Rosen-Roback Framework

Urban Economics

Ignacio Sarmiento-Barbieri

Universidad de los Andes

September 17, 2025

Rosen-Roback

- The Rosen-Roback model Roback (1982) is a cousin of the monocentric city model that is particularly useful for comparing one location to another.
  - For example, we expect that climate change will affect the attractiveness, and maybe the productivity of locations differently.
  - Can we infer these values from cross-location differences in rent, wages, and climate?

Rosen-Roback

Set up

- 3 Sectors:
  - Consumers of Housing (homogeneous)
  - The production sector
  - The construction sector
- Assumption cities are small, and exogenous amount of land $\bar{L}$ in each city

Rosen Roback Framework

Spatial equilibrium

> *The high mobility of labor leads urban economists to assume a spatial equilibrium, where elevated New York incomes do not imply that New Yorkers are better off. Instead, welfare levels are equalized across space and high incomes are offset by negative urban attributes such as high prices or low amenities.*

Glaeser and Gottlieb (JEL 2009)

Rosen Roback Framework

Three Simultaneous Equilibria

- Individuals are optimally choosing which city to live in
  - There is a group of homogeneous individuals
  - Some of them are living in different cities
  - Their utility level is the same in all those cities
- Firms earn zero expected profits
  - Free entry of firms
  - Firm profits are equalized across cities
- The construction sector operates optimally
  - Free entry, zero profit for builders
  - Construction profits are equalized across cities

Rosen Roback Framework

Housing consumption

$$maxU(C,H) = \theta C^{1-\alpha} H^{\alpha} \tag{1}$$

$$st \tag{2}$$

$$W = C + p_H H \tag{3}$$

For spatial equilibrium to hold, the indirect utility must equal a reservation utility level $\bar{u}$

$$\bar{u} = \theta \alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \frac{W}{p_H^{\alpha}} \tag{4}$$

Rosen Roback Framework

Production Sector

Cobb-Douglas production function with constant returns to scale:

$$y = AN^{\beta}K^{\gamma}\bar{Z}^{\zeta} \tag{5}$$

$$st \tag{6}$$

$$WN + p_k K + p_z Z \tag{7}$$

$\beta + \gamma + \zeta = 1$
The competitive wage in each city is

$$W = \beta\left(\left(\frac{\gamma}{p_k}\right)^{\gamma} A\left(\frac{\bar{Z}}{N}\right)^{\zeta}\right)^{\frac{1}{1-\gamma}} \tag{8}$$

Rosen Roback Framework

Construction sector

- Housing supply is the product of land L, (here exogoenous) and building height h
- Height is built with tradable capital at a convex cost

$$\varphi p_K \left(\frac{h^{\delta}}{\delta}\right) \tag{9}$$

for $\varphi > 0$ and $\delta > 1$

Free entry to developers

$$max \left\{ p_H h - \varphi p_k \left(\frac{h}{\delta}\right)^{\delta} \right\}$$

$$h^* = \delta \left(\frac{p_H}{\varphi p_k}\right)^{\frac{1}{\delta - 1}}$$

Rosen Roback Framework

Three Simultaneous Equilibria

- Individual optimal location choice

$$\bar{u} = \theta \alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \frac{W}{p_H^{\alpha}}$$

- Firms labor demand

$$W = \beta \left(\left(\frac{\gamma}{p_k} \right)^{\gamma} A \left(\frac{\bar{Z}}{N} \right)^{\zeta} \right)^{\frac{1}{1-\gamma}} \tag{10}$$

- Housing market equilibrium

$$p_H = \left[\varphi p_k \left(\frac{\alpha}{\delta} \frac{WN}{\bar{L}} \right)^{\delta-1} \right]^{\frac{1}{\delta}}$$

Rosen Roback Framework

Three Simultaneous Equilibria

- Individual optimal Location choice

$$logW - \alpha log(p_H) + log\theta = log(\bar{u}) + k_1$$

- Firms labor demand

$$(1-\gamma)logW + \zeta(logN - log\bar{Z}) - logA = k_2 - \gamma logp_k$$

- Housing Market equilibrium

$$\delta logp_H - (\delta - 1)(logW + logN - log\bar{L}) - log\varphi = logp_k + k_3$$

Rosen Roback Framework

Three Simultaneous Equilibria

Endogenous variables

1. N
2. w
3. p_H

Exogenous parameters

1. θ
2. $\tilde{A} = A\bar{Z}^{\zeta}$
3. $\tilde{L} = \bar{L}\varphi^{\frac{-1}{(\delta-1)}}$
4. $\bar{u}$
5. p_k

Rosen Roback Framework

Solve

With the definitions, rearrange (2)

$$(1-\gamma)\log(W)+\zeta\log(N)=k_2'+\log(\tilde{A}) \tag{11}$$

rearrange (3)

$$\delta\log(p_H)-(\delta-1)\log(W)-(\delta-1)\log(N)=k_3'-(\delta-1)\log(\tilde{L}) \tag{12}$$

rearrange (1)

$$-\alpha\log(p_H)+\log(W)=k_1'-\log(\theta) \tag{13}$$

Rosen Roback Framework

Solve

$$\begin{pmatrix} 0 & (1-\gamma) & \zeta \\ \delta & -(\delta-1) & -(\delta-1) \\ -\alpha & 1 & 0 \end{pmatrix} \begin{pmatrix} \log(p_H) \\ \log(W) \\ \log(N) \end{pmatrix} = \begin{pmatrix} k_3' + \log(\tilde{A}) \\ k_2' - (\delta-1)\log(\tilde{L}) \\ k_1' - \log(\theta) \end{pmatrix} \tag{14}$$

Rosen Roback Framework

Equilibrium Solution

1. Equilibrium wages

$$logW = k_w + \frac{(\delta - 1)\alpha(log\tilde{A} - \zeta log\tilde{L}) - \delta\zeta log\theta}{\beta(\delta - 1)\alpha + \delta\zeta}$$

2. Equilibrium housing prices

$$logp_H = k_p + \frac{(\delta - 1)(log\tilde{A} + \beta log\theta - \zeta log\tilde{L})}{\beta(\delta - 1)\alpha + \delta\zeta}$$

3. Equilibrium population

$$logN = k_N + \frac{[\delta(1 - \alpha) + \alpha]log\tilde{A} + (\beta + \zeta)[\delta log\theta + (\delta - 1)\alpha log\tilde{L}]}{\beta(\delta - 1)\alpha + \delta\zeta}$$

$$\log\left(\frac{N}{\bar{L}}\right) = k_N + \frac{(\delta(1-\alpha)+\alpha)\left(\log(\tilde{A}) + \zeta\log(\bar{L})\right) + (\beta+\zeta)\left(\delta\log(\theta) - \alpha\log(\psi)\right)}{\alpha(\delta-1)\beta + \zeta\delta}$$

Rosen Roback Framework

As an example of how this framework can be used, we can look to the predictions of the model when we change an exogenous variable X

$$log\theta = k_\theta + \xi_\theta X + \epsilon_\theta$$

$$log\tilde{A} = k_A + \xi_A X + \epsilon_A$$

$$log\tilde{L} = k_L + \xi_L X + \epsilon_L$$

Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- In the US, fastest growing areas have warm climates, something similar in Europe (what about Latam?)
- These areas, in the south and west of US, are known as the “sunbelt”
- The growth of the Sunbelt, which is among the most striking, studied, and debated trends in regional economics over the last fifty years

Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- If we look across metropolitan areas, the relationship between January temperature and size is:

$$log(Population2000) = \underset{(0.2)}{12.2} + \underset{(0.005)}{0.017} JanuaryTemp$$

- and with growth

$$\log\left(\frac{\text{Population 2000}}{\text{Population 1990}}\right) = 0.016 + 0.003 \times \text{January Temperature}$$

Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

Why has population growth shifted to sunbelt?

1. Changes in amenities? E.g the advent of air conditioning made South more comfortable (Borts and Stein, 1964; Mueser and Graves, 1995)
2. Has productivity increased ? Barro and Sala-i-Martin (1991) and Caselli and Coleman (2001)
3. Has land availability increased? Are people attracted to cheap housing, made possible by pro-building policies?

Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- To distinguish between the different explanations, we can use the above framework.
- If X is January temperature, we want to see the effect of the Sunbelt (warm winters) on $\theta, \tilde{A}, \tilde{L}$

$$log\theta = k_\theta + \xi_\theta X + \epsilon_\theta$$

$$log\tilde{A} = k_A + \xi_A X + \epsilon_A$$

$$log\tilde{L} = k_L + \xi_L X + \epsilon_L$$

Rosen Roback Framework

If we replace in th equilibrium conditions, we have reduced forms

$$logw = k_w + \xi_w X + \epsilon_w$$

$$logp_H = k_p + \xi_p X + \epsilon_p$$

$$logN = k_N + \xi_N X + \epsilon_N$$

where

$$\xi_w = \frac{(\delta - 1)\alpha(\xi_A - \zeta\xi_L) - \delta\zeta\xi_\theta}{\beta(\delta - 1)\alpha + \delta\zeta}$$

$$\xi_p = \frac{(\delta - 1)(\xi_A + \beta\xi_\theta - \zeta\xi_L)}{\beta(\delta - 1)\alpha + \delta\zeta}$$

$$\xi_N = \frac{[\delta(1 - \alpha) + \alpha]\xi_A + (\beta + \zeta)[\delta\xi_\theta + (\delta - 1)\alpha\xi_L]}{\beta(\delta - 1)\alpha + \delta\zeta}$$

Rosen Roback Framework

Solving and inverting we get

$$\xi_\theta = \alpha\xi_p - \xi_w$$

$$\xi_A = \zeta\xi_N + (1-\gamma)\xi_w$$

$$\xi_L = \xi_N + \xi_W - \frac{\delta}{\delta - 1}\xi_p$$

Example Spatial Equilibrium

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

$$\log(Population2000) = \underset{(0.2)}{12.2} + \underset{(0.005)}{0.017}\,JanuaryTemp$$

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)
Dependent variable	Log wage	Log house value
Year:	2000	2000
Mean January temperature	−0.19	0.60
	[0.06]	[0.31]
Mean January temperature × year 2000		
Year 2000 dummy		
Individual controls	Yes	—
Housing controls	—	Yes
MSA fixed effects	—	—
N	1,590,467	2,341,976
R^2	0.29	0.36

Example Spatial Equilibrium

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

Changes

$$log(\frac{Population2000}{Population1990}) = \underset{(0.14)}{0.016} + \underset{(0.0004)}{0.003} \; JanuaryTemp$$

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable	Log wage	Log house value		Log wage	Log house value	
Year:	2000	2000		1990, 2000	1990, 2000	
Mean January temperature	−0.19 [0.06]	0.60 [0.31]				
Mean January temperature × year 2000				−0.001 [0.05]	−0.43 [0.11]	
Year 2000 dummy				0.25 [0.02]	0.62 [0.06]	
Individual controls	Yes	—		Yes	—	
Housing controls	—	Yes		—	Yes	
MSA fixed effects	—	—		Yes	Yes	
N	1,590,467	2,341,976		2,950,850	4,245,315	
R^2	0.29	0.36		0.27	0.60	