Prediction and Linear Regression

Big Data y Machine Learning para Economía Aplicada

Ignacio Sarmiento-Barbieri

Universidad de los Andes

Agenda

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1. Prediction and loss functions
2. GitHub

Getting serious about prediction

$$y = f(X) + u \tag{1}$$

- Interest on predicting Y
- Model? We treat $f()$ as a black box, and any approximation $\hat{f}()$ that yields a good prediction is good enough (*"Whatever works, works..."*).
- How do we measure "what works"?

Getting serious about prediction

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- Interest on predicting Y
- Model? We treat $f()$ as a black box, and any approximation $\hat{f}()$ that yields a good prediction is good enough (*"Whatever works, works..."*).
- How do we measure "what works"?
- Formal statistics can help figure out this: what is a good prediction.

Minimizing our losses

- A very common loss function in a regression setting is the squared loss $L(d) = d^2$
- Under this loss function the expected prediction loss is the mean squared error (MSE)

- **Result**: The best prediction of Y at any point $X = x$ is the conditional mean, when best is measured using a square error loss

Minimizing our losses

- Prediction problem solved if we knew $f^* = E[y|X = x]$

Minimizing our losses

- Prediction problem solved if we knew $f^* = E[y|X = x]$
- But we have to settle for an estimate: $\hat{f}(x)$
- The EMSE of this

$$E(y - \hat{y})^2 = E(f(X) + u - \hat{f}(X))^2 \tag{2}$$

Reducible and irreducible error

$$E(y-\hat{y})^2 = \underbrace{[f(X)-\hat{f}(X)]^2}_{Reducible} + \underbrace{Var(u)}_{Irreducible} \tag{3}$$

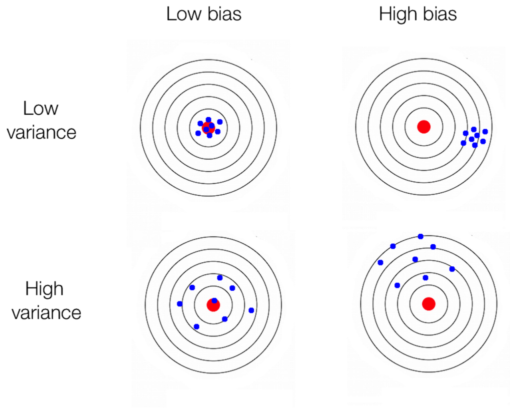
- The focus the is on techniques for estimating f with the aim of minimizing the reducible error
- It is important to keep in mind that the irreducible error will always provide an upper bound on the accuracy of our prediction for y
- This bound is almost always unknown in practice

Bias/Variance Decomposition

Recall that

- $Bias(\hat{f}(X)) = E(\hat{f}(X)) - f = E(\hat{f}(X) - f(X))$
- $Var(\hat{f}(X)) = E(\hat{f}(X) - E(\hat{f}(X)))^2$

Bias/Variance Decomposition



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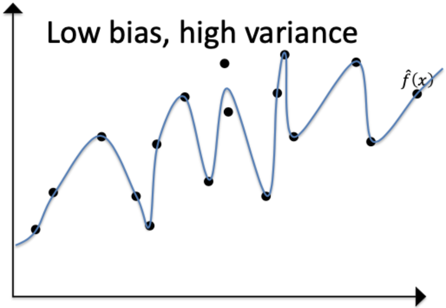
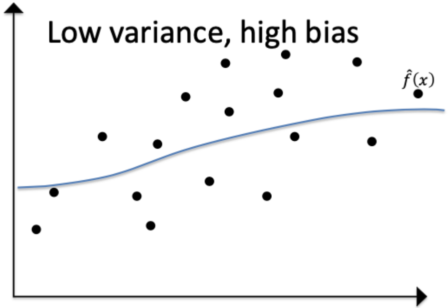
Result (very important!)

$$EMSE = Bias^2(\hat{f}(X)) + V(\hat{f}(X)) + \underbrace{Var(u)}_{Irreducible} \tag{4}$$

HW: Proof

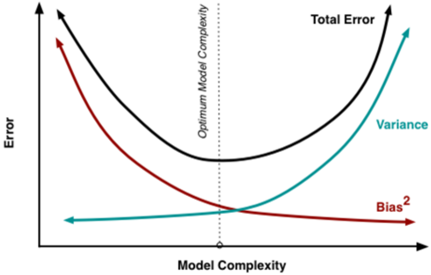
Bias/Variance Decomposition





The Bias-Variance Trade-Off

$$EMSE = Bias^2(\hat{f}(X)) + V(\hat{f}(X)) + \underbrace{Var(u)}_{Irreducible} \tag{5}$$



Source: https://tinyurl.com/y4lvjxpc

- The best kept secret: tolerating some bias is possible to reduce $V(\hat{f}(X))$ and lower MSE

Prediction and linear regression

- The goal is to predict y given another variables X.
- We assume that the link between y and X is given by the simple model:

$$y = f(X) + u \tag{6}$$

- we just learned that under a squared loss we need to approximate $E[y|X = x]$

Prediction and linear regression

- As economists we know that we can approximate $E[y|X = x]$ with a linear regression

$$f(X) = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p \tag{7}$$

- The problem boils down to estimating βs
- We can estimate these using
  - OLS
  - MLE
  - MM

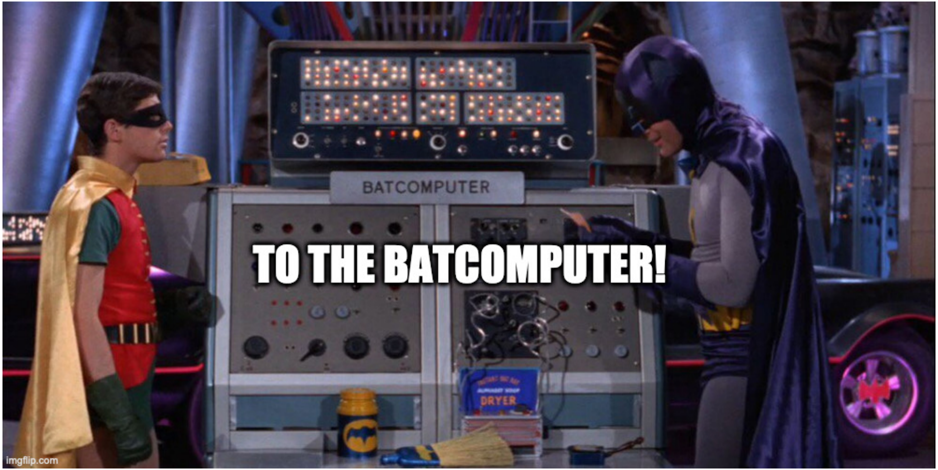


photo from https://www.dailydot.com/parsec/batman-1966-labels-tumblr-twitter-vine/

Prediction and linear regression

- And the Bias-Variance Trade-Off?

Prediction and linear regression

- And the Bias-Variance Trade-Off?
- Under the classical assumptions the OLS estimator is unbiased, hence

$$
\begin{aligned}
E(X\hat{\beta}) &= E(\hat{\beta}_1 + \hat{\beta}_2 X_2 + \cdots + \hat{\beta}_p X_p) && (8)\\
&= E(\hat{\beta}_1) + E(\hat{\beta}_2) X_2 + \cdots + E(\hat{\beta}_p) X_p && (9)\\
&= X\beta && (10)
\end{aligned}
$$

- Then,
  - $MSE(\hat{f})$ reduces to just $V(\hat{f})$

Complexity and the variance/bias trade off

- When the focus switches from estimating f to predicting Y,
- f plays a secondary role, as just a tool to improve the prediction based on X.
- Predicting Y involves *learning* f, that is, f is no longer taken as given, as in the classical view.
- Now it implies an iterative process where initial choices for f are revised in light of potential improvements in predictive performance.
- Model choice or learning involves choosing both f and a strategy to estimate it ($\hat{f}$), guided by predictive performance.

Complexity and the variance/bias trade off

- Classical econometrics, model choice involves deciding between a smaller and a larger linear model.
- Consider the following competing models for y:

$$y = \beta_1 X_1 + u_1$$

$$y = \beta_1 X_1 + \beta_2 X_2 + u_2$$

- $\hat{\beta}_1^{(1)}$ the OLS estimator of regressing y on X_1
- Prediction is:

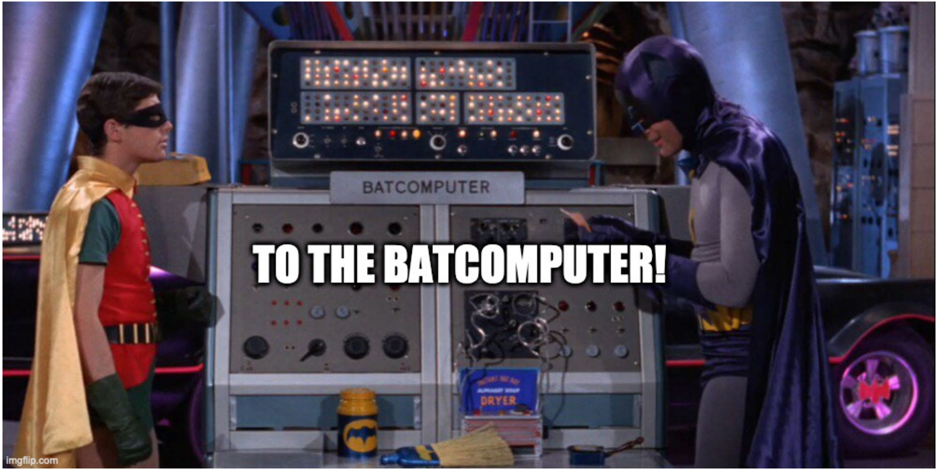
$$\hat{y}^{(1)} = \hat{\beta}_1^{(1)} X_1$$

- $\hat{\beta}_1^{(2)}$ and $\hat{\beta}_2^{(2)}$ the OLS estimators of β_1 and β_2 of regressing Y on X_1 and X_2.
- Prediction is:

$$\hat{y}^{(2)} = \hat{\beta}_1^{(2)} X_1 + \hat{\beta}_2^{(2)} X_2$$

Complexity and the variance/bias trade off

- An important discussion in classical econometrics is that of omission of relevant variables vs. inclusion of irrelevant ones.
  - If model (1) is true then estimating the larger model (2) leads to inefficient though unbiased estimators due to unnecessarily including X_2.
  - If model (2) holds, estimating the smaller model (1) leads to a more efficient but biased estimate if X_1 is also correlated with the omitted regressor X_2.
- This discussion of small vs large is always with respect to a model that is supposed to be true.
- But in practice the true model is unknown!!!



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Complexity and the variance/bias trade off

- Choosing between models involves a *bias/variance trade off*
- Classical econometrics tends to solve this dilemma abruptly,
  - requiring unbiased estimation, and hence favoring larger models to avoid bias
- In this simple setup, larger models are 'more complex', hence more complex models are less biased but more inefficient.
- Hence, in this very simple framework complexity is measured by the number of explanatory variables.
- A central idea in machine learning is to generalize the idea of complexity,
  - Optimal level of complexity, that is, models whose bias and variance led to minimum MSE.

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Collaboration on GitHub

- This is what I'm going to do (and want you to practice)
  - Partner 1: Invite Partner 2 to join you as a collaborator on your GitHub repo
  - Partner 2: Clone Partner 1's repo to your local machine. Make some edits (e.g. delete lines of text and add your own). Stage, commit and push these changes.
  - Partner 1: Make your own changes to the same file on your local machine. Stage, commit and then try to push them (*after* pulling from the GitHub repo first).

Collaboration time

... and we are back

- Did Partner 1 encounter a ‘merge conflict’ error?
- Git is protecting P1 by refusing the merge. It wants to make sure that you don’t accidentally overwrite all of your changes by pulling P2’s version of the file.

Collaboration time

```
Some text here.
<<<<<<< HEAD
Text added by Partner 2.
=======
Text added by Partner 1.
>>>>>>> 814e09178910383c128045ce67a58c9c1df3f558.
More text here.
```

Collaboration time

- Fixing these conflicts is a simple matter of (manually) editing the file.
  - Delete the lines of the text that you don't want.
  - Then, delete the special Git merge conflict symbols.

- Once that's done, you should be able to stage, commit, pull and finally push your changes to the GitHub repo without any errors.
  - P1 gets to decide what to keep because they fixed the merge conflict.
  - The full commit history is preserved, so P2 can always recover their changes if desired.
  - Another solution is using `branches`

Review

- This Week: The predictive paradigm and linear regression
  - Machine Learning is all about prediction
  - ML targets something different than causal inference, they can complement each other
  - ML best kept secret: tolerating some bias is possible to reduce $V(\hat{f}(X))$ and lower MSE
- Next Week: Out of sample prediction. Overfit, Resampling Methods, Webscrapping