Problem. Consider a curved second-order `line3` element lying on the x-y plane defined by the corner nodes

$$\mathbf{x}_1 = [0, 1]$$
$$\mathbf{x}_2 = [1, 0]$$

with the mid-edge node located at

$$\mathbf{x}_3 = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

Using the shape functions for $\xi \in [-1 : 1]$

$$h_1(\xi) = \frac{\xi \cdot (\xi - 1)}{2}$$
$$h_2(\xi) = \frac{\xi \cdot (\xi + 1)}{2}$$
$$h_3(\xi) = (1 + \xi) \cdot (1 - \xi)$$

compute the Lebesgue measure (i.e. length) of the element. Share the details and any code you used.

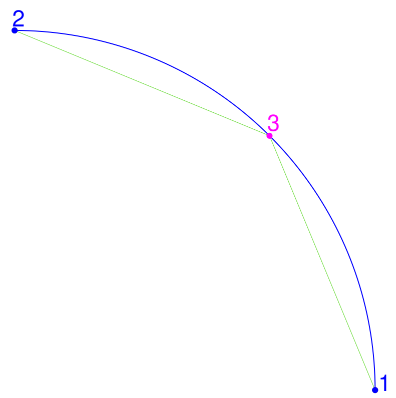

Figure 1: Illustration of the `line3` element.

Notes

- The real length of the curve is $\pi/2 \approx 1.5708$.
- If the element had been `line2` (and hence there was no mid-edge node) then the length would have been $\sqrt{2} \approx 1.4142$.
- If the element had been assumed to be composed of two straight edges, one from node 1 to node 3 and another one from node 3 to node 2 then the length would have been ≈ 1.5307.