Rosen-Roback Framework

Urban Economics

Ignacio Sarmiento-Barbieri

Universidad de los Andes

September 18, 2024

Rosen-Roback

- The Rosen-Roback model Roback (1982) is a cousin of the monocentric city model that is particularly useful for comparing one location to another.
  - For example, we expect that climate change will affect the attractiveness, and maybe the productivity of locations differently.
  - Can we infer these values from cross-location differences in rent, wages, and climate?

Rosen-Roback

Set up

- 3 Sectors:
  - Consumers of Housing (homogeneous)
  - The production sector
  - The construction sector
- Assumption cities are small, and exogenous amount of land $\bar{L}$ in each city

Rosen Roback Framework

Spatial equilibrium

> *The high mobility of labor leads urban economists to assume a spatial equilibrium, where elevated New York incomes do not imply that New Yorkers are better off. Instead, welfare levels are equalized across space and high incomes are offset by negative urban attributes such as high prices or low amenities.*

Glaeser and Gottlieb (JEL 2009)

Rosen Roback Framework

Three Simultaneous Equilibria

- Individuals are optimally choosing which city to live in
  - There is a group of homogeneous individuals
  - Some of them are living in different cities
  - Their utility level is the same in all those cities
- Firms earn zero expected profits
  - Free entry of firms
  - Firm profits are equalized across cities
- The construction sector operates optimally
  - Free entry, zero profit for builders
  - Construction profits are equalized across cities

Rosen Roback Framework

Housing consumption

$$maxU(C,H) = \theta C^{1-\alpha} H^{\alpha} \tag{1}$$

$$st \tag{2}$$

$$W = C + p_H H \tag{3}$$

For spatial equilibrium to hold, the indirect utility must equal a reservation utility level $\bar{u}$

$$\bar{u} = \theta \alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \frac{W}{p_H^{\alpha}} \tag{4}$$

Rosen Roback Framework

Production Sector

Cobb-Douglas production function with constant returns to scale:

$$y = AN^{\beta}K^{\gamma}\bar{Z}^{\zeta} \tag{5}$$

$$st \tag{6}$$

$$WN + p_k K + p_z Z \tag{7}$$

$\beta + \gamma + \zeta = 1$

The competitive wage in each city is

$$W = \beta \left(\left(\frac{\gamma}{p_k} \right)^{\gamma} A \left(\frac{\bar{Z}}{N} \right)^{\zeta} \right)^{\frac{1}{1-\gamma}} \tag{8}$$

Rosen Roback Framework

Construction sector

- Housing supply is the product of land L, (here exogoenous) and building height h
- Height is built with tradable capital at a convex cost

$$\varphi p_K \left(\frac{h^{\delta}}{\delta} \right) \tag{9}$$

for $\varphi > 0$ and $\delta > 1$

Free entry to developers

$$max \left\{ p_H h - \varphi p_k \left(\frac{h}{\delta} \right)^{\delta} \right\}$$

$$h^* = \delta \left(\frac{p_H}{\varphi p_k} \right)^{\frac{1}{\delta - 1}}$$

Rosen Roback Framework

Three Simultaneous Equilibria

- Individual optimal location choice

$$\bar{u} = \theta\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\frac{W}{p_H^{\alpha}}$$

- Firms labor demand

$$W = \beta\left(\left(\frac{\gamma}{p_k}\right)^{\gamma} A\left(\frac{\bar{Z}}{N}\right)^{\zeta}\right)^{\frac{1}{1-\gamma}} \tag{10}$$

- Housing market equilibrium

$$p_H = \left[\varphi p_k\left(\frac{\alpha}{\delta}\frac{WN}{\bar{L}}\right)^{\delta-1}\right]^{\frac{1}{\delta}}$$

Rosen Roback Framework

Three Simultaneous Equilibria

- Individual optimal Location choice

$$logW - \alpha log(p_H) + log\theta = log(\bar{u}) + k_1$$

- Firms labor demand

$$(1-\gamma)logW + \zeta(logN - log\bar{Z}) - logA = k_2 - \gamma logp_k$$

- Housing Market equilibrium

$$\delta logp_H - (\delta - 1)(logW + logN - log\bar{L}) - log\varphi = logp_k + k_3$$

Rosen Roback Framework

Three Simultaneous Equilibria

Endogenous variables

1. N
2. w
3. p_H

Exogenous parameters

1. θ
2. $\tilde{A} = A\bar{Z}^{\zeta}$
3. $\tilde{L} = \bar{L}\varphi^{\frac{-1}{(\delta-1)}}$
4. $\bar{u}$
5. p_k

Rosen Roback Framework

Solve

With the definitions, rearrange (2)

$$(1-\gamma)\log(W)+\zeta\log(N)=k_2'+\log(\tilde{A}) \tag{11}$$

rearrange (3)

$$\delta\log(p_H)-(\delta-1)\log(W)-(\delta-1)\log(N)=k_3'-(\delta-1)\log(\tilde{L}) \tag{12}$$

rearrange (1)

$$-\alpha\log(p_H)+\log(W)=k_1'-\log(\theta) \tag{13}$$

Rosen Roback Framework

Solve

$$\begin{pmatrix} 0 & (1-\gamma) & \zeta \\ \delta & -(\delta-1) & -(\delta-1) \\ -\alpha & 1 & 0 \end{pmatrix} \begin{pmatrix} \log(p_H) \\ \log(W) \\ \log(N) \end{pmatrix} = \begin{pmatrix} k_3' + \log(\tilde{A}) \\ k_2' - (\delta-1)\log(\tilde{L}) \\ k_1' - \log(\theta) \end{pmatrix} \tag{14}$$

Rosen Roback Framework

Equilibrium Solution

1. Equilibrium wages

$$logW = k_w + \frac{(\delta-1)\alpha(log\tilde{A} - \zeta log\tilde{L}) - \delta\zeta log\theta}{\beta(\delta-1)\alpha + \delta\zeta}$$

2. Equilibrium housing prices

$$logp_H = k_p + \frac{(\delta-1)(log\tilde{A} + \beta log\theta - \zeta log\tilde{L})}{\beta(\delta-1)\alpha + \delta\zeta}$$

3. Equilibrium population

$$logN = k_N + \frac{[\delta(1-\alpha)+\alpha]log\tilde{A} + (\beta+\zeta)[\delta log\theta + (\delta-1)\alpha log\tilde{L}]}{\beta(\delta-1)\alpha + \delta\zeta}$$

$$\log\left(\frac{N}{\bar{L}}\right) = k_N + \frac{(\delta(1-\alpha)+\alpha)\left(\log(\tilde{A}) + \zeta\log(\bar{L})\right) + (\beta+\zeta)\left(\delta\log(\theta) - \alpha\log(\psi)\right)}{\alpha(\delta-1)\beta + \zeta\delta}$$

Rosen Roback Framework

As an example of how this framework can be used, we can look to the predictions of the model when we change an exogenous variable X

$$log\theta = k_\theta + \xi_\theta X + \epsilon_\theta$$

$$log\tilde{A} = k_A + \xi_A X + \epsilon_A$$

$$log\tilde{L} = k_L + \xi_L X + \epsilon_L$$

Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- In the US, fastest growing areas have warm climates, something similar in Europe (what about Latam?)
- These areas, in the south and west of US, are known as the "sunbelt"
- The growth of the Sunbelt, which is among the most striking, studied, and debated trends in regional economics over the last fifty years

Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- If we look across metropolitan areas, the relationship between January temperature and size is:

$$log(Population2000) = \underset{(0.2)}{12.2} + \underset{(0.005)}{0.017} JanuaryTemp$$

- and with growth

$$\log\left(\frac{\text{Population 2000}}{\text{Population 1990}}\right) = 0.016 + 0.003 \times \text{January Temperature}$$

Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

Why has population growth shifted to sunbelt?

1. Changes in amenities? E.g the advent of air conditioning made South more comfortable (Borts and Stein, 1964; Mueser and Graves, 1995)
2. Has productivity increased ? Barro and Sala-i-Martin (1991) and Caselli and Coleman (2001)
3. Has land availability increased? Are people attracted to cheap housing, made possible by pro-building policies?

Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- To distinguish between the different explanations, we can use the above framework.
- If X is January temperature, we want to see the effect of the Sunbelt (warm winters) on $\theta, \tilde{A}, \tilde{L}$

$$log\theta = k_\theta + \xi_\theta X + \epsilon_\theta$$

$$log\tilde{A} = k_A + \xi_A X + \epsilon_A$$

$$log\tilde{L} = k_L + \xi_L X + \epsilon_L$$

Rosen Roback Framework

If we replace in th equilibrium conditions, we have reduced forms

$$logw = k_w + \xi_w X + \epsilon_w$$

$$logp_H = k_p + \xi_p X + \epsilon_p$$

$$logN = k_N + \xi_N X + \epsilon_N$$

where

$$\xi_w = \frac{(\delta-1)\alpha(\xi_A - \zeta\xi_L) - \delta\zeta\xi_\theta}{\beta(\delta-1)\alpha + \delta\zeta}$$

$$\xi_p = \frac{(\delta-1)(\xi_A + \beta\xi_\theta - \zeta\xi_L)}{\beta(\delta-1)\alpha + \delta\zeta}$$

$$\xi_N = \frac{[\delta(1-\alpha)+\alpha]\xi_A + (\beta+\zeta)[\delta\xi_\theta + (\delta-1)\alpha\xi_L]}{\beta(\delta-1)\alpha + \delta\zeta}$$

Rosen Roback Framework

Solving and inverting we get

$$\xi_\theta = \alpha\xi_p - \xi_w$$

$$\xi_A = \zeta\xi_N + (1-\gamma)\xi_w$$

$$\xi_L = \xi_N + \xi_W - \frac{\delta}{\delta - 1}\xi_p$$

Example Spatial Equilibrium

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

$$\underset{}{log(Population2000)} = \underset{(0.2)}{12.2} + \underset{(0.005)}{0.017}JanuaryTemp$$

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)
Dependent variable	Log wage	Log house value
Year:	2000	2000
Mean January temperature	−0.19	0.60
	[0.06]	[0.31]
Mean January temperature × year 2000		
Year 2000 dummy		
Individual controls	Yes	—
Housing controls	—	Yes
MSA fixed effects	—	—
N	1,590,467	2,341,976
R^2	0.29	0.36

Example Spatial Equilibrium

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

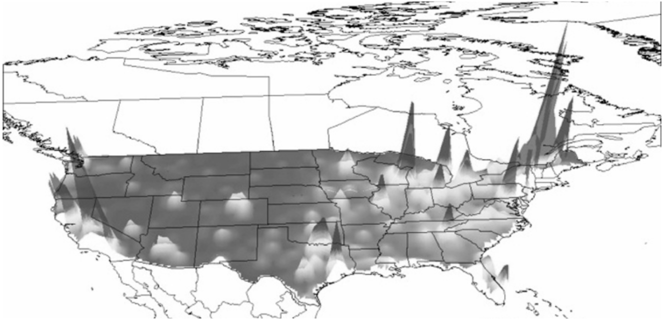
Changes

$$log(\frac{Population2000}{Population1990}) = \underset{(0.14)}{0.016} + \underset{(0.0004)}{0.003}\ JanuaryTemp$$

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable	Log wage	Log house value		Log wage	Log house value	
Year:	2000	2000		1990, 2000	1990, 2000	
Mean January temperature	−0.19	0.60				
	[0.06]	[0.31]				
Mean January temperature × year 2000				−0.001	−0.43	
				[0.05]	[0.11]	
Year 2000 dummy				0.25	0.62	
				[0.02]	[0.06]	
Individual controls	Yes	—		Yes	—	
Housing controls	—	Yes		—	Yes	
MSA fixed effects	—	—		Yes	Yes	
N	1,590,467	2,341,976		2,950,850	4,245,315	
R^2	0.29	0.36		0.27	0.60	

Agglomeration Economies



Agglomeration Economies

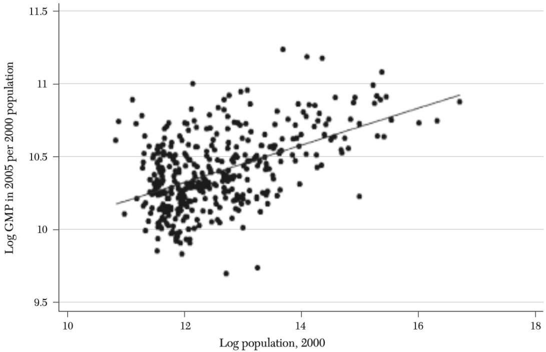


Figure 1. Productivity and City Size

Notes: Units of observation are Metropolitan Statistical Areas under the 2006 definitions. Population is from the Census, as described in the Data Appendix. Gross Metropolitan Product is from the Bureau of Economic Analysis.

The regression line is log *GMP per capita* $= 0.13$ [0.01] $\times$ log *population* $+ 8.8$ [0.1]. $R^2 = 0.25$ and $N = 363$.

Agglomeration Economies

- Why do we see such a remarkable clustering of human activity in a small number of urban areas?
- Model above: cities may form because some places have innate advantages in productivity, housing supply or amenities.
- Or it may be because clusters of people endogenously increase productivity, housing supply or amenities (agglomeration effects)

Evidence of Agglomeration Economies

- Three strategies to identify agglomeration economies

  1. Show there is too much spatial concentration to be random (Duranton and Overman, 2005)
  2. Compare productivity over space (Greenstone, 2010)
  3. Compare wages and rents across space (Quantitative Spatial Models, Ahlfeldt et al, 2015)

Spatial Concentration

Extremes of Localization and Dispersion

(c) Other Agricultural and Forestry Machinery (SIC2932)

(d) Machinery for Textile, Apparel and Leather Production (SIC2954)

Spatial Concentration

Ambiguous Cases

(a) Basic Pharmaceuticals
(SIC2441)

(b) Pharmaceutical Preparations
(SIC2442)

Most Localized

sic92	Industry	Γ or Ψ
Most localised		
2214	Publishing of Sound Recordings	0.470
1711	Preparation and Spinning of Cotton-type Fibres	0.411
2231	Reproduction of Sound Recordings	0.403
1760	Manufacture of Knitted and Crocheted Fabrics	0.321
1713	Preparation and Spinning of Worsted-type Fibres	0.319
2861	Manufacture of Cutlery	0.314
1771	Manufacture of Knitted and Crocheted Hosiery	0.290
1810	Manufacture of Leather Clothes	0.203
1822	Manufacture of Other Outerwear	0.181
2211	Publishing of Books	0.178

Most Dispersed

Most dispersed		
1520	Processing and Preserving of Fish and Fish Products	0.200
3511	Building and Repairing of Ships	0.113
1581	Manufacture of Bread, Fresh Pastry Goods and Cakes	0.094
2010	Saw Milling and Planing of Wood, Impregnation of Wood	0.082
2932	Other Agricultural and Forestry Machinery	0.067
1551	Operation of Dairies and Cheese Making	0.064
1752	Manufacture of Cordage, Rope, Twine and Netting	0.062
3615	Manufacture of Mattresses	0.050
1571	Manufacture of Prepared Feeds for Farm Animals	0.049
2030	Manufacture of Builders' Carpentry and Joinery	0.047

Measuring Agglomeration Economies Through Productivity

- The most direct approach
  - Measure productivity from output, then relate it to density

Measuring Agglomeration Economies Through Productivity

- The most direct approach
  - Measure productivity from output, then relate it to density
- Problems with this approach?

Measuring Agglomeration Economies Through Productivity

- The most direct approach
  - Measure productivity from output, then relate it to density
- Problems with this approach?
  - Natural advantages make a region more productive
  - Greater productivity attracts workers and firms

Measuring Agglomeration Economies Through Productivity

- The most direct approach
  - Measure productivity from output, then relate it to density
- Problems with this approach?
  - Natural advantages make a region more productive
  - Greater productivity attracts workers and firms
- Can you think on an ideal experiment?

MDP Greenstone, Hornbeck, and Moretti (2010)

- Use new plant openings, and compare results of those counties where plants opened up vs those that didn't
- What is the model?
  - New plants choose their location to maximize profits

MDP Greenstone, Hornbeck, and Moretti (2010)

- Use new plant openings, and compare results of those counties where plants opened up vs those that didn't
- What is the model?
  - New plants choose their location to maximize profits
  - Places without new plants are not a valid control group

MDP Greenstone, Hornbeck, and Moretti (2010)

- Regular feature in the corporate real estate journal Site Selection Stories about the location choice of large new plants
- Gradual narrowing down of potential counties to 2 or 3 finalists
- The 1 or 2 losers in the shortlist provide a control group
  - Almost as attractive as the winning county
  - Yet, they did not receive the treatment

MDP Greenstone, Hornbeck, and Moretti (2010)

- Plant-level regression

$$log(Y) = log(A) + \beta_1 \log(L) + \beta_2 \log(K_B) + \beta_3 \log(K_E) + \beta_4 \log(M) \tag{15}$$

- where

$$log(A) = \delta_1 Winner + \delta_2 Post + \delta_3 Winner \times Post \tag{16}$$

MDP Greenstone, Hornbeck, and Moretti (2010)