Inference and Hypothesis Testing for One Proportion - Solved Problems

Inference and Hypothesis Testing for One Proportion

Problem 1: Confidence Interval for a Population Proportion A survey was conducted to estimate the proportion of people who support a new environmental policy. Out of 500 randomly selected respondents, 320 expressed support for the policy. Construct a 95% confidence interval for the true proportion of supporters.

Solution

To construct a 95% confidence interval for a population proportion, we use the formula:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where:

- $\hat{p} = \frac{320}{500} = 0.64$ is the sample proportion,
- $z_{\alpha/2} = 1.96$ for a 95% confidence level,
- $n = 500$ is the sample size.

The margin of error is:

$$1.96 \times \sqrt{\frac{0.64(1-0.64)}{500}} = 1.96 \times \sqrt{\frac{0.64 \times 0.36}{500}} = 1.96 \times 0.0215 = 0.0421.$$

Thus, the 95% confidence interval is:

$$0.64 \pm 0.0421 = (0.5979, 0.6821).$$

Therefore, the 95% confidence interval for the proportion of people who support the policy is approximately (0.598, 0.682).

Problem 2: Hypothesis Test for a Population Proportion A medical researcher claims that 80% of people are willing to participate in a new health study. To test this claim, a random sample of 200 people is surveyed, and 150 people indicate their willingness to participate. Perform a hypothesis test at the 5% significance level to determine if the true proportion is different from the claimed 80%.

(a) State the null and alternative hypotheses.

(b) Compute the test statistic.

(c) Determine the p-value.

(d) State your conclusion.

Solution

(a) The null and alternative hypotheses are:

$$H_0 : p = 0.80 \quad \text{(the true proportion is 80\%)},$$

$$H_a : p \neq 0.80 \quad \text{(the true proportion is different from 80\%)}.$$

This is a two-tailed test.

(b) The test statistic for a population proportion is given by:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}},$$

where:

- $\hat{p} = \frac{150}{200} = 0.75$ is the sample proportion,
- $p_0 = 0.80$ is the claimed proportion,
- $n = 200$ is the sample size.

Substituting these values, we get:

$$z = \frac{0.75 - 0.80}{\sqrt{\frac{0.80(1-0.80)}{200}}} = \frac{-0.05}{\sqrt{0.0008}} = \frac{-0.05}{0.02828} = -1.77.$$

(c) To find the p-value, we look at the standard normal distribution. The p-value for a two-tailed test with $z = -1.77$ is:

$$p = 2 \times P(Z < -1.77) = 2 \times 0.03855 = 0.077.$$

(d) Since the p-value (0.077) is greater than the significance level ($\alpha = 0.05$), we fail to reject the null hypothesis. There is not enough evidence to conclude that the true proportion of people willing to participate is different from 80%.