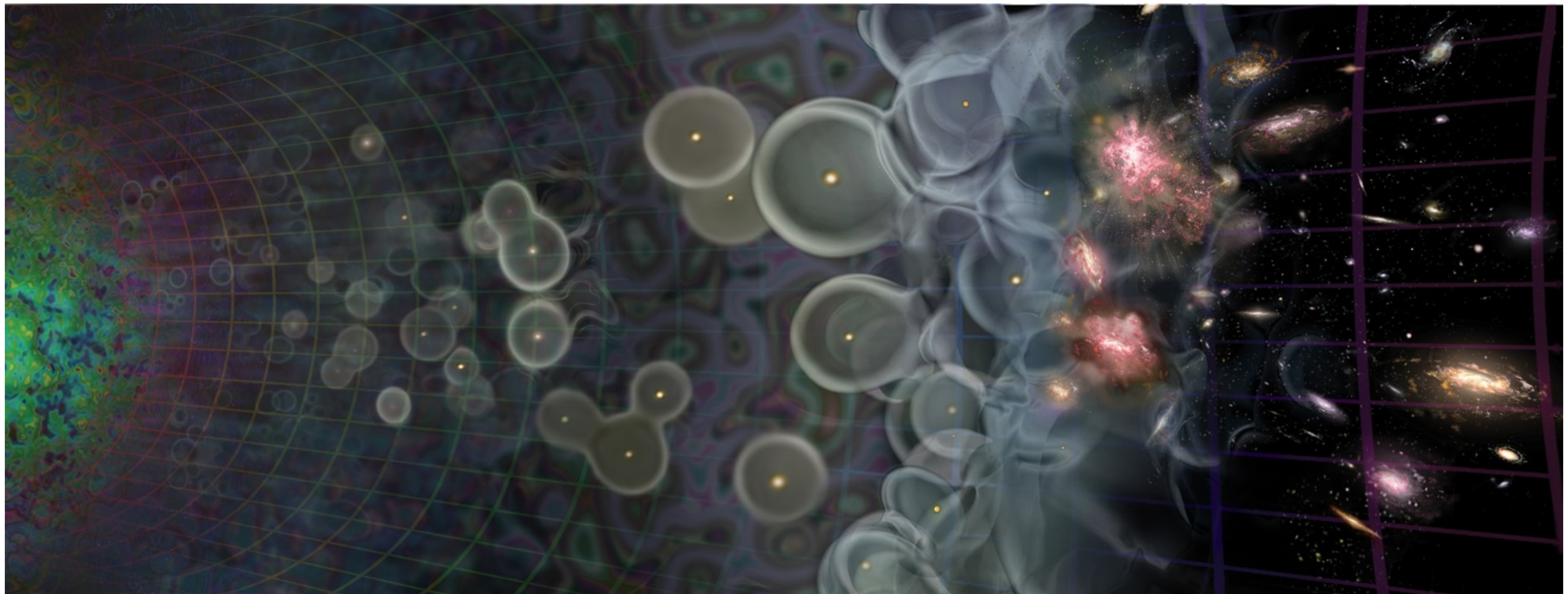

Curso de cosmologia

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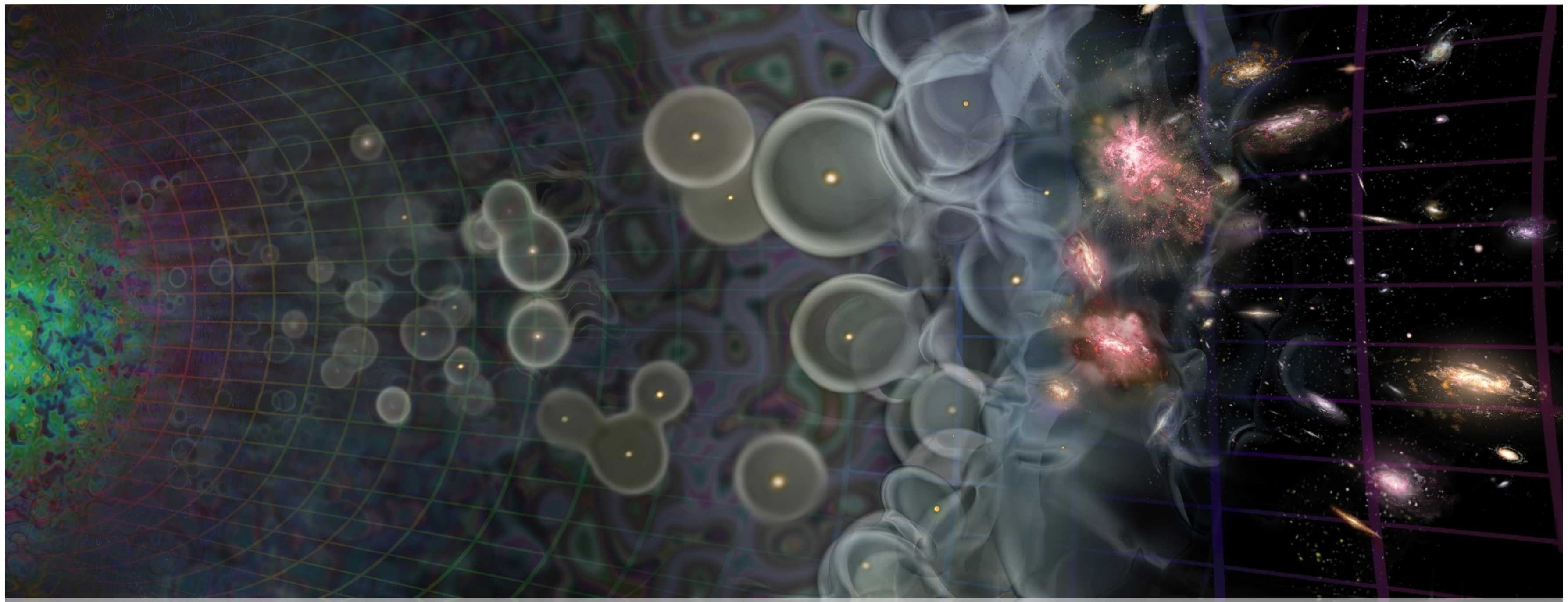
Results from last lecture

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3}$$

from Einstein's equations

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

from continuity equation $\nabla_\mu T^{\mu\nu}=0$



Densities Evolution

Densities Evolution

$$\omega = \frac{p}{\rho} \quad \Rightarrow \quad p = \omega\rho$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \quad \Rightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+\omega) = 0$$

$$\frac{\dot{\rho}}{\rho} + 3\frac{\dot{a}}{a}(1+\omega) = 0 \quad \Rightarrow \quad \frac{d}{dt}ln(\rho) + 3(1+\omega)\frac{d}{dt}ln(a) = 0$$

Densities Evolution

$$\frac{\dot{\rho}}{\rho} + 3\frac{\dot{a}}{a}(1+\omega) = 0 \quad \Rightarrow \quad \frac{d}{dt}ln(\rho) + 3(1+\omega)\frac{d}{dt}ln(a) = 0$$

$$\frac{d}{dt}ln(\rho) = -3(1+\omega)\frac{d}{dt}ln(a) \quad \Rightarrow \quad \frac{d}{dt}ln(\rho) = \frac{d}{dt}ln(a^{-3(1+\omega)})$$

$$\rho \propto a^{-3(1+\omega)}$$

$$\boxed{\rho = \rho_0 a^{-3(1+\omega)}}$$

Densities Evolution

$$\boxed{\rho = \rho_0 a^{-3(1+\omega)}}$$

Dark matter

Collision-less so there is no pressure => p=0 => ω=0
the density decrease with the volume which increase in a^3

$$\omega_m = 0 \quad \Longleftrightarrow \quad \rho_m(a) = \rho_{m,0} \times a^{-3}$$

Radiation

the number density decrease with the volume which increase in a^3
but the photon loose energy with the redshift with a so in a^{-4}

$$\omega_R = 1/3 \quad \Longleftrightarrow \quad \rho_R(a) = \rho_{R,0} \times a^{-4}$$

Densities Evolution

$$\boxed{\rho = \rho_0 a^{-3(1+\omega)}}$$

cosmological constant

ρ=constant so -3(1+w)=0 => w=-1

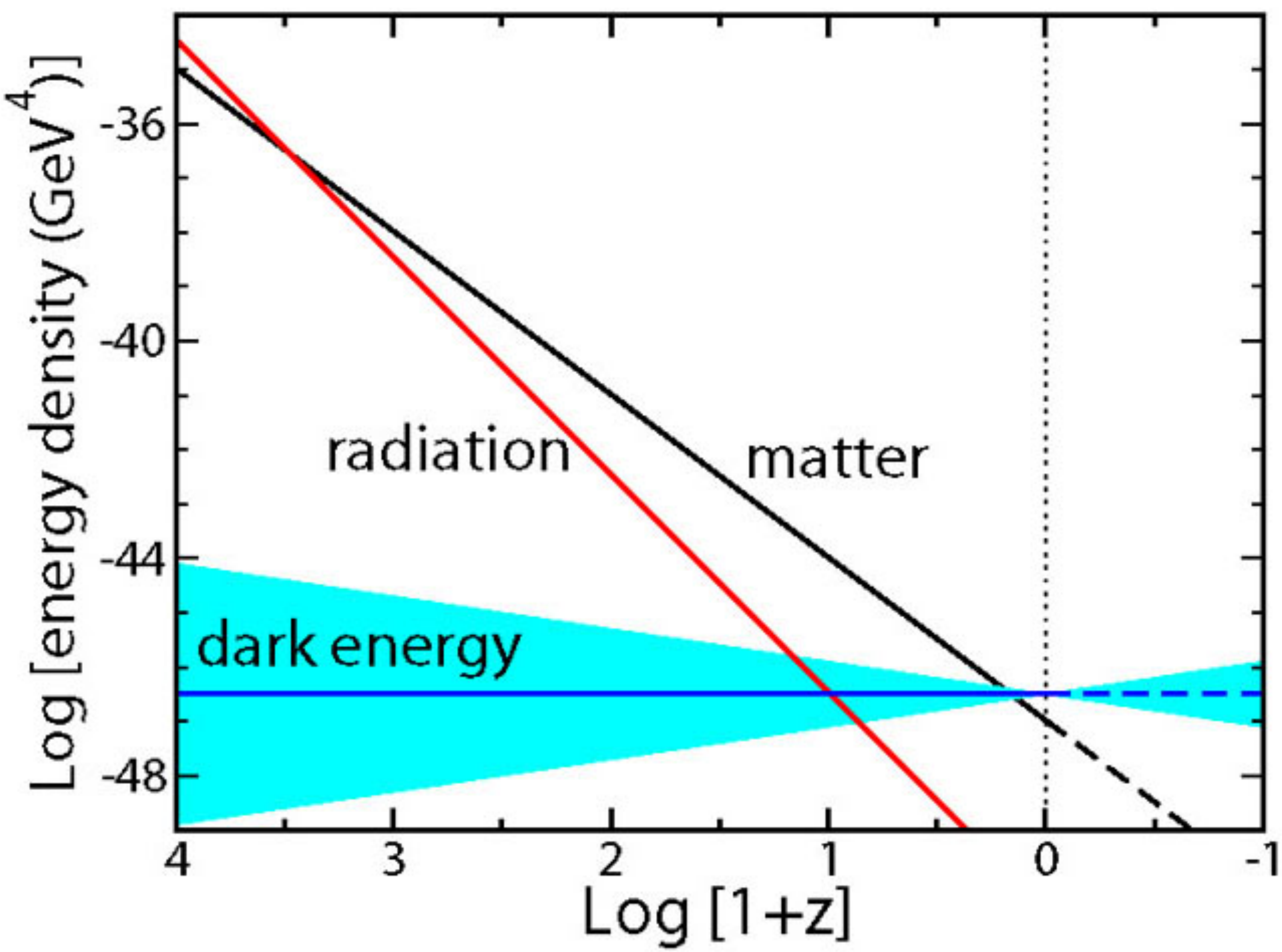
$$\omega_\Lambda = -1 \quad \Longleftrightarrow \quad \rho_\Lambda(a) = Cte$$

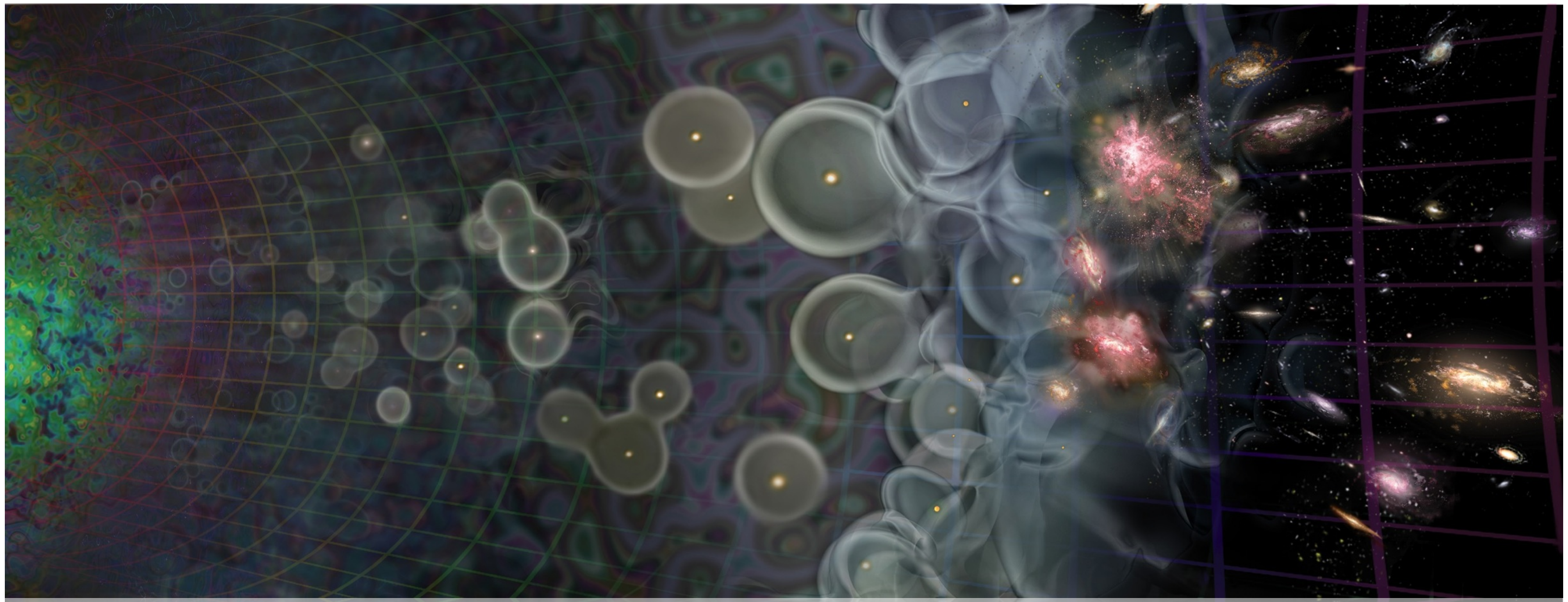
Energia obscura (general condition)

acceleration of the Universe expanding

$$\omega_{DE} < -1/3$$

Densities Evolution





Scale factor Evolution

Scale factor Evolution

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} \qquad + \qquad \rho \propto a^{-3(1+\omega)}$$

$$a \propto t^n$$

$$\dot{a} \propto t^{n-1}$$

$$\frac{\dot{a}}{a} \propto t^{-1}$$

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 \propto t^{-2}}$$

$$\boxed{\rho \propto t^{-3(1+\omega)\times n}}$$

$$\boxed{n = \frac{2}{3(1+\omega)}}$$

Scale factor Evolution

$$\boxed{a(t) \propto t^{\frac{2}{3(1+\omega)}}}$$

Radiation era

$$\omega_R = 1/3 \quad \Rightarrow \quad a(t) \propto t^{1/2}$$

Matter era

$$\omega_m = 0 \quad \Rightarrow \quad a(t) \propto t^{2/3}$$

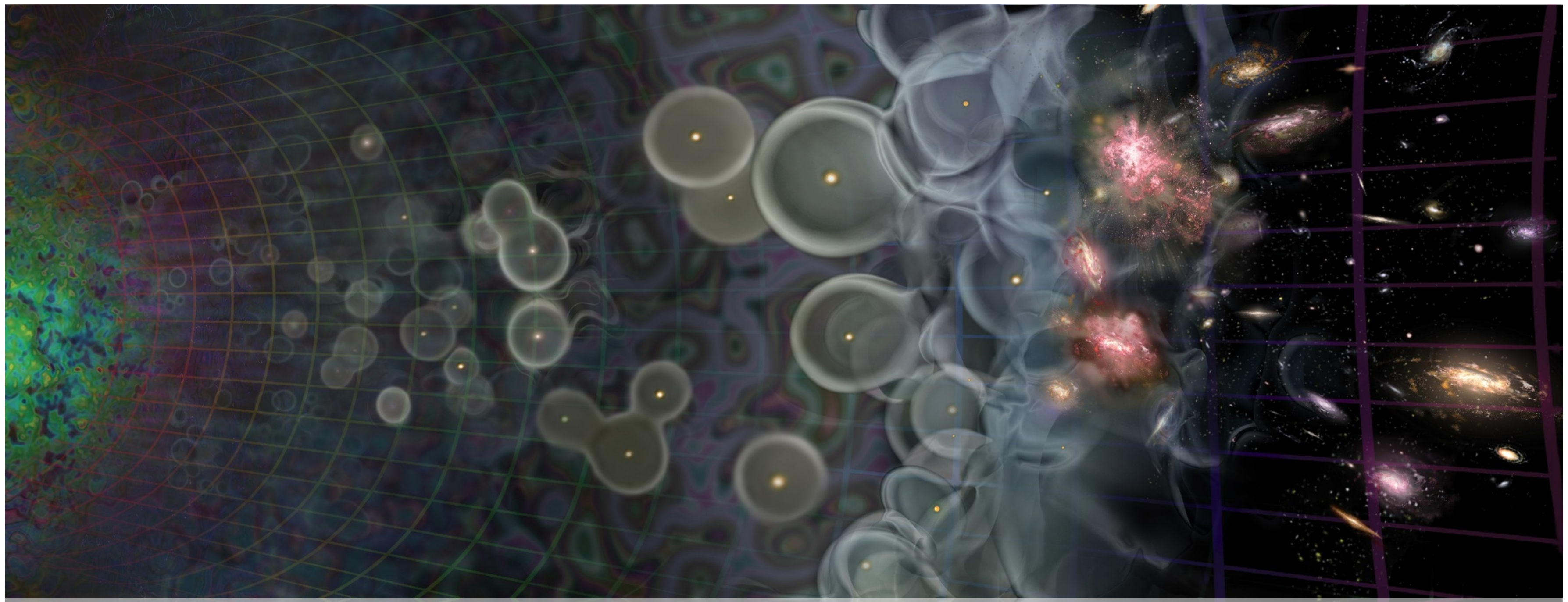
Scale factor Evolution

Cosmological Cte era

$$\rho_\Lambda = cte \Leftrightarrow \dot{\rho_\Lambda} = 0$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_\Lambda}{3} = cte$$

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho_\Lambda}{3}} \quad \Rightarrow \quad \boxed{a(t) \propto \exp^{\sqrt{\frac{8\pi G\rho_\Lambda}{3}}t}}$$



Distances

Calculation for the distances

$$\lambda_{obs} = \lambda_{emit}(1 + \frac{v}{c}) = \lambda_{emit}(1 + z)$$

$$\lambda_{obs} = \lambda_{emit}\frac{a(t_{obs})}{a(t_{emit})}\,, \quad a(t_{obs}) = 1 \Rightarrow a(t) = \frac{1}{1 + z(t)}$$

Calculation for the distances

$$ds^2 = -c^2 dt^2 + a^2(t) d\chi^2 \,, \qquad \gamma \Rightarrow ds^2 = 0$$

$$c^2 dt^2 = a^2(t) d\chi^2 \quad \Rightarrow \quad \frac{c^2}{a^2(t)} dt^2 = d\chi^2$$

$$d\chi = \frac{c}{a(t)} dt \quad \Rightarrow \quad \chi(t_{emis}) = c \int_{t_{emis}}^{t_0} \frac{dt}{a(t)}$$

Calculation for the distances

Cambio de variable con el factor de escala

$$H(t) = \frac{\dot{a}(t)}{a(t)}\ ;\ \dot{a} = \frac{da}{dt} \qquad \Rightarrow \qquad dt = \frac{da}{a(t)H(t)}$$

$$\chi(a_{emis}) = c\int_{a_{emis}}^{a(t_0)=1} \frac{da}{a^2 H(a)}$$

Calculation for the distances

Cambio de variable con el redshift

$$a = \frac{1}{1+z} \quad \Rightarrow \quad da = \frac{-dz}{(1+z)^2}$$

$$\chi(z_{emis}) = -c \int_{z_{emis}}^{z(t_0)=0} \frac{dz(1+z)^2}{(1+z)^2 H(z)}$$

$$\boxed{\chi(z_{emis}) = c \int_0^{z_{emis}} \frac{dz}{H(z)}}$$

Calculation for the distances

Expresión con la función de
Hubble standard

$$H(z) = H_0\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}$$

$$\boxed{\chi(z_{emis}) = \frac{c}{H_0}\int_0^{z_{emis}} \frac{dz}{\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}}}$$

Distances in Cosmology

- **Distance co-mobile**, est la distance instantanée d'un objet à un redshift z.

$$\chi = a_0 r(z) = \int_0^z \frac{dz'}{H(z')}$$

Paramètres Cosmologiques

- **Distance de luminosité**. Définie en termes du flux de un objet avec une luminosité connue.

$$D_L = \chi(1+z) \qquad F = \frac{L}{4\pi\chi^2}\frac{1}{(1+z)^2}$$

ralentissement des horloges

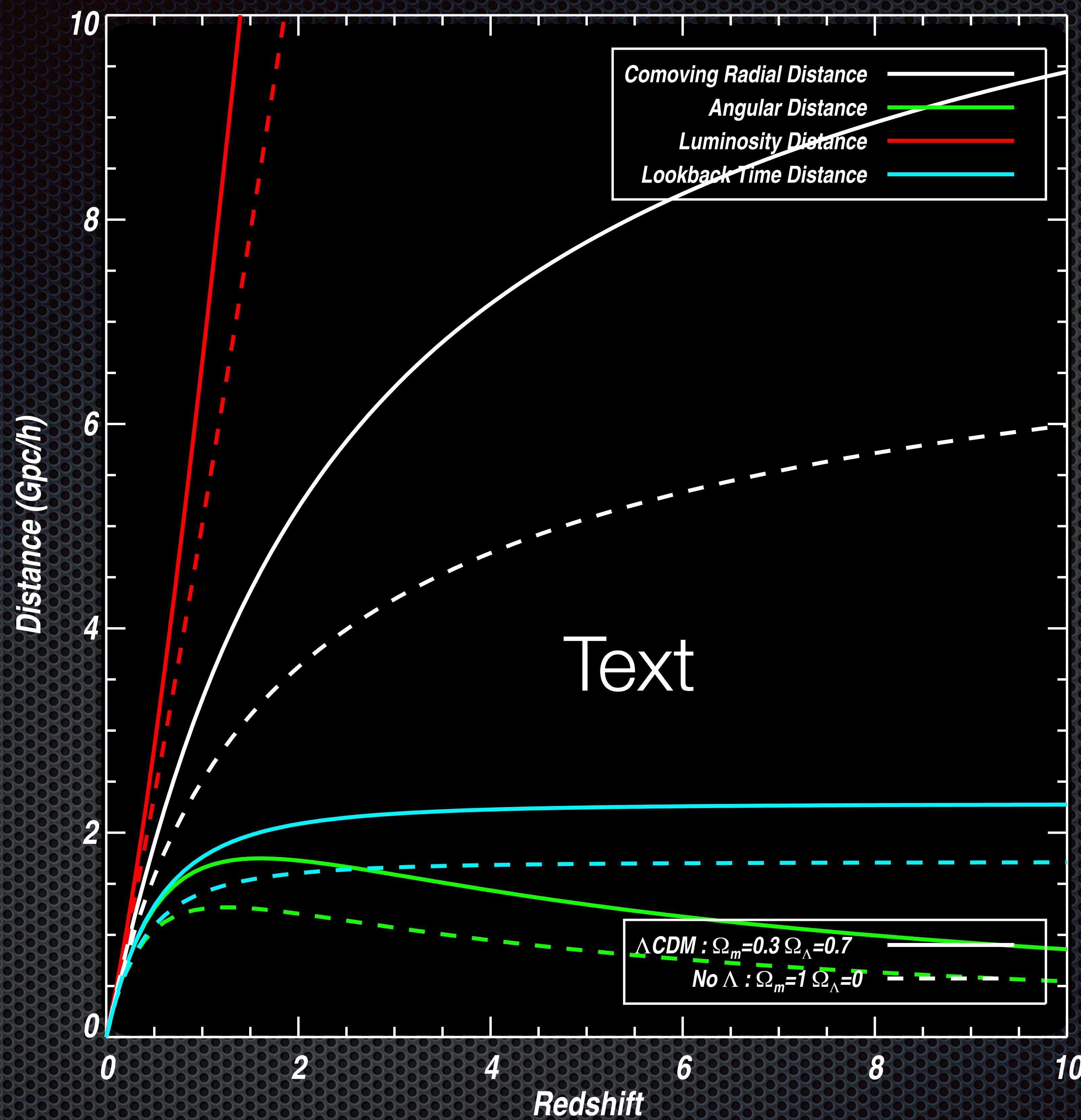
redshift des photons

=>objets semblent plus loin que leur distance propre !!

- **Distance angulaire**, donnée pour l'angle sous-tendu pour un objet de taille physique L.

$$D_A = \frac{\chi}{(1+z)}$$

photons émis quand l'objet était plus proche=>objets semblent plus proche



Comoving Radial Distance
Angular Distance
Luminosity Distance
Lookback Time Distance
Text
ΛCDM : Ω_m=0.3 Ω_Λ=0.7
No Λ : Ω_m=1 Ω_Λ=0
Distance (Gpc/h)
Redshift
0
2
4
6
8
10