Fundamental diagram in the context of the Social Force Model

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I. INTRODUCTION

$$J = v(\rho)\rho \tag{1}$$

II. BACKGROUND

A. The Social Force Model

Our research was carried out in the context of the "social force model" (SFM) proposed by Helbing and co-workers [?]. This model states that human motion is caused by the desire of people to reach a certain destination, as well as other environmental factors. The pedestrians behavioral pattern in a crowded environment can be modeled by three kind of forces: the "desire force", the "social force" and the "granular force".

The "desire force" represents the pedestrian's own desire to reach a specific target position at a desired velocity v_d. But, in order to reach the desired target, he (she) needs to accelerate (decelerate) from his (her) current velocity $\mathbf{v}^{(i)}(t)$. This acceleration (or deceleration) represents a "desire force" since it is motivated by his (her) own willingness. The corresponding expression for this forces is

$$\mathbf{f}_d^{(i)}(t) = m_i \, \frac{v_d^{(i)} \, \mathbf{e}_d^{(i)}(t) - \mathbf{v}^{(i)}(t)}{\tau} \tag{2}$$

where m_i is the mass of the pedestrian i. $\mathbf{e}_d$ corresponds to the unit vector pointing to the target position and τ is a constant related to the relaxation time needed to reach his (her) desired velocity. Its value is determined experimentally. For simplicity, we assume that v_d remains constant during an evacuation process and is the same for all individuals, but $\mathbf{e}_d$ changes according to the current position of the pedestrian. Detailed values for m_i and τ can be found in Refs. [? ?].

The "social force" represents the psychological tendency of two pedestrians, say i and j, to stay away from each other by a repulsive interaction force

$$\mathbf{f}_s^{(ij)} = A_i \, e^{(r_{ij}-d_{ij})/B_i} \mathbf{n}_{ij} \tag{3}$$

where (ij) means any pedestrian-pedestrian pair, or pedestrian-wall pair. A_i and B_i are fixed values, d_{ij} is the distance between the center of mass of the pedestrians i and j and the distance $r_{ij} = r_i + r_j$ is the sum of the pedestrians radius. $\mathbf{n}_{ij}$ means the unit vector in the $\vec{ji}$ direction.

Any two pedestrians touch each other if their distance d_{ij} is smaller than r_{ij}. In this case, an additional force is included in the model, called the "granular force". This force is considered be a linear function of the relative (tangential) velocities of the contacting individuals. Its mathematical expression reads

$$\mathbf{f}_g^{(ij)} = \kappa \, (r_{ij} - d_{ij}) \, \Theta(r_{ij} - d_{ij}) \, \Delta\mathbf{v}^{(ij)} \cdot \mathbf{t}_{ij} \tag{4}$$

where κ is a fixed parameter. The function $\Theta(r_{ij} - d_{ij})$ is zero when its argument is negative (that is, $r_{ij} < d_{ij}$) and equals unity for any other case (Heaviside function). $\Delta\mathbf{v}^{(ij)} \cdot \mathbf{t}_{ij}$ represents the difference between the tangential velocities of the sliding bodies (or between the individual and the walls).

The above forces actuate on the pedestrians dynamics by changing his (her) current velocity. The equation of motion for pedestrian i reads

$$m_i \, \frac{d\mathbf{v}^{(i)}}{dt} = \mathbf{f}_d^{(i)} + \sum_{j=1}^{N} \mathbf{f}_s^{(ij)} + \sum_{j=1}^{N} \mathbf{f}_g^{(ij)} \tag{5}$$

where the subscript j represents all the other pedestrians (excluding i) and the walls.

B. Fundamental Diagram

III. NUMERICAL SIMULATIONS

IV. RESULTS

A. Fundamental diagram in the original model

In Fig 1, we show the fundamental diagram (density vs. flow) for different corridor widths. We can distinguish the two typical branches of the fundamental diagram. In the free flow branch ($\rho < 5$), the flow increases linearly with the density since there are no collisions between pedestrians. This behavior holds for all the corridor's width. In the congested branch ($\rho > 5$), we have two different scenarios:

- i) In narrow corridors (say $w < 10$) we can see that the flow reduces as the density increases. This resembles the traditional behavior of the fundamental diagram reported in the literature.
- ii) In wide corridors (say $w > 15$) we see that the flow increases with density. This contradicts the typical behavior of the fundamental diagram.

Since this discrepancy only appears in the congested branch, we can argue that the the friction force is playing a fundamental role in the flow reduction.

In order to satisfy the fundamental diagram reported in the literature, It is necessary that the flow at the maximum density ($\rho_{max} = 9$) be less than the flow at $\rho = 5$. That is: $J(\rho = 9) < J(\rho = 5)$. From the flow definition in Ec. 1 we can derive

$$v(\rho_{max}) < \frac{5v_d}{\rho_{max}}$$
$$v(\rho_{max}) < \frac{5}{9}v_d$$

As our desired velocity is fixed $v_d = 1$ m/s, we conclude that the speed at the maximum density has to be bounded by $v(\rho_{max}) \lesssim 0.5$ m/s in order to satisfy the qualitative behavior of the fundamental diagram reported in the literature.

The above reasoning is consistent with the speed-density results shown in Fig. 1. Notice that the speed values in $\rho_{max} = 9$ is higher than $v = 0.5$ m/s for $w = 15$ m and $w = 22$ m. Whereas the speed value for ρ_{max} is lower than $v = 0.5$ m/s for narrow corridors ($w < 15$)

B. Speed profile

In Fig. 3 we show the speed profile (speed vs. y-position) of the pedestrians in the corridor. When the density is low, pedestrians achieve the desired velocity ($v = v_d = 1$ m/s), leading to a cruising velocity profile for every position in the corridor. For higher densities, the speed profile shifts to a parabola-like shape. Pedestrians near near the walls are the one with the lower speed. The speed increases when moving away from the wall until it reaches the maximum in the middle of the corridor. This behavior suggests that the friction that the wall exerts on pedestrians, is playing a fundamental role in the speed profile. We did some simulations removing the walls and adding periodic boundary conditions in the y-coordinate. This yield

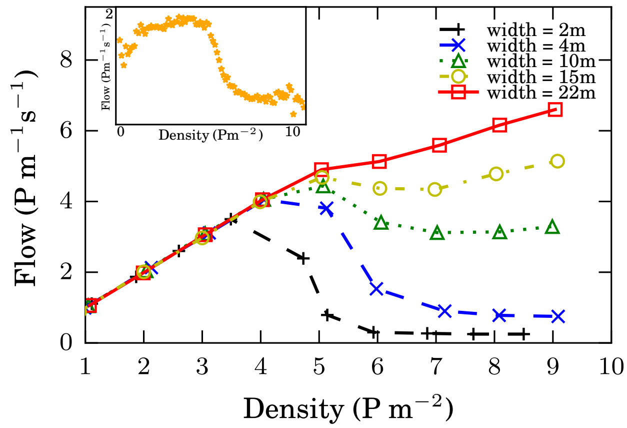

FIG. 1. Flow as a function of the density for different widths. Initially, pedestrians were randomly distributed along the corridor. The measurements were taken in the middle of the corridor once the system reached the stationary state. The length of the corridor was 28 m in all cases with periodic boundary conditions in the x direction.

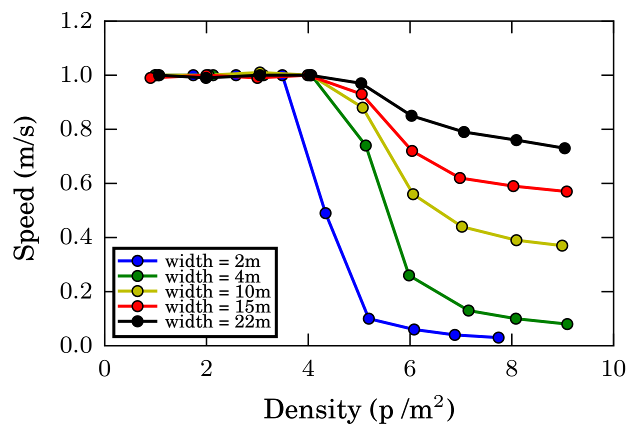

FIG. 2. Speed as a function of the density for different widths. Initially, pedestrians were randomly distributed along the corridor. The measurements were taken in the middle of the corridor once the system reached the stationary state. The length of the corridor was 28 m in all cases with periodic boundary conditions in the x direction.

to constant-cruising speed profiles for all the densities, confirming that the walls are a necessary condition in order to produce a parabola-shaped speed profile.

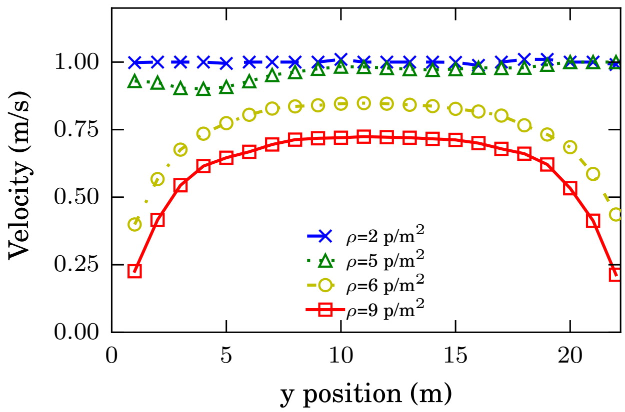

FIG. 3. Speed profile (speed vs y-position) for different densities. The desired velocity was $v_d = 1$ m/s. The measurements were taken in $x = 14$ m once the system reached the stationary state. The length of the corridor was 28 m in all cases with periodic boundary conditions in the x direction.

In Fig. 4 we show the speed profile for different widths, the x-axis is the y-position normalized by the with of the corridor. The density chosen was $\rho = 6$ since we wanted to study a situation in which pedestrians collide with each other, remember that when $\rho < 5$ pedestrians avoid collisions. We can see the parabola shape in which lower velocities are in the areas near the walls. The inset of the figure shows the same data normalized by the maximum speed value for each corridor width. It is interesting that once normalized, all the data follow the same pattern. This suggests that regardless the scale of the corridor, the speed profile yields the same behavior.

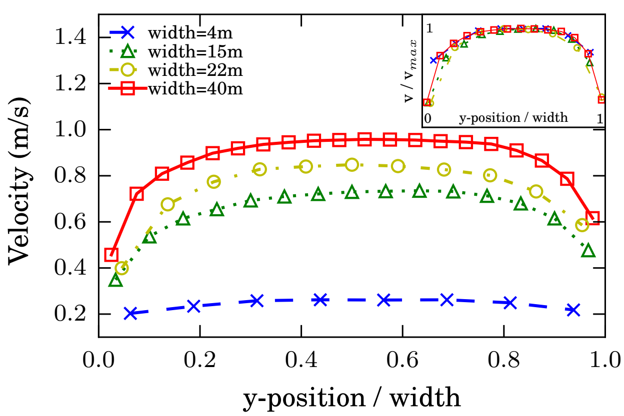

FIG. 4. Speed profile (speed vs y-position) for different corridors width.

C. Friction modification

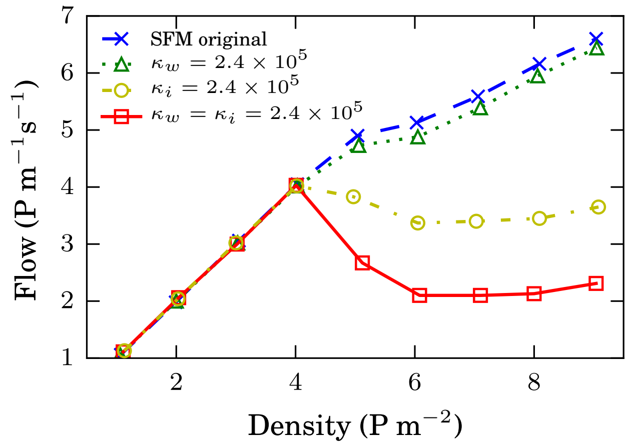

FIG. 5.

V. CONCLUSIONS

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Appendix A: