Rosen-Roback Framework

Urban Economics

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September 26, 2023

Rosen-Roback

Set up

- 3 Sectors:
  - Consumers of Housing (homogeneous)
  - The production sector
  - The construction sector
- Assumption cities are small, and exogenous amount of land $\bar{L}$ in each city

Recap: Rosen Roback Framework

Three Simultaneous Equilibria

- Individuals are optimally choosing which city to live in
  - There is a group of homogeneous individuals
  - Some of them are living in different cities
  - Their utility level is the same in all those cities
- Firms earn zero expected profits
  - Free entry of firms
  - Firm profits are equalized across cities
- The construction sector operates optimally
  - Free entry, zero profit for builders
  - Construction profits are equalized across cities

Recap: Rosen Roback Framework

Housing consumption

$$maxU(C,H) = \theta C^{1-\alpha} H^{\alpha} \tag{1}$$

$$st \tag{2}$$

$$W = C + p_H H \tag{3}$$

Sol

$$H^* = \alpha \frac{W}{p_H} \tag{4}$$

$$C^* = (1-\alpha) W \tag{5}$$

Recap: Rosen Roback Framework

Housing consumption

Indirect Utility

$$V = \theta \alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \frac{W}{p_H^{\alpha}} \tag{6}$$

For spatial equilibrium to hold, this quantity must equal a reservation utility level $\bar{u}$

$$\bar{u} = \theta \alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \frac{W}{p_H^{\alpha}} \tag{7}$$

Recap: Rosen Roback Framework

Production Sector

Cobb-Douglas production function with constant returns to scale:

$$y = AN^{\beta}K^{\gamma}\bar{Z}^{\zeta} \tag{8}$$

$$st \tag{9}$$

$$C = WN + p_k K + p_z Z \tag{10}$$

$\beta + \gamma + \zeta = 1$
The competitive wage in each city is

$$W = \beta \left(\left(\frac{\gamma}{p_k} \right)^{\gamma} A \left(\frac{\bar{Z}}{N} \right)^{\zeta} \right)^{\frac{1}{1-\gamma}} \tag{11}$$

Recap: Rosen Roback Framework

Construction sector

- Housing supply is the product of land L, (here exogoenous) and building height h
- Height is built with tradable capital at a convex cost

$$\varphi p_K \left(\frac{h}{\delta}^{\delta}\right) \tag{12}$$

for $\varphi > 0$ and $\delta > 1$

Free entry to developers

$$max\left\{p_H h - \varphi p_k \left(\frac{h}{\delta}\right)^{\delta}\right\}$$

$$h^* = \delta\left(\frac{p_H}{\varphi p_k}\right)^{\frac{1}{\delta-1}}$$

Recap: Rosen Roback Framework

Three Simultaneous Equilibria

- Individual optimal location choice

$$\bar{u} = \theta\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\frac{W}{p_H^{\alpha}}$$

- Firms labor demand

$$W = \beta\left(\left(\frac{\gamma}{p_k}\right)^{\gamma} A\left(\frac{\bar{Z}}{N}\right)^{\zeta}\right)^{\frac{1}{1-\gamma}} \tag{13}$$

- Housing market equilibrium

$$p_H^* = \left[\varphi p_k\left(\frac{\alpha}{\delta}\frac{WN}{\bar{L}}\right)^{\delta-1}\right]^{\frac{1}{\delta}}$$

Recap: Rosen Roback Framework

Three Simultaneous Equilibria

- Individual optimal Location choice

$$logW - \alpha log(p_H) + log\theta = log(\bar{u}) + k_1$$

- Firms labor demand

$$(1-\gamma)logW + \zeta(logN - log\bar{Z}) - logA = k_2 - \gamma logp_k$$

- Housing Market equilibrium

$$\delta logp_H - (\delta - 1)(logw + logN - log\bar{L}) - log\varphi = logp_k + k_3$$

Recap: Rosen Roback Framework

Spatial Equilibrium 3 conditions

Three endogenous variables

1. N
2. W
3. p_H

Three exogenous

1. θ
2. $\tilde{A} = A\bar{Z}^{\zeta}$
3. $\tilde{L} = \bar{L}\varphi^{\frac{-1}{(\delta-1)}}$

Two economy wide characteristics

1. $\bar{u}$
2. p_k

Recap: Rosen Roback Framework
Solve

$$\begin{pmatrix} 0 & (1-\gamma) & \zeta \\ \delta & -(\delta-1) & -(\delta-1) \\ -\alpha & 1 & 0 \end{pmatrix} \begin{pmatrix} \log(p_H) \\ \log(W) \\ \log(N) \end{pmatrix} = \begin{pmatrix} k+\log(\tilde{A}) \\ k-(\delta-1)\log(\tilde{L}) \\ k-\log(\theta) \end{pmatrix} \tag{14}$$

The determinant of

$$\begin{pmatrix} 0 & (1-\gamma) & \zeta \\ \delta & -(\delta-1) & -(\delta-1) \\ -\alpha & 1 & 0 \end{pmatrix} = \alpha(\delta-1)\beta+\zeta\delta \tag{15}$$

Recap: Rosen Roback Framework

Equilibrium Solution

1. Equilibrium wages

$$logW = k_w + \frac{(\delta-1)\alpha(log\tilde{A} - \zeta log\tilde{L}) - \delta\zeta log\theta}{\beta(\delta-1)\alpha + \delta\zeta}$$

2. Equilibrium housing prices

$$logp_H = k_p + \frac{(\delta-1)(log\tilde{A} + \beta log\theta - \zeta log\tilde{L})}{\beta(\delta-1)\alpha + \delta\zeta}$$

3. Equilibrium population

$$logN = k_N + \frac{[\delta(1-\alpha)+\alpha]log\tilde{A} + (\beta+\zeta)[\delta log\theta + (\delta-1)\alpha log\tilde{L}]}{\beta(\delta-1)\alpha + \delta\zeta}$$

$$\log\left(\frac{N}{\bar{L}}\right) = k_N + \frac{(\delta(1-\alpha)+\alpha)\left(\log(\tilde{A}) + \zeta\log(\bar{L})\right) + (\beta+\zeta)\left(\delta\log(\theta) - \alpha\log(\psi)\right)}{\alpha(\delta-1)\beta + \zeta\delta} \tag{16}$$

Recap: Rosen Roback Framework

As an example of how this framework can be used, we can look to the predictions of the model about the relationship between an exogenous variable X and prices, wages and density.
Let's assume

$$log\theta = k_\theta + \xi_\theta X + \epsilon_\theta$$

$$log\tilde{A} = k_A + \xi_A X + \epsilon_A$$

$$log\tilde{L} = k_L + \xi_L X + \epsilon_L$$

Recap: Rosen Roback Framework

If we replace in th equilibrium conditions

$$logW = k_w + \xi_w X + \epsilon_w$$

$$logp_H = k_p + \xi_p X + \epsilon_p$$

$$logN = k_p + \xi_N X + \epsilon_N$$

where

$$\xi_W = \frac{(\delta - 1)\alpha(\xi_A - \zeta\xi_L) - \delta\zeta\xi_\theta}{\beta(\delta - 1)\alpha + \delta\zeta}$$

$$\xi_p = \frac{(\delta - 1)(\xi_A + \beta\xi_\theta - \zeta\xi_L)}{\beta(\delta - 1)\alpha + \delta\zeta}$$

$$\xi_N = \frac{[\delta(1 - \alpha) + \alpha]\xi_A + (\beta + \zeta)[\delta\xi_\theta + (\delta - 1)\alpha\xi_L]}{\beta(\delta - 1)\alpha + \delta\zeta}$$

Recap: Rosen Roback Framework

Solving and inverting we get

$$\xi_\theta = \alpha \xi_p - \xi_w$$

$$\xi_A = \zeta \xi_N + (1-\gamma)\xi_w$$

$$\xi_L = \xi_N + \xi_W - \frac{\delta}{\delta - 1}\xi_p$$

Recap: Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

In the US, fastest growing areas have warm climates, something similar in Europe (what about Latam?) These areas, in the south and west of US, are known as the "sunbelt"

If we look across metropolitan areas, the relationship between January temperature and size is:

$$log(Population2000) = \underset{(0.2)}{12.2} + \underset{(0.005)}{0.017} JanuaryTemp$$

more if we look at changes

$$log(\frac{Population2000}{Population1990}) = \underset{(0.14)}{0.016} + \underset{(0.0004)}{0.003} JanuaryTemp$$

Recap: Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

Why has population growth shifted to sunbelt?

1. Changes in amenities? E.g the advent of air conditioning made South more comfortable
2. Has productivity increased ?
3. Has land availability increased? Are people attracted to cheap housing, made possible by pro-building policies?

Recap: Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- We can use the spatial equilibrium model
- Glaeser and Gottileb run regressions of population, wages, and house values on temperature with controls
- Combine coefficients using the model to look at effect of temperature on amenities, productivity, and land availability

Example Spatial Equilibrium

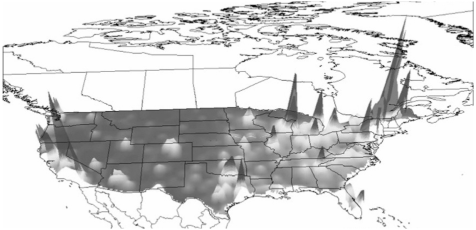
An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable	Log wage	Log house value	Log real wage	Log wage	Log house value	Log real wage
Year:	2000	2000	2000	1990, 2000	1990, 2000	1990, 2000
Mean January temperature	−0.19	0.60	−0.33			
	[0.06]	[0.31]	[0.10]			
Mean January temperature × year 2000				−0.001	−0.43	0.19
				[0.05]	[0.11]	[0.03]
Year 2000 dummy				0.25	0.62	0.06
				[0.02]	[0.06]	[0.02]
Individual controls	Yes	—	Yes	Yes	—	Yes
Housing controls	—	Yes	—	—	Yes	—
MSA fixed effects	—	—	—	Yes	Yes	Yes
N	1,590,467	2,341,976	1,590,467	2,950,850	4,245,315	2,950,850
R^2	0.29	0.36	0.21	0.27	0.60	0.26

Notes: Individual-level data are from the Census Public Use Microdata Sample, as described in the Data Appendix. Metropolitan-area population is from the Census, as also described in the Data Appendix. Mean January temperature is from the City and County Data Book, 1994, and is measured in hundreds of degrees Fahrenheit. Real wage is controlled for with median house value, also from the Census as described in the Data Appendix. Individual controls include age and education. Location characteristics follow Metropolitan Statistical Areas under the 1999 definitions, using Primary Metropolitan Statistical Areas rather than Consolidated Metropolitan Statistical Areas where applicable and New England County Metropolitan Areas where applicable. Standard errors are clustered by

Agglomeration Economies



Agglomeration Economies

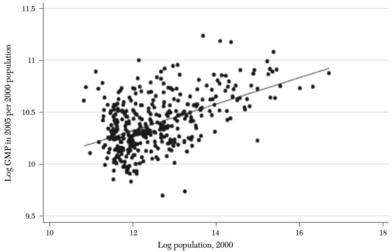


Figure 1. Productivity and City Size

Notes: Units of observation are Metropolitan Statistical Areas under the 2006 definitions. Population is from the Census, as described in the Data Appendix. Gross Metropolitan Product is from the Bureau of Economic Analysis.

The regression line is log *GMP per capita* = 0.13 [0.01] × log *population* + 8.8 [0.1]. $R^2 = 0.25$ and $N = 363$.

Agglomeration Economies

- Why do we see such a remarkable clustering of human activity in a small number of urban areas?
- Model above: cities may form because some places have innate advantages in productivity, housing supply or amenities.
- Or it may be because clusters of people endogenously increase productivity, housing supply or amenities (agglomeration effects)
  - Example: Los Angeles. In its early history, prosperous retirees came to enjoy the climate (an innate amenity). Also: restaurants and theather endogenously emerged with the infux of population.
  - But: if cities were driven by amenities, then real wages should be lower in big urban areas. This is not true. People require a wage premium to locate in big cities.
  - Can cities be driven by innate advantage in supplying housing, or because density makes it easier to build? No. It is more expensive to build vertically than horizontally. Housing supply is more expensive in bigger areas.

Evidence of Agglomeration Economies

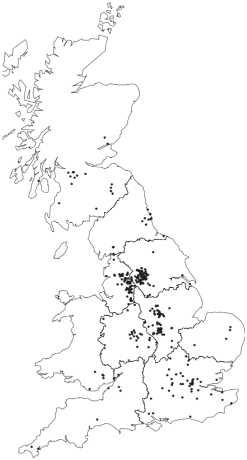
- Three strategies to identify agglomeration economies

  1. Show there is too much spatial concentration to be random (Duranton and Overman, 2005)
  2. Compare productivity over space (Greenstone, 2010)
  3. Compare wages and rents across space (Quantitative Spatial Models, Ahlfeldt et al, 2015)

Spatial Concentration

Extremes of Localization and Dispersion

(c) Other Agricultural and Forestry Machinery (SIC2932)



(d) Machinery for Textile, Apparel and Leather Production (SIC2954)

Spatial Concentration

Ambiguous Cases

(a) Basic Pharmaceuticals (SIC2441)

(b) Pharmaceutical Preparations (SIC2442)

Most Localized

sic92	Industry	Γ or Ψ
Most localised		
2214	Publishing of Sound Recordings	0.470
1711	Preparation and Spinning of Cotton-type Fibres	0.411
2231	Reproduction of Sound Recordings	0.403
1760	Manufacture of Knitted and Crocheted Fabrics	0.321
1713	Preparation and Spinning of Worsted-type Fibres	0.319
2861	Manufacture of Cutlery	0.314
1771	Manufacture of Knitted and Crocheted Hosiery	0.290
1810	Manufacture of Leather Clothes	0.203
1822	Manufacture of Other Outerwear	0.181
2211	Publishing of Books	0.178

Most Dispersed

Most dispersed		
1520	Processing and Preserving of Fish and Fish Products	0.200
3511	Building and Repairing of Ships	0.113
1581	Manufacture of Bread, Fresh Pastry Goods and Cakes	0.094
2010	Saw Milling and Planing of Wood, Impregnation of Wood	0.082
2932	Other Agricultural and Forestry Machinery	0.067
1551	Operation of Dairies and Cheese Making	0.064
1752	Manufacture of Cordage, Rope, Twine and Netting	0.062
3615	Manufacture of Mattresses	0.050
1571	Manufacture of Prepared Feeds for Farm Animals	0.049
2030	Manufacture of Builders' Carpentry and Joinery	0.047

Measuring Agglomeration Economies Through Productivity

- The most direct approach
  - Measure productivity from output, then relate it to density

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- The most direct approach
  - Measure productivity from output, then relate it to density
- Problems with this approach?
  - Natural advantages make a region more productive
  - Greater productivity attracts workers and firms
- Can you think on an ideal experiment?

MDP Greenstone, Hornbeck, and Moretti (2010)

- Use new plant openings, and compare results of those counties where plants opened up vs those that didn't
- What is the model?
  - New plants choose their location to maximize profits

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- Use new plant openings, and compare results of those counties where plants opened up vs those that didn't
- What is the model?
  - New plants choose their location to maximize profits
  - Places without new plants are not a valid control group

MDP Greenstone, Hornbeck, and Moretti (2010)

- Regular feature in the corporate real estate journal Site Selection Stories about the location choice of large new plants
- Gradual narrowing down of potential counties to 2 or 3 finalists
- The 1 or 2 losers in the shortlist provide a control group
  - Almost as attractive as the winning county
  - Yet, they did not receive the treatment

MDP Greenstone, Hornbeck, and Moretti (2010)

TABLE 1
THE MILLION DOLLAR PLANT SAMPLE

	(1)
Sample MDP openings:[a]	
Across all industries	47
Within same two-digit SIC	16
Across all industries:	
Number of loser counties per winner county:	
1	31
2+	16
Reported year − matched year:[b]	
−2 to −1	20
0	15
1 to 3	12
Reported year of MDP location:	
1981–85	11
1986–89	18
1990–93	18
MDP characteristics, 5 years after opening:[c]	
Output ($1,000s)	452,801
	(901,690)
Output, relative to county output 1 year prior	.086
	(.109)
Hours of labor (1,000s)	2,986
	(6,789)

MDP Greenstone, Hornbeck, and Moretti (2010)

- Plant-level regression

$$log(Y) = log(A) + \beta_1 \log(L) + \beta_2 \log(K_B) + \beta_3 \log(K_E) + \beta_4 \log(M) \tag{17}$$

- where

$$log(A) = \delta_1 Winner + \delta_2 Post + \delta_3 Winner \times Post \tag{18}$$

MDP Greenstone, Hornbeck, and Moretti (2010)

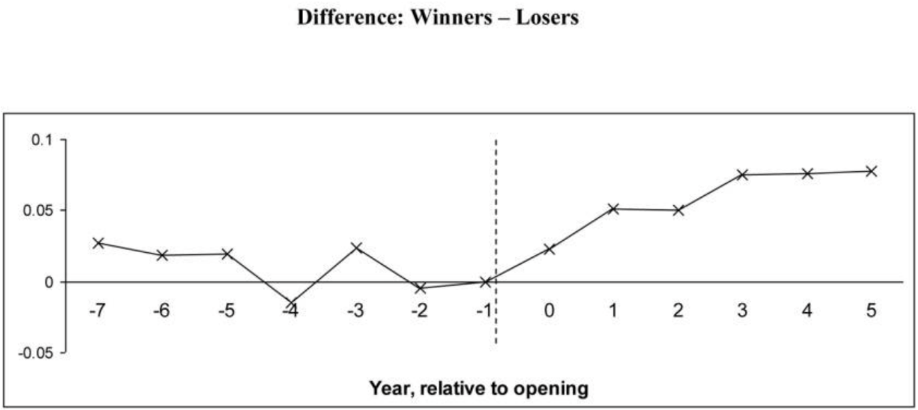

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TABLE 9
CHANGES IN COUNTIES' NUMBER OF PLANTS, TOTAL OUTPUT, AND SKILL-ADJUSTED WAGES FOLLOWING AN MDP OPENING

	A. CENSUS OF MANUFACTURES		B. CENSUS OF POPULATION
	Dependent Variable: Log(Plants) (1)	Dependent Variable: Log(Total Output) (2)	Dependent Variable: Log(Wage) (3)
Difference-in-difference	.1255**	.1454	.0268*
	(.0550)	(.0900)	(.0139)
R^2	.9984	.9931	.3623
Observations	209	209	1,057,999