MH3500 Statistics

Tutorial 3 AY2022/23 Semester 2

Problem 3.1

For each $n \in \{10, 100, 1000, 10000\}$, use CLT to approximate the following probabilities and compare the approximations with the exact probabilities.

a) $\Pr(\sum_{i=1}^{n} X_i \leq n)$ where $X_1, \dots, X_n \sim \text{Gamma}(1,1)$

b) $\Pr\left(\sum_{i=1}^{n} X_i \leq \frac{101n}{200}\right)$ where $X_1, \dots, X_n \sim \text{Bernoulli}(0.5)$

Hint: first, we need to identify the distribution of $\sum_{i=1}^{n} X_i$.

Problem 3.2

A battery manufacturer claims that the run time of a new "ultra strong" laptop battery has a population mean of 40 hours with a standard deviation of 5 hours. A random sample $X_1, \dots, X_n$ of size $n = 100$ is taken.

a) Find an approximation to $Pr(\bar{X} \leq 36.7)$.

b) If the claim of the manufacturer were true, would an average run time of 36.7 among 100 batteries be unusually short?

c) If you observed an average run time of 36.7 among 100 batteries, would you find the claim of the manufacturer plausible?

d) Answer parts a) to c) with 36.7 replaced by 39.8.

Problem 3.3

Let $X_1, \dots, X_n$ be an i.i.d. random sample drawn from $U(a, \ b)$ (uniform distribution on the interval $(a, \ b)$), where a and b are unknown parameters.

a) Find the method of moments estimators for a and b.

b) The following observations for $x_1, \dots, x_{30}$ for $X_1, \dots, X_{30}$ are given with $n = 30$. What are the estimates for a and b that you get from the method of moments estimators for these data?

$$15.4, 71.4, 13.8, 16.4, 56.9, \quad 18.7, 83.6, 83.6, 75.0, 23.5,$$
$$69.4, 56.7, 97.6, 68.4, 82.0, \quad 50.8, 48.9, 84.3, 17.5, 22.0,$$
$$25.6, 45.2, 84.8, 82.3, 15.4, \quad 45.9, 57.4, 47.5, 69.1, 66.5$$

[In fact, these observations are drawn from $U(10, \ 100)$]