Quantitative Spatial Economics and Agglomeration Economies

Urban Economics

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October 29, 2025

The economics of density: evidence from the Berlin wall

- Ahlfeldt et al. (2015) tackle the question of how strong are agglomeration and dispersion forces in a city.
- We already know that this is a difficult question, and they point out two reasons why this is the case.
  1. Endogeneity: it is hard to disentangle the strength of agglomeration forces from simple differences in location fundamentals. For this you would need exogenous variation in agglomeration.
  2. It is hard to bring the traditional urban models to data on cities.
- Ahlfeldt et al. (2015) leverage a great source of exogenous variation to identify the effects of agglomeration: the Berlin Wall.

The economics of density: evidence from the Berlin wall
Data

- Data on land prices, workplace employment, residence employment and bilateral travel times
  - Data for Greater Berlin in 1936 and 2006
  - Data for West Berlin in 1986
  - Land prices: official assessed land value of a representative undeveloped property or the fair market value of a developed property if it were not developed
  - Geographical Information Systems (GIS) data on:
    - landarea,landuse,building density, proximity to U-Bahn(underground) and S-Bahn (suburban) stations, schools, parks, lakes, canals and rivers, Second World War destruction, location of government buildings and urban regeneration programs

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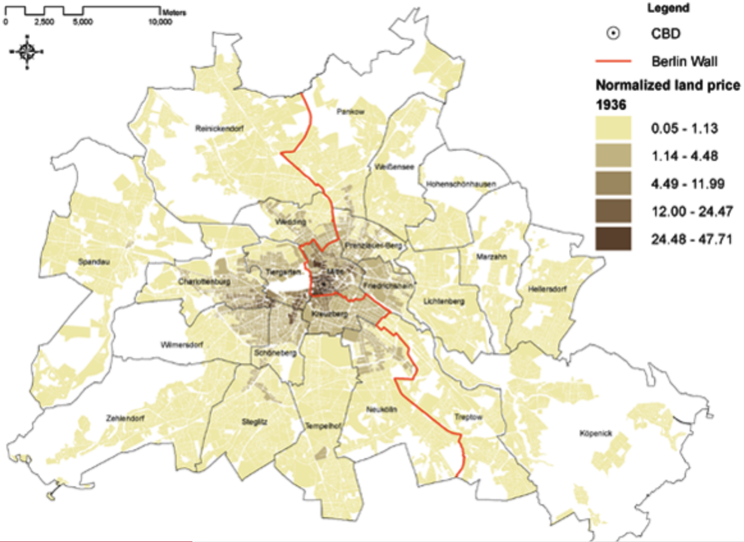
Historical Background



Qualitative Predictions for Division

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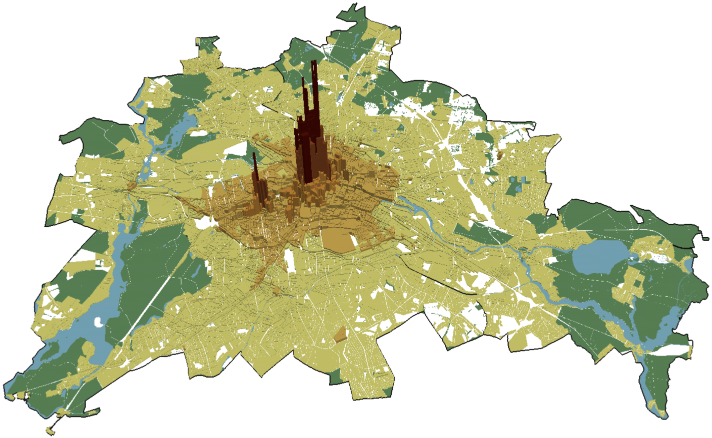
Reduced form evidence

Berlin 1936



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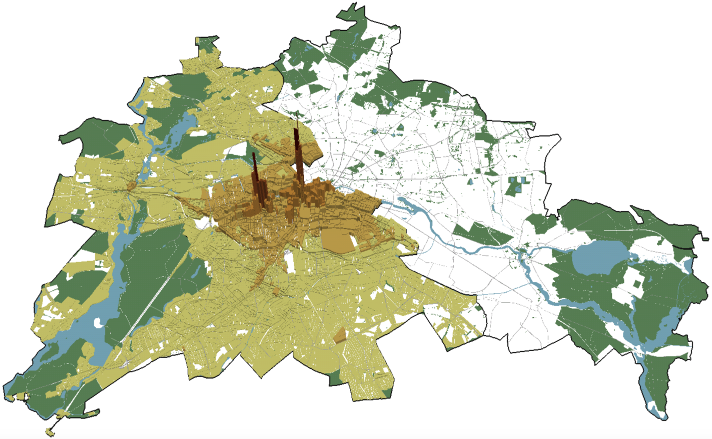
Reduced form evidence

West Berlin 1936



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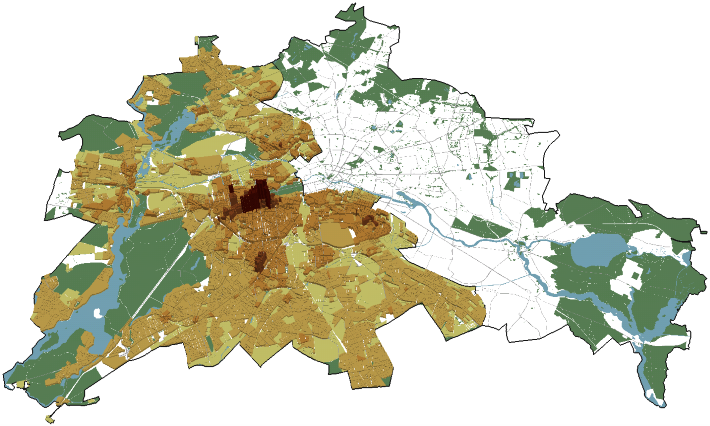
Reduced form evidence

West Berlin 1986



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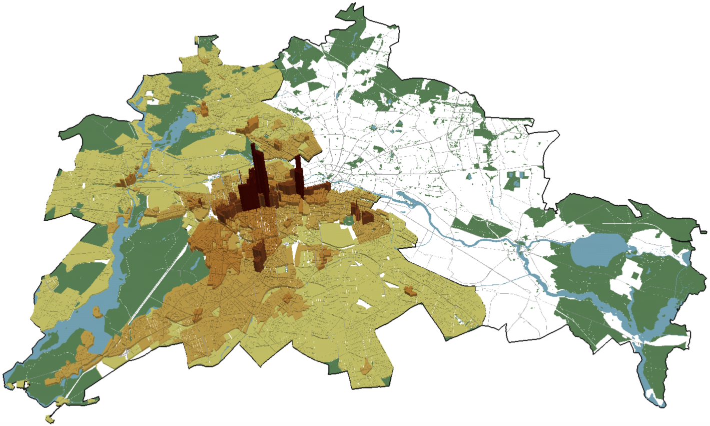
Reduced form evidence

West Berlin 2006



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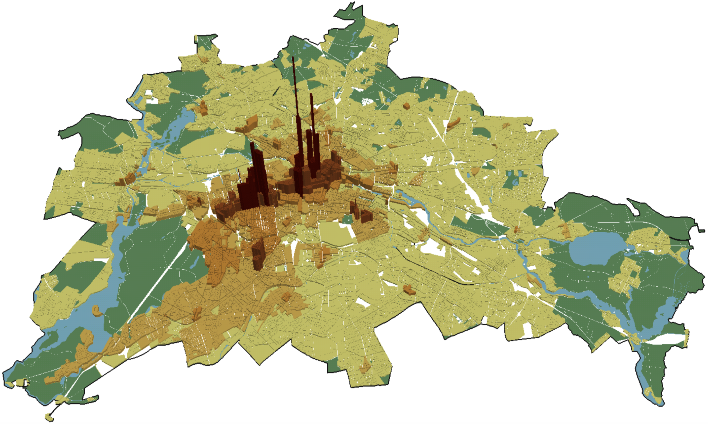
Reduced form evidence

Berlin 2006



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Reduced form evidence

TABLE I

BASELINE DIVISION DIFFERENCE-IN-DIFFERENCE RESULTS (1936–1986)[a]

	(1) $\Delta \ln Q$	(2) $\Delta \ln Q$	(3) $\Delta \ln Q$	(4) $\Delta \ln Q$	(5) $\Delta \ln Q$	(6) $\Delta \ln$ EmpR	(7) $\Delta \ln$ EmpR	(8) $\Delta \ln$ EmpW	(9) $\Delta \ln$ EmpW
CBD 1	−0.800***	−0.567***	−0.524***	−0.503***	−0.565***	−1.332***	−0.975***	−0.691*	−0.639*
	(0.071)	(0.071)	(0.071)	(0.071)	(0.077)	(0.383)	(0.311)	(0.408)	(0.338)
CBD 2	−0.655***	−0.422***	−0.392***	−0.360***	−0.400***	−0.715**	−0.361	−1.253***	−1.367***
	(0.042)	(0.047)	(0.046)	(0.043)	(0.050)	(0.299)	(0.280)	(0.293)	(0.243)
CBD 3	−0.543***	−0.306***	−0.294***	−0.258***	−0.247***	−0.911***	−0.460**	−0.341	−0.471**
	(0.034)	(0.039)	(0.037)	(0.032)	(0.034)	(0.239)	(0.206)	(0.241)	(0.190)
CBD 4	−0.436***	−0.207***	−0.193***	−0.166***	−0.176***	−0.356**	−0.259	−0.512***	−0.521***
	(0.022)	(0.033)	(0.033)	(0.030)	(0.026)	(0.145)	(0.159)	(0.199)	(0.169)
CBD 5	−0.353***	−0.139***	−0.123***	−0.098***	−0.100***	−0.301***	−0.143	−0.436***	−0.340***
	(0.016)	(0.024)	(0.024)	(0.023)	(0.020)	(0.110)	(0.113)	(0.151)	(0.124)
CBD 6	−0.291***	−0.125***	−0.094***	−0.077***	−0.090***	−0.360***	−0.135	−0.280**	−0.142
	(0.018)	(0.019)	(0.017)	(0.016)	(0.016)	(0.100)	(0.089)	(0.130)	(0.116)
Inner Boundary 1–6			Yes	Yes	Yes		Yes		Yes
Outer Boundary 1–6			Yes	Yes	Yes		Yes		Yes
Kudamm 1–6				Yes	Yes		Yes		Yes
Block Characteristics					Yes		Yes		Yes
District Fixed Effects		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6,260	6,260	6,260	6,260	6,260	5,978	5,978	2,844	2,844
R^2	0.26	0.51	0.63	0.65	0.71	0.19	0.43	0.12	0.33

[a] Q denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1–CBD6 are six 500 m distance grid cells for distance from the pre-war CBD. Inner Boundary 1–6 are six 500 m grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1–6 are six 500 m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1–6 are six 500 m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A.2 of the Technical Data Appendix. Block characteristics include the log distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley (1999)). * significant at 10%; ** significant at 5%; *** significant at 1%.

The economics of density: evidence from the Berlin wall

Reduced form evidence

TABLE II

BASELINE REUNIFICATION DIFFERENCE-IN-DIFFERENCE RESULTS (1986–2006)[a]

	(1) $\Delta \ln Q$	(2) $\Delta \ln Q$	(3) $\Delta \ln Q$	(4) $\Delta \ln Q$	(5) $\Delta \ln Q$	(6) $\Delta \ln$ EmpR	(7) $\Delta \ln$ EmpR	(8) $\Delta \ln$ EmpW	(9) $\Delta \ln$ EmpW
CBD 1	0.398***	0.408***	0.368***	0.369***	0.281***	1.079***	1.025***	1.574***	1.249**
	(0.105)	(0.090)	(0.083)	(0.081)	(0.088)	(0.307)	(0.297)	(0.479)	(0.517)
CBD 2	0.290***	0.289***	0.257***	0.258***	0.191**	0.589*	0.538*	0.684**	0.457
	(0.111)	(0.096)	(0.090)	(0.088)	(0.087)	(0.315)	(0.299)	(0.326)	(0.334)
CBD 3	0.122***	0.120***	0.110***	0.115***	0.063**	0.340*	0.305*	0.326	0.158
	(0.037)	(0.033)	(0.032)	(0.032)	(0.028)	(0.180)	(0.158)	(0.216)	(0.239)
CBD 4	0.033***	0.031	0.030	0.034	0.017	0.110	0.034	0.336**	0.261
	(0.013)	(0.023)	(0.022)	(0.021)	(0.020)	(0.068)	(0.066)	(0.161)	(0.185)
CBD 5	0.025***	0.018	0.020	0.020	0.015	−0.012	−0.056	0.114	0.066
	(0.010)	(0.015)	(0.014)	(0.014)	(0.013)	(0.056)	(0.057)	(0.118)	(0.131)
CBD 6	0.019**	−0.000	−0.000	−0.003	0.005	0.060	0.053	0.049	0.110
	(0.009)	(0.012)	(0.012)	(0.012)	(0.011)	(0.039)	(0.041)	(0.095)	(0.098)
Inner Boundary 1–6			Yes	Yes	Yes		Yes		Yes
Outer Boundary 1–6			Yes	Yes	Yes		Yes		Yes
Kudamm 1–6				Yes	Yes		Yes		Yes
Block Characteristics					Yes		Yes		Yes
District Fixed Effects		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	7,050	7,050	7,050	7,050	7,050	6,718	6,718	5,602	5,602
R^2	0.08	0.32	0.34	0.35	0.43	0.04	0.07	0.03	0.06

[a] Q denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1–CBD6 are six 500 m distance grid cells for distance from the pre-war CBD. Inner Boundary 1–6 are six 500 m grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1–6 are six 500 m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1–6 are six 500 m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A.4 of the Technical Data Appendix. Block characteristics include the log distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley (1999)). * significant at 10%; ** significant at 5%; *** significant at 1%.

The economics of density: evidence from the Berlin wall

Model

Assumptions:

- Open city.
- In the agricultural area utility is $\bar{U}$.
- The city is in discrete space, and there are S blocks indexed by $i = 1, \ldots, S$
- There is L_i floor space in every block, and these blocks can be assigned to residential or commercial usage.
- θ_i is the endogenous fraction of every block that is dedicated to commercial use.
- The city produces a single numeraire good that is costlessly traded.
- The blocks are connected, and there are H (endogenous) workers that can freely move within the city.

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Model: Workers

$$U_{ijo} = \frac{B_i}{d_{ij}} \left(\frac{c_{ijo}}{\beta} \right)^{\beta} \left(\frac{l_{ijo}}{1-\beta} \right)^{(1-\beta)} z_{ijo} \tag{1}$$

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Model: Workers

$$z_{ijo} \sim Frechet \tag{2}$$

$$F(z_{ijo}) = exp(-T_i E_j z_{ijo}^{-\epsilon}) \tag{3}$$

with

- scale parameters:
  - $T_i > 0$ determines the average utility derived from living in block i
  - $E_j > 0$ determines the average utility derived from working in block j
- the shape parameter $\epsilon > 1$ controls the dispersion of idiosyncratic utility.

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Model: Workers choice

- After observing her realizations for idiosyncratic utility for each pair of residence and employment blocks,
- each worker chooses where to live and work to maximize her utility, taking as given residential amenities, goods prices, factor prices, and the location decisions of other workers and firms.
- The indirect utility

$$u_{ijo} = \frac{B_i}{d_{ij}} \left(\frac{w_j}{Q_i^{(1-\beta)}} \right) z_{ijo} \tag{4}$$

- Therefore, workers sort across pairs of residence and employment blocks depending on their idiosyncratic preferences and the characteristics of these locations

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Model: Workers choice

- Since z_{ijo} is Frechet, then u_{ijo} is also Frechet

$$\Phi_{ij} = T_i E_j \left(d_{ij} Q_i^{(1-\beta)} \right)^{-\epsilon} \left(B_i w_j \right)^{\epsilon} \tag{5}$$

- The individuals will choose to live in i and work in j with some probability, which is the probability of $u_{ijo} = max_{r,s} u_{rso}$

$$\pi_{ij} = \frac{\Phi_{ij}}{\sum_{r=1}^{S} \sum_{s=1}^{S} \Phi_{ij}} = \frac{\Phi_{ij}}{\Phi} \tag{6}$$

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Model: Workers choice

- This completely characterizes the solution of the household's problem and the spatial distribution of households.
- We can now find the fraction of people who reside in i,

$$\pi_{Rj} = \sum_j \pi_{ij} \tag{7}$$

- The fraction of people who work in j,

$$\pi_{Mj} = \sum_i \pi_{ij} \tag{8}$$

- The probability of working on a particular place j conditional on living in i is

$$\pi_{ij|i} = \frac{\pi_{ij}}{\sum_s \pi_{is}} \tag{9}$$

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Model: Workers choice

- Using these conditional commuting probabilities, we obtain the following commuting market clearing condition that equates the measure of workers employed in block j (H_{Mj}) with the measure of workers choosing to commute to block j:

$$H_{Mj} = \sum_{i} \pi_{ij|i} H_{Ri} \tag{10}$$

- Adding up then requires that the workers in each place j, H_{Mj} , equal the sum of the residents in each place, H_{Ri}, times the probability that they work in j:

Expected worker income conditional on living in block i is equal to the wages in all possible employment locations weighted by the probabilities of commuting to those locations conditional on living in i:

$$\mathbb{E}[w_j \mid i] = \sum_{j=1}^{S} \frac{E_j \left(\frac{w_j}{d_{ij}}\right)^{\varepsilon}}{\sum_{s=1}^{S} E_s \left(\frac{w_s}{d_{is}}\right)^{\varepsilon}} w_j. \tag{11}$$

Therefore, expected worker income is high in blocks that have low commuting costs (low d_{is}) to high-wage employment locations.

Finally, population mobility implies that the expected utility from moving to the city is equal to the reservation level of utility in the wider economy ($\bar{U}$):

$$\mathbb{E}[u] = \gamma \left[\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta} \right)^{-\varepsilon} (B_r w_s)^{\varepsilon} \right]^{\frac{1}{\varepsilon}} = \bar{U}, \tag{12}$$

where $\mathbb{E}$ is the expectations operator and the expectation is taken over the distribution for the idiosyncratic component of utility. $\gamma = \Gamma\left(\frac{\varepsilon-1}{\varepsilon}\right)$ and $\Gamma(\cdot)$ is the Gamma function.

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Model: Firm choice

- Firms are assumed to have CRS Cobb-Douglas production functions over workers H_{Mj} and commercially-used land L_{Mj} , for which they pay wages w_j and rents q_j :

$$y_j = A_j H_{Mj}^{\alpha} L_{Mj} 1 - \alpha \tag{13}$$

From the first-order conditions of the firm's problem:

$$H_{Mj} = \left(\frac{\alpha A_j}{w_j} \right)^{\frac{1}{1-\alpha}} \tag{14}$$

$$q_j = (1-\alpha) \left(\frac{\alpha}{w_j} \right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}} \tag{15}$$

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Land Market Clearing: Overview

Land market clearing ensures no arbitrage between commercial and residential uses of floor space after accounting for land use regulations.

- If one use is consistently more profitable, all space would switch to that use.
- The share of floor space used commercially is denoted by θ_i.

The commercial share of floor space, θ_i, depends on the relationship between the commercial price q_i and the adjusted residential price $\xi_i Q_i$:

$$\theta_i = \begin{cases} 1 & \text{if} \quad q_i > \xi_i Q_i, \\ [0,1] & \text{if} \quad q_i = \xi_i Q_i, \\ 0 & \text{if} \quad q_i < \xi_i Q_i. \end{cases} \tag{16}$$

Observed Floor Prices and No-Arbitrage Condition

- The observed price of floor space in the data is:

$$\mathbb{Q}_i = \max\{q_i, Q_i\}.$$

- This ensures the observed price reflects the highest value from either commercial or residential use.

$$\begin{aligned}
\mathbb{Q}_i &= q_i, & q_i &> \xi_i Q_i, & \theta_i &= 1, \\
\mathbb{Q}_i &= q_i, & q_i &= \xi_i Q_i, & \theta_i &\in [0,1], \\
\mathbb{Q}_i &= Q_i, & q_i &< \xi_i Q_i, & \theta_i &= 0.
\end{aligned}$$

Cobb-Douglas Production Function

The production of floor space follows a Cobb-Douglas form:

$$L_i = M_i^{\mu} K_i^{1-\mu}$$

- L_i: Floor space produced in block i.
- M_i: Capital input used in block i.
- K_i: Land used in block i.
- $\mu \in (0, 1)$: Share parameter indicating the importance of capital relative to land.

Dual Cost Function for Floor Space

The corresponding dual cost function for producing floor space is:

$$\mathbb{Q}_i = \mu^{-\mu}(1-\mu)^{-(1-\mu)} P^{\mu} \mathbb{R}_i^{1-\mu}$$

- $\mathbb{Q}_i = \max\{q_i, Q_i\}$: Maximum of commercial and residential floor prices.
- P: Common price of capital across locations.
- $\mathbb{R}_i$: Price of land in block i.

- Using utility maximization for each worker and taking espectaion over the distribution for idiosyncratic utility, the residential market clearing condition:

$$E[l_i]H_{Ri} = (1-\beta)\frac{E(w_s|i)H_{Ri}}{Q_i} = (1-\theta)L_i \tag{17}$$

- Residential land market clearing implies that the demand for residential floor space equals the supply of floor space allocated to residential use in each location

Commercial Market Clearing Condition

Using first-order conditions for profit maximization, the commercial market clearing condition is:

$$\left(\frac{(1-\alpha)A_j}{q_j}\right)^{1/\alpha} H_{Mj} = \theta_j L_j.$$

- A_j: Productivity of firms.
- q_j: Price of commercial floor space.
- H_{Mj}: Number of workers employed in commercial activities.
- $\theta_j L_j$: Total supply of commercial floor space in block j.

Total Floor Space Allocation

- Both residential and commercial markets must clear for total floor space to be fully utilized.
- Total demand for floor space is:

$$(1-\theta_i)L_i + \theta_i L_i = L_i = \phi_i K_i^{1-\mu}.$$

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Model: Equilibrium

$$\bar{U} = \gamma \left[\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta} \right)^{-\epsilon} (B_r w_s)^{\epsilon} \right]^{\frac{1}{\epsilon}} \tag{18}$$

$$\pi_{Ri} = \frac{\sum_{s=1}^{S} T_i E_s \left(d_{is} Q_i^{1-\beta} \right)^{-\epsilon} (B_i w_s)^{\epsilon}}{\sum_{s=1}^{S} \sum_{r=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta} \right)^{-\epsilon} (B_r w_s)^{\epsilon}} \tag{19}$$

$$\pi_{Mi} = \frac{\sum_{s=1}^{S} T_r E_i \left(d_{ri} Q_r^{1-\beta} \right)^{-\epsilon} (B_r w_i)^{\epsilon}}{\sum_{s=1}^{S} \sum_{r=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta} \right)^{-\epsilon} (B_r w_s)^{\epsilon}} \tag{20}$$

$$\theta_i L_i = \left(\frac{(1-\alpha) A_i}{q_i} \right)^{\frac{1}{\alpha}} H_{Mi} \tag{21}$$

$$(1-\theta_i) L_i = (1-\beta) \left[\frac{\sum_{s=1}^{S} E_s \left(\frac{w_s}{d_{is}} \right)^{\epsilon}}{\sum_{s=1}^{S} \sum_{r=1}^{S} E_r \left(\frac{w_r}{d_{ir}} \right)^{\epsilon}} \right] \frac{H_{Ri}}{Q_i} \tag{22}$$

$$q_i = (1-\alpha) \left(\frac{\alpha}{w_i} \right)^{\frac{\alpha}{1-\alpha}} A_i^{\frac{1}{1-\alpha}} \tag{23}$$

$$\theta_i = \begin{cases} 1 & \text{if } q_i > \xi_i Q_i, \\ [0,1] & \text{if } q_i = \xi_i Q_i, \\ 0 & \text{if } q_i < \xi_i Q_i. \end{cases} \tag{24}$$

Proposition 1: Existence of a Unique Equilibrium

Proposition 1

Assuming exogenous, finite, and strictly positive location characteristics, there exists a unique general equilibrium vector:

$$\{\pi_M, \pi_R, H, Q, q, w, \theta\}.$$

- All location characteristics must be exogenous and strictly positive.
- Zero residents result if $B_i = 0$, and zero employment results if $w_j = 0$.

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Model:Composites

- **Composites for Productivity:**

$$\tilde{A}_i = A_i E_i^{\alpha/\varepsilon}$$

- **Composites for Residential Amenities:**

$$\tilde{B}_i = B_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta}$$

- **Composite for Wages:**

$$\tilde{w}_i = w_i E_i^{1/\varepsilon}$$

- **Composite for Land Use Density:**

$$\tilde{\phi}_i = \phi_i(\varphi_i, E_i^{1/\varepsilon}, \xi_i)$$

Note: These composites provide unique mappings between observed variables and unobserved location characteristics. $\zeta_{Ri} = 1$ for specialized residential blocks; $\zeta_{Ri} = \xi_i$ for mixed-use blocks.

The economics of density: evidence from the Berlin wall

Model: Equilibrium and no agglomeration

	(1) ln Bilateral Commuting Probability 2008	(2) ln Bilateral Commuting Probability 2008	(3) ln Bilateral Commuting Probability 2008	(4) ln Bilateral Commuting Probability 2008
Travel Time ($-\kappa\varepsilon$)	-0.0697*** (0.0056)	-0.0702*** (0.0034)	-0.0771*** (0.0025)	-0.0706*** (0.0026)
Estimation	OLS	OLS	Poisson PML	Gamma PML
More than 10 Commuters		Yes	Yes	Yes
Fixed Effects	Yes	Yes	Yes	Yes
Observations	144	122	122	122
R-squared	0.8261	0.9059	-	-

Note: Gravity equation estimates based on representative micro survey data on commuting for Greater Berlin for 2008. Observations are bilateral pairs of 12 workplace and residence districts (post 2001 Bezirke boundaries). Travel time is measured

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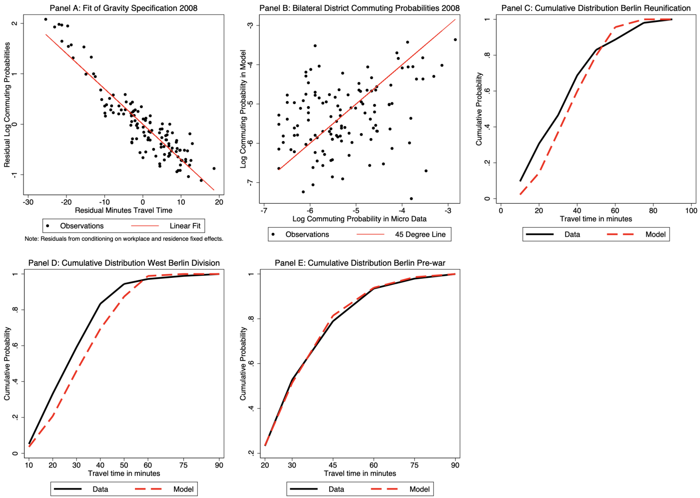
Model: Equilibrium and no agglomeration



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Model: Equilibrium and no agglomeration

	(1)	(2)	(3)	(4)	(5)	(6)
	Δ ln A	Δ ln B	Δ ln A	Δ ln B	Δ ln QC	Δ ln QC
	1936-86	1936-86	1986-2006	1986-2006	1936-1986	1986-2006
CBD 1	-0.207***	-0.347***	0.261***	0.203***	-0.229***	0.065***
	(0.049)	(0.070)	(0.073)	(0.054)	(0.020)	(0.014)
CBD 2	-0.260***	-0.242***	0.144**	0.109**	-0.184***	0.065***
	(0.032)	(0.053)	(0.056)	(0.058)	(0.008)	(0.009)
CBD 3	-0.138***	-0.262***	0.077***	0.059**	-0.177***	0.043***
	(0.021)	(0.037)	(0.024)	(0.026)	(0.012)	(0.009)
CBD 4	-0.131***	-0.154***	0.057***	0.010	-0.189***	0.048***
	(0.016)	(0.023)	(0.015)	(0.008)	(0.010)	(0.009)
CBD 5	-0.095***	-0.126***	0.028**	-0.014*	-0.188***	0.055***
	(0.014)	(0.013)	(0.013)	(0.007)	(0.012)	(0.012)
CBD 6	-0.061***	-0.117***	0.023**	0.001	-0.170***	0.035***
	(0.015)	(0.015)	(0.010)	(0.005)	(0.009)	(0.009)
Counterfactuals					Yes	Yes
Agglomeration Effects					No	No
Observations	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

- To model agglomeration, Ahlfeldt et al. (2015) make residential and workplace amenities depend on the density of employment and residency:

$$B_i = b_i \left(\sum_s e^{-\rho \tau_{js}} \left(\frac{H_{Ms}}{K_s} \right) \right)^{\eta} \tag{25}$$

$$A_i = a_j \left(\sum_s e^{-\delta \tau_{js}} \left(\frac{H_{Ms}}{K_s} \right) \right)^{\lambda} \tag{26}$$

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Model:Composites

- **Composites for Productivity:**

$$\tilde{A}_i = A_i E_i^{\alpha/\varepsilon}, \quad \tilde{a}_i = a_i E_i^{\alpha/\varepsilon}$$

- **Composites for Residential Amenities:**

$$\tilde{B}_i = B_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta}, \quad \tilde{b}_i = b_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta}$$

- **Composite for Wages:**

$$\tilde{w}_i = w_i E_i^{1/\varepsilon}$$

- **Composite for Land Use Density:**

$$\tilde{\phi}_i = \phi_i(\varphi_i, E_i^{1/\varepsilon}, \xi_i)$$

Note: These composites provide unique mappings between observed variables and unobserved location characteristics. $\zeta_{Ri} = 1$ for specialized residential blocks; $\zeta_{Ri} = \xi_i$ for mixed-use blocks.

Proposition 2

Proposition 2:

- (i) Given known values for the parameters $\{\alpha, \beta, \mu, \epsilon, \kappa\}$ and the observed data $\{Q, \mathbf{H}_M, \mathbf{H}_R, \mathbf{K}, \tau\}$, there exist unique vectors of the unobserved location characteristics $\{\tilde{A}^*, \tilde{B}^*, \tilde{\varphi}^*\}$ that are consistent with the data being an equilibrium of the model.
- (ii) Given known values for the parameters $\{\alpha, \beta, \mu, \epsilon, \kappa, \lambda, \delta, \eta, \rho\}$ and the observed data $\{Q, \mathbf{H}_M, \mathbf{H}_R, \mathbf{K}, \tau\}$, there exist unique vectors of the unobserved location characteristics $\{\tilde{a}^*, \tilde{b}^*, \tilde{\varphi}^*\}$ that are consistent with the data being an equilibrium of the model.

Algorithm Overview: Nested-Fixed Point Estimation

Algorithm Description from the Paper:

1. Fix a starting value of the parameters $\lambda, \delta, \eta, \rho$.
2. For these values, find the values of a_i, b_i that rationalize the data as being an equilibrium of the model.
    - This is the **"inner loop"**.
3. With these estimates of the location fundamentals $\hat{a}_i, \hat{b}_i$ in hand, use the identification condition to obtain estimates $\hat{\lambda}, \hat{\delta}, \hat{\eta}, \hat{\rho}$.
4. Find new values of the location fundamentals $\hat{a}_i, \hat{b}_i$ for the updated parameter values.
    - This is the **"outer loop"**.
5. Iterate until convergence in $\hat{\lambda}, \hat{\delta}, \hat{\eta}, \hat{\rho}$.

Method: Use the **Generalized Method of Moments (GMM)** to estimate parameters in the outer loop.

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Structural Estimation

Assumed Parameter		Source	Value
Residential land	$1-\beta$	Morris-Davis (2008)	0.25
Commercial land	$1-\alpha$	Valentinyi-Herrendorf (2008)	0.20
Fréchet Scale	T	(normalization)	1
Expected Utility	$\bar{u}$	(normalization)	1000

Estimated Parameter	
Production externalities elasticity	λ
Production externalities decay	δ
Residential externalities elasticity	η
Residential externalities decay	ρ
Commuting semi-elasticity	$\nu = \epsilon\kappa$
Commuting heterogeneity	ϵ

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Structural Estimation

TABLE V

GENERALIZED METHOD OF MOMENTS (GMM) ESTIMATION RESULTS[a]

	(1) Division Efficient GMM	(2) Reunification Efficient GMM	(3) Division and Reunification Efficient GMM
Commuting Travel Time Elasticity ($\kappa\varepsilon$)	0.0951***	0.1011***	0.0987***
	(0.0016)	(0.0016)	(0.0016)
Commuting Heterogeneity (ε)	6.6190***	6.7620***	6.6941***
	(0.0939)	(0.1005)	(0.0934)
Productivity Elasticity (λ)	0.0793***	0.0496***	0.0710***
	(0.0064)	(0.0079)	(0.0054)
Productivity Decay (δ)	0.3585***	0.9246***	0.3617***
	(0.1030)	(0.3525)	(0.0782)
Residential Elasticity (η)	0.1548***	0.0757**	0.1553***
	(0.0092)	(0.0313)	(0.0083)
Residential Decay (ρ)	0.9094***	0.5531	0.7595***
	(0.2968)	(0.3979)	(0.1741)

[a]Generalized Method of Moments (GMM) estimates. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley (1999)). * significant at 10%; ** significant at 5%; *** significant at 1%.

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Structural Estimation

TABLE VI

EXTERNALITIES AND COMMUTING COSTS[a]

	(1) Production Externalities $(1 \times e^{-\delta\tau})$	(2) Residential Externalities $(1 \times e^{-\rho\tau})$	(3) Utility After Commuting $(1 \times e^{-\kappa\tau})$
0 minutes	1.000	1.000	1.000
1 minute	0.696	0.468	0.985
2 minutes	0.485	0.219	0.971
3 minutes	0.338	0.102	0.957
5 minutes	0.164	0.022	0.929
7 minutes	0.079	0.005	0.902
10 minutes	0.027	0.001	0.863
15 minutes	0.004	0.000	0.802
20 minutes	0.001	0.000	0.745
30 minutes	0.000	0.000	0.642

[a] Proportional reduction in production and residential externalities with travel time and proportional reduction in utility from commuting with travel time. Travel time is measured in minutes. Results are based on the pooled efficient GMM parameter estimates: $\delta = 0.3617$, $\rho = 0.7595$, $\kappa = 0.0148$.

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Counterfactuals

TABLE VII

COUNTERFACTUALS[a]

	(1) $\Delta \ln QC$ 1936–1986	(2) $\Delta \ln QC$ 1936–1986	(3) $\Delta \ln QC$ 1936–1986	(4) $\Delta \ln QC$ 1936–1986
CBD 1	−0.836***	−0.613***	−0.467***	−0.821***
	(0.052)	(0.032)	(0.060)	(0.051)
CBD 2	−0.560***	−0.397***	−0.364***	−0.624***
	(0.034)	(0.025)	(0.019)	(0.029)
CBD 3	−0.455***	−0.312***	−0.336***	−0.530***
	(0.036)	(0.030)	(0.030)	(0.036)
CBD 4	−0.423***	−0.284***	−0.340***	−0.517***
	(0.026)	(0.019)	(0.022)	(0.031)
CBD 5	−0.418***	−0.265***	−0.351***	−0.512***
	(0.032)	(0.022)	(0.027)	(0.039)
CBD 6	−0.349***	−0.222***	−0.304***	−0.430***
	(0.025)	(0.016)	(0.022)	(0.029)
Counterfactuals	Yes	Yes	Yes	Yes
Agglomeration Effects	Yes	Yes	Yes	Yes
Observations	6,260	6,260	6,260	6,260
R^2	0.11	0.13	0.07	0.13