Problem Statement

A company wants to test if a new training program increases the average productivity of its employees. They conduct a study with a sample of 100 employees who underwent the new training program and another sample of 120 employees who did not. The sample mean productivity score for the trained employees is 85 with a standard deviation of 10, while the sample mean for the untrained employees is 80 with a standard deviation of 12. Use a significance level of 0.05 to determine if the new training program significantly increases productivity.

Solution

Step 1: State the Hypotheses

$$H_0 : \mu_1 \leq \mu_2 \quad \text{(The training program does not increase productivity)}$$
$$H_1 : \mu_1 > \mu_2 \quad \text{(The training program increases productivity)}$$

Step 2: Present the Data

	Trained Employees	**Untrained Employees**
Sample Mean ($\bar{X}$)	85	80
Standard Deviation (s)	10	12
Sample Size (n)	100	120

Table 1: Summary of the data

Step 3: Calculate the Test Statistic

The formula for the test statistic is:

$$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Plugging in the values:

$$z = \frac{85 - 80}{\sqrt{\frac{10^2}{100} + \frac{12^2}{120}}} = \frac{5}{\sqrt{1 + 1.2}} = \frac{5}{\sqrt{2.2}} \approx 3.37$$

Step 4: Rejection Region

The critical value for a one-sided test at the 0.05 significance level is:

$$z_{0.05} = 1.645$$

Rejection Criteria

Reject H_0 if $z > 1.645$.
Since $z \approx 3.37$ and $3.37 > 1.645$, we reject the null hypothesis.

Step 5: P-value Approach

The p-value for the test statistic $z \approx 3.37$ can be found using the standard normal distribution.

$$\text{p-value} = P(Z > 3.37) = 1 - P(Z \leq 3.37)$$

Using a standard normal table or calculator, we find:

$$P(Z \leq 3.37) \approx 0.9997$$

$$\text{p-value} = 1 - 0.9997 = 0.0003$$

Since the p-value $0.0003 < 0.05$, we reject the null hypothesis.