

# Fundamental diagram in the context of the Social Force Model

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## I. INTRODUCTION

$$J = v(\rho)\rho \quad (1)$$

## II. BACKGROUND

### A. The Social Force Model

Our research was carried out in the context of the “social force model” (SFM) proposed by Helbing and co-workers [? ]. This model states that human motion is caused by the desire of people to reach a certain destination, as well as other environmental factors. The pedestrians behavioral pattern in a crowded environment can be modeled by three kind of forces: the “desire force”, the “social force” and the “granular force”.

The “desire force” represents the pedestrian’s own desire to reach a specific target position at a desired velocity  $v_d$ . But, in order to reach the desired target, he (she) needs to accelerate (decelerate) from his (her) current velocity  $\mathbf{v}^{(i)}(t)$ . This acceleration (or deceleration) represents a “desire force” since it is motivated by his (her) own willingness. The corresponding expression for this forces is

$$\mathbf{f}_d^{(i)}(t) = m_i \frac{v_d^{(i)} \mathbf{e}_d^{(i)}(t) - \mathbf{v}^{(i)}(t)}{\tau} \quad (2)$$

where  $m_i$  is the mass of the pedestrian  $i$ .  $\mathbf{e}_d$  corresponds to the unit vector pointing to the target position and  $\tau$  is a constant related to the relaxation time needed to reach his (her) desired velocity. Its value is determined experimentally. For simplicity, we assume that  $v_d$  remains constant during an evacuation process and is the same for all individuals, but  $\mathbf{e}_d$  changes according to the current position of the pedestrian. Detailed values for  $m_i$  and  $\tau$  can be found in Refs. [? ? ].

The “social force” represents the psychological tendency of two pedestrians, say  $i$  and  $j$ , to stay away from each other by a repulsive interaction force

$$\mathbf{f}_s^{(ij)} = A_i e^{(r_{ij}-d_{ij})/B_i} \mathbf{n}_{ij} \quad (3)$$

where  $(ij)$  means any pedestrian-pedestrian pair, or pedestrian-wall pair.  $A_i$  and  $B_i$  are fixed values,  $d_{ij}$  is the distance between the center of mass of the pedestrians  $i$  and  $j$  and the distance  $r_{ij} = r_i + r_j$  is the sum of the pedestrians radius.  $\mathbf{n}_{ij}$  means the unit vector in the  $\vec{j}$  direction.

Any two pedestrians touch each other if their distance  $d_{ij}$  is smaller than  $r_{ij}$ . In this case, an additional force is included in the model, called the “granular force”. This force is considered be a linear function of the relative (tangential) velocities of the contacting individuals. Its mathematical expression reads

$$\mathbf{f}_g^{(ij)} = \kappa (r_{ij} - d_{ij}) \Theta(r_{ij} - d_{ij}) \Delta \mathbf{v}^{(ij)} \cdot \mathbf{t}_{ij} \quad (4)$$

where  $\kappa$  is a fixed parameter. The function  $\Theta(r_{ij} - d_{ij})$  is zero when its argument is negative (that is,  $r_{ij} < d_{ij}$ ) and equals unity for any other case (Heaviside function).  $\Delta \mathbf{v}^{(ij)} \cdot \mathbf{t}_{ij}$  represents the difference between the tangential velocities of the sliding bodies (or between the individual and the walls).

The above forces actuate on the pedestrians dynamics by changing his (her) current velocity. The equation of motion for pedestrian  $i$  reads

$$m_i \frac{d\mathbf{v}^{(i)}}{dt} = \mathbf{f}_d^{(i)} + \sum_{j=1}^N \mathbf{f}_s^{(ij)} + \sum_{j=1}^N \mathbf{f}_g^{(ij)} \quad (5)$$

where the subscript  $j$  represents all the other pedestrians (excluding  $i$ ) and the walls.

## B. Fundamental Diagram

### III. NUMERICAL SIMULATIONS

#### IV. RESULTS

##### A. Fundamental diagram in the original model

In Fig 1, we show the fundamental diagram (density vs. flow) for different corridor widths. We can distinguish the two typical branches of the fundamental diagram. In the free flow branch ( $\rho < 5$ ), the flow increases linearly with the density since there are no collisions between pedestrians. This behavior holds for all the corridor's width. In the congested branch ( $\rho > 5$ ), we have two different scenarios:

- i) In narrow corridors (say  $w < 10$ ) we can see that the flow reduces as the density increases. This resembles the traditional behavior of the fundamental diagram reported in the literature.
- ii) In wide corridors (say  $w > 15$ ) we see that the flow increases with density. This contradicts the typical behavior of the fundamental diagram.

Since this discrepancy only appears in the congested branch, we can argue that the friction force is playing a fundamental role in the flow reduction.

In order to satisfy the fundamental diagram reported in the literature, It is necessary that the flow at the maximum density ( $\rho_{max} = 9$ ) be less than the flow at  $\rho = 5$ . That is:  $J(\rho = 9) < J(\rho = 5)$ . From the flow definition in Ec. 1 we can derive

$$v(\rho_{max}) < \frac{5v_d}{\rho_{max}}$$

$$v(\rho_{max}) < \frac{5}{9}v_d$$

As our desired velocity is fixed  $v_d = 1$  m/s, we conclude that the speed at the maximum density has to be bounded by  $v(\rho_{max}) \lesssim 0.5$  m/s in order to satisfy the qualitative behavior of the fundamental diagram reported in the literature.

The above reasoning is consistent with the speed-density results shown in Fig. 1. Notice that the speed values in  $\rho_{max} = 9$  is higher than  $v = 0.5$  m/s for  $w = 15$  m and  $w = 22$  m. Whereas the speed value for  $\rho_{max}$  is lower than  $v = 0.5$  m/s for narrow corridors ( $w < 15$ )

##### B. Speed profile

In Fig. 3 we show the speed profile (speed vs. y-location) of the pedestrians in the corridor. When the

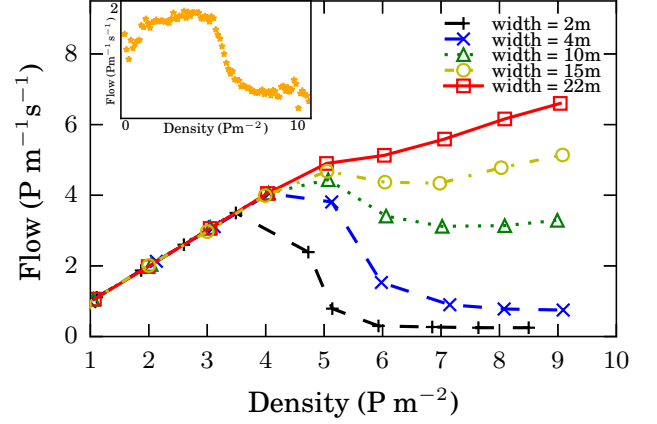


FIG. 1: Flow as a function of the density for different widths. Initially, pedestrians were randomly distributed along the corridor. The measurements were taken in the middle of the corridor once the system reached the stationary state. The length of the corridor was 28 m in all cases with periodic boundary conditions in the x direction.

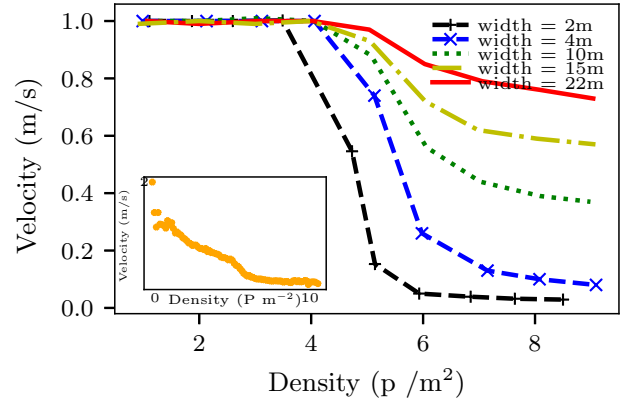


FIG. 2: Speed as a function of the density for different widths. Initially, pedestrians were randomly distributed along the corridor. The measurements were taken in the middle of the corridor once the system reached the stationary state. The length of the corridor was 28 m in all cases with periodic boundary conditions in the x direction.

density is low, pedestrians achieve the desired velocity ( $v = v_d = 1$  m/s), leading to a cruising velocity profile for every position in the corridor. For higher densities, the speed profile shifts to a parabola-like shape. Pedestrians near the walls are the one with the lower speed. The speed increases when moving away from the wall until it reaches the maximum in the middle of the corridor. This behavior suggests that the friction that the wall exerts on pedestrians, is playing a fundamental role in the speed

profile. We did some simulations removing the walls and adding periodic boundary conditions in the y-coordinate. This yield to constant-cruising speed profiles for all the densities, confirming that the walls are a necessary condition in order to produce a parabola-shaped speed profile.

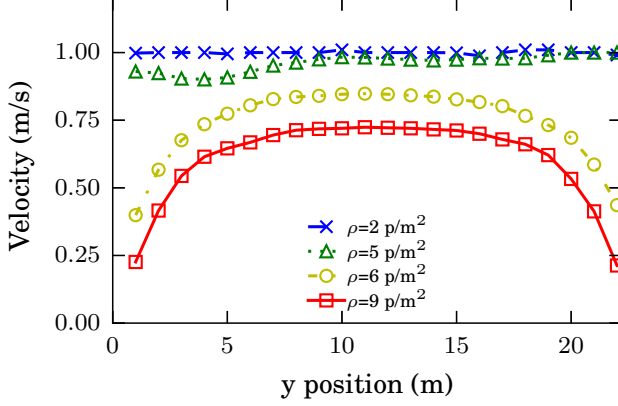


FIG. 3: Velocity profile (velocity vs y-position) for different densities (see legend for the corresponding densities). The simulated corridor was 28 m length. Pedestrians walk from left to right with periodic boundary condition in the x-direction. Initially, pedestrians were randomly distributed, the corridor width was  $w = 22$  m in all the cases.

In Fig. 4 we show the speed profile for different widths, the x-axis is the y-position normalized by the width of the corridor. The density chosen was  $\rho = 6$  since we wanted to study a situation in which pedestrians collide with each other, remember that when  $\rho < 5$  pedestrians manage to avoid collisions. We can see the parabola shape in which lower velocities are placed in the areas near the walls. The inset of the figure shows the same data normalized by the maximum speed value correspondent to each corridor width. It is interesting that once normalized, all the data follow the same pattern. This suggests that regardless the scale of the corridor, the speed profile exhibit a fundamental behavior.

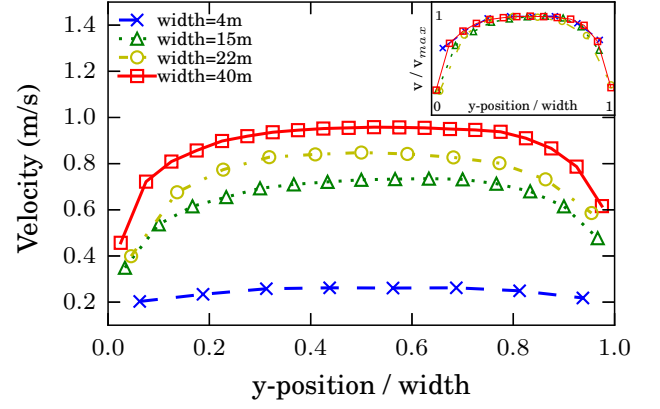


FIG. 4: Velocity profile (velocity vs y-position) for different corridors width (see legend for the corresponding widths). The simulated corridor was 28 m length. Pedestrians walk from left to right with periodic boundary condition in the x-direction. Initially, pedestrians were randomly distributed, the density was  $\rho = 6$  in all the cases. The inset shows the same data with velocity normalized by the maximum value corresponding to each dataset.

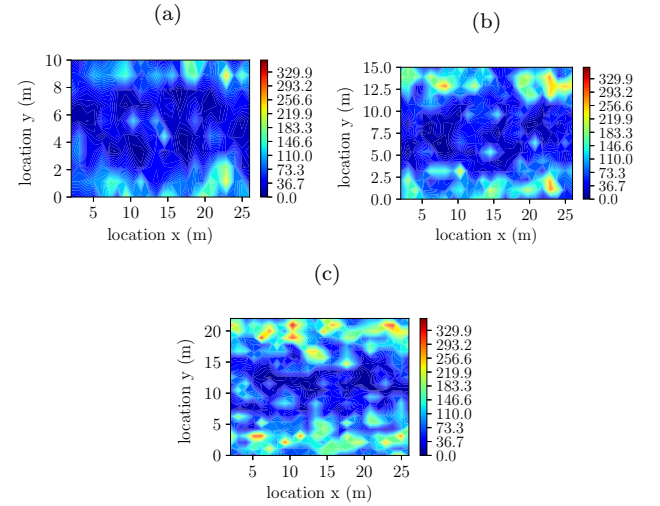


FIG. 5

### C. Work done by friction force

We have shown how the velocity takes different values along the corridor. We also analyzed the work done by the friction between pedestrians in the corridor. The work exerted was numerically calculated for each pedestrian with the Simpson rule. The sampling rate was ( $\Delta t = 0.05$  s). Once all the pedestrians had its work calculated, we proceed to create a squared grid in the corridor. The cells were 1 m length, and we associated the

### D. Friction modification

As already mentioned, the results shown so far indicate that friction may be the key to improving the fundamental diagram. We want to make clear that improving the fundamental diagram means obtaining the congested branch reported by different authors, in particular in the Jamarat study of Helbing et al. where the congested branch reaches a plateau in a 22 m width corridor. The original version of the Social Force Model proposes the same friction coefficient for the pedestrian-pedestrian interaction and the pedestrian-wall interaction. The value

proposed was  $\kappa = 2.4 \times 10^4$ . This value is widely used by the pedestrian dynamics community despite it lacks a rigorous foundation.

We tested how the system reacts to the modification of the friction value. Fig. 6 shows the flow vs density for different friction values. The triangles correspond to the increment of the wall friction in one order of magnitude (now  $\kappa_w = 2.4 \times 10^5$ ). In this case we left the pedestrian-pedestrian friction unchanged. We can see that the flow reduces a little bit, but this is not enough to produce the negative slope that characterizes the congested regime.

The circles correspond to a modification of the friction between pedestrians without changing the value of the wall friction. We increased the pedestrian-pedestrian friction by a factor of ten ( $\kappa_i = 2.4 \times 10^5$ ). Here we see a significant reduction of the flow. The qualitative behavior resembles the fundamental diagram reported by Helbing et al. with a well defined congested branch for the greatest densities.

We also tested the case where both friction coefficients are ten times the value of the original model (now  $\kappa_w = \kappa_i = 2.4 \times 10^5$ ). The squared symbols represent this scenario. As expected, the flow reduces significantly respect the original case (cross symbol plot). Interestingly, the reduction of the flow is more than the reduction due to the increment of  $\kappa_i$  plus the reduction of the flow due to  $\kappa_w$ . This behavior is indicative that the superposition principle does not hold in this system. This phenomena is explained by the non-linearity of the equation of motion.

This finding allows us to affirm that the friction plays a crucial role in the functional behavior of the fundamental diagram. The increment of both individual friction and wall friction are determinant in order to achieve a congested branch. Other authors attain the congested branch modifying different aspects of the model. Parisi et al. impose zero desired velocity once pedestrians are close enough ??, Johansson increases the relaxation time in order to slow down the net-time headway ?? and more recently, Kwak et al. ?? induce the jamming transition by an attraction. Our results show that the empirical behavior for the fundamental diagram can be achieved by properly increasing the friction coefficients. In appendix we discuss about how the modification of the relaxation time and the increment of the friction coefficient yield the same effect, since they affect the same term in the reduced-in-units equation of motion.

We argue that in real scenarios, a combination of all these factors may be the cause of the marked flow reduction that portray the fundamental diagram. The pedestrian's path can be very complex even if it is a simple enclosure (straight corridor) and the target is well defined (unidirectional flow). Beyond the complexities given by the internal motivations of pedestrians, we strongly suggest studying and modeling coefficients of friction between individuals and friction with the walls. These two parameters have shown to be very important in the dynamics and deserve a closer inspection in future research.

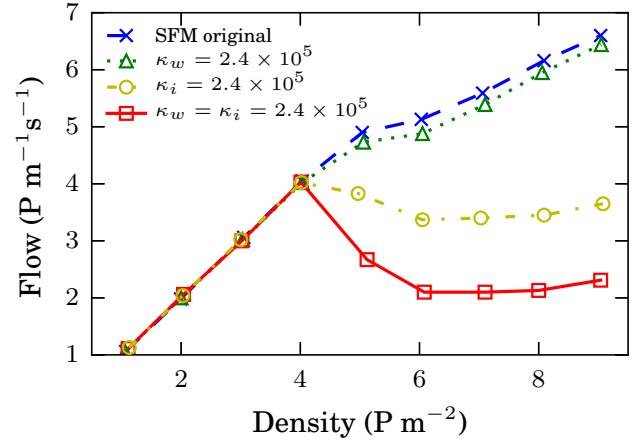


FIG. 6: Fundamental diagram (flow vs density) for different friction coefficient (see legend for the corresponding values). The simulated corridor was 28 m length. Pedestrians walk from left to right with periodic boundary condition in the x-direction. Initially, pedestrians were randomly distributed. For each density we measure the flow once the system reaches the stationary state.

## V. CONCLUSIONS

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## Appendix A: