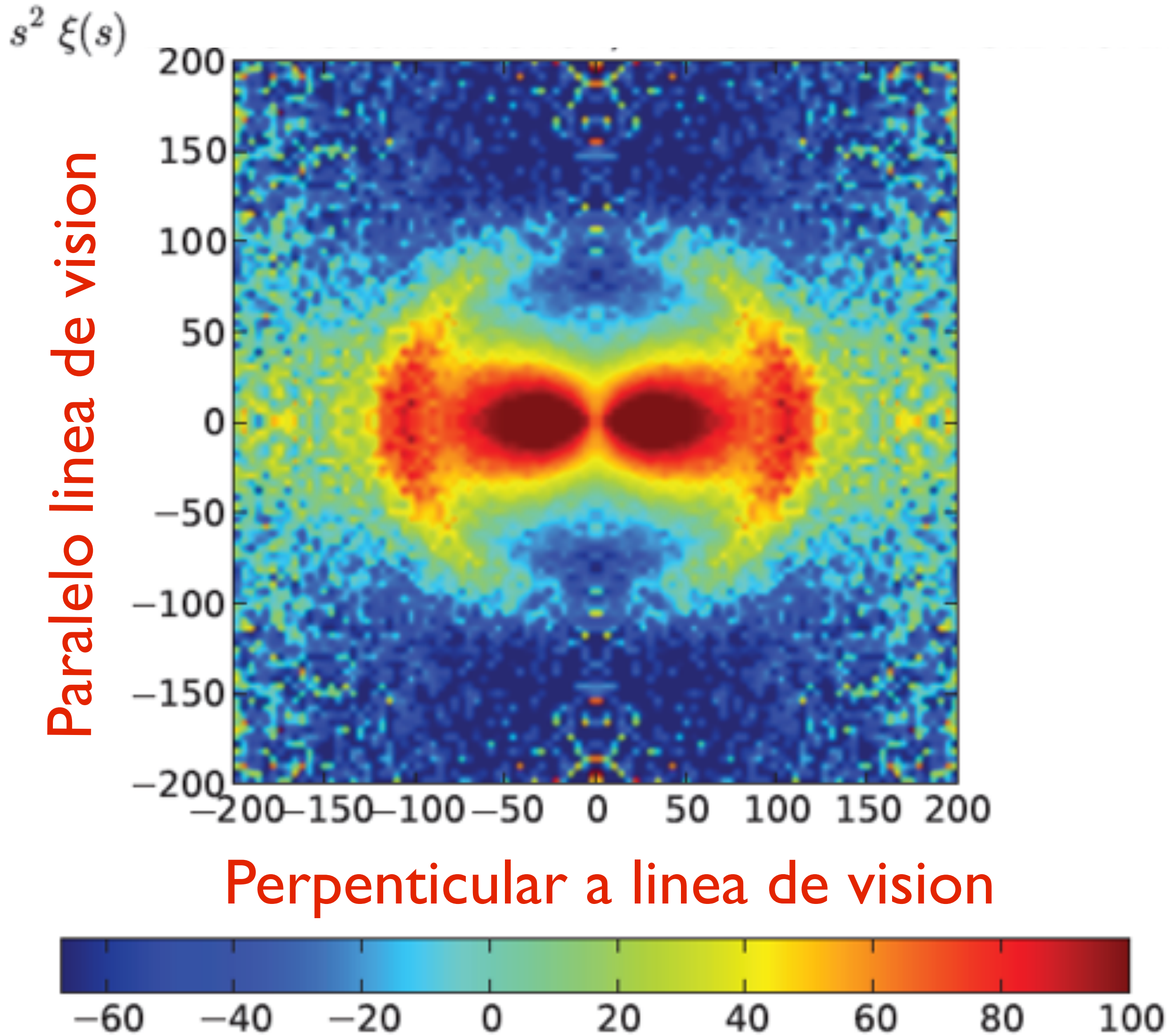


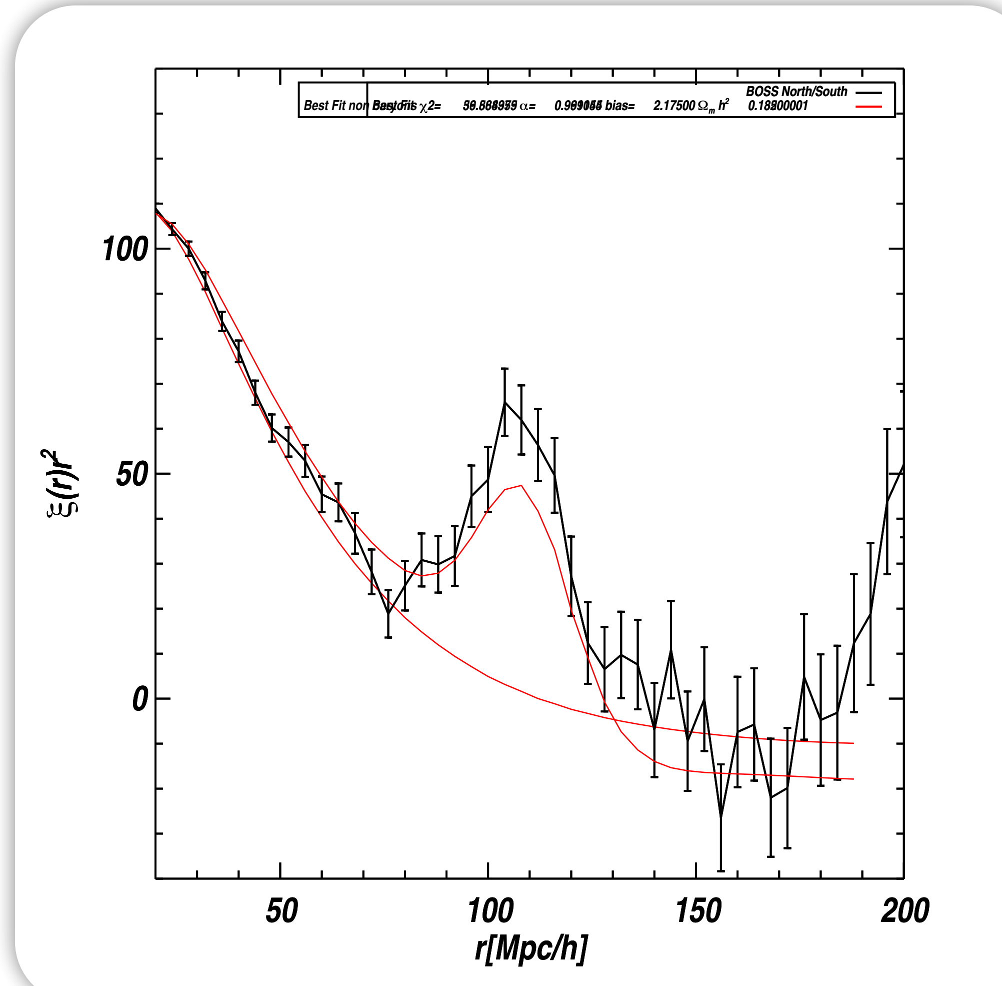




$$\xi(\mathbf{r}') = \sum_{\ell'=0}^{\infty} \xi_{\ell'}(r') L_{\ell'}(\mu'),$$

II. Correlation Function isotropic & anisotropic





Funciona de correlación promediada angularmente

$$\boxed{\xi(\boldsymbol{r}') = \sum_{\ell'=0}^{\infty} \xi_{\ell'}(r')L_{\ell'}(\mu'),}$$

https://arxiv.org/abs/1610.03506

Mon. Not. R. Astron. Soc. **000**, 1–29 (2016)   Printed 13 October 2016   (MN LaTeX style file v2.2)

The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: theoretical systematics and Baryon Acoustic Oscillations in the galaxy correlation function

$$\boxed{\xi(\boldsymbol{r}') = \sum_{\ell'=0}^{\infty} \xi_{\ell'}(r')L_{\ell'}(\mu'),}$$

LZ-2D

$$r^2 = r_{||}^2 + r_{\perp}^2. \tag{1}$$

We denote θ the angle between the galaxy pair separation and the LOS direction, and we define $\mu = \cos\theta$ so that:

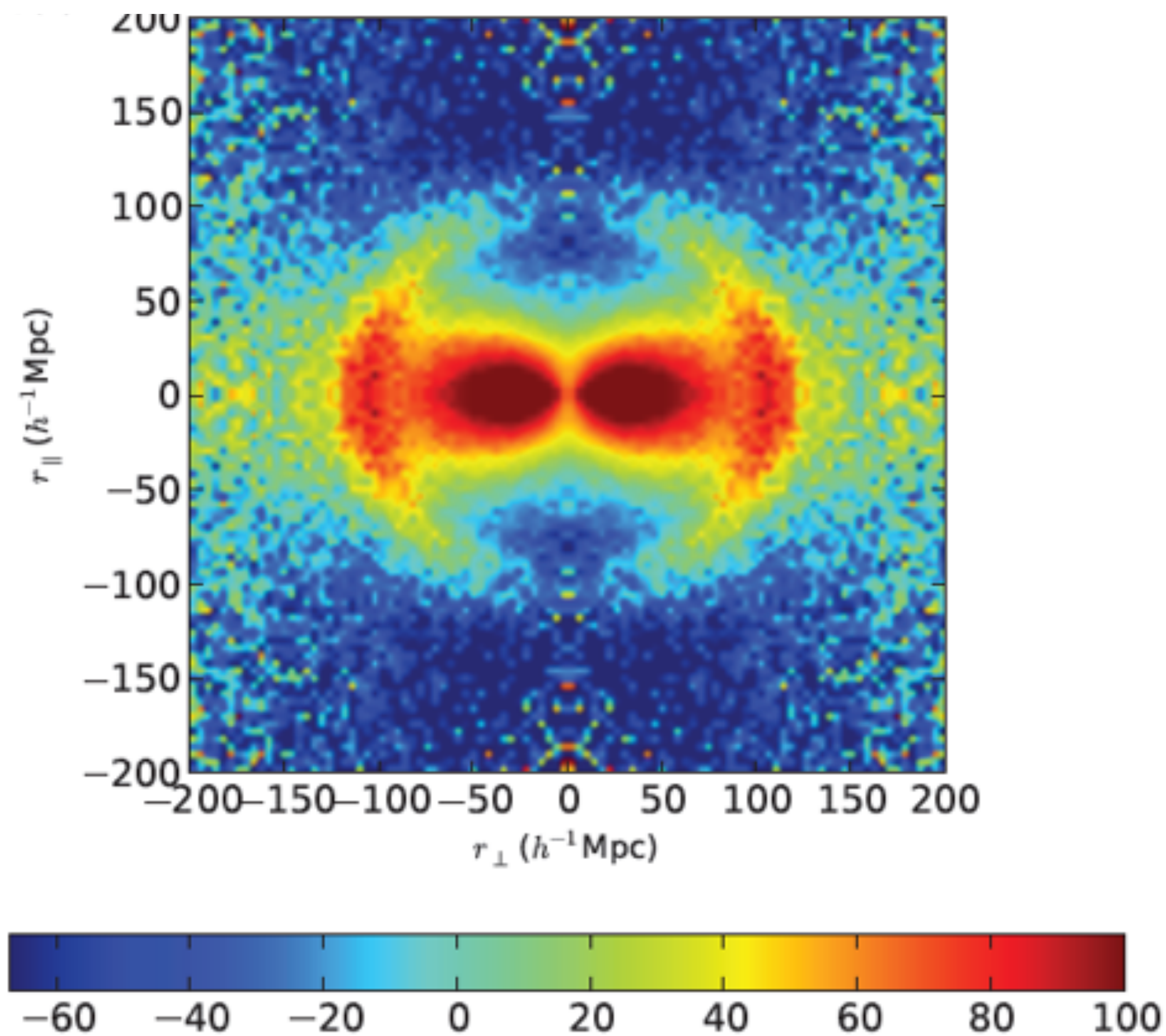
$$\mu^2 = \cos^2\theta = \frac{r_{||}^2}{r^2}. \tag{2}$$

The 2D-correlation function $\xi(r,\mu)$ (for the pre-reconstructed case) is then computed using Landy-Szalay estimator (Landy & Szalay 1993) that reads as follows:

$$\xi(r,\mu) = \frac{DD(r,\mu) - 2DR(r,\mu) + RR(r,\mu)}{RR(r,\mu)}, \tag{3}$$

II. 2PCF anisotropic

Paralell line-of-sight



Perpenticular line-of-sight

Multipoles

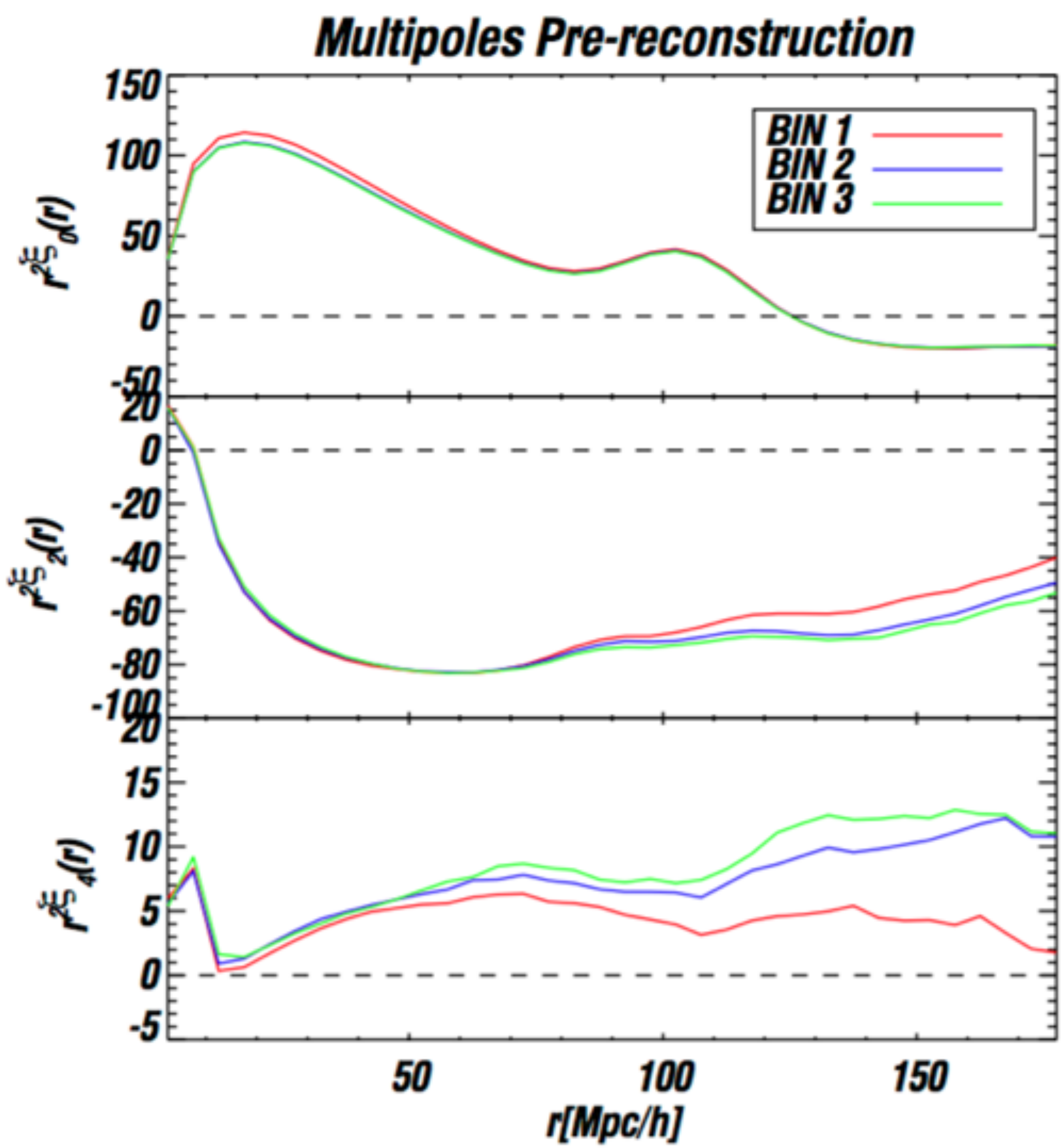
2.5.1 *Multipoles*

The multipoles are Legendre moments of the 2D correlation function $\xi(r,\mu)$. They can be computed through the following equation:

$$\xi_\ell(r) = \frac{2\ell+1}{2}\int_{-1}^{+1} d\mu\, \xi(r,\mu)\, L_\ell(\mu), \tag{4}$$

where $L_\ell(\mu)$ is the ℓ-th order Legendre polynomial. We focus primarily on the monopole and the quadrupole ($\ell = 0$ and $\ell = 2$), although we will have a discussion on hexadecapole ($\ell = 4$) in this work.

Multipoles



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Omega-Estimator

2.5.3 ω-Estimator

As in Xu et al. (2010), we define ω_l as the redshift space correlation function, $\xi_s(r, \mu)$, convolved with a compact and compensated filter $W_\ell(r, \mu, r_c)$ as a function of characteristic scale r_c:

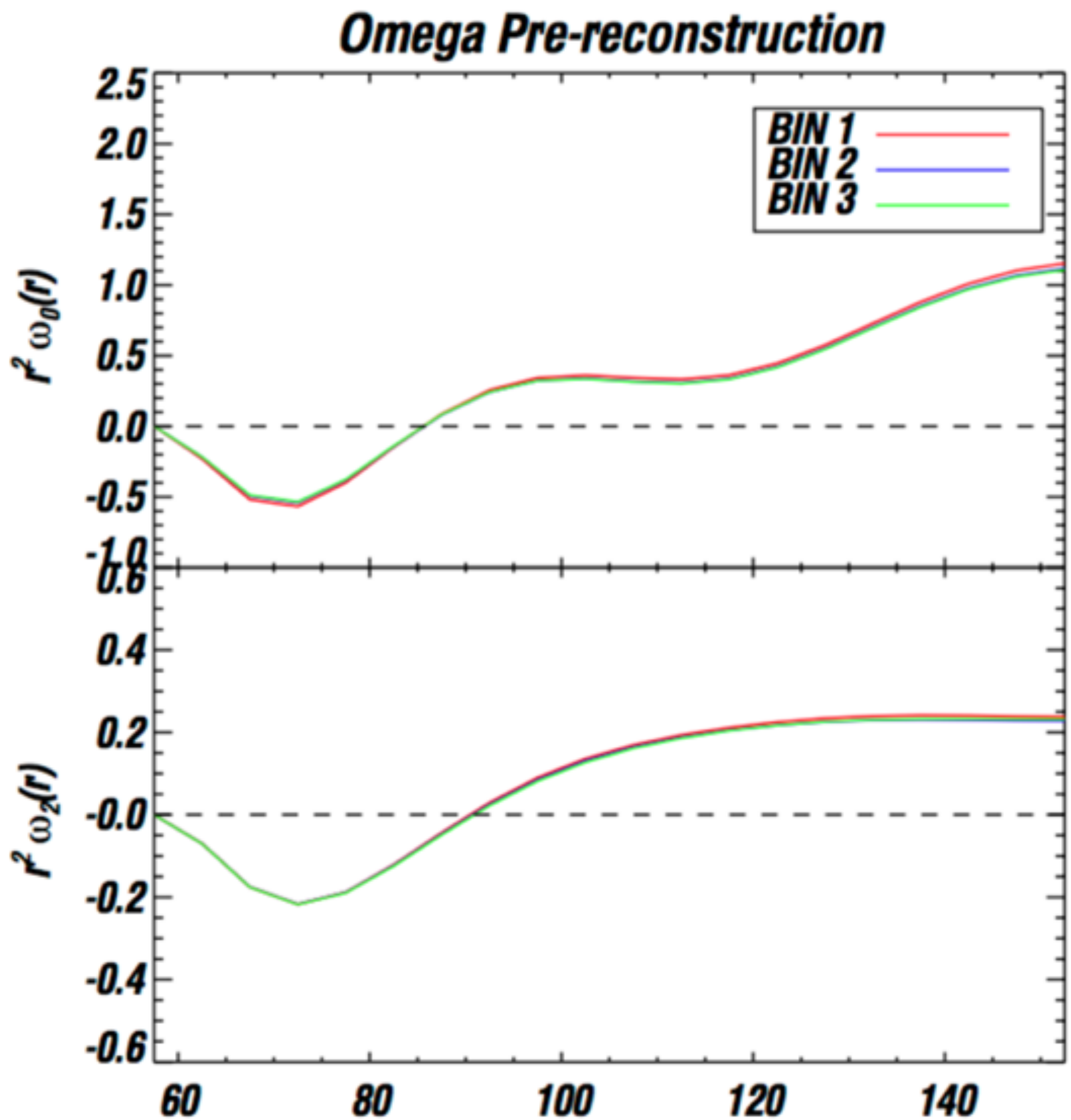
$$\omega_l = i^\ell \int d^3r \, \xi_s(r, \mu) \, W_\ell(r, \mu, r_c) \, , \tag{8}$$

where we have taken advantage of the orthogonality of the Legendre polynomials and set $W_\ell(r, \mu, r_c) = W_\ell(r, r_c) \, L_\ell(\mu)$. Following Padmanabhan et al. (2007) and Xu et al. (2010), we define a smooth, low order, compensated filter independent of ℓ:

$$W_\ell(x) = (2x)^2 (1-x)^2 \left(\frac{1}{2} - x\right) \frac{1}{r_c^3}, \tag{9}$$

where $x = (r/r_c)^3$. This choice of filter gives the ω_l statistic several advantages. By design, ω_l probes a narrow range of scales near the BAO feature and is not sensitive to large scale fluctuations or to poorly measured or modelled large scale modes (Xu et al. 2010).

Omega-Estimator



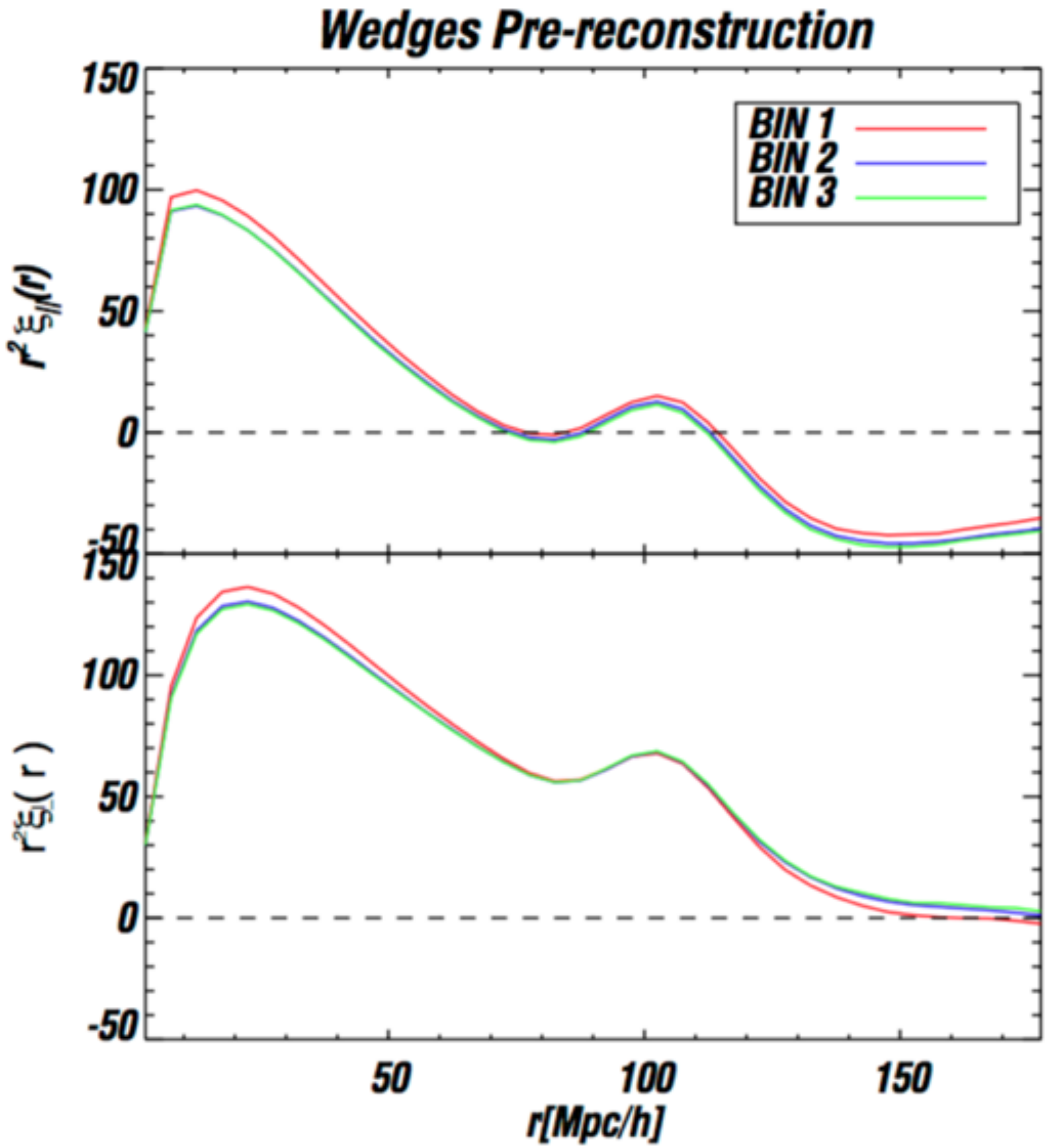
Clustering Wedges

2.5.2 *Clustering Wedges*

The clustering wedges are an integral of the correlation function over a range of μ:

$$\xi_{\Delta\mu}(r) = \frac{1}{\Delta\mu} \int_{\mu_{min}}^{\mu_{min}+\Delta\mu} d\mu \, \xi(r,\mu). \tag{5}$$

Clustering Wedges



https://arxiv.org/abs/1607.03147

The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological implications of the configuration-space clustering wedges

Anisotropic Power Spectrum

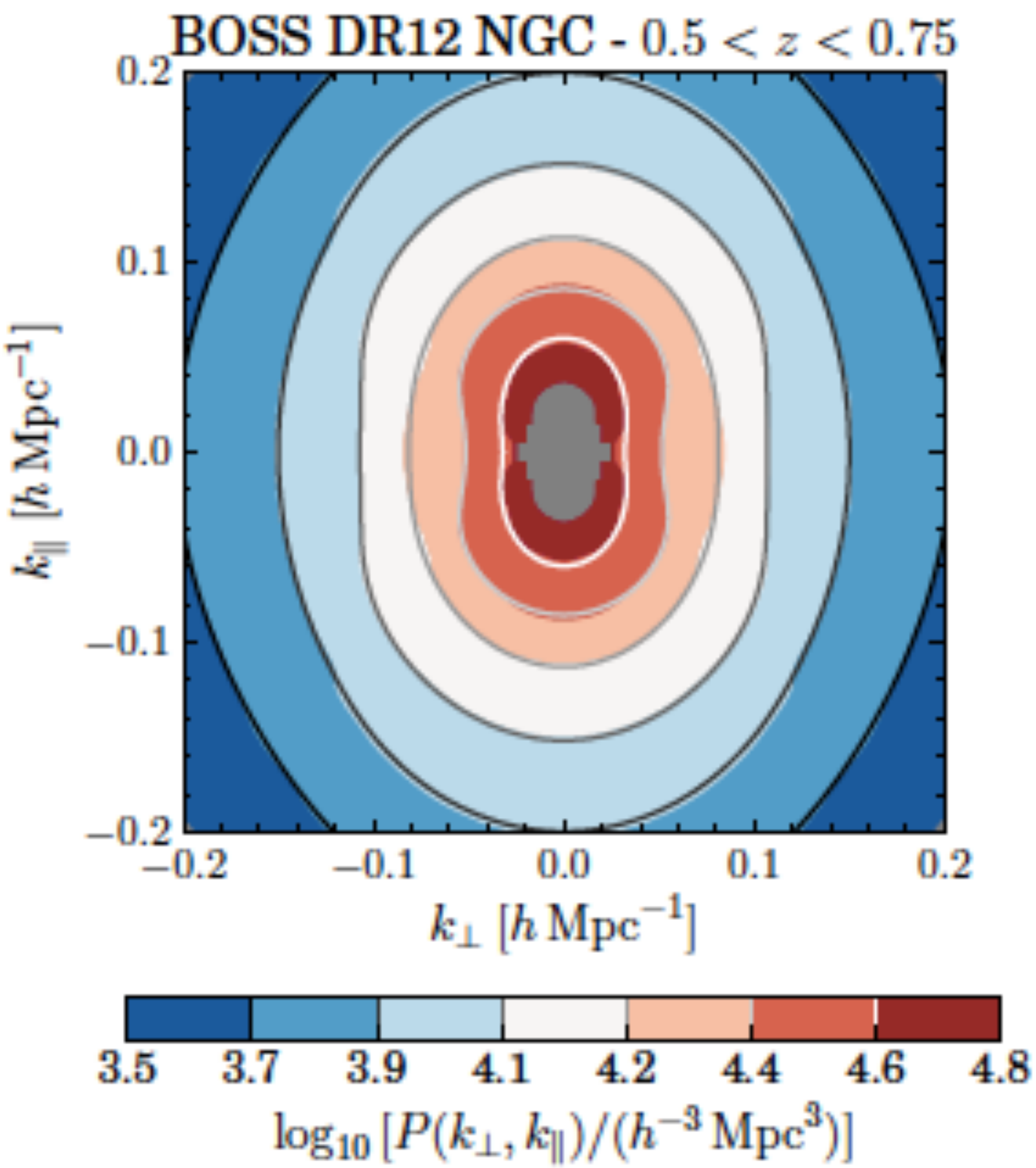
https://arxiv.org/pdf/1607.03149v1.pdf

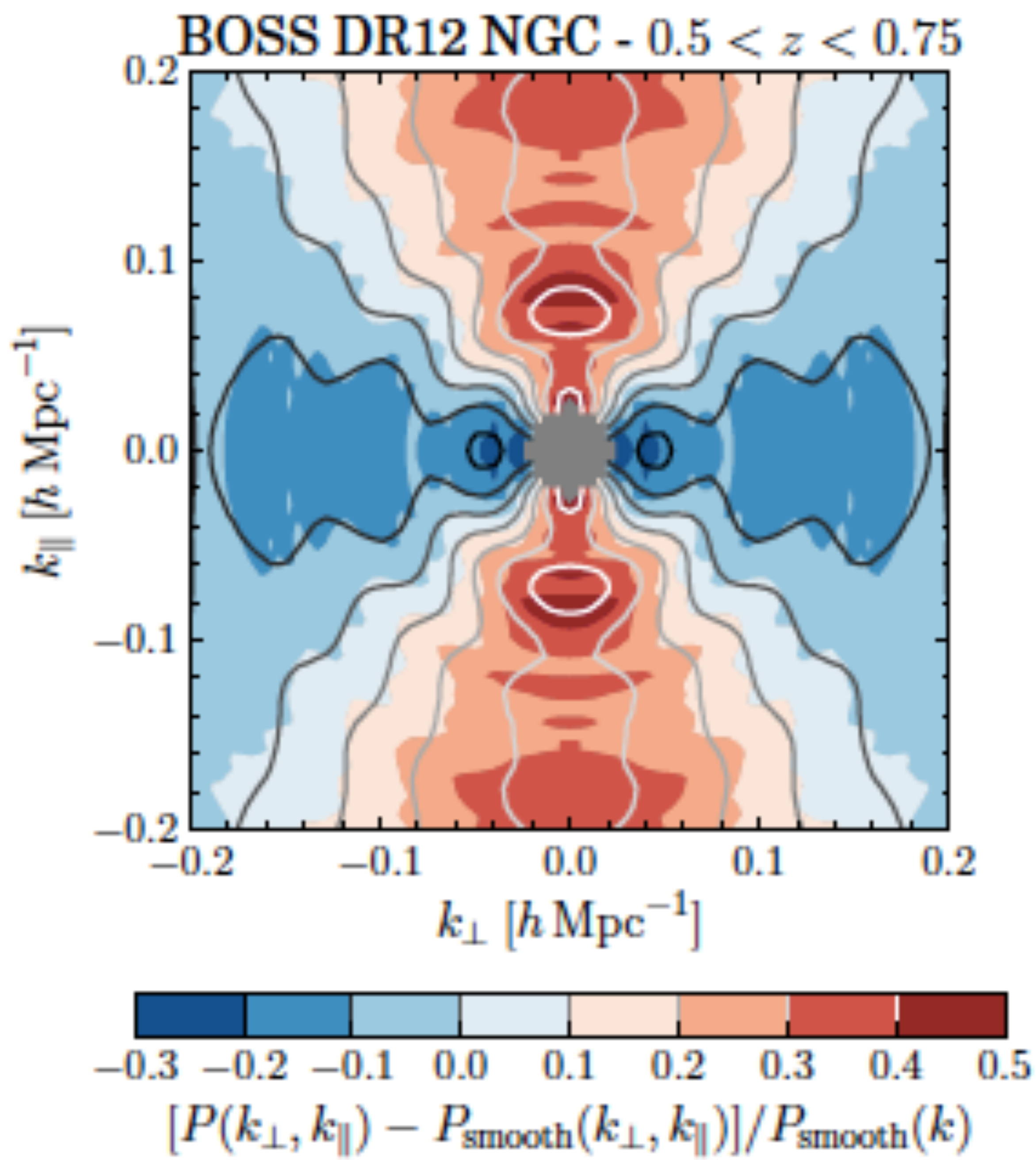
Mon. Not. R. Astron. Soc. **000**, 1–25 (2013)   Printed 13 July 2016   (MN LaTeX style file v2.2)

The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: Baryon Acoustic Oscillations in Fourier-space

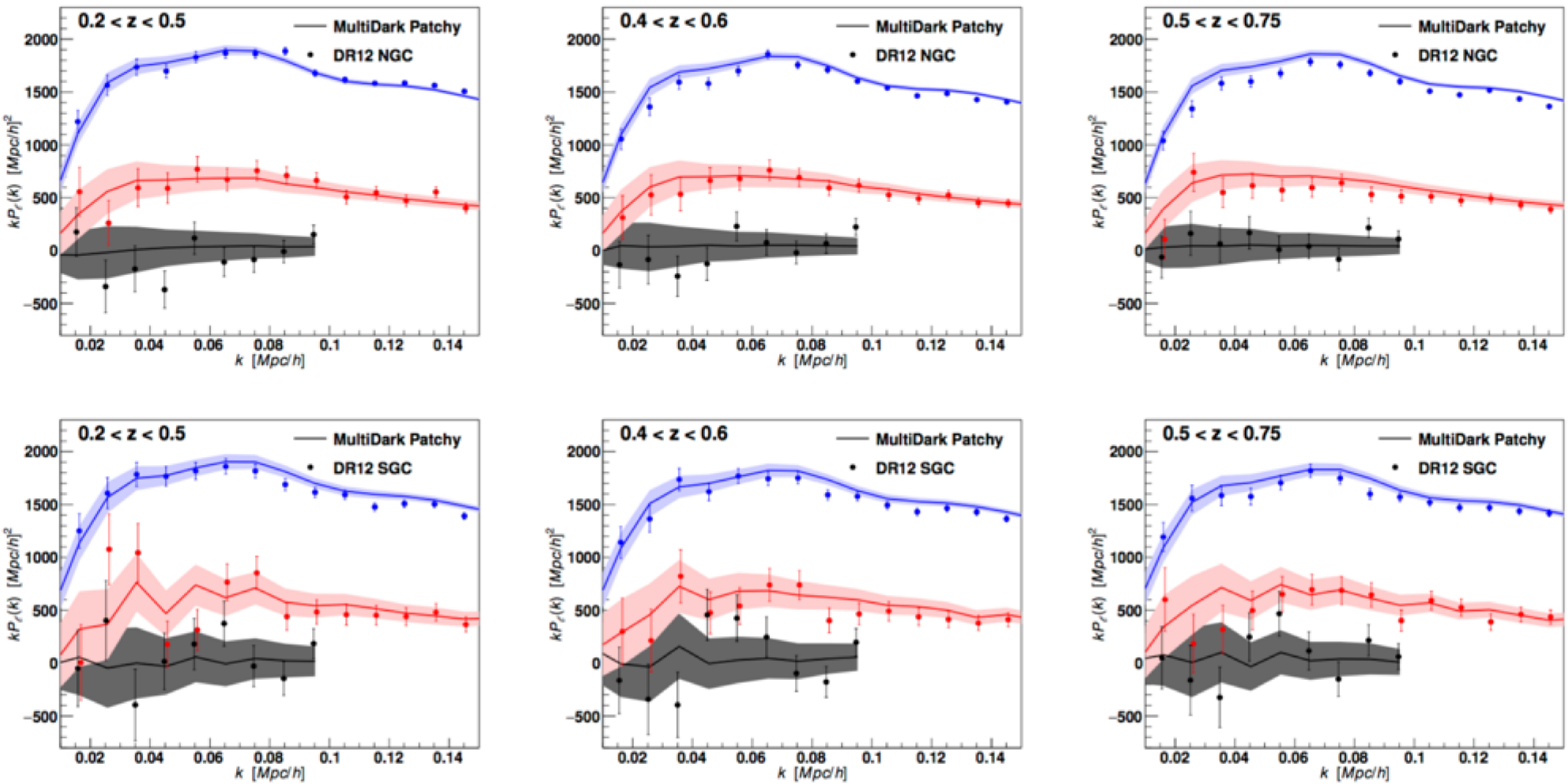
- Emiliano Sefusatti, Martin Crocce, Roman Scoccimarro, Hugh Couchman, Accurate Estimators of Correlation Functions in Fourier Space, https://arxiv.org/abs/1512.07295

2D Power Spectrum

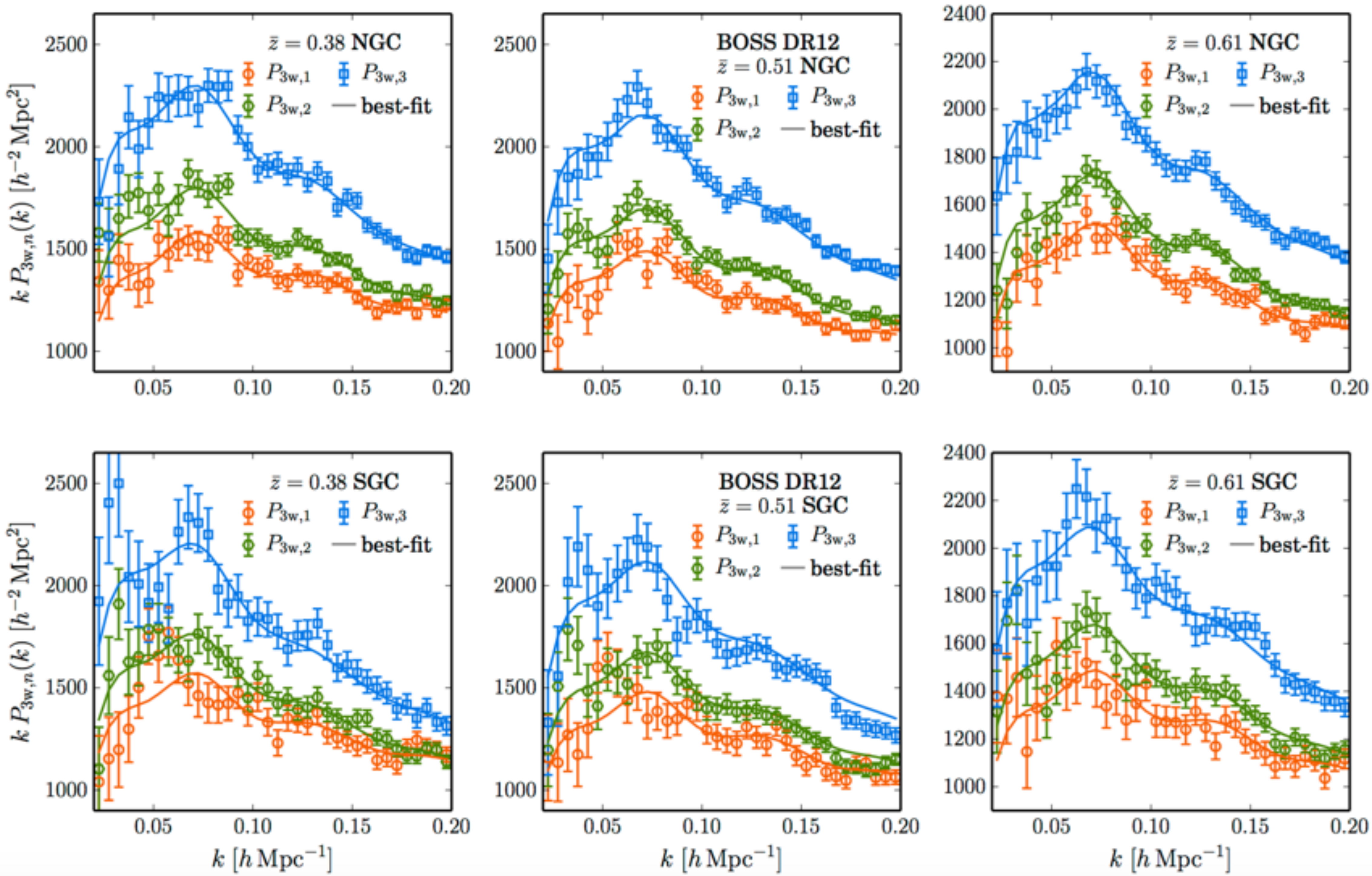




Multipoles in FS

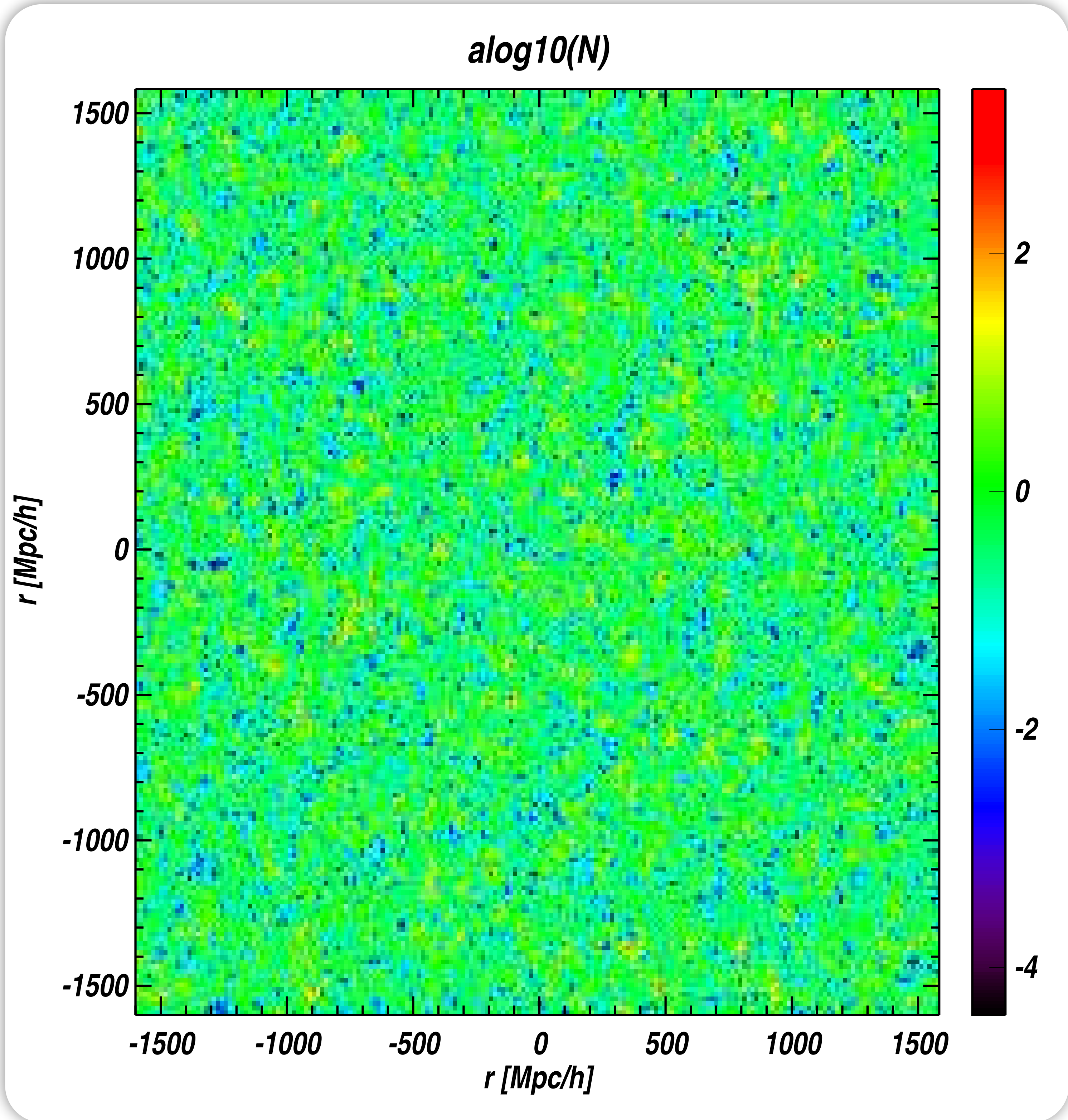


Wedges in FS



$\bar{z} = 0.38$ NGC
BOSS DR12
$\bar{z} = 0.51$ NGC
$\bar{z} = 0.61$ NGC
$\bar{z} = 0.38$ SGC
BOSS DR12
$\bar{z} = 0.51$ SGC
$\bar{z} = 0.61$ SGC
$P_{3w,1}$
$P_{3w,2}$
$P_{3w,3}$
best-fit
$k\,P_{3w,n}(k)\ [h^{-2}\,\mathrm{Mpc}^2]$
$k\ [h\,\mathrm{Mpc}^{-1}]$

Gaussian Random Field



Gaussian Random Fields

- A random field is said to be Gaussian if the distribution of the field values at an arbitrary set of N points is an N-variate Gaussian.

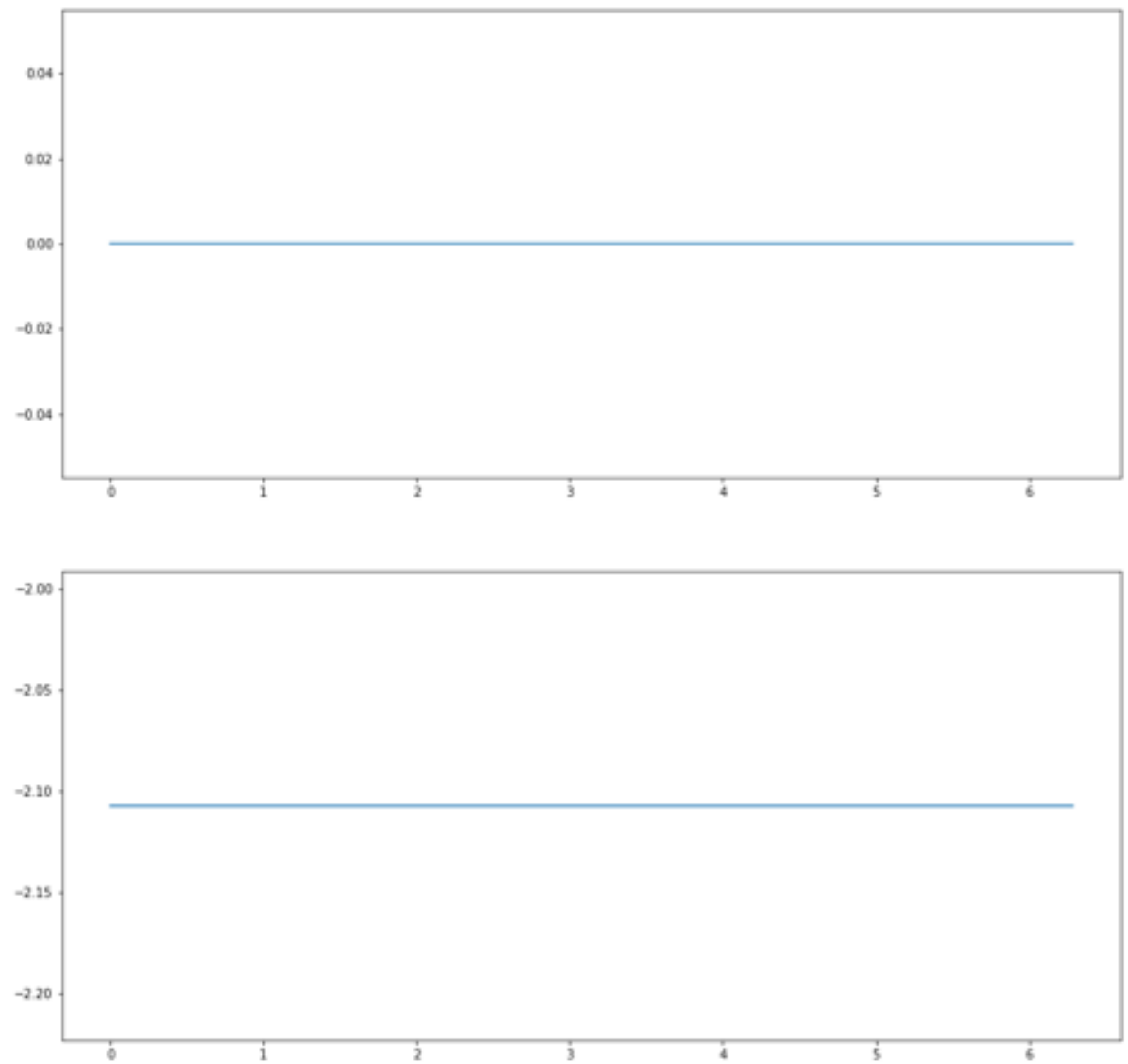
$$\mathcal{P}(\delta_1, \delta_2, ..., \delta_N) = \frac{\exp(-Q)}{[(2\pi)^N \det(\mathcal{C})]^{1/2}} \qquad Q \equiv \frac{1}{2}\sum_{i,j} \delta_i \, (\mathcal{C}^{-1})_{ij}\delta_j \qquad \mathcal{C}_{ij} = \langle \delta_i \delta_j \rangle \equiv \xi(r_{ij})$$

- A random field is said to be Gaussian if the distribution of the field values at an arbitrary set of N points is an N-variate Gaussian: δ(x) where we have defined the two-point correlation function.

- $\xi(x)=<\delta(x)\delta(x+r)>$

- As you can see, **for Gaussian random field the N-point probability function is completely specified by the two-point correlation function**

Gaussian Field 1D



Variance of the density Field

Mean Density Field $< \delta >= \frac{1}{V} \int \delta(\vec{x}) d^3\vec{x}$

Variance Density Field $\sigma^2 =< \delta^2 >= \frac{1}{V} \int \delta^2(\vec{x}) d^3\vec{x}$

$$\xi(r) =< \delta(\vec{x})\delta(\vec{x}+\vec{r}) >= \frac{1}{(2\pi)^3} \int P(k) e^{+i\vec{k}\vec{r}} d^3\vec{k}$$

- The power spectrum is defined as

$$\boxed{\langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2) \rangle = (2\pi)^3 P(k_1) \delta^D(\boldsymbol{k}_1 + \boldsymbol{k}_2)}$$

The Variance of the Density Field

Recall that the assumption of ergodicity implies that $\langle\delta\rangle = \frac{1}{V}\int \delta(\vec{x})\,\mathrm{d}^3\vec{x}$

where V is the volume of the Universe over which we assume it to be periodic.

Similarly, we have that the variance of the density field can be written as

$$\boxed{\sigma^2 = \langle\delta^2\rangle = \frac{1}{V}\int \delta^2(\vec{x})\,\mathrm{d}^3\vec{x}}$$

Recall that $\xi(r) = \langle\delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle = \frac{1}{(2\pi)^3}\int P(k)e^{+i\vec{k}\cdot\vec{r}}\mathrm{d}^3\vec{k}$, from which it is clear that

$$\boxed{\sigma^2 = \xi(0) = \frac{1}{(2\pi)^3}\int P(k)\mathrm{d}^3\vec{k} = \frac{1}{2\pi^2}\int P(k)\,k^2\,\mathrm{d}k = \int \Delta^2(k)\frac{\mathrm{d}k}{k}}$$

where $\Delta^2(k) = \frac{k^3}{2\pi^2}P(k)$ is the unitless power spectrum.

A power law is assumed: $P(k)=V\,Ak^n$

Initial conditions and linear power spectrum

- preferred ansatz for the initial power spectrum of the form

$$\mathcal{P}(k) \propto k^{n_s}$$

- with ns the spectral index. The virtue of this ansatz is that it does not introduce any particular length scale. If ns = 1 then this spectrum is called the Harrison-Peebles-Zel'dovich spectrum and has the preference of being scale invariant

- A further assumption about the initial perturbation is that it constitutes a realization of a Gaussian random field which, again, is probably the simplest and most natural choice.

- where ε and η are the slow-roll parameters evaluated at the time when the mode k exits the horizon.