Agglomeration Economies

Urban Economics

Ignacio Sarmiento-Barbieri

Universidad de los Andes

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Agenda

1. Quantitative Spatial Models
2. Frechet Shocks

Quantitative Spatial Models

- One of the most striking economic features of our world is the uneven distribution of economic activity across space.
- These rich patterns of the concentration of economic activity can be explained by a three-way interaction between natural advantages, agglomeration forces, and dispersion forces.
- The complexity of modeling these forces in spatial equilibrium has meant that the traditional theoretical literature on cities focused on stylized settings, such as a monocentric city or the Rosen Roback framework.
  - These models, however, **cannot** capture the rich internal variation in patterns of economic activity within and between real world locations

Quantitative Spatial (Urban) Models

- Although traditional models in urban economics explain certain features of the data, their simplifying assumptions olimit their usefulness for empirical work.
- These simplifying assumptions abstract from empirically relevant differences in natural advantage across locations, such as access to natural harbors or green parks.
- To address these limitations, recent quantitative spatial (urban) models allow for empirically relevant differences in natural advantage while also incorporating agglomeration forces.
- These models are designed to connect directly to observed data on cities.

Introduction to a basic quantitative spatial model

- Consider a city (or coutry), embedded in a wider economy.
- The city/country consists of a set of discrete locations (blocks/cities).
- These locations are populated by workers, who are mobile between locations and the larger economy.
- Workers have idiosyncratic preferences for living and working in different locations within the city/country).
- They consider all the personal, work-related, or amenity-related reasons and pick the locations that yields the highest utility.

Introduction to a basic quantitative spatial model

- We begin with a twist to Rosen-Roback; between cities
- Rosen–Roback key insight is that any local shock to the demand or supply of labor in a city is, in equilibrium, fully capitalized in the price of land.
- As a consequence, shocks to a local economy do not affect worker welfare.
- This rule's outs some interesting questions

From Traditional Rosen-Roback to Quantitative Models

Limitations of the classical Rosen-Roback model:

- Assumes perfect worker mobility
- Extreme prediction: $U_A = U_B$ in equilibrium (equalized utility)
- All workers are indifferent between cities
- Cannot explain bidirectional flows between cities

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What do we observe in the data?

- People move in both directions simultaneously
- Not everyone responds equally to changes in w and r
- Heterogeneity in location preferences

The Solution: Idiosyncratic Shocks

Key modification:
Add idiosyncratic location preferences:

$$V_i^c = w_c - r_c + A_c + \varepsilon_i^c$$

where ε_i^c captures personal reasons for preferring city c:

- Family
- Social connections
- Personal tastes
- Personal history

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Implication: Not all workers are marginal. Only those with $\varepsilon_i^A \approx \varepsilon_i^B$ are indifferent.

Introduction to a basic quantitative spatial model

- The goal of the model is to clarify what happens to wages, costs of housing and worker utility when a local economy experiences a shock to labor demand or labor supply.
- An example of a shock
  - to labor demand is an increase in productivity.
  - to labor supply is an increase in amenities.

Introduction to a basic quantitative spatial model

- We'll work through a 2 location case to develop intuition, but it can be easily extended to n locations
- We assume that workers and firms are mobile across cities, but worker mobility is not necessarily infinite, because workers have **idiosyncratic preferences for certain locations**.

Rosen-Roback model: Exogenous Prices

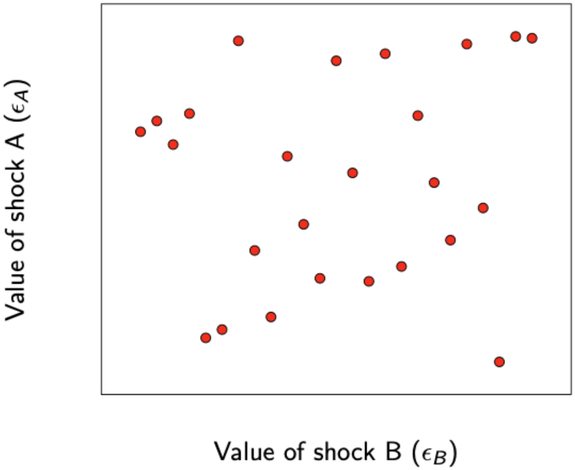
- Assume wages, rents, amenities are exogenous
- Person i's indirect utility of being in A:

$$V_A^i = w_A - r_A + A_A + \epsilon_A^i \tag{1}$$

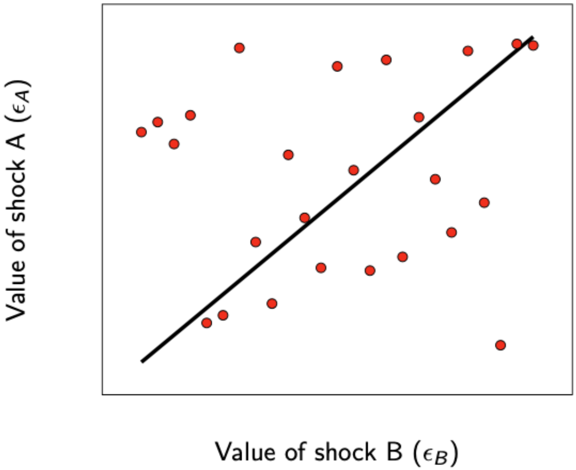
- Person i's indirect utility of being in B:

$$V_B^i = w_B - r_B + A_B + \epsilon_B^i \tag{2}$$

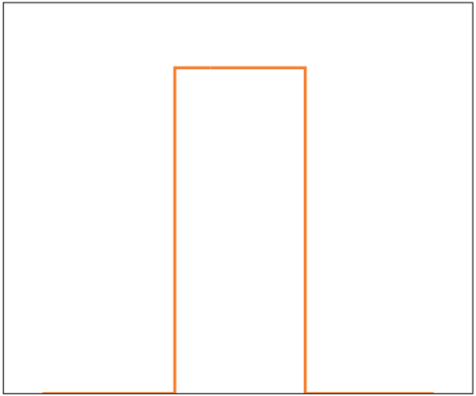
Rosen-Roback model: Exogenous Prices



Rosen-Roback model: Exogenous Prices

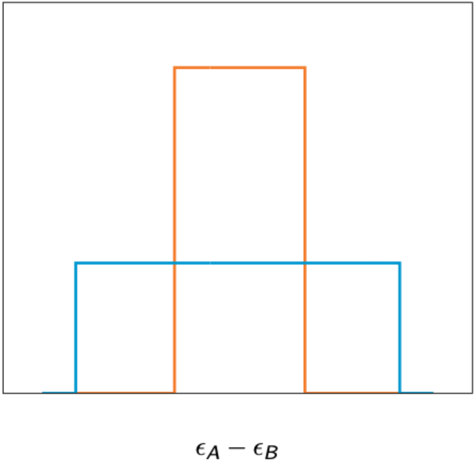


Rosen-Roback model: Exogenous Prices



$\epsilon_A - \epsilon_B$

Rosen-Roback model: Exogenous Prices



Closing the Model: We Need Firms and Housing

So far: We have modeled worker decisions

- Utility with idiosyncratic preferences
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But for general equilibrium we need:

1. Demand side: Where do wages w_c come from?
   - We need firms that hire workers
   - Marginal product of labor

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But for general equilibrium we need:

1. Demand side: Where do wages w_c come from?
   - We need firms that hire workers
   - Marginal product of labor
2. Housing market: How are rents r_c determined?
   - We need housing supply
   - Construction sector

Firms

- The production function for firms in city c is Cobb–Douglas with constant returns to scale, so that

$$\ln y_c = X_c - hN_c + (1-h)K_c \tag{3}$$

- where X_c is a city especific productivity shifter.
- there is an international capital market, and that capital is infinitely supplied at a given price i

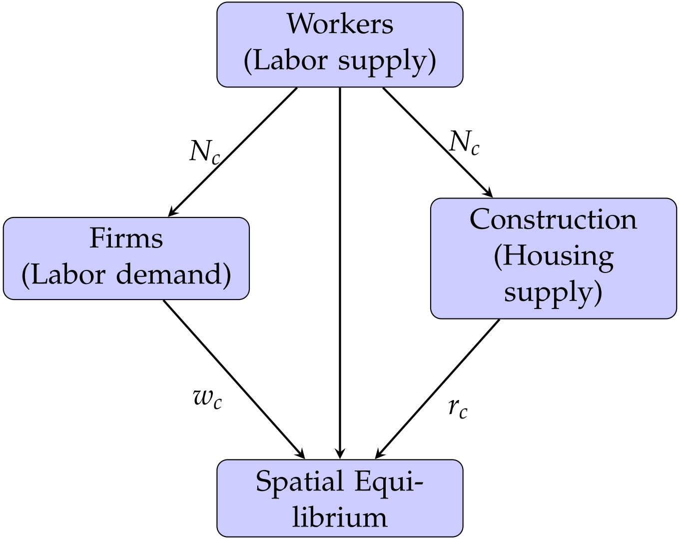
Construction

- The supply of housing is

$$r_c = z + k_c N_c \tag{4}$$

- number of housing units in city c is assumed to be equal to the number of workers.
- k_c is the elasticity of the supply of housing

Complete Model Structure



Equilibrium

- Local labor market supply

$$w_B = w_A + (r_B - r_A) + (A_A - A_B) + s\frac{(N_B - N_A)}{N} \tag{5}$$

- Labor demand

$$w_c = X_c - (1-h)N_c + (1-h)K_c + ln(h) \tag{6}$$

- Local housing demand

$$r_B = r_A + (w_B - w_A) + (A_B - A_A) - s\frac{(N_B - N_A)}{N} \tag{7}$$

- Housing supply

$$r_c = z + k_c N_c \tag{8}$$

Effect of a labor demand shock on wages and prices

- Two periods
  - Period 1: cities are identical
  - Period 2: TFP increases in b: $X_{B2} = X_{B1} + \Delta$ where $\Delta > 0$
- Workers are more productive in B than A.

Effect of a labor demand shock on wages and prices

Change in nominal wages?

$$w_{B2} - w_{B1} = \Delta \tag{9}$$

$$w_{A2} - w_{A1} = 0 \tag{10}$$

Effect of a labor demand shock on wages and prices

Change in population?

$$w_{B2} = w_{A2} + (r_{B2} - r_{A2}) + s\frac{(N_{B2} - N_{A2})}{N} \tag{11}$$

$$w_{B1} = w_{A1} + (r_{B1} - r_{A1}) + s\frac{(N_{B1} - N_{A1})}{N} \tag{12}$$

$$\Delta = k_B N_{B2} - k_A N_{A2} - k_B N_{B1} + k_A N_{A1} + s\frac{(N_{B2} - N_{A2} - N_{B1} + N_{A1})}{N} \tag{13}$$

$$(N_{B2} - N_{B1}) = \frac{N}{N(k_B + k_A) + 2s}\Delta \geq 0 \tag{14}$$

Effect of a labor demand shock on wages and prices

Change in housing markets?

- In B

$$r_{B2} - r_{B1} = \frac{Nk_B}{N\left(k_B + k_A\right) + 2s}\Delta \geq 0 \tag{15}$$

- In A

$$r_{A2} - r_{A1} = \frac{-k_A N}{N\left(k_B + k_A\right) + 2s}\Delta \leq 0 \tag{16}$$

Effect of a labor demand shock on wages and prices

Change in Real wages?

- In B

$$(w_{B2} - w_{B1}) - (r_{B2} - r_{B1}) = \frac{Nk_A + 2s}{N\left(k_B + k_A\right) + 2s}\Delta \geq 0 \tag{17}$$

- In A

$$(w_{A2} - w_{A1}) - (r_{A2} - r_{A1}) = \frac{k_A N}{N\left(k_B + k_A\right) + 2s}\Delta \geq 0 \tag{18}$$

Effect of a labor demand shock on wages and prices

Marginal worker?

$$V_c^i = w_c - r_c + A_c + \epsilon_c^i \tag{19}$$

- The change in the relative preference for city a of the marginal worker is equal a

$$\left(\epsilon_{A2}^i - \epsilon_{B2}^i\right) - \left(\epsilon_{A1}^i - \epsilon_{B1}^i\right) = \frac{2s}{N\left(k_B + k_A\right) + 2s}\Delta \geq 0 \tag{20}$$

- The marginal worker in period 2 is different from the marginal worker in period 1.
- Since city b offers higher real wages in period 2, the new marginal worker in period 2 has stronger preferences for city a.

Spatial Equilibrium with Agglomeration Economies

- Consider the case where there are agglomeration economies so that the productivity of firms in a locality is an endogenous function of the level of economic activity in that locality.
- This amounts to endogenizing the city-specific productivity shifter.
- Eg. productivity in a locality is a function of the number of workers in that locality

$$X_c = f(N_c) \tag{21}$$

- with $f' > 0$
- Decisions of workers generates a positive externality.

Spatial Equilibrium with Agglomeration Economies

- Assume

$$X_c = x_c + \gamma N_c \tag{22}$$

- The MPL

$$w_c = x_c + (\gamma - (1-h))\, N_c + (1-h)\, K_c + ln(h) \tag{23}$$

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Spatial Equilibrium with Agglomeration Economies

Change in nominal wages?

$$w_{B2} - w_{B1} = \frac{h\left(N\left(k_B + k_A\right) + 2s\right) - \gamma N}{h\left(N\left(k_B + k_A\right) + 2s\right) - 2\gamma N}\Delta \geq \Delta \geq 0 \tag{24}$$

$$\frac{\partial\left(w_{B2} - w_{B1}\right)}{\partial\gamma} = \frac{Nh\left(N\left(k_B + k_A\right) + 2s\right)}{\left(h\left(N\left(k_B + k_A\right) + 2s\right) - 2\gamma N\right)^2}\Delta \geq 0 \tag{25}$$

Spatial Equilibrium with Agglomeration Economies

Change in population?

$$(N_{B2} - N_{B1}) = \frac{Nh}{h\left(N\left(k_B + k_A\right) + 2s\right) - 2\gamma N}\Delta \geq 0 \tag{26}$$

Spatial Equilibrium with Agglomeration Economies

Change in housing markets?

$$r_{B2} - r_{B1} = \frac{hNk_B}{h\left(N\left(k_B + k_A\right) + 2s\right) - 2\gamma N}\Delta \geq 0 \tag{27}$$

Spatial Equilibrium with Agglomeration Economies

Change in Real wages?

$$(w_{B2} - w_{B1}) - (r_{B2} - r_{B1}) = \frac{(k_A N + 2s)\, h - \gamma N}{h\,(N\,(k_B + k_A) + 2s) - 2\gamma N}\Delta \tag{28}$$

$$\frac{\partial\,(w_{B2} - w_{B1}) - (r_{B2} - r_{B1})}{\partial \gamma} = \frac{Nh\,(N\,(k_A - k_B) + 2s)}{(\gamma N - 2hs - k_B N h - k_A N h)^2}\Delta \tag{29}$$

Agenda

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Empirical Implementation: The Challenge

Problem: With n locations, the model becomes very complex

- Each worker compares utility across n locations
- System of n simultaneous equilibrium equations
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How do we take this model to the data?

We need:

1. Analytical expressions for location probabilities
2. Simple aggregation from individual decisions to flows
3. Method to estimate model parameters

The Solution: Frechet Distribution

$$U_{ijo} = \frac{B_i}{d_{ij}} \left(\frac{c_{ijo}}{\beta}\right)^{\beta} \left(\frac{l_{ijo}}{1-\beta}\right)^{(1-\beta)} z_{ijo} \tag{30}$$

$$z_{ijo} \sim Frechet \tag{31}$$

$$F(z_{ijo}) = exp(-T_i E_j z_{ijo}^{-\epsilon})) \tag{32}$$

with

- scale parameters:
  - $T_i > 0$ determines the average utility derived from living in block i
  - $E_j > 0$ determines the average utility derived from working in block j
- the shape parameter $\epsilon > 1$ controls the dispersion of idiosyncratic utility.

The Solution: Frechet Distribution

Why Frechet?

1. Extreme value: Appropriate for modeling "max" of utilities

$$\max_j \left\{ U_{ij}^o = \frac{B_i}{d_{ij}^\tau} \left(\frac{c_{ij}^o}{\beta} \right)^\beta \left(\frac{l_{ij}^o}{1-\beta} \right)^{1-\beta} z_{ij}^o \right\}$$

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2. Stability property: The maximum of Frechets is Frechet
3. Closed form for probabilities:

$$\pi_{ij} = \Pr[\text{worker lives in } i, \text{ works in } j] = \frac{T_i E_j (\cdots)^\epsilon}{\sum_{i'j'} T_{i'} E_{j'} (\cdots)^\epsilon}$$

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4. Easy estimation: Multinomial logit structure

Frechet in Action: Graphical Intuition

Parameter ϵ controls dispersion:

- ϵ **low** (high dispersion):
  - Very heterogeneous preferences
  - Lower response to changes in w or r
  - Limited mobility

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Scale parameters T_i and E_j:

- T_i: Residential amenities of location i
- E_j: Productivity/job opportunities in j

Frechet Magic