Intro to Quantitative Spatial Models (cont.)

Urban Economics

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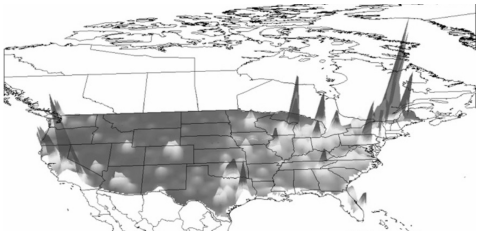
Spatial Distribution

- Why do we see such a remarkable clustering of human activity in a small number of urban areas?
- Cities exist because they are areas with high levels of productivity, which might occur because people come to places that are innately more productive or because density itself enhances productivity because of agglomeration economies



Spatial Distribution: TFP

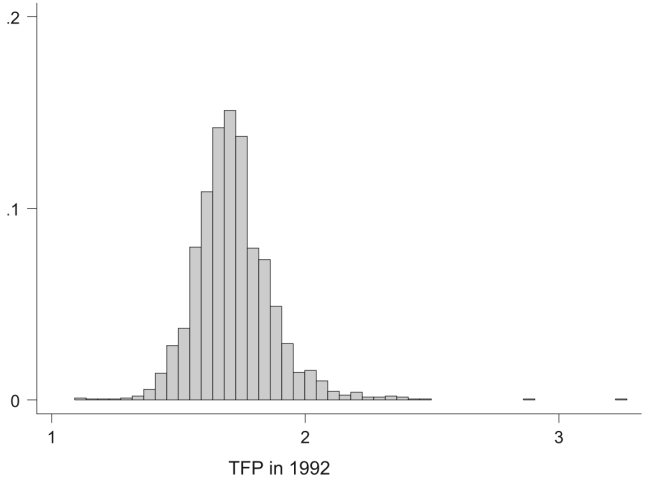


Figure 5 ***Distribution of total factor productivity in manufacturing establishments, by county.***

Introduction to a basic quantitative spatial model

- We begin with a twist to Rosen-Roback
  - We'll work through n = 2 case to develop intuition, but it can be easily extended to n locations
  - Can be used for other applications (trade, commuting, etc.)

Set up

- Assume wages, rents, amenities are exogenous
- Two cities A and B
- Person i's indirect utility of being in A:

$$V_A^i = w_A - r_A + A_A + \epsilon_A^i \tag{1}$$

- Person i's indirect utility of being in B:

$$V_B^i = w_B - r_B + A_B + \epsilon_B^i \tag{2}$$

$$\epsilon_A^i - \epsilon_B^i \sim U[-s, s] \tag{3}$$

Equilibrium

- Local labor market supply

$$w_B = w_A + (r_B - r_A) + (A_A - A_B) + s\frac{(N_B - N_A)}{N} \tag{4}$$

- Labor demand

$$w_c = X_c - (1-h)N_c + (1-h)K_c + ln(h) \tag{5}$$

- Local housing demand

$$r_B = r_A + (w_B - w_A) + (A_B - A_A) - s\frac{(N_B - N_A)}{N} \tag{6}$$

- Housing supply

$$r_c = z + k_c N_c \tag{7}$$

Effect of a labor demand shock on wages and prices

- Two periods
  - Period 1: cities are identical
  - Period 2: TFP increases in b: $X_{B2} = X_{B1} + \Delta$ where $\Delta > 0$
- Workers are more productive in B than A.

Effect of a labor demand shock on wages and prices

Change in nominal wages?

$$w_{B2} - w_{B1} = \Delta \tag{8}$$

$$w_{A2} - w_{A1} = 0 \tag{9}$$

Effect of a labor demand shock on wages and prices

Change in population?

$$w_{B2} = w_{A2} + (r_{B2} - r_{A2}) + s\frac{(N_{B2} - N_{A2})}{N} \tag{10}$$

$$w_{B1} = w_{A1} + (r_{B1} - r_{A1}) + s\frac{(N_{B1} - N_{A1})}{N} \tag{11}$$

$$\Delta = k_B N_{B2} - k_A N_{A2} - k_B N_{B1} + k_A N_{A1} + s\frac{(N_{B2} - N_{A2} - N_{B1} + N_{A1})}{N} \tag{12}$$

$$(N_{B2} - N_{B1}) = \frac{N}{N(k_B + k_A) + 2s}\Delta \geq 0 \tag{13}$$

Effect of a labor demand shock on wages and prices

Change in housing markets?

- In B

$$r_{B2} - r_{B1} = \frac{N k_B}{N\left(k_B + k_A\right) + 2s}\Delta \geq 0 \tag{14}$$

- In A

$$r_{A2} - r_{A1} = \frac{-k_A N}{N\left(k_B + k_A\right) + 2s}\Delta \leq 0 \tag{15}$$

Effect of a labor demand shock on wages and prices

Change in Real wages?

- In B

$$(w_{B2} - w_{B1}) - (r_{B2} - r_{B1}) = \frac{Nk_A + 2s}{N(k_B + k_A) + 2s}\Delta \geq 0 \tag{16}$$

- In A

$$(w_{A2} - w_{A1}) - (r_{A2} - r_{A1}) = \frac{k_A N}{N(k_B + k_A) + 2s}\Delta \geq 0 \tag{17}$$

Effect of a labor demand shock on wages and prices

Incidence?

Effect of a labor supply shock on wages and prices

Change in Real wages?

- Two periods
  - Period 1 both cities are identical
  - Period 2 amenity increases in B: $A_{B2} = A_{B1} + \Delta'$ where $\Delta' > 0$

Effect of a labor supply shock on wages and prices

Spatial Equilibrium with Agglomeration Economies

- Consider the case where there are agglomeration economies so that the productivity of firms in a locality is an endogenous function of the level of economic activity in that locality.
- This amounts to endogenizing the city-specific productivity shifter.
- Eg. productivity in a locality is a function of the number of workers in that locality

$$X_c = f(N_c) \tag{18}$$

- with $f' > 0$
- Decisions of workers generates a positive externality.

Spatial Equilibrium with Agglomeration Economies

- Assume

$$X_c = x_c + \gamma N_c \tag{19}$$

- The MPL

$$w_c = x_c + (\gamma - (1-h))\, N_c + (1-h)\, K_c + ln(h) \tag{20}$$

Spatial Equilibrium with Agglomeration Economies

- Two periods
  - Period 1 both cities are identical
  - Period 2 amenity increases in B: $X_{B2} = X_{B1} + \Delta$ where $\Delta > 0$

Spatial Equilibrium with Agglomeration Economies

Agglomeration and Empirics

Ciccone and Hall (1996)

Agglomeration and Empirics

Ciccone and Hall (1996)

TABLE 1—ESTIMATION RESULTS

Instrument	Density elasticity, θ (standard error)	Education elasticity, η (standard error)	R^2
None (NLLS)	1.052	0.410	0.551
	(0.008)	(0.396)	
Eastern seaboard	1.055	0.460	0.548
	(0.017)	(0.51)	
Railroad in 1860	1.061	0.330	0.537
	(0.011)	(0.450)	
Population in 1850	1.060	0.350	0.539
	(0.015)	(0.510)	
Population density in 1880	1.051	0.530	0.549
	(0.019)	(0.550)	
All	1.06	0.060	0.536
	(0.01)	(0.82)	

Notes: The equation estimated is (24). The data are value added for 46 states and Washington DC. For the 46 states we have used data on employment and average years of education at the county level.