Modelling a single elimination tournament bracket

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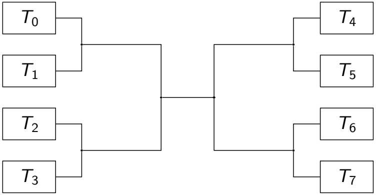
The problem

Problem

Consider a single elimination tournament bracket of $N = 2^k$ teams.
Compute the probability of each team winning the tournament.



$$\begin{pmatrix} 1 & p_{01} & p_{02} & \dots & p_{07} \\ p_{10} & 1 & p_{12} & \dots & p_{17} \\ p_{20} & p_{21} & 1 & \dots & p_{27} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{70} & p_{71} & p_{72} & \dots & 1 \end{pmatrix}$$

$$\begin{aligned} P(T_0 \text{ win}) &= P(T_0 \text{ win } R_1) \cdot P(T_0 \text{ win } R_2) \cdot P(T_0 \text{ win } R_3) \\ &= p_{01} \cdot (p_{02} \cdot p_{23} + p_{03} \cdot p_{32}) \\ &\quad \cdot p_{04} \cdot (p_{45} \cdot (p_{46} \cdot p_{67} + p_{47} \cdot p_{76})) + \dots \end{aligned}$$

General formula

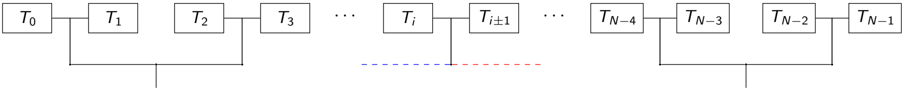
We denote

$$P_{i,r} \equiv \text{probability of team } i \text{ reaches round } r$$

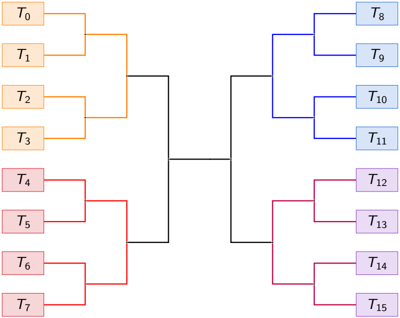
Setting $P_{i,0} = 1$, we have

$$P_{i,r} = P_{i,r-1} \cdot \sum_{j \in Opp(i,r)} p_{ij} \cdot P_{j,r-1}$$

where $Opp(i,r)$ are the possible opponents of team i in round r.



Determining the opponents



In round r, teams are grouped in blocks of 2^r teams: $B_0, B_1, \ldots$.

Then, $T_i \in B_j$ if and only if,

$$2^r j \leq i < 2^r(j+1),$$

that is,

$$\left\lfloor \frac{i}{2^r} \right\rfloor = j$$

For a team $T_i \in B_j$, the possible opponents for that round belong to "*the other half of the bracket*":