Quiz 4 – Solutions

Question 1: Discrete Random Variable

The die has faces: {1, 2, 2, 3, 3, 4}, so the probability distribution is:

$$P(1) = \frac{1}{6}, \quad P(2) = \frac{2}{6}, \quad P(3) = \frac{2}{6}, \quad P(4) = \frac{1}{6}$$

We define $Z = X_1 + X_2$. The maximum value of Z is $4 + 4 = 8$.

$$P(Z = 8) = P(X_1 = 4, X_2 = 4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(Z > 7) = \boxed{\frac{1}{36}}$$

Question 2: Bayes' Rule

Given:

$$P(G) = 0.002, \quad P(E \mid G) = 0.75, \quad P(E \mid G^c) = 0.05$$

Using Bayes' Rule:

$$P(G \mid E) = \frac{P(E \mid G)P(G)}{P(E \mid G)P(G) + P(E \mid G^c)P(G^c)}$$

$$= \frac{0.75 \cdot 0.002}{0.75 \cdot 0.002 + 0.05 \cdot 0.998} = \frac{0.0015}{0.0514} \approx \boxed{0.0292}$$

Question 3: Binomial and Complement Rule

Let $Y \sim \text{Binomial}(n = 5, p = 0.6)$, where Y is the number of votes in favor.

We are asked:

$$P(\text{At least 2 vote against}) = P(Y \leq 3)$$

This can be computed directly:

$$P(Y \leq 3) = \sum_{k=0}^{3} \binom{5}{k} (0.6)^k (0.4)^{5-k}$$

$$= 0.01024 + 0.0768 + 0.2304 + 0.3456 = \boxed{0.6630}$$

Note: Complement Rule Alternatively, we can apply the complement rule:

$$P(Y \leq 3) = 1 - P(Y > 3) = 1 - P(Y \in \{4, 5\}) = 1 - P(Y = 4) - P(Y = 5)$$

$$P(Y = 4) = \binom{5}{4}(0.6)^4(0.4)^1 = 5 \cdot 0.1296 \cdot 0.4 = 0.2592$$

$$P(Y = 5) = \binom{5}{5}(0.6)^5(0.4)^0 = 1 \cdot 0.07776 = 0.07776$$

$$P(Y \leq 3) = 1 - (0.2592 + 0.07776) = 1 - 0.33696 = \boxed{0.6630}$$

Both methods yield the same result.