Agglomeration Economies

Urban Economics

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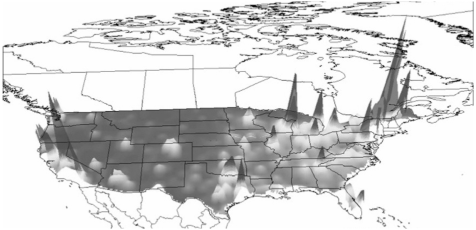
October 8, 2025

Agenda

Agglomeration Economies

- Why do we see such a remarkable clustering of human activity in a small number of urban areas?



Evidence of Agglomeration Economies

- Three strategies to identify agglomeration economies

  1. Show there is too much spatial concentration to be random (Duranton and Overman, 2005)
  2. Compare productivity over space (Greenstone et al., 2010)
  3. Compare wages and rents across space (Quantitative Spatial Models, Ahlfeldt et al, 2015)

Agenda

Spatial Concentration

Extremes of Localization and Dispersion

(c) Other Agricultural and Forestry Machinery (SIC2932)

(d) Machinery for Textile, Apparel and Leather Production (SIC2954)

Spatial Concentration
Ambiguous Cases

(a) Basic Pharmaceuticals
(SIC2441)

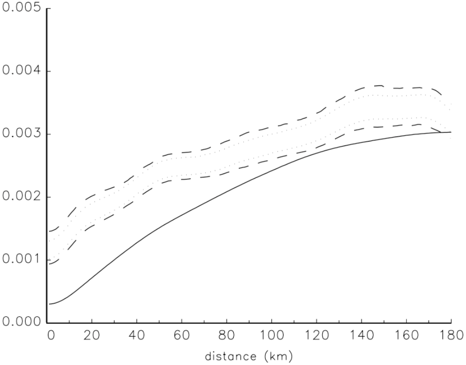
(b) Pharmaceutical Preparations
(SIC2442)

Duranton & Overman Methodology

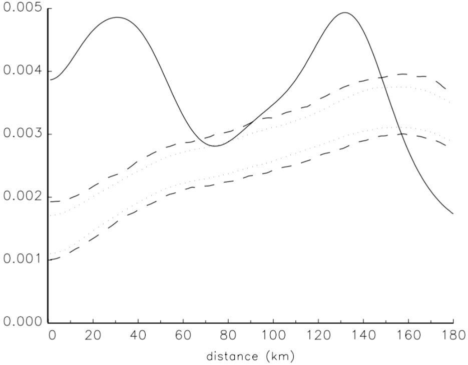
1. **Select Relevant Establishments:**
   - Choose establishments based on industry and size.
   - Consider different thresholds to assess robustness (e.g., include only those contributing to 90% of employment).
2. **Compute Bilateral Distances:**
   - Calculate Euclidean distances between all pairs of establishments.
   - Use Kernel Density Estimation (KDE) to estimate the density of these distances.

Spatial Concentration

K Density Estimates



(c) Other Agricultural and Forestry Machinery (SIC2932)

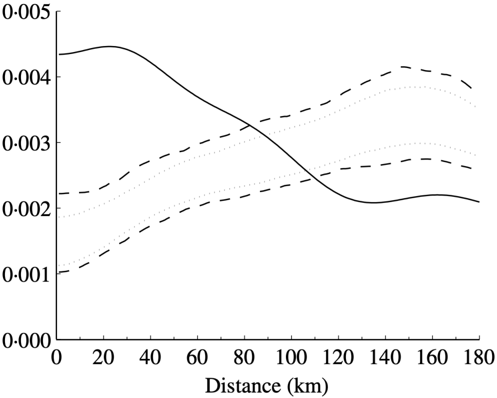


(d) Machinery for Textile, Apparel and Leather Production (SIC2954)

Figure 2. *K*-density, local confidence intervals and global confidence bands for four illustrative industries

Spatial Concentration

Ambiguous Cases



(a) Basic Pharmaceuticals
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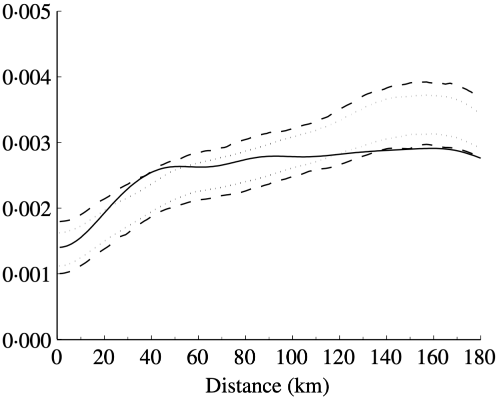

(b) Pharmaceutical Preparations
(SIC2442)

Duranton & Overman Methodology

4. **Generate Counterfactuals:**
   - Randomly assign establishments to locations, maintaining the number of establishments and industrial concentration.
   - Create 1,000 simulations to construct a baseline for comparison.
5. **Statistical Significance:**
   - Compare actual densities with simulated counterfactuals to determine if localization is significant.
   - Use local and global confidence intervals to assess statistical significance of localization.
6. **Localization Metrics:**
   - Define indices for localization (γ) and dispersion (ψ) at each distance.
   - Determine global localization or dispersion based on these indices over all distances.

Agenda

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Univariate density estimation: parametric vs nonparametric methods

- Let $f = f(.)$ be the density function of the random variable X
- Let $x_1,\ x_2,\ \dots x_n$ be a random sample of X. Then, $x_i \sim f\ iid$
- How can we estimate the density in a particular point x_0, then $f(x_0)$?

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Parametric methods

- They assume a particular functional form for f.

Parametric methods

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- Example:

$$f(x_o) = \phi(x_o) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x_o-\mu}{\sigma}\right)^2\right] \tag{1}$$

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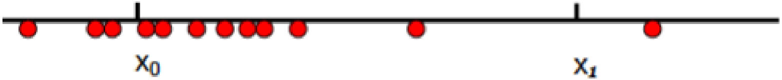
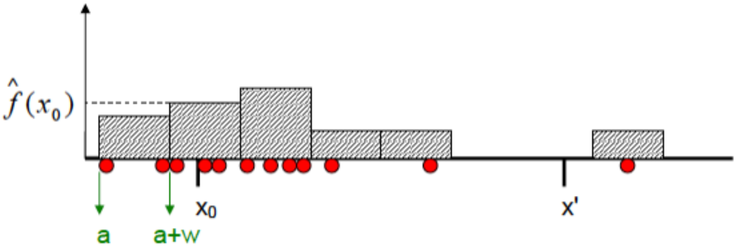
Non-parametric methods

- They seek to estimate $f\left(x_o\right))$ without assuming a particular functional form, only assuming certain regularity conditions of the density (smoothness, differentiability)



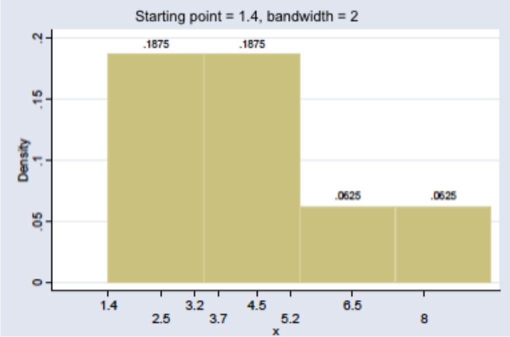
A rudimentary non-parametric estimator: the histogram



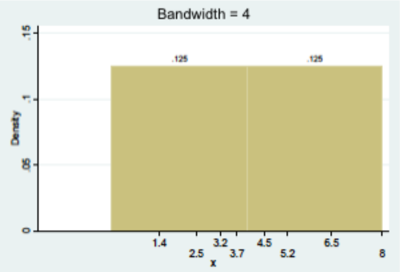
- The height of the bars is an estimator of the density at any point in the interval

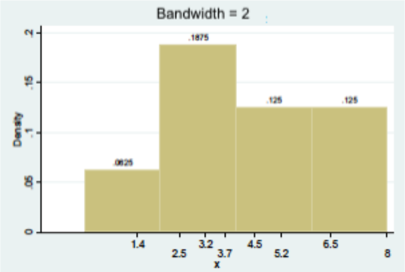
$$\hat{f}\left(x_o\right) = \frac{Nber.\,obs\ interval}{n \ \times \ w} \tag{2}$$

Histogram problems: (1) depends on starting point

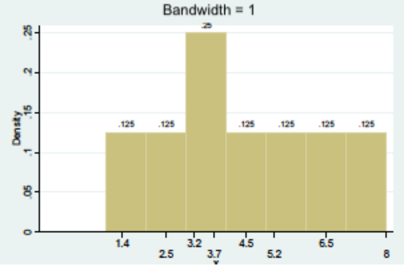


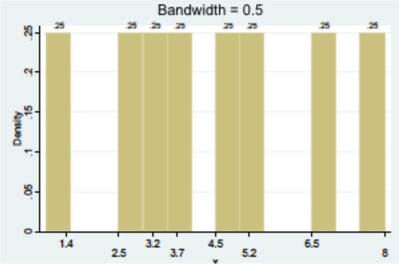
Histogram problems: (2) depends on the bandwidth





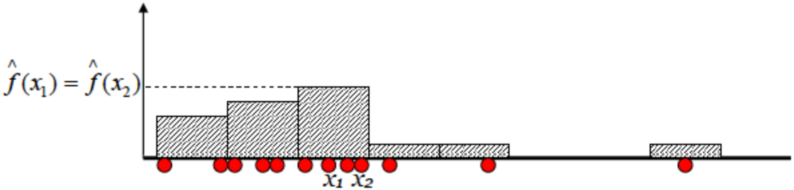
Histogram problems: (2) depends on the bandwidth





Histogram problems: (3) is discontinuous at the ends of the interval

- Note that $\hat{f}\ (x_1) = \hat{f}\ (x_2)$
- But $\hat{f}\ (x_2 + \epsilon) = \frac{1}{4}\hat{f}\ (x_2)$ for any $\epsilon > 0$



Another non-parametric estimator: the naive estimator

Discrete case

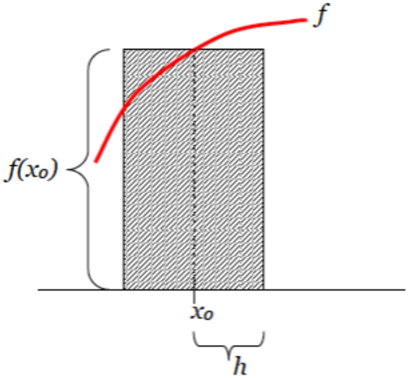
- X is a discrete RV, *iid*
- Objective: estimate $\Pr(X = x_o) = f(x_o)$
- The naive estimator comes from:

$$\hat{f}(x_o) = \frac{\# \, x_i = x_o}{n} = \frac{1}{n} \sum_{i=1}^{n} I(x_i = x_o) \tag{3}$$

Another non-parametric estimator: the naive estimator

Continuous case

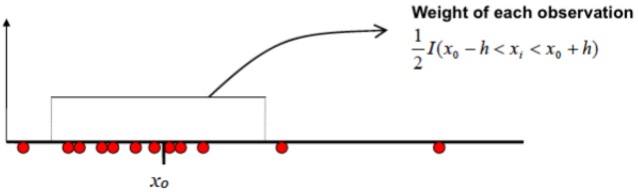
- X is a continuous RV ($Pr\left(X = x_0\right)$) we evaluate the probability that X is "close" to x_0.
- We say that x_i is close to x_0 if x_i belongs to the interval $(x_o - \ h, \ x_o + \ h)$



The naive estimator

Notice:

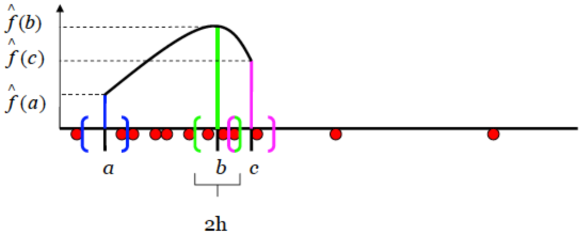
- Every possible point x_o is the center of an interval
- Observations that are within that interval (less than one h away from x_o) are weighted by 1⁄2



The naive estimator

Notice:

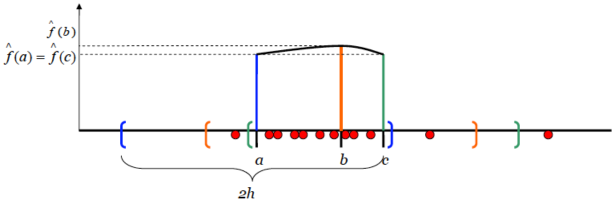
- To estimate the density at points a, b and c we construct intervals around them
- Then, unlike what happens in the histograms, the intervals of the naive estimator overlap (the initial point no longer matters)



The naive estimator

Problems

- It depends on h: the larger the bandwidth, the more distant observations from x_0 are used to estimate f(x_0).The higher h, the smoother the estimated density (h=smoothing parameter)



- The weight $1/2I(.)$ is discontinuous at the limits of each interval, generating discontinuities in the estimated density.
- The weight treats observations very close to x_0 in the same way as others somewhat further away, as long as they belong to the interval of length 2h around x_0

The weighted average estimator or kernel method

- The kernel estimator is a generalization of the naive estimator that overcomes some of the deficiencies of the latter.
- The weight $1/2I(.)$ of the naive estimator is replaced by a new weight K(.):

$$\hat{f}\ (x_o) = \frac{1}{n\ \times h}\sum_{i=1}^{n} K\left(\frac{x_i - x_0}{h}\right) \tag{4}$$

- Rosenblatt-Parzen density estimation approach

The weighted average estimator or kernel method

- The weights or kernels are known functions that satisfy:
  1. $K(\phi) \geq 0$
  2. $\int_{-\infty}^{\infty} K\ (\phi)\ d\phi = 1$
  3. $K(\phi) = K(-\ \phi)$
  4. $E(K(\phi)) = 0$

The weighted average estimator or kernel method

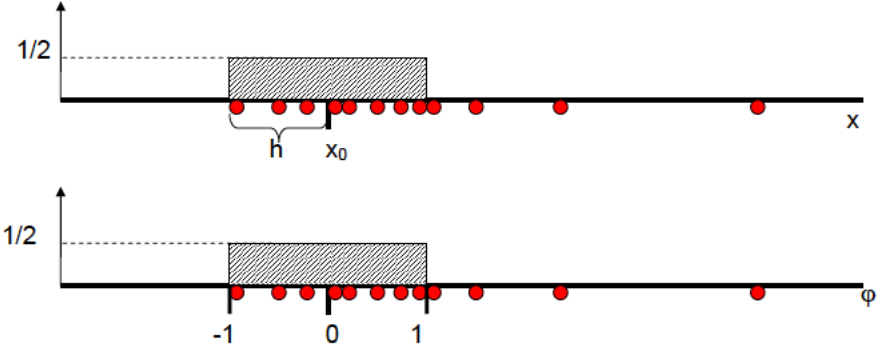
Example 1: rectangular or uniform kernel

$$K(\phi) = \begin{cases} \frac{1}{2} & |\phi| < 1 \\ 0 & o.w. \end{cases} \tag{5}$$

where $\phi = \frac{x_i - x_o}{h}$ In this case, the Kernels estimator matches the naive estimator. Note that the weight is similar to a uniform density function on the interval (-1, 1)

The weighted average estimator or kernel method

Example 1: rectangular or uniform kernel



The weighted average estimator or kernel method

Example 2: Gaussian kernel

- Gaussian

$$K(\phi) = \frac{1}{\sqrt{2\pi}} e^{-\phi^2/2} \tag{6}$$

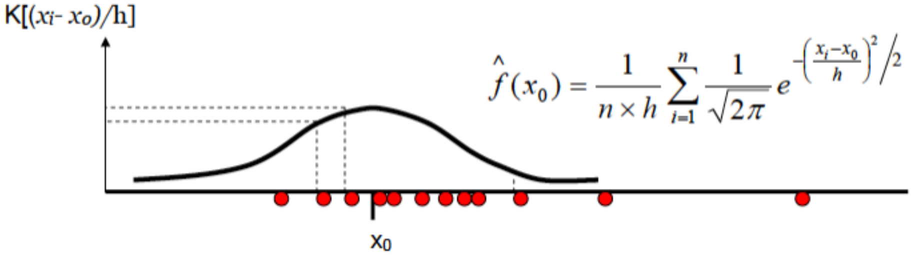
- The kernel function corresponds to the standard normal density function that satisfies the above assumptions.
- For this type of kernel, the density estimator is given by:

$$\hat{f}(x_o) = \frac{1}{n \times h} \sum_{i=1}^{n} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x_i - x_o}{h}\right)^2/2} \tag{7}$$

- Important: we don't assume normal distribution for any variable. The functional form is just used to weight the sample observations

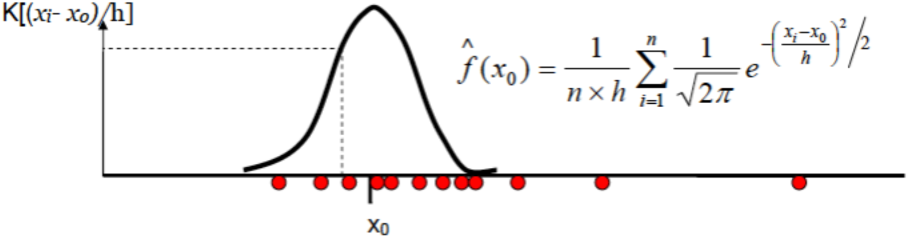
The weighted average estimator or kernel method

Example 2: Gaussian kernel



The weighted average estimator or kernel method

Example 2: Gaussian kernel (smaller h)



The weighted average estimator or kernel method

Other Kernels

Kernel type	Formula	Support
Gaussian or normal	$\frac{1}{\sqrt{2\pi}}\exp(-\varphi^2/2)$	$\varphi \in (-\infty, \infty)$
Epanechnikov	$\frac{3}{4\sqrt{5}}(1-\frac{1}{5}\varphi^2)I(\lvert\varphi\rvert<\sqrt{5})$	$\varphi \in \left(-\sqrt{5}, \sqrt{5}\right)$
Epanechnikov modificado	$\frac{3}{4}(1-\varphi^2)I(\lvert\varphi\rvert<1)$	$\varphi \in (-1, 1)$
Triangular	$(1-\lvert\varphi\rvert)I(\lvert\varphi\rvert<1)$	$\varphi \in (-1, 1)$
Uniform or rectangular	$\frac{1}{2}I(\lvert\varphi\rvert<1)$	$\varphi \in (-1, 1)$

The weighted average estimator or kernel method

Properties: Bias

- The kernel estimator is generally biased.
- The approximate expression for the bias is given by:

$$bias[\hat{f}(x_o)] \approx \frac{h^2}{2} f''(x_o) \int_{-\infty}^{\infty} K(\phi)\phi^2 d\phi \tag{8}$$

- The approximate expression for the asymptotic variance of the kernel estimator is given by:

$$variance[\hat{f}(x_o)] \approx \frac{1}{n \times h} f(x_o) \int_{-\infty}^{\infty} K^2(\phi) d\phi \tag{9}$$

The weighted average estimator or kernel method

Bandwidth Selection

$$MSE(\hat{f}(x)) = Var(\hat{f}(x)) + \left(bias(\hat{f}(x))\right)^2 \tag{10}$$

The weighted average estimator or kernel method

Bandwidth Selection

$$MSE(\hat{f}(x)) = Var(\hat{f}(x)) + \left(bias(\hat{f}(x))\right)^2 \tag{10}$$

- We obtain a bandwidth which globally balances bias and variance by minimizing MSE with respect to h, i.e.,

$$h_{opt} = \left(\frac{\int K^2(z)dz}{(\int z^2K(z)dz)^2 \int f'(x)^2 dx}\right)^{-1/5} n^{-1/5} \tag{11}$$

The weighted average estimator or kernel method

Bandwidth Selection

- There are two popular approaches to bandwidth selection,
  1. Rules-of-thumb,

The weighted average estimator or kernel method

Rule-of-Thumb

- The rule-of-thumb for choosing the bandwidth makes assumtions about f and K
- For example: under a gaussian density and kernel

$$h_{opt} = 1.059\sigma n^{-1/5} \tag{12}$$

- There are two popular approaches to bandwidth selection,
  1. Rules-of-thumb,
  2. cross-validation methods,

Agenda

Duranton and Overman

1. **Select Relevant Establishments:**
   - Choose establishments based on industry and size.
   - Consider different thresholds to assess robustness (e.g., include only those contributing to 90% of employment).
2. **Compute Bilateral Distances:**
   - Calculate Euclidean distances between all pairs of establishments.
   - Use Kernel Density Estimation (KDE) to estimate the density of these distances.

Duranton and Overman

Density estimates

$$\hat{K}(d) = \frac{1}{n(n-1)h} \sum_{i=1}^{n-1} \sum_{j=1+1}^{n} K\left(\frac{d - d_{i,j}}{h}\right) \tag{13}$$

- $d_{i,j}$ the observed distance between establishments i and j
- Gaussian kernel
- Section 3.4.2 of Silverman (1986): $h = 1.059\sigma n^{-1/5}$; $h = 0.79IQR\sigma n^{-1/5}$; $h = 0.79min\{\sigma, IQR/1.34\}n^{-1/5}$

Duranton and Overman

3. **Generate Counterfactuals:**
   - Randomly assign establishments to locations, maintaining the number of establishments and industrial concentration.
   - Create 1,000 simulations to construct a baseline for comparison.

(a) Basic Pharmaceuticals (SIC2441)

(b) Pharmaceutical Preparations (SIC2442)

Duranton and Overman

4 Statistical Significance and Localization Metrics

Local confidence intervals are constructed for each distance d by comparing the observed K-density, $\hat{K}(d)$, with the distribution of simulated K-densities from random assignments.

$$\bar{K}_A(d) = \text{5th percentile of } \hat{K}_{\text{sim}}(d), \tag{14}$$

$$\underline{\mathrm{K}}_A(d) = \text{95th percentile of } \hat{K}_{\text{sim}}(d), \tag{15}$$

Interpretation:

- If $\hat{K}(d) > \bar{K}_A(d)$, it indicates significant **localization** at distance d.
- If $\hat{K}(d) < \underline{\mathrm{K}}_A(d)$, it indicates significant **dispersion** at distance d.

Duranton and Overman

4 **Statistical Significance and Localization Metrics**

Local confidence intervals are constructed for each distance d by comparing the observed K-density, $\hat{K}(d)$, with the distribution of simulated K-densities from random assignments.
We can also define an index of localization:

$$\gamma_A(d) \equiv \max\left(\hat{K}_A(d) - \bar{K}_A(d), 0\right), \tag{16}$$

as well as an index of dispersion:

$$\psi_A(d) \equiv \max\left(\bar{K}_A(d) - \hat{K}_A(d), 0\right). \tag{17}$$

Interpretation:

- To reject the hypothesis of randomness at distance d because of localization (dispersion), we only need $\gamma_A(d) > 0$ ($\psi_A(d) > 0$).
- The exact value of these two indices does not matter. However, the indices do indicate how much localization and dispersion there is at any level of distance.

Duranton and Overman

4 **Statistical Significance and Localization Metrics**

Global confidence bands to make statements about the overall location patterns of an industry.

- Denote $\bar{\bar{K}}_A(d)$ **the upper confidence band of industry** A.
- This band is hit by 5% of our simulations between 0 and 180 km.
- If

$$\hat{K}_A(d) > \bar{\bar{K}}_A(d)$$

for at least one $d \in [0, 180]$, this industry is said to exhibit **global localization** (at a 5% confidence level).

Duranton and Overman

4 **Statistical Significance and Localization Metrics**

Global confidence bands to make statements about the overall location patterns of an industry.

- An industry which is very localized at short distances can show dispersion at larger distances.
- Then an industry is said to exhibit **global dispersion** (at a 5% confidence level) when

$$\hat{K}_A(d) < \underline{\underline{K}}_A(d)$$

for at least one $d \in [0, 180]$ **and** the industry does not exhibit localization.

Duranton and Overman

4 **Statistical Significance and Localization Metrics**

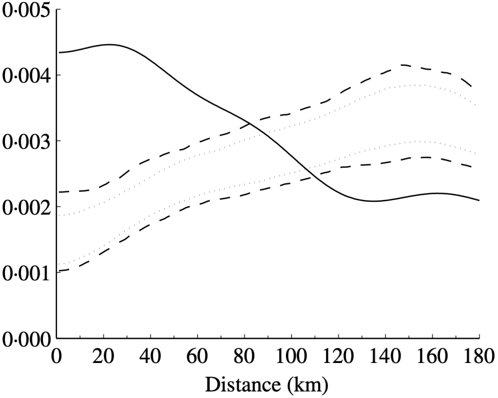
We can also define an index of global localization:

$$\gamma_A(d) \equiv \max\left(\hat{K}_A(d) - \bar{\bar{K}}_A(d), 0\right), \tag{18}$$

as well as an index of global dispersion:

$$\psi_A(d) \equiv \begin{cases} \max\left(\underline{\underline{K}}_A(d) - \hat{K}_A(d), 0\right) \text{ if } \sum_{d=0}^{d=180} = 0 \\ 0 \text{ o.w.} \end{cases} \tag{19}$$

Spatial Concentration



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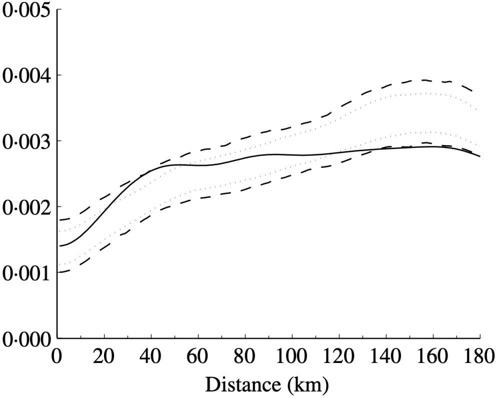

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