SHDR User Manual

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1 Description

This manual aims to be a quick guide to using the codes in the repository https://github.com/manufrn/SHDR. For a detailed explanation of fitting the method, please refer to the original work by González-Pola et al. (2007).

The SHDR algorithm fits an upper oceanic profile to a physically inspired analytical form. SHDR stands for *Sharp Homogenization/Diffusive Retreat*. The thermocline/pycnocline is assumed to be a result of diffusive processes, and the following analytical form can be found to describe an upper oceanic profile:

$$f(z) = \begin{cases} a_1 & \text{if } z < D_1, \\ a_3 + b_3(z - D_1) + a_2 e^{\left(-b_2(z-D_1) - c_2(z-D_1)^2\right)} & \text{if } z > D_1. \end{cases} \tag{1}$$

For simplicity, here on we will use the nomenclature relative to a temperature profile, but in principle the method can be applied to temperature, salinity and density profiles.

The first part of equation 1 defines the mixed layer and the second the seasonal and permanent thermoclines. The seasonal thermocline is parameterized as a combination of exponential and gaussian decays. These decay to a straight line that describes the permanent thermocline. The different parameters in equation 1 are presented graphically in figure 1, and can be interpreted as:

- D_1 - Mixed layer depth.
- a_1 - Temperature of the mixed layer depth.
- b_2 - Exponential decay coefficient at the seasonal thermocline.
- c_2 - Gaussian decay coefficient at the seasonal thermocline.
- b_3 - Gradient of the permanent thermocline.
- a_2 - Temperature difference between the mixed layer and the base of the thermocline.
- a_3 - Temperature at the base of the thermocline.

The optimization method used by SHDR is a differential evolution algorithm. To find the combination of parameters that minimize the mean squared error (MSE) for a given profile, a population of individuals is randomly generated. Each individual represents a certain combination of the parameters in equation 1. By means of crossing and mutations, the "genetics" of the individuals that perform best are favored, and the MSE is progressively minimized. A description of the differential evolution algorithm as implemented in SHDR can be found in the González-Pola et al. (2007).

2 Using SHDR

The source code of SHDR is very simple. All the routines needed to use SHDR are included in the file `SHDR.py`. To use it, download it and place it in your working directory. This file is meant to be used as a module. It contains two main functions: `fit_profile` and `fit_time_series`, which should be imported from the module in your preferred Python programming environment.

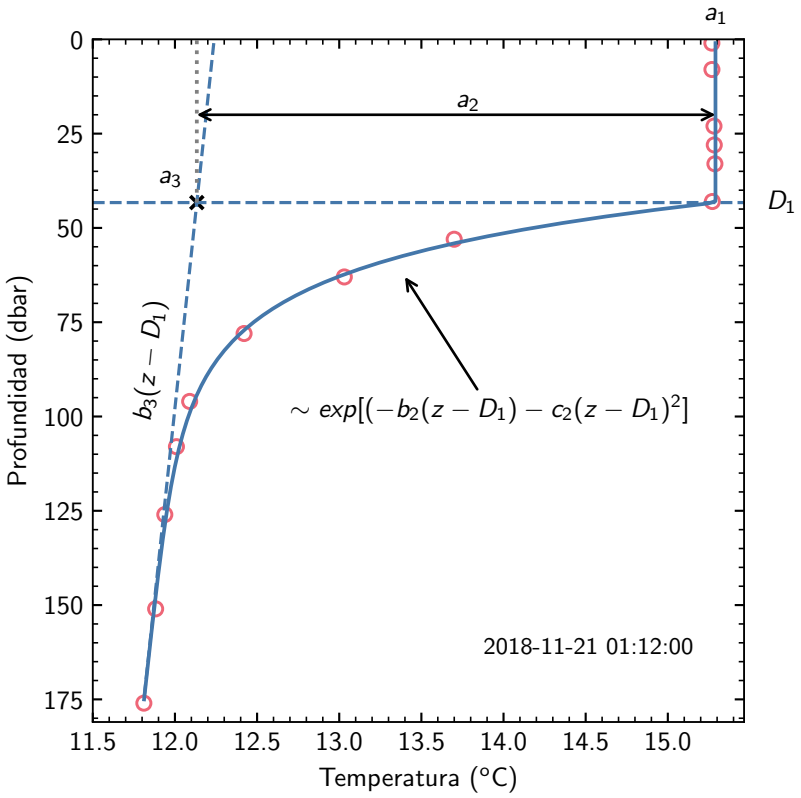

Figure 1: Temperature profile and SHDR fit ideal profile and parameters.

The `examples` jupyter notebook in the code repository contains scripts and notebooks showcasing different use cases of the SHDR module. Here we will present the functions, their use case, arguments and output.

The main function in SHDR is `fit_profile`. It performs the differential evolution search to find the optimal parameters of equation 1 that minimize the MSE for a given profile:

`fit_profile(y, z, **opts)`.

- `y` - 1-D array defining the values of the profile.
- `z` - 1-D array defining the depths of the profile.
- `opts` - Optional arguments. Described bellow.

`fit_profile` returns an array containing the optimal parameters for the profile in the following order: [`D1, b2, c2, b3, a2, a1, a3, em`]. `em` is the MSE error of the fit.

`fit_profile` only fits a single profile. For datasets containing multiple profiles, iterate through them with `fit_profile` and store the results as convenient. For time series datasets, the `SHDR.py` contains a specific function to simplify this process (see section 2.2).

2.1 Optional arguments

The `fit_profile` function has different optional arguments. For most use cases you only need to be aware of a few:

- `max_depth`: Maximum depth in the profile to be considered for the fitting. Measurements bellow this depth are not passed to the algorithm. Use this argument to specify the depth that approximately defines the upper ocean in your dataset. Default value is 450 (dbar or m).
- `only_mld`: If `True`, only the D_1 parameter is returned. Defaults to `False`.
- `delta_coding`: If `True`, after the differential evolution has reached a solution, a whole new generation is set to evolve, this time within the surroundings of the first solution. This allows for a refined search to ensure the fit completely minimizes the MSE. Defaults to `False`
- `min_depth`: Minimum depth that a profile must reach to perform the fit. For profiles that don't reach this depth, an array of `NaNs` is returned as solution. Defaults to 50 (dbar or m).
- `min_obs`: Minimum number of measurements in the profile to perform the fit. If a profile has less measurements, an array of `NaNs` is returned. Defaults to 10.

For a more advanced usage, the parameters of the differential evolution algorithm can be modified. The default values should reproduce consistent results for most CTD profiles:

- `CR` - Cross probability. Default: 0.7
- `FF` - Mutation factor. Default: 0.7
- `num_generations` - Maximum number of generations. Default: 1200
- `num_individuals` - Number of individuals in each generation. Default: 60
- `tol` - tolerance to stop evolution. Default: 0.00025

2.2 Fitting a time series

As many datasets are organized as time series, the `SHDR` module includes a function called `fit_time_series` that performs the fit and organizes the results for data structured as a time series. Its optional parameters are the same as above. Its main parameters are:

```
fit_time_series(time, variable, depth, lat=None, lon=None, save=None,
                **opts)
```

- `time` - 1D Array defining the timestamps of the measurements
- `variable` - 2D Array containing the measurements. The first dimension of the array must be time, and the same length as `time`
- `depth` - 2D Array defining the depths of the measurements. The first dimension must be temporal.
- `lat` - Optional 1D Array defining the latitude of each measurement.
- `lon` - Optional 1D Array defining the longitude of each measurement.
- `save` - Defines a relative path to store the results as a `.csv` file. If this argument is not passed, the results won't be stored.

`fit_time_series` returns a Pandas DataFrame containing the results of the fit. The DataFrame columns are: [`time, lat, lon, D1, b2, c2, b3, a2, a1, a3, em`]. The stored `.csv` follows the same structure.

3 Exploring SHDR results

The code repository contains another file: `SHDR_utils.py`. This file incorporates routines to analyze the output of SHDR:

- `fit_function(z, params)`. Returns the value at depth z of an analytical profile following equation 1 with parameters `params`. The `params` argument must be a `np.ndarray` containing the output of `fit_profile` or a `pd.Series` containing a specific profile fit of `fit_time_series`.
- `compute_stratification(params, alpha=0.05)`. Returns the G_α stratification index as defined in González-Pola et al. for a given combination of parameters.
- `time_series_stratification(time_series_fit, alpha=0.05)`. Wrapper to apply `compute_stratification` to a fitted time series. The `time_series_fit` argument can be a path where the `.csv` file of the fit is stored or the output of `time_series_fit`. In the first case, an extra column containing the stratification index is added to selected results file. Otherwise a `pd.Series` object is returned containing the stratification for each measurement.

Important note: the previous functions can't be used if the fit was performed with the option `only_MLD=True`.

References

González-Pola, C., Fernández-Díaz, J. M., & Lavín, A. (2007). Vertical structure of the upper ocean from profiles fitted to physically consistent functional forms. *Deep Sea Research Part I: Oceanographic Research Papers*, *54*(11), 1985–2004. https://doi.org/10.1016/j.dsr.2007.08.007

A Differential evolution algorithm

The differential evolution algorithm implemented in SHDR is as follows following. Firstly, an initial population is generated randomly following a uniform distribution inside the limits for each parameter. Each generation g evolves according to the following algorithm:

1. The MSE is computed for each individual, and the best is detected, $\mathbf{x}_{b,g}$.
2. For each individual $\mathbf{x}_{i,g}$, two individuals $\mathbf{x}_{k_1,g}, \mathbf{x}_{k_2,g}$ are selected randomly with $i \neq k_1 \neq k_2$, and its mutation $\mathbf{y}_i$ is computed:

$$\mathbf{y}_{i,g} = (1-\mu)\mathbf{x}_{i,g} + \mu\mathbf{x}_{b,g} + F(\mathbf{x}_{k_1,g} - \mathbf{x}_{k_2,g}), \tag{2}$$

   where $FF \in (0,1)$ is the mutation factor and $\mu \in (0,1)$ the weight of the best individual.
3. In the next generation, the individual will evolve from $\mathbf{x}$ a $\mathbf{y}$ following

$$\mathbf{x}_{i,g+1} = \begin{cases} \mathbf{y}_{i,g} & \text{if } r_i < CR, \\ \mathbf{x}_{i,g} & \text{otherwise,} \end{cases} \tag{3}$$

   where $CR \in (0,1)$ is the cross probability $r_i \sim U[0,1)$.
4. If a parameter of a new individual excedes the limits, it is returned to the closest limit.
5. If $g+1 = g_{\max}$ with $g_{\max}$ the maximun number of generation, or if the mean $\overline{em} < \text{tol} \cdot \sigma_{em}$, where e_m is the MSE, σ_{em} is the standard deviation of the population MSE, and tol is a tolerance previously defined, the interation is halted and the best individual of the new generation $g+1$ is taken as solution. Otherwise, ..