# Quantitative Spatial Economics and Agglomeration Economies
## Urban Economics

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# The economics of density: evidence from the Berlin wall

► Ahlfeldt et al. (2015) tackle the question of how strong are agglomeration and dispersion forces in a city?

► We already know that this is a difficult question, and they point out two reasons why this is the case.

  1 Endogeneity: it is hard to disentangle the strength of agglomeration forces from simple differences in location fundamentals. For this you would need exogenous variation in agglomeration.

  2 It is hard to bring the traditional urban models to data on cities.

► Ahlfeldt et al. (2015) leverage a great source of exogenous variation to identify the effects of agglomeration: the Berlin Wall.

# The economics of density: evidence from the Berlin wall

Historical Background
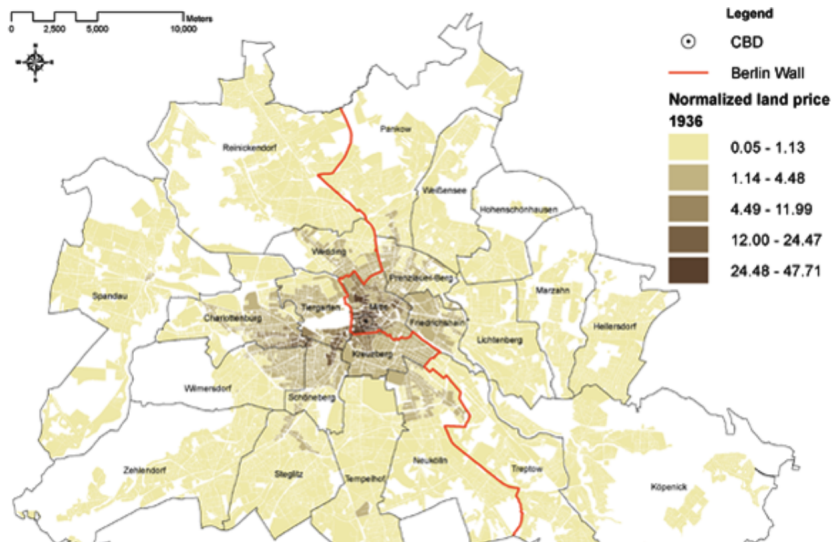
# The economics of density: evidence from the Berlin wall
### Model

Assumptions:

- ▶ Open city.
- ▶ In the agricultural area utility is $\bar{U}$.
- ▶ The city is in discrete space, and there are S blocks indexed by $i = 1, \dots, S$
- ▶ There is $L_i$ floor space in every block, and these blocks can be assigned to residential or commercial usage.
- ▶ $\theta_i$ is the endogenous fraction of every block that is dedicated to commercial use.
- ▶ The city produces a single numeraire good that is costlessly traded.
- ▶ The blocks are connected, and there are H (endogenous) workers that can freely move within the city.

# The economics of density: evidence from the Berlin wall
## Model: Workers

$$U_{ijo} = \frac{B_i}{d_{ij}} \left( \frac{c_{ijo}}{\beta} \right)^{\beta} \left( \frac{l_{ijo}}{1-\beta} \right)^{(1-\beta)} z_{ijo} \tag{1}$$
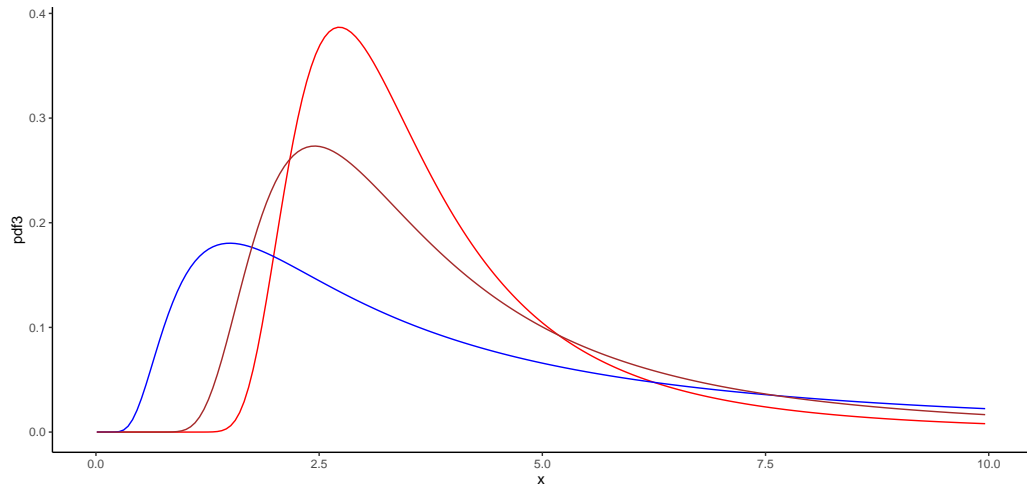
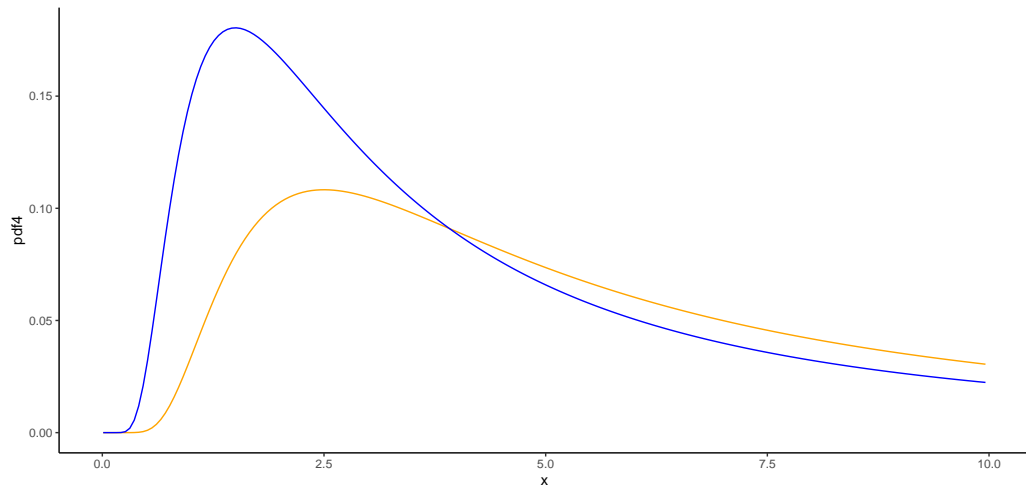# The economics of density: evidence from the Berlin wall
## Model: Workers

$$z_{ijo} \sim Frechet \tag{2}$$

▶ General Frechet

$$F(x) = exp(-Tx^{-\theta})) \tag{3}$$

$$E(x) = \Gamma \left( \frac{\theta - 1}{\theta} \right) T^{\frac{1}{\theta}} \qquad (4)$$

# The economics of density: evidence from the Berlin wall
## Model: Workers

$$z_{ijo} \sim Frechet \tag{5}$$

$$F(z_{ijo}) = exp(-T_i E_j z_{ijo}^{-\epsilon})) \tag{6}$$

with

▶ $T_i > 0$ scale parameter determines the average utility derived from living in block i

▶ $E_j > 0$ scale parameter determines the average utility derived from working in block j

▶ the shape parameter $\epsilon > 1$ controls the dispersion of idiosyncratic utility.

# The economics of density: evidence from the Berlin wall
## Model: Workers choice

- ▶ After observing her realizations for idiosyncratic utility for each pair of residence and employment blocks,

- ▶ each worker chooses where to live and work to maximize her utility, taking as given residential amenities, goods prices, factor prices, and the location decisions of other workers and firms.

- ▶ The indirect utility

$$V_{ijo} = \frac{B_i}{d_{ij}} \left( \frac{w_j}{Q_i^{1-\beta)}} \right) z_{ijo} \tag{7}$$

- ▶ Therefore, workers sort across pairs of residence and employment blocks depending on their idiosyncratic preferences and the characteristics of these locations

# The economics of density: evidence from the Berlin wall

## Model: Workers choice

▶ Since $z_{ijo}$ is Frechet, then $V_{ijo}$, with shape parameter

$$\Phi_{ij} = T_i E_j \left( d_{ij} Q_i^{(1-\beta)} \right)^{-\epsilon} \left( B_i w_j \right)^{\epsilon} \tag{8}$$

▶ The individuals will choose to live in $i$ and work in $j$ with some probability, which is the probability of $V_{ijo} = max_{r,s} V_{rso}$ , which is

$$\pi_{ij} = \frac{\Phi_{ij}}{\sum_{r=1}^{S} \sum_{s=1}^{S} \Phi_{ij}} = \frac{\Phi_{ij}}{\Phi} \tag{9}$$

# The economics of density: evidence from the Berlin wall

Model: Workers choice

▶ This completely characterizes the solution of the household's problem and the spatial distribution of households.

▶ We can now find the fraction of people who reside in i,

$$\pi_{Rj} = \sum_j \pi_{ij} \tag{10}$$

▶ The fraction of people who work in j,

$$\pi_{Mj} = \sum_i \pi_{ij} \tag{11}$$

# The economics of density: evidence from the Berlin wall

Model: Workers choice

▶ The probability of working on a particular place j conditional on living in i is

$$\pi_{ij|i} = \frac{\pi_{ij}}{\sum_s \pi_{is}} \tag{12}$$

▶ Adding up then requires that the workers in each place j, $H_{Mj}$, equal the sum of the residents in each place, $H_{Ri}$, times the probability that they work in j:

$$H_{Mj} = \sum_i \pi_{ij|i} H_{Ri} \tag{13}$$

▶ Firms are assumed to have CRS Cobb-Douglas production functions over workers $H_{Mj}$ and commercially-used land $L_{Mj}$, for which they pay wages $w_j$ and rents $q_j$:

$$y_j = A_j H_{Mj}^{\alpha} L_{Mj} 1 - \alpha \tag{14}$$

From the first-order conditions of the firm's problem:

$$H_{Mj} = \left( \frac{\alpha A_j}{w_j} \right)^{\frac{1}{1-\alpha}} \tag{15}$$

$$q_j = (1 - \alpha) \left( \frac{\alpha}{w_j} \right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} \tag{16}$$

# The economics of density: evidence from the Berlin wall
## Model:Land Markets

▶ Floor space is supplied proportionally to land K at each location

$$L_i = \phi_i K_i^{1-\mu} \tag{17}$$

▶ The expected demand of land from each individual ($l_i$):

$$E[l_i]H_{Ri} = (1-\beta)\frac{E(w_s|i)H_{Ri}}{Q_i} = (1-\theta)L_i \tag{18}$$

▶ Residential land market clearing implies that the demand for residential floor space equals the supply of floor space allocated to residential use in each location

# The economics of density: evidence from the Berlin wall
## Model: Equilibrium

An equilibrium here requires:

- ▶ Firms maximize profits and choose optimal locations

- ▶ Households maximize utility and choose optimal locations

- ▶ Land markets clear

- ▶ Labor markets clear

# The economics of density: evidence from the Berlin wall

Model: Equilibrium and agglomeration

▶ To model agglomeration, Ahlfeldt et al. (2015) make residential and workplace amenities depend on the density of employment and residency:

$$B_i = b_i \left( \sum_s e^{-\rho \tau_{js}} \left( \frac{H_{Ms}}{K_s} \right) \right)^{\eta} \tag{19}$$

$$A_i = a_j \left( \sum_s e^{-\delta \tau_{js}} \left( \frac{H_{Ms}}{K_s} \right) \right)^{\lambda} \tag{20}$$

# The economics of density: evidence from the Berlin wall

Reduced form evidence



Panel A: Berlin 1936

# The economics of density: evidence from the Berlin wall
Reduced form evidence



Panel B: West Berlin 1936

# The economics of density: evidence from the Berlin wall

Reduced form evidence



**Panel C: West Berlin 1986**

# The economics of density: evidence from the Berlin wall

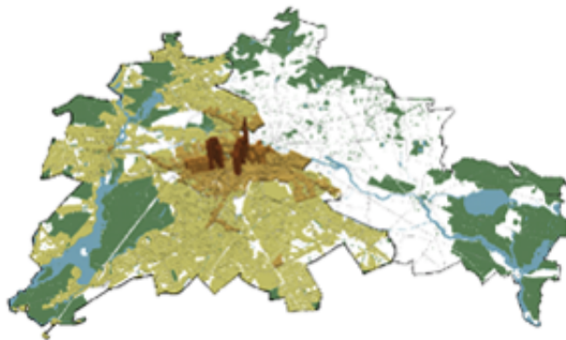Reduced form evidence



Panel D: Berlin 2006

# The economics of density: evidence from the Berlin wall
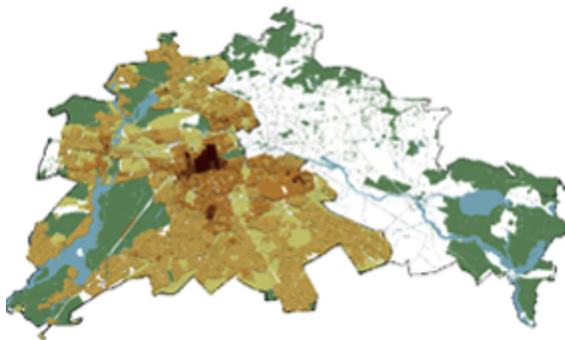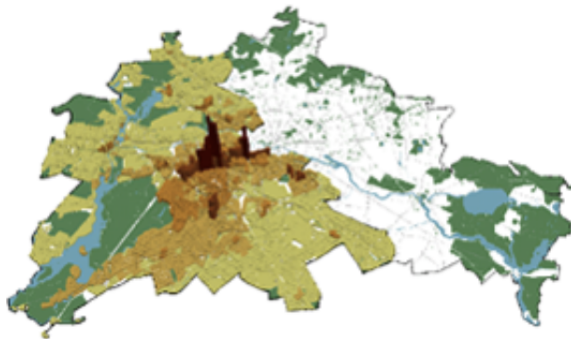Reduced form evidence



**Panel E: West Berlin 2006**

# The economics of density: evidence from the Berlin wall

## Reduced form evidence

TABLE I

BASELINE DIVISION DIFFERENCE-IN-DIFFERENCE RESULTS (1936–1986)[a]

| | (1) $\Delta \ln Q$ | (2) $\Delta \ln Q$ | (3) $\Delta \ln Q$ | (4) $\Delta \ln Q$ | (5) $\Delta \ln Q$ | (6) $\Delta \ln \text{EmpR}$ | (7) $\Delta \ln \text{EmpR}$ | (8) $\Delta \ln \text{EmpW}$ | (9) $\Delta \ln \text{EmpW}$ |
|---|---|---|---|---|---|---|---|---|---|
| CBD 1 | −0.800*** | −0.567*** | −0.524*** | −0.503*** | −0.565*** | −1.332*** | −0.975*** | −0.691* | −0.639* |
| | (0.071) | (0.071) | (0.071) | (0.071) | (0.077) | (0.383) | (0.311) | (0.408) | (0.338) |
| CBD 2 | −0.655*** | −0.422*** | −0.392*** | −0.360*** | −0.400*** | −0.715*** | −0.361 | −1.253*** | −1.367*** |
| | (0.042) | (0.047) | (0.046) | (0.043) | (0.050) | (0.299) | (0.280) | (0.293) | (0.243) |
| CBD 3 | −0.543*** | −0.306*** | −0.294*** | −0.258*** | −0.247*** | −0.911*** | −0.460** | −0.341 | −0.471** |
| | (0.034) | (0.039) | (0.037) | (0.032) | (0.034) | (0.239) | (0.206) | (0.241) | (0.190) |
| CBD 4 | −0.436*** | −0.207*** | −0.193*** | −0.166*** | −0.176*** | −0.356** | −0.259 | −0.512*** | −0.521*** |
| | (0.022) | (0.033) | (0.033) | (0.030) | (0.026) | (0.145) | (0.159) | (0.199) | (0.169) |
| CBD 5 | −0.353*** | −0.139*** | −0.123*** | −0.098*** | −0.100*** | −0.301*** | −0.143 | −0.436*** | −0.340*** |
| | (0.016) | (0.024) | (0.024) | (0.023) | (0.020) | (0.110) | (0.113) | (0.151) | (0.124) |
| CBD 6 | −0.291*** | −0.125*** | −0.094*** | −0.077*** | −0.090*** | −0.360*** | −0.135 | −0.280** | −0.142 |
| | (0.018) | (0.019) | (0.017) | (0.016) | (0.016) | (0.100) | (0.089) | (0.130) | (0.116) |
| | | | | | | | | | |
| Inner Boundary 1–6 | | | Yes | Yes | Yes | | Yes | | Yes |
| Outer Boundary 1–6 | | | Yes | Yes | Yes | | Yes | | Yes |
| Kudamm 1–6 | | | Yes | Yes | Yes | | Yes | | Yes |
| Block Characteristics | | | | | Yes | | Yes | | Yes |
| District Fixed Effects | | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| | | | | | | | | | |
| Observations | 6,260 | 6,260 | 6,260 | 6,260 | 6,260 | 5,978 | 5,978 | 2,844 | 2,844 |
| $R^2$ | 0.26 | 0.51 | 0.63 | 0.65 | 0.71 | 0.19 | 0.43 | 0.12 | 0.33 |

[a]$Q$ denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1–CBD6 are six 500 m distance grid cells for distance from the pre-war CBD. Inner Boundary 1–6 are six 500 m grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1–6 are six 500 m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1–6 are six 500 m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A.2 of the Technical Data Appendix. Block characteristics include the log distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors are in parentheses (Conley (1999)). * significant at 10%, ** significant at 5%, *** significant at 1%.

# The economics of density: evidence from the Berlin wall

## Reduced form evidence

TABLE II

BASELINE REUNIFICATION DIFFERENCE-IN-DIFFERENCE RESULTS (1986–2006)[a]

| | (1) $\Delta \ln Q$ | (2) $\Delta \ln Q$ | (3) $\Delta \ln Q$ | (4) $\Delta \ln Q$ | (5) $\Delta \ln Q$ | (6) $\Delta \ln$ EmpR | (7) $\Delta \ln$ EmpR | (8) $\Delta \ln$ EmpW | (9) $\Delta \ln$ EmpW |
|---|---|---|---|---|---|---|---|---|---|
| CBD 1 | 0.398*** | 0.408*** | 0.368*** | 0.369*** | 0.281*** | 1.079*** | 1.025*** | 1.574*** | 1.249** |
| | (0.105) | (0.090) | (0.083) | (0.081) | (0.088) | (0.307) | (0.297) | (0.479) | (0.517) |
| CBD 2 | 0.290*** | 0.289*** | 0.257*** | 0.258*** | 0.191** | 0.589* | 0.538* | 0.684** | 0.457 |
| | (0.111) | (0.096) | (0.090) | (0.088) | (0.087) | (0.315) | (0.299) | (0.326) | (0.334) |
| CBD 3 | 0.122*** | 0.120*** | 0.110*** | 0.115*** | 0.063** | 0.340* | 0.305* | 0.326 | 0.158 |
| | (0.037) | (0.033) | (0.032) | (0.032) | (0.028) | (0.180) | (0.158) | (0.216) | (0.239) |
| CBD 4 | 0.033*** | 0.031 | 0.030 | 0.034 | 0.017 | 0.110 | 0.034 | 0.336** | 0.261 |
| | (0.013) | (0.023) | (0.022) | (0.021) | (0.020) | (0.068) | (0.066) | (0.161) | (0.185) |
| CBD 5 | 0.025*** | 0.018 | 0.020 | 0.020 | 0.015 | −0.012 | −0.056 | 0.114 | 0.066 |
| | (0.010) | (0.015) | (0.014) | (0.014) | (0.013) | (0.056) | (0.057) | (0.118) | (0.131) |
| CBD 6 | 0.019** | −0.000 | −0.000 | −0.003 | 0.005 | 0.060 | 0.053 | 0.049 | 0.110 |
| | (0.009) | (0.012) | (0.012) | (0.012) | (0.011) | (0.039) | (0.041) | (0.095) | (0.098) |
| Inner Boundary 1–6 | | | Yes | Yes | Yes | | Yes | | Yes |
| Outer Boundary 1–6 | | Yes | Yes | Yes | Yes | | Yes | | Yes |
| Kudamm 1–6 | | | Yes | Yes | Yes | | Yes | | Yes |
| Block Characteristics | | | | | Yes | | Yes | | Yes |
| District Fixed Effects | | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 7,050 | 7,050 | 7,050 | 7,050 | 7,050 | 6,718 | 6,718 | 5,602 | 5,602 |
| $R^2$ | 0.08 | 0.32 | 0.34 | 0.35 | 0.43 | 0.04 | 0.07 | 0.03 | 0.06 |

[a] $Q$ denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1–CBD6 are six 500 m distance grid cells for distance from the pre-war CBD. Inner Boundary 1–6 are six 500 m distance grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1–6 are six 500 m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1–6 are six 500 m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A.4 of the Technical Data Appendix. Block characteristics include the log distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley (1999)). * significant at 10%; ** significant at 5%; *** significant at 1%.

# The economics of density: evidence from the Berlin wall
Structural Estimation