Rosen-Roback Framework

Urban Economics

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Rosen-Roback

- The Rosen-Roback model Roback (1982) is a cousin of the monocentric city model that is particularly useful for comparing one location to another.
  - For example, we expect that climate change will affect the attractiveness, and maybe the productivity of locations differently.
  - Can we infer these values from cross-location differences in rent, wages, and climate?

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Set up

- 3 Sectors:
  - Consumers of Housing (homogeneous)
  - The production sector
  - The construction sector
- Assumption cities are small, and exogenous amount of land $\bar{L}$ in each city

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Spatial equilibrium

The high mobility of labor leads urban economists to assume a spatial equilibrium, where elevated New York incomes do not imply that New Yorkers are better off. Instead, welfare levels are equalized across space and high incomes are offset by negative urban attributes such as high prices or low amenities.

Glaeser and Gottlieb (JEL 2009)

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Three Simultaneous Equilibria

- Individuals are optimally choosing which city to live in
  - There is a group of homogeneous individuals
  - Some of them are living in different cities
  - Their utility level is the same in all those cities
- Firms earn zero expected profits
  - Free entry of firms
  - Firm profits are equalized across cities
- The construction sector operates optimally
  - Free entry, zero profit for builders
  - Construction profits are equalized across cities

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Housing consumption

$$maxU(C,H) = \theta C^{1-\alpha} H^{\alpha} \tag{1}$$

$$st \tag{2}$$

$$W = C + p_H H \tag{3}$$

For spatial equilibrium to hold, the indirect utility must equal a reservation utility level $\bar{u}$

$$\bar{u} = \theta \alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \frac{W}{p_H^{\alpha}} \tag{4}$$

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Production Sector

Cobb-Douglas production function with constant returns to scale:

$$y = AN^{\beta}K^{\gamma}\bar{Z}^{\zeta} \tag{5}$$

$$st \tag{6}$$

$$WN + p_k K + p_z Z \tag{7}$$

$\beta + \gamma + \zeta = 1$

The competitive wage in each city is

$$W = \beta\left(\left(\frac{\gamma}{p_k}\right)^{\gamma} A\left(\frac{\bar{Z}}{N}\right)^{\zeta}\right)^{\frac{1}{1-\gamma}} \tag{8}$$

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Construction sector

- Housing supply is the product of land L, (here exogoenous) and building height h
- Height is built with tradable capital at a convex cost

$$\varphi p_K \left(\frac{h}{\delta}^{\delta} \right) \tag{9}$$

for $\varphi > 0$ and $\delta > 1$

Free entry to developers

$$max \left\{ p_H h - \varphi p_k \left(\frac{h}{\delta} \right)^{\delta} \right\}$$

$$h^* = \delta \left(\frac{p_H}{\varphi p_k} \right)^{\frac{1}{\delta - 1}}$$

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Three Simultaneous Equilibria

- Individual optimal location choice

$$\bar{u} = \theta\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\frac{W}{p_H^{\alpha}}$$

- Firms labor demand

$$W = \beta\left(\left(\frac{\gamma}{p_k}\right)^{\gamma} A\left(\frac{\bar{Z}}{N}\right)^{\zeta}\right)^{\frac{1}{1-\gamma}} \tag{10}$$

- Housing market equilibrium

$$p_H = \left[\varphi p_k\left(\frac{\alpha}{\delta}\frac{WN}{\bar{L}}\right)^{\delta-1}\right]^{\frac{1}{\delta}}$$

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Three Simultaneous Equilibria

- Individual optimal Location choice

$$logW - \alpha log(p_H) + log\theta = log(\bar{u}) + k_1$$

- Firms labor demand

$$(1-\gamma)logW + \zeta(logN - log\bar{Z}) - logA = k_2 - \gamma logp_k$$

- Housing Market equilibrium

$$\delta logp_H - (\delta - 1)(logW + logN - log\bar{L}) - log\varphi = logp_k + k_3$$

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Three Simultaneous Equilibria

Endogenous variables

1. N
2. w
3. p_H

Exogenous parameters

1. θ
2. $\tilde{A} = A\bar{Z}^{\zeta}$
3. $\tilde{L} = \bar{L}\varphi^{\frac{-1}{(\delta-1)}}$
4. $\bar{u}$
5. p_k

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Solve

With the definitions, rearrange (2)

$$(1-\gamma)\log(W)+\zeta\log(N)=k_2'+\log(\tilde{A}) \tag{11}$$

rearrange (3)

$$\delta\log(p_H)-(\delta-1)\log(W)-(\delta-1)\log(N)=k_3'-(\delta-1)\log(\tilde{L}) \tag{12}$$

rearrange (1)

$$-\alpha\log(p_H)+\log(W)=k_1'-\log(\theta) \tag{13}$$

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Solve

$$\begin{pmatrix} 0 & (1-\gamma) & \zeta \\ \delta & -(\delta-1) & -(\delta-1) \\ -\alpha & 1 & 0 \end{pmatrix} \begin{pmatrix} \log(p_H) \\ \log(W) \\ \log(N) \end{pmatrix} = \begin{pmatrix} k_3' + \log(\tilde{A}) \\ k_2' - (\delta-1)\log(\tilde{L}) \\ k_1' - \log(\theta) \end{pmatrix} \tag{14}$$

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Equilibrium Solution

1. Equilibrium wages

$$logW = k_w + \frac{(\delta-1)\alpha(log\tilde{A} - \zeta log\tilde{L}) - \delta\zeta log\theta}{\beta(\delta-1)\alpha + \delta\zeta}$$

2. Equilibrium housing prices

$$logp_H = k_p + \frac{(\delta-1)(log\tilde{A} + \beta log\theta - \zeta log\tilde{L})}{\beta(\delta-1)\alpha + \delta\zeta}$$

3. Equilibrium population

$$logN = k_N + \frac{[\delta(1-\alpha)+\alpha]log\tilde{A} + (\beta+\zeta)[\delta log\theta + (\delta-1)\alpha log\tilde{L}]}{\beta(\delta-1)\alpha + \delta\zeta}$$

$$\log\left(\frac{N}{\bar{L}}\right) = k_N + \frac{(\delta(1-\alpha)+\alpha)\left(\log(\tilde{A}) + \zeta\log(\bar{L})\right) + (\beta+\zeta)\left(\delta\log(\theta) - \alpha\log(\psi)\right)}{\alpha(\delta-1)\beta + \zeta\delta}$$

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As an example of how this framework can be used, we can look to the predictions of the model when we change an exogenous variable X

$$log\theta = k_\theta + \xi_\theta X + \epsilon_\theta$$

$$log\tilde{A} = k_A + \xi_A X + \epsilon_A$$

$$log\tilde{L} = k_L + \xi_L X + \epsilon_L$$

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An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- In the US, fastest growing areas have warm climates, something similar in Europe (what about Latam?)
- These areas, in the south and west of US, are known as the "sunbelt"
- The growth of the Sunbelt, which is among the most striking, studied, and debated trends in regional economics over the last fifty years

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An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- If we look across metropolitan areas, the relationship between January temperature and size is:

$$log(Population2000) = \underset{(0.2)}{12.2} + \underset{(0.005)}{0.017} JanuaryTemp$$

- and with growth

$$\log\left(\frac{\text{Population 2000}}{\text{Population 1990}}\right) = 0.016 + 0.003 \times \text{January Temperature}$$

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An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

Why has population growth shifted to sunbelt?

1. Changes in amenities? E.g the advent of air conditioning made South more comfortable (Borts and Stein, 1964; Mueser and Graves, 1995)
2. Has productivity increased ? Barro and Sala-i-Martin (1991) and Caselli and Coleman (2001)
3. Has land availability increased? Are people attracted to cheap housing, made possible by pro-building policies?

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An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- To distinguish between the different explanations, we can use the above framework.
- If X is January temperature, we want to see the effect of the Sunbelt (warm winters) on $\theta, \tilde{A}, \tilde{L}$

$$log\theta = k_\theta + \xi_\theta X + \epsilon_\theta$$

$$log\tilde{A} = k_A + \xi_A X + \epsilon_A$$

$$log\tilde{L} = k_L + \xi_L X + \epsilon_L$$

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If we replace in th equilibrium conditions, we have reduced forms

$$logw = k_w + \xi_w X + \epsilon_w$$

$$logp_H = k_p + \xi_p X + \epsilon_p$$

$$logN = k_N + \xi_N X + \epsilon_N$$

where

$$\xi_w = \frac{(\delta-1)\alpha(\xi_A - \zeta\xi_L) - \delta\zeta\xi_\theta}{\beta(\delta-1)\alpha + \delta\zeta}$$

$$\xi_p = \frac{(\delta-1)(\xi_A + \beta\xi_\theta - \zeta\xi_L)}{\beta(\delta-1)\alpha + \delta\zeta}$$

$$\xi_N = \frac{[\delta(1-\alpha)+\alpha]\xi_A + (\beta+\zeta)[\delta\xi_\theta + (\delta-1)\alpha\xi_L]}{\beta(\delta-1)\alpha + \delta\zeta}$$

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Solving and inverting we get

$$\xi_\theta = \alpha\xi_p - \xi_w$$

$$\xi_A = \zeta\xi_N + (1-\gamma)\xi_w$$

$$\xi_L = \xi_N + \xi_W - \frac{\delta}{\delta - 1}\xi_p$$

Example Spatial Equilibrium

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

$$\log(Population2000) = \underset{(0.2)}{12.2} + \underset{(0.005)}{0.017} JanuaryTemp$$

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)
Dependent variable	Log wage	Log house value
Year:	2000	2000
Mean January temperature	−0.19	0.60
	[0.06]	[0.31]
Mean January temperature × year 2000		
Year 2000 dummy		
Individual controls	Yes	—
Housing controls	—	Yes
MSA fixed effects	—	—
N	1,590,467	2,341,976
R^2	0.29	0.36

Example Spatial Equilibrium

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

Changes

$$log(\frac{Population2000}{Population1990}) = \underset{(0.14)}{0.016} + \underset{(0.0004)}{0.003}\ JanuaryTemp$$

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable	Log wage	Log house value		Log wage	Log house value	
Year:	2000	2000		1990, 2000	1990, 2000	
Mean January temperature	−0.19	0.60				
	[0.06]	[0.31]				
Mean January temperature × year 2000				−0.001	−0.43	
				[0.05]	[0.11]	
Year 2000 dummy				0.25	0.62	
				[0.02]	[0.06]	
Individual controls	Yes	—		Yes	—	
Housing controls	—	Yes		—	Yes	
MSA fixed effects	—	—		Yes	Yes	
N	1,590,467	2,341,976		2,950,850	4,245,315	
R^2	0.29	0.36		0.27	0.60	

The Value of Climate Amenities: Hedonic vs Discrete Choice

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The value of climate amenities: A comparison of hedonic and discrete choice approaches

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The Hedonic Framework - Roback (1982)

$$MWTP_a \equiv \frac{V_a}{V_W} = H\frac{dr}{da} - \frac{dW}{da} \text{ and } \frac{MWTP_a}{W} \equiv \frac{V_a}{V_W}\frac{1}{W}$$

$$\ln w_{mj} = \gamma^0 + \boldsymbol{X}^w_{mj}\boldsymbol{\Gamma}^{X,0} + \boldsymbol{A}_j\Gamma^{A,0} + \nu^0_{mj}$$

$$\ln P_{ij} = \delta^0 + \boldsymbol{X}^P_{ij}\boldsymbol{\Delta}^{X,0} + \boldsymbol{A}_j\boldsymbol{\Delta}^{A,0} + \eta^0_{ij}$$

Albouy's Modification

$$\frac{MWTP_a}{\bar{m}} \equiv \frac{\partial QOL_j}{\partial a} = \left(s_H + \gamma s_O\right)\frac{d\ln(p_{j,H})}{da} - (1-\tau)s_w \frac{d\ln(w_j)}{da}$$

$$\ln w_{mj} = \boldsymbol{X}^w_{mj}\boldsymbol{\Gamma}^{X,1} + \lambda^w_j + \nu^1_{mj}$$

$$\ln P_{ij} = \boldsymbol{X}^P_{ij}\boldsymbol{\Delta}^{X,1} + \lambda^P_j + \eta^1_{ij}$$

$$QOL_j \equiv 0.33\lambda^P_j - 0.51\lambda^w_j = \boldsymbol{A}_j\boldsymbol{\theta} + \xi_j$$

Hedonic Estimation Strategy

$$\boldsymbol{QOL}_j = \boldsymbol{A}_j\boldsymbol{\theta} + f\left(\boldsymbol{Z}_j\right) + \xi_j$$

Discrete Choice Framework

$$V_{ij} = \alpha\left(Y_{ij} - P_{ij}\right) + \boldsymbol{A}_j\boldsymbol{\beta}_i + \boldsymbol{MC}_{ij} \ + \ \xi_j + \varepsilon_{ij}.$$

Discrete Choice Estimation

$$V_{ij} = \alpha\left(\hat{Y}_{ij} - \hat{P}_{ij}\right) + WT_j\beta_i^{WT} + ST_j\beta_i^{ST} + MC_{ij} + \delta_j \ + \varepsilon_{ij}$$

$$P(i\ selectes\ j) = \int_{-\infty}^{\infty} \frac{\exp\left(V_{ij}\left(\alpha, \boldsymbol{\beta}_i, \boldsymbol{\pi}\right)\right)}{\sum_k \exp\left(V_{ik}\left(\alpha, \boldsymbol{\beta}_i, \boldsymbol{\pi}\right)\right)} f(\boldsymbol{\beta}|\boldsymbol{\mu}, \boldsymbol{\Sigma})d\beta$$

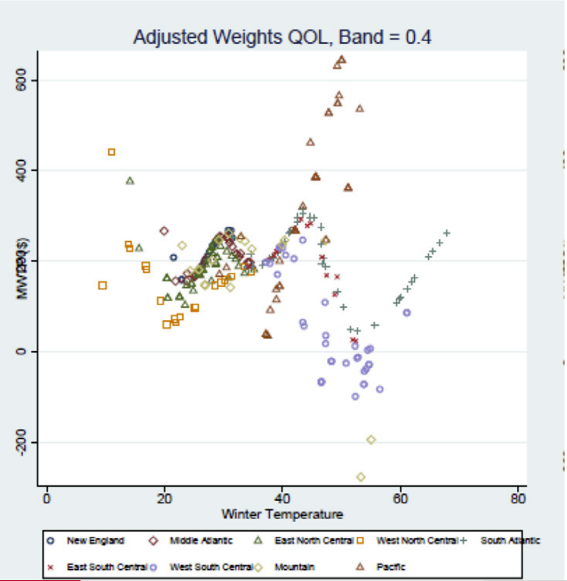
Data

Variable	N	Mean	Std. dev.	Minimum	Maximum	Median
Avg. winter temperature (°F)	284	37.339	12.158	9.442	67.922	34.996
Avg. summer temperature (°F)	284	73.309	5.817	60.848	89.733	72.517
Annual snowfall (inches)	284	20.360	21.366	0.000	84.050	18.050
Summer precipitation (inches)	284	10.966	5.057	0.440	23.300	11.932
July relative humidity (%)	284	66.246	10.891	22.500	78.000	70.500
Annual sunshine (% of possible sunshine in 24 h)	284	60.764	8.323	43.000	78.000	58.000
Avg. elevation (miles)	284	0.197	0.273	0.000	1.620	0.130
Distance to coast (miles)	284	141.096	169.592	0.009	824.451	91.025
Visibility > 10 miles (% of hours)	284	46.053	19.541	5.000	85.500	45.500
Mean $PM_{2.5}$ (micrograms/cubic meter)	284	12.829	2.884	5.382	19.535	12.818
Population density (persons per square mile)	284	471.767	983.041	5.400	13,043.600	259.050
Violent crime rate (number of violent crimes per 1000 persons)	284	4.560	2.214	0.069	12.330	4.349
Park area (square miles)	284	192.908	584.303	0.000	5477.564	24.893
Transportation score	284	50.370	29.181	0.000	100.000	50.280
Education score	284	51.230	29.322	0.000	100.000	51.130
Arts score	284	51.137	29.055	0.000	100.000	51.140
Healthcare score	284	49.201	28.657	0.000	98.300	49.430
Recreation score	284	53.342	28.386	0.000	100.000	54.245

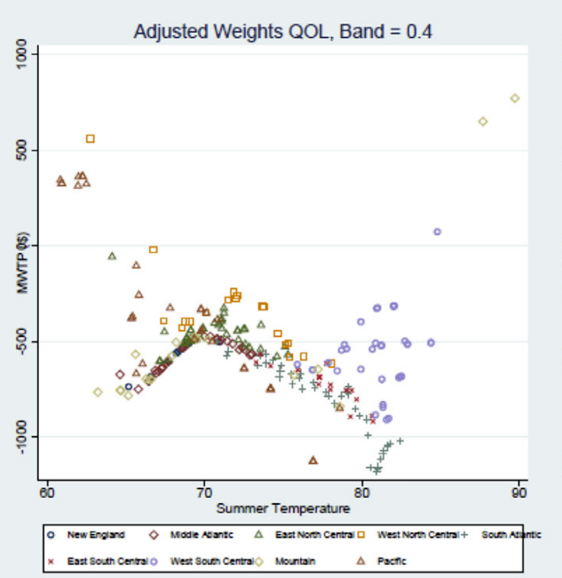
Results - Mean MWTP

	Wage regression (1)	Housing cost regression (2)	QOL regression Traditional Weights (3)	QOL regression Adjusted weights (4)
Variable	Coef. (Std. err.)	Coef. (Std. err.)	Coef. (Std. err.)	Coef. (Std. err.)
Avg. winter temperature	–0.0030	–0.0001	0.0030	0.0015
	(0.0008)	(0.0020)	(0.0006)	(0.0005)
Avg. summer temperature	–0.0010	–0.0172	–0.0033	–0.0052
	(0.0015)	(0.0040)	(0.0010)	(0.0009)
July humidity	–0.0007	0.0020	0.0012	0.0010
	(0.0007)	(0.0016)	(0.0005)	(0.0003)
Annual snowfall	–0.0010	–0.0022	0.0004	–0.0002
	(0.0003)	(0.0007)	(0.0002)	(0.0002)
Ln(summer precipitation)	–0.0247	–0.0475	0.0128	–0.0031
	(0.0111)	(0.0283)	(0.0080)	(0.0067)
Annual sunshine	0.0004	0.0089	0.0019	0.0028
	(0.0009)	(0.0022)	(0.0006)	(0.0005)
No. of obs. (MSAs)	284	284	284	284
Adjusted R-squared	0.71	0.74	0.50	0.59

Results - Mean MWTP



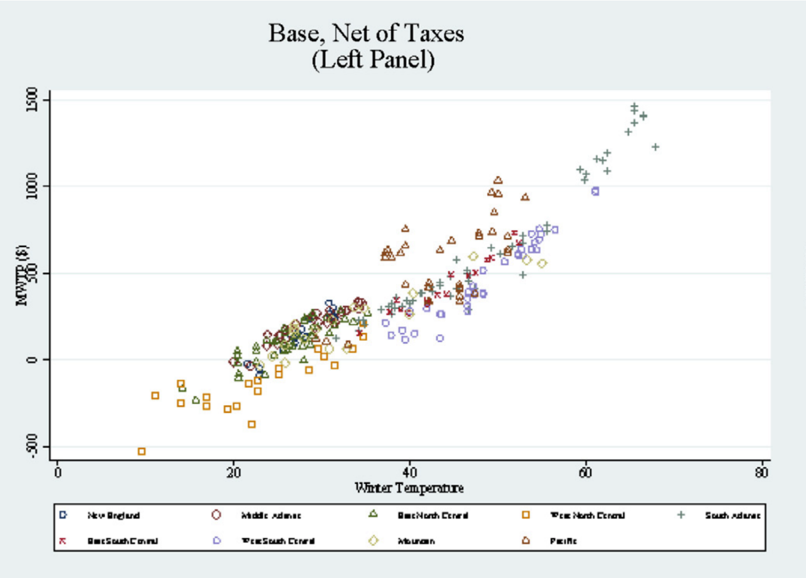
Results - Mean MWTP



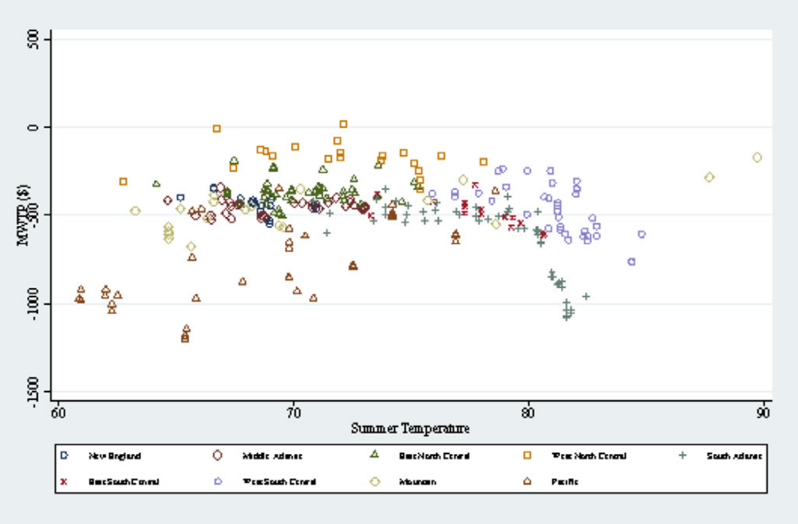
Results - Discrete /choice

	M.1: No tax adjustments		M.2: With tax adjustments		M.3: No tax adjustments + Omit moving costs		M.4: With tax adjustments moving costs	
Panel A: 1st stage estimates								
Variable	Coef. (Std. err.)		Coef. (Std. err.)		Coef. (Std. err.)		Coef. (Std. err.)	
Std. dev.: avg. winter temperature	0.0588		0.0592		0.0011		0.0032	
	(0.0026)		(0.0026)		(0.0128)		(0.0097)	
Std. dev.: avg. summer temperature	0.0592		0.0612		0.0352		0.0525	
	(0.0068)		(0.0066)		(0.0215)		(0.0174)	
Correlation coefficient	–0.6893		–0.6993		0.8614		–0.9433	
	(0.0827)		(0.0776)		(0.2756)		(0.1297)	
Panel B: 2nd stage estimates								
Variable	Coef. (Std. err.)	MWTP (Std. err.)	Coef. (Std. err.)	MWTP (Std. err.)	Coef. (Std. err.)	MWTP (Std. err.)	Coef. (Std. err.)	MWTP (err.)
Mean: avg. winter temperature	0.0209	$518	0.0210	$382	0.0184	$491	0.0171	$326
	(0.0058)	($144)	(0.0057)	($104)	(0.0055)	($146)	(0.0055)	($104)
Mean: avg. summer temperature	–0.0253	–$627	–0.0286	–$522	–0.0145	–$386	–0.0178	–$339
	(0.0100)	($249)	(0.0098)	($180)	(0.0108)	($288)	(0.0110)	($209)

Results - Discrete /choice



Results - Discrete /choice



Results - Discrete /choice

Census region	Northeast		South			Midwest		West		All
Census division	NE	MA	SA	WSC	ESC	ENC	WNC	M	P	All
PANEL A: Baseline Values (1970 to 2000)										
Share of population	5%	15%	19%	11%	3%	17%	4%	6%	19%	100%
ST	69	71	78	81	77	71	71	74	71	74
WT	28	30	48	49	43	27	22	37	47	39
MWTP for ST: Hedonic	(471)	(495)	(675)	(533)	(658)	(478)	(464)	(318)	(573)	(536)
MWTP for ST: Discrete Choice	(460)	(454)	(621)	(475)	(484)	(385)	(211)	(430)	(762)	(522)
MWTP for WT: Hedonic	182	191	169	75	198	181	190	100	234	174
MWTP for WT: Discrete Choice	195	242	636	490	393	141	(80)	303	653	395
PANEL B: Projected Values under SRES Scenarios (2020 to 2050)										
Change in ST (A2)	3.1	3.1	3.0	5.2	4.7	3.6	4.1	3.7	3.4	3.6
Change in WT (A2)	1.9	2.2	2.1	2.2	2.2	2.0	1.9	2.7	1.9	2.1
Change in ST (B1)	2.8	2.5	2.7	5.5	4.3	3.3	3.9	3.7	3.1	3.3
Change in WT (B1)	4.5	5.1	3.1	3.0	3.0	3.7	3.6	2.9	2.0	3.4
WTP (A2): Hedonic	(1096)	(1132)	(1676)	(2592)	(2649)	(1369)	(1557)	(935)	(1560)	(1573)
WTP (A2): Discrete Choice	(1033)	(879)	(436)	(1230)	(1367)	(1084)	(1048)	(952)	(1212)	(969)
WTP (B1): Hedonic	(476)	(277)	(1260)	(2664)	(2257)	(908)	(1199)	(958)	(1412)	(1207)
WTP (B1): Discrete Choice	(381)	102	112	(1077)	(936)	(699)	(1057)	(786)	(909)	(518)