Introduction to Statistical Methods in Political Science

Quiz 4

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Question 1 - Discrete Random Variable and Probability Calculation

Consider a non-standard six-sided die with faces $\{1, 2, 2, 3, 3, 4\}$. The face "2" appears twice, as does "3", while "1" and "4" each appear once. You roll this die twice. Let X_1 be the outcome of the first roll and X_2 be the outcome of the second roll.
Define $Z = X_1 + X_2$, as the *random variable* representing the sum of the outcomes of the two rolls.

Question: Compute $P(Z > 7)$.

Question and Correct Answer

- Question 1. What is the probability that Z is larger than 7?
  - $1/36 \leftarrow$ **Correct Answer**
  - $15/36$
  - 50%
  - 0%
  - 6.25%

Question 2 - Bayes' Rule

A burglary has been committed in a residential neighborhood. The police have identified a suspect but do not have definitive proof. Initially, based on past crime data, only **1 in 500** (0.2%) of individuals in similar circumstances have actually committed a burglary.

A rare type of shoeprint is found at the crime scene, and the suspect owns the same type of shoes (*Evidence*). From previous forensic studies, we know:

- If a person **did** commit the crime, the probability that their shoeprint would be found is $P(\textit{Evidence} \mid \textit{Guilty}) = 0.75$.
- If a person **did not** commit the crime, the probability that they own the same rare shoe type is $P(\textit{Evidence} \mid \textit{Not Guilty}) = 0.05$.

Question: Using **Bayes' Rule**, compute the updated probability that the suspect actually committed the crime given that their shoeprint type was found at the scene – i.e., compute $P(\textit{Guilty} \mid \textit{Evidence})$.

Question and Correct Answer

- Question 2. What is the probability that the subject is truly guilty, given that their shoeprint type was found at the scene? (hint: use Bayes' rule).
  - 2.9% $\leftarrow$ **Correct Answer**
  - 0.2%
  - 75%
  - 93.75%
  - 70%
  - 40%
  - 1.5%

Question 3 - Binomial Distribution and Complement Rule

A 5-member congressional committee is voting on an amendment to a given bill. Each member independently supports the amendment with probability $p = 0.60$. Let Y be the number of members (out of 5) who vote in favor.

Questions:

- Compute the probability that at least two committee members vote in favor of the amendment, i.e., $P(Y \geq 2)$.

Question and Correct Answer

- Question 3. What is the probability that at least two committee members will vote against the amendment?
  - 66.30% ← **Correct Answer**
  - 34.56%
  - 68.26%
  - 80%
  - 16%
  - 40%
  - 33.70%