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DU_NE

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2	Contest (1)	
2	template.cpp	17 lines
2	<pre>#include <bits/stdc++.h> using namespace std; #define rep(i, a, b) for(int i = a; i<(b); ++i) #define all(x) begin(x), end(x) #define sz(x) (int)(x).size() typedef long long ll; typedef pair<int, int> pii; typedef vector<int> vi; int main() { cin.tie(0)->sync_with_stdio(0); cin.exceptions(cin.failbit); #ifdef ONPC cerr << endl << "finished in " << clock() * 1.0 / CLOCKS_PER_SEC << " sec" << endl; #endif }</pre>	
3	.bashrc	3 lines
6	<pre>alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \ -fsanitize=undefined,address' xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps =<</pre>	
6	.vimrc	6 lines
9	<pre>set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul sy on im jk <esc> im kj <esc> no ; : " Select region and then type :Hash to hash your selection. " Useful for verifying that there aren't mistypes. ca Hash w !cpp -dD -P -fpreprocessed \ tr -d '[:space:]' \ \ md5sum \ cut -c-6</pre>	
11	hash.sh	3 lines
11	<pre># Hashes a file, ignoring all whitespace and comments. Use for # verifying that code was correctly typed. cpp -dD -P -fpreprocessed tr -d '[:space:]' md5sum cut -c-6</pre>	
13	stress.sh	10 lines
13	<pre>#!/bin/bash ["\$#" -ne 3] && echo "Usage: \$0 test_file brute_file mycode_file" && exit 1 g++ -O2 \$1 -o test && g++ -O2 \$2 -o brute && g++ -O2 \$3 -o mycode for i in {1..10000}; do ./test > tests.txt ./brute < tests.txt > correct.txt ./mycode < tests.txt > myans.txt diff -q correct.txt myans.txt >/dev/null { echo -e "\e[31 mTest \$i: WA\e[0m"; cat tests.txt; break; } echo -e "\e[32mTest \$i: AC\e[0m" done</pre>	
16		
18		

```
interactiveStress.py
19 lines
import subprocess, random
def generate_permutation(n): return random.sample(range(1, n +
1), n)
def handle_queries(hidden, n, max_q=6666):
    process = subprocess.Popen(["./solve"], stdin=subprocess.
PIPE, stdout=subprocess.PIPE, text=True)
    process.stdin.write(f"{n}\n"); process.stdin.flush()
    for _ in range(max_q):
        query = process.stdout.readline().strip().split()
        if query[0] == "1":
            print("Correct!" if list(map(int, query[1:])) ==
hidden else "Wrong!")
            break
        matches = sum(p == h for p, h in zip(map(int, query
[1:]), hidden))
        process.stdin.write(f"{matches}\n"); process.stdin.
flush()
    else: print("Query limit exceeded!")
    process.terminate()

n = 1000
hidden_permutation = generate_permutation(n)
print("Hidden permutation:", hidden_permutation)
handle_queries(hidden_permutation, n)

makefile
10 lines
# runs by make run file=filename, use *tab*
CC = g++
CFLAGS = -fsanitize=address -std=c++17 -Wall -Wextra -Wshadow -
DONPC -O2
all:
%: %.cpp
$(CC) $(CFLAGS) -o "$@" "$<"
run: $(file)
./$(file)
clean:
find . -type f -executable -delete
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix} \Rightarrow \begin{matrix} x = \frac{ed - bf}{ad - bc} \\ y = \frac{af - ec}{ad - bc} \end{matrix}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where A'_i is A with the i 'th column replaced by b .

2.2 Recurrences

If $a_n = c_1a_{n-1} + \cdots + c_ka_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k - c_1x^{k-1} - \cdots - c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1r_1^n + \cdots + d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1n + d_2)r^n$.

2.3 Trigonometry

$$\begin{aligned}\sin(v+w) &= \sin v \cos w + \cos v \sin w \\ \cos(v+w) &= \cos v \cos w - \sin v \sin w\end{aligned}$$

$$\begin{aligned}\tan(v+w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}\end{aligned}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$\begin{aligned}a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi)\end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b+c} \right)^2 \right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$

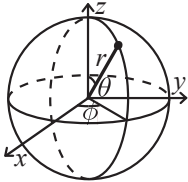
2.4.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned}x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \text{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \text{atan2}(y, x)\end{aligned}$$

2.5 Derivatives/Integrals

$$\begin{aligned}\frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1+x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \text{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1)\end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$\begin{aligned}1 + 2 + 3 + \cdots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \cdots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \cdots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \cdots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}\end{aligned}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \cdots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x . It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y ,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $U(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Data structures (3)

OrderStatisticTree.h

Description: ...
Time: $\mathcal{O}(\log N)$
`<ext/pb.ds/assoc.container.hpp>`, `<ext/pb.ds/tree.policy.hpp>` d41d8c, 14 lines
`using namespace __gnu_pbds;`

`#define ordered_set tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>`
`#define ordered_pair_set tree<pair<int, int>, null_type, less<pair<int, int>>, rb_tree_tag, tree_order_statistics_node_update>`
`ordered_set os;`
`// Example using ordered_set`
`os.insert(5);os.insert(1);os.insert(10);os.insert(3);`
`cout << "2nd smallest element: " << *os.find_by_order(2) << endl;` `// Output: 5`
`cout << "Elements less than 6: " << os.order_of_key(6) << endl;`
`// Output: 3`
`// Example using ordered_pair_set`
`ordered_pair_set ops;`
`ops.insert({1, 100});ops.insert({2, 200});ops.insert({1, 150});`
`ops.insert({3, 250});`
`cout << "1st smallest pair: (" << ops.find_by_order(0)->first << ", " << ops.find_by_order(0)->second << ")" << endl;`
`// Output: (1, 100)`
`cout << "Pairs less than (2, 150): " << ops.order_of_key({2, 150}) << endl;` `// Output: 2`

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).
`<bits/extc++.h>` d41d8c, 6 lines
`struct chash {`
`const uint64_t C = uint64_t(4e18 * acos(0)) | 71;`
`ll operator()(ll x) const { return __builtin_bswap64(x * C); }`
`};`

`__gnu_pbds::gp_hash_table<ll, int, chash> h;`

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$ d41d8c, 113 lines
`struct Segtree {`
`// 0 base indexing`
`int n;`
`vector<ll> tree;`
`ll merge(ll x, ll y) {`

`return x + y;`
`}`
`void build(vector<ll> &a, int node, int l, int r) {`
`if(l == r) {`
`tree[node] = a[l];`
`return;`
`}`
`int mid = l + ((r - l) >> 1);`
`build(a, (node << 1)+1, l, mid);`
`build(a, (node << 1)+2, mid+1, r);`
`tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);`
`}`
`void update(int i, ll value, int node, int l, int r) {`
`if(l == i && r == i) {`
`tree[node] = value;`
`return;`
`}`
`int mid = l + ((r-l) >> 1);`
`if(i <= mid)update(i, value, (node << 1)+1, l, mid);`
`else update(i, value, (node << 1)+2, mid+1, r);`
`tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);`
`}`
`void update(int i, int value) {`
`update(i, value, 0, 0, n-1);`
`}`
`ll query(int i, int j, int node, int l, int r) {`
`if(l > j || r < i) return 0;`
`if(l >= i && r <= j)return tree[node];`
`int mid = l + ((r - l) >> 1);`
`return merge(query(i, j, (node << 1)+1, l, mid), query(i, j, (node << 1)+2, mid+1, r));`
`}`
`ll query(int i, int j) {`
`return query(i, j, 0, 0, n-1);`
`}`
`void init(vector<ll> &a, int _n) {`
`n = _n;`
`int size = 1;`
`while(size < n) size = size << 1;`
`tree.resize((size << 1)-1);`
`build(a, 0, 0, n-1);`
`}`
`} st;`
`struct Segtree {`
`// 0 base indexing`
`int n;`
`vector<ll> tree, lazy;`

`ll merge(ll x, ll y) {`
`return x + y;`
`}`
`void push(int node, int l, int r) {`
`int a = (node << 1)+1, b = (node << 1)+2;`
`int mid = l + ((r-l) >> 1);`
`tree[a]+=(mid-l+1)*lazy[node], tree[b]+=(r-(mid+1)+1)*lazy[node];`
`lazy[a]+=lazy[node], lazy[b]+=lazy[node];`
`lazy[node] = 0;`
`}`
`void build(vector<ll> &a, int node, int l, int r) {`
`if(l == r) {`
`tree[node] = a[l];`
`return;`
`}`
`int mid = l + ((r-l) >> 1);`
`build(a, (node << 1)+1, l, mid);`
`build(a, (node << 1)+2, mid+1, r);``tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);`
`}`
`void build(vector<ll> &a) {`
`build(a, 0, 0, n-1);`
`}`
`void update(int i, int j, ll value, int node, int l, int r) {`
`{`
`if(l > j || r < i)return;`
`if(l >= i && r <= j) {`
`lazy[node]+=value;`
`tree[node]+=(r-l+1)*value;`
`return;`
`}`
`if(lazy[node])push(node, l, r);`
`int mid = l + ((r-l) >> 1);`
`update(i, j, value, (node << 1)+1, l, mid);`
`update(i, j, value, (node << 1)+2, mid+1, r);`
`tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);`
`}`
`void update(int i, int j, ll value) {`
`update(i, j, value, 0, 0, n-1);`
`}`
`ll query(int i, int j, int node, int l, int r) {`
`if(l > j || r < i)`
`return 0;`
`if(l >= i && r <= j)`
`return tree[node];`

`if(lazy[node]) push(node, l, r);`
`int mid = l + ((r-l) >> 1);`
`return merge(query(i, j, (node << 1)+1, l, mid), query(i, j, (node << 1)+2, mid+1, r));`
`}`
`ll query(int i, int j) {`
`return query(i, j, 0, 0, n-1);`
`}`
`void init(vector<ll> &a, int _n) {`
`n = _n;`
`int size = 1;`
`while(size < n) size = size << 1;`
`tree.resize((size << 1)-1);`
`lazy.assign((size << 1)-1, 0);`
`build(a, 0, 0, n-1);`
`}`
`} st;`

LazySegmentTree.h

Description: Segment tree with lazy propagation
Usage: update(l, 0, n - 1, ql, qr, val), query(l, 0, n - 1, ql, qr)
Time: $\mathcal{O}(\log N)$ d41d8c, 66 lines
`struct Segtree {`
`// 0 base indexing`
`int n;`
`vector<ll> tree, lazy;`

`ll merge(ll x, ll y) {`
`return x + y;`
`}`
`void push(int node, int l, int r) {`
`int a = (node << 1)+1, b = (node << 1)+2;`
`int mid = l + ((r-l) >> 1);`
`tree[a]+=(mid-l+1)*lazy[node], tree[b]+=(r-(mid+1)+1)*lazy[node];`
`lazy[a]+=lazy[node], lazy[b]+=lazy[node];`
`lazy[node] = 0;`

```

}
void build(vector<ll> &a, int node, int l, int r) {
    if(l == r) {
        tree[node] = a[l];
        return;
    }
    int mid = l + ((r-l) >> 1);
    build(a, (node << 1)+1, l, mid);
    build(a, (node << 1)+2, mid+1, r);
    tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);
}
void build(vector<ll> &a) {
    build(a, 0, 0, n-1);
}
void update(int i, int j, ll value, int node, int l, int r)
{
    if(l > j || r < i) return;
    if(l >= i && r <= j) {
        lazy[node] += value;
        tree[node] += (r-l+1)*value;
        return;
    }
    if(lazy[node]) push(node, l, r);
    int mid = l + ((r-l) >> 1);
    update(i, j, value, (node << 1)+1, l, mid);
    update(i, j, value, (node << 1)+2, mid+1, r);
    tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);
}
void update(int i, int j, ll value) {
    update(i, j, value, 0, 0, n-1);
}
ll query(int i, int j, int node, int l, int r) {
    if(l > j || r < i)
        return 0;
    if(l >= i && r <= j)
        return tree[node];

    if(lazy[node]) push(node, l, r);
    int mid = l + ((r-l) >> 1);
    return merge(query(i, j, (node << 1)+1, l, mid), query(
        i, j, (node << 1)+2, mid+1, r));
}
ll query(int i, int j) {
    return query(i, j, 0, 0, n-1);
}
void init(vector<ll> &a, int _n) {
    n = _n;
    int size = 1;
    while(size < n) size = size << 1;
    tree.resize((size << 1)-1);
    lazy.assign((size << 1)-1, 0);
    build(a, 0, 0, n-1);
}
} st;

```

PersistentSegtree.h

Description: Persistent Segment Tree

d41d8c, 76 lines

```

struct persistentSegtree {
    // 0 base indexing
    ll data;
    persistentSegtree *left, *right;

    ll merge(ll x, ll y) {
        return x + y;
    }
    void build(vector<ll> &a, int l, int r) {

```

```

        if(l == r) {
            data = a[l];
            return;
        }
        int mid = l + ((r - l) >> 1);
        left = new persistentSegtree();
        right = new persistentSegtree();
        left->build(a, l, mid);
        right->build(a, mid+1, r);
        data = merge(left->data, right->data);
    }
    persistentSegtree* update(int i, ll value, int l, int r) {
        if(l > i || r < i) return this;
        if(l == i && r == i) {
            persistentSegtree *rslt = new persistentSegtree();
            rslt->data = value;
            return rslt;
        }
        int mid = l + ((r-l) >> 1);
        persistentSegtree *rslt = new persistentSegtree();

        rslt->left = left->update(i, value, l, mid);
        rslt->right = right->update(i, value, mid+1, r);
        rslt->data = merge(rslt->left->data, rslt->right->data);
    }

    return rslt;
}
ll query(int i, int j, int l, int r) {
    if(l > j || r < i) return 0;
    if(l >= i && r <= j) return data;
    int mid = l + ((r - l) >> 1);
    return merge(left->query(i, j, l, mid), right->query(i,
        j, mid+1, r));
}
} *roots[N];
int main() { // Idea from Mahmudul Yeamim
    int tt = 1;
    while(tt--) {
        int n, q, k = 0;
        cin >> n >> q;
        vector<ll> a(n);
        for(int i = 0; i < n; i++) {
            cin >> a[i];
        }
        roots[0] = new persistentSegtree();
        roots[k++] -> build(a, 0, n-1);
        while(q--) {
            int type;
            cin >> type;
            if(type == 1) {
                int _k, i;
                ll x;
                cin >> _k >> i >> x;
                --_k;
                roots[_k] = roots[_k] -> update(--i, x, 0, n-1);
            } else if(type == 2) {
                int _k, i, j;
                cin >> _k >> i >> j;
                cout << roots[--_k] -> query(--i, --j, 0, n-1) <<
                    "\n";
            } else {
                int _k;
                cin >> _k;
                roots[k++] = roots[--_k];
            }
        }
    }
    return 0;
}

```

```

}

```

UnionFind.h

Description: Disjoint-set data structure.

Time: $\mathcal{O}(\alpha(N))$

d41d8c, 17 lines

```

void make_set(int v) {
    parent[v] = v;
    Size[v] = 1;
}
int find_set(int v) {
    if (v == parent[v]) return v;
    return parent[v] = find_set(parent[v]);
}
void union_sets(int a, int b) {
    a = find_set(a);
    b = find_set(b);
    if (a != b) {
        if(Size[a] < Size[b]) swap(a, b);
        parent[b] = a;
        Size[a] += Size[b];
    }
}

```

UnionFindRollback.h

Description: 2D prefix with update

d41d8c, 34 lines

2DPrefix.h

Description: 2D prefix with update

Usage: SubMatrix<int> m(matrix);

m.sum(0, 0, 2, 2); // top left 4 elements

Time: $\mathcal{O}(N^2 + Q)$

d41d8c, 34 lines

```

void update(vector<vector<ll>>& grid, int x1, int y1, int x2,
    int y2, int val) {
    grid[x1][y1] += val;
    if (x2 + 1 < n) grid[x2 + 1][y1] += val;
    if (y2 + 1 < m) grid[x1][y2 + 1] += val;
    if (x2 + 1 < n && y2 + 1 < m) grid[x2 + 1][y2 + 1] += val;
}
vector<vector<ll>> calculate(vector<vector<ll>> &grid) {
    vector<vector<ll>> ans(n, vector<ll>(m, 0));
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            ans[i][j] = grid[i][j];
            if(i > 0) ans[i][j] += ans[i - 1][j];
            if(j > 0) ans[i][j] += ans[i][j - 1];
            if(i > 0 && j > 0) ans[i][j] += ans[i - 1][j - 1];
        }
    }
    return ans;
}
template<class T> struct SubMatrix {
    vector<vector<T>> p;
    SubMatrix(const vector<vector<T>>& v) {
        int R = v.size(), C = v[0].size();
        p.assign(R + 1, vector<T>(C + 1, 0));

        for (int r = 0; r < R; ++r) {
            for (int c = 0; c < C; ++c) {
                p[r + 1][c + 1] = v[r][c] + p[r][c + 1] + p[r + 1][c] - p[r][c];
            }
        }
    }
    T sum(int u, int l, int d, int r) {
        return p[d + 1][r + 1] - p[u][r + 1] - p[d + 1][l] + p[u][l];
    }
}

```

```
    }
};

Matrix.h
Description: Basic operations on square matrices.
Usage: Matrix<int, 3, 3> A;
A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};
vector<int> vec = {1,2,3};
vec = (A^N) * vec;
```

d41d8c, 34 lines

```
template<class T, int N, int M> struct Matrix {
    typedef Matrix Mx;
    array<array<T, M>, N> d{};
    // Matrix multiplication
    template<int P>
    Matrix<T, N, P> operator*(const Matrix<T, M, P>& m) const {
        Matrix<T, N, P> a;
        for (int i = 0; i < N; i++)
            for (int j = 0; j < P; j++)
                for (int k = 0; k < M; k++)
                    a.d[i][j] += d[i][k] * m.d[k][j];
        return a;
    }
    // Matrix-vector multiplication
    vector<T> operator*(const vector<T>& vec) const {
        vector<T> ret(N, 0);
        for (int i = 0; i < N; i++)
            for (int j = 0; j < M; j++)
                ret[i] += d[i][j] * vec[j];
        return ret;
    }
    // Matrix exponentiation
    Matrix<T, N, N> operator^(ll p) const {
        static_assert(N == M); assert(p >= 0);
        Matrix<T, N, N> a, b(*this);
        for (int i = 0; i < N; i++) a.d[i][i] = 1; // Identity matrix
        while (p) {
            if (p & 1) a = a * b;
            b = b * b;
            p >>= 1;
        }
        return a;
    }
};
```

```
CHT.h
Description: Container where you can add lines of the form kx+m, and query minimum values at points x. Useful for dynamic programming (“convex hull trick”).
Time: O(log N)
```

d41d8c, 38 lines

```
struct Line {
    // m = slope, c = intercept
    ll m, c;
    Line(ll a, ll b) : m(a), c(b) {}
};

struct CHT {
    // SayeefMahmud
    vector<Line> lines;

    bool bad(Line l1, Line l2, Line l3) {
        __int128 a = (__int128)(l2.c - l1.c) * (l2.m - l3.m);
        __int128 b = (__int128)(l3.c - l2.c) * (l1.m - l2.m);
        return a >= b;
    }

    void add(Line line) {
        lines.push_back(line);
        int sz = lines.size();
```

```
        while (sz >= 3 && bad(lines[sz - 3], lines[sz - 2], lines[sz - 1])) {
            lines.erase(lines.end() - 2);
            sz--;
        }
    }

    ll query(ll x) {
        int l = 0, r = lines.size() - 1;
        ll ans = LLONG_MAX;
        while (l <= r) {
            int mid1 = l + (r - l) / 3;
            int mid2 = r - (r - l) / 3;
            ans = min(ans, min(lines[mid1].m * x + lines[mid1].c, lines[mid2].m * x + lines[mid2].c));
            if (lines[mid1].m * x + lines[mid1].c <= lines[mid2].m * x + lines[mid2].c) {
                r = mid2 - 1;
            } else {
                l = mid1 + 1;
            }
        }
        return ans;
    }
};
```

```
Treap.h
Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: O(log N)
```

```
FenwickTree.h
Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
```

d41d8c, 26 lines

```
struct FenwickTree {
    // 0 base indexing
    vector<int> bit;
    int n;
    FenwickTree(int n) {
        this->n = n;
        bit.assign(n, 0);
    }
    FenwickTree(vector<int> const &a) : FenwickTree(a.size()) {
        for (size_t i = 0; i < a.size(); i++)
            add(i, a[i]);
    }
    int sum(int r) {
        int ret = 0;
        for (; r >= 0; r = (r & (r + 1)) - 1)
            ret += bit[r];
        return ret;
    }
    int sum(int l, int r) {
        return sum(r) - sum(l - 1);
    }
    void add(int idx, int delta) {
        for (; idx < n; idx = idx | (idx + 1))
            bit[idx] += delta;
    }
};
```

```
FenwickTree2d.h
Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
```

d41d8c, 36 lines

```
struct FenwickTree2D {
```

```
    // 0 base indexing
    vector<vector<int>> bit;
    int n, m;
    FenwickTree2D(int n, int m) {
        this->n = n;
        this->m = m;
        bit.assign(n, vector<int>(m, 0));
    }
    FenwickTree2D(vector<vector<int>>& matrix) : FenwickTree2D(matrix.size(), matrix[0].size()) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                add(i, j, matrix[i][j]);
            }
        }
    }
    int sum(int x, int y) {
        int ret = 0;
        for (int i = x; i >= 0; i = (i & (i + 1)) - 1) {
            for (int j = y; j >= 0; j = (j & (j + 1)) - 1) {
                ret += bit[i][j];
            }
        }
        return ret;
    }
    int sum(int x1, int y1, int x2, int y2) {
        return sum(x2, y2) - sum(x2, y1 - 1) - sum(x1 - 1, y2) + sum(x1 - 1, y1 - 1);
    }
    void add(int x, int y, int delta) {
        for (int i = x; i < n; i = i | (i + 1)) {
            for (int j = y; j < m; j = j | (j + 1)) {
                bit[i][j] += delta;
            }
        }
    }
};
```

```
RMQ.h
Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
Usage: RMQ rmq(values);
rmq.query(inclusive, exclusive);
Time: O(|V| log |V| + Q)
```

d41d8c, 26 lines

```
struct RMQ {
    // 0-base indexing
    int n, logN;
    vector<vector<int>> st;
    vector<int> lg;

    void init(const vector<int>& array) {
        n = array.size();
        logN = ceil(log2(n));
        st.resize(logN, vector<int>(n));
        lg.resize(n + 1);
        lg[1] = 0;
        for (int i = 2; i <= n; i++)
            lg[i] = lg[i / 2] + 1;
        copy(array.begin(), array.end(), st[0].begin());
        for (int i = 1; i < logN; i++) {
            for (int j = 0; j + (1 << i) <= n; j++) {
                st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
            }
        }
    }
    int query(int L, int R) {
        int i = lg[R - L + 1];
```

```

        return min(st[i][L], st[i][R - (1 << i) + 1]);
    }
} ST;

```

MoQueries.h

Description: ...

d41d8c, 48 lines

```

// 0-base indexing
void add(int x) {
    if(!freq[x]) distinct++;
    freq[x]++;
}
void remove(int x) {
    freq[x]--;
    if(!freq[x]) distinct--;
}
void adjust(int &curr_l, int &curr_r, int L, int R) {
    while(curr_l > L) {
        curr_l--;
        add(a[curr_l]);
    }
    while(curr_r < R) {
        curr_r++;
        add(a[curr_r]);
    }
    while(curr_l < L) {
        remove(a[curr_l]);
        curr_l++;
    }
    while(curr_r > R) {
        remove(a[curr_r]);
        curr_r--;
    }
}
void solve(vector<array<int, 3>> &queries) {
    // const int BLOCK_SIZE = sqrt(queries.size()) + 1;
    const int BLOCK_SIZE = 555;
    sort(queries.begin(), queries.end(), [&](const array<int, 3>& a, const array<int, 3>& b) {
        int blockA = a[0] / BLOCK_SIZE;
        int blockB = b[0] / BLOCK_SIZE;
        if (blockA != blockB)
            return blockA < blockB;
        return a[1] < b[1];
    });
    auto[L, R, id] = queries[0];
    int curr_l = L, curr_r = R;
    distinct = 1;
    freq[a[curr_l]]++;
    vector<int> ans(queries.size());
    for(auto [L, R, id] : queries) {
        adjust(curr_l, curr_r, L, R);
        ans[id] = distinct;
    }
    for(auto x : ans) cout << x << "\n";
}

```

Inversion.h

Description: ...

Time: $\mathcal{O}(\log N)$

<bits/stdc++.h>

d41d8c, 78 lines

```

// https://codeforces.com/contest/1983/problem/D
using namespace std;
int inversion_count = 0;

```

```

void merge(vector<int> &v, int l, int r, int mid) {
    int l_sz = mid - 1 + 1;
    int r_sz = r - (mid + 1) + 1;
    int L[l_sz], R[r_sz];

```

```

for(int i = 0; i < l_sz; i++)L[i] = v[i+1];
for(int i = 0; i < r_sz; i++)R[i] = v[i+mid+1];
int l_i = 0, r_i = 0;

```

```

for(int i = 1; i <= r && l_i < l_sz && r_i < r_sz; i++) {
    if(L[l_i] <= R[r_i])l_i++;
    else if(L[l_i] > R[r_i]){
        inversion_count+=(l_sz-1)-l_i+1;
        r_i++;
    }
}
l_i = 0, r_i = 0;
for(int i = 1; i <= r; i++) {
    if(r_i == r_sz) {
        v[i] = L[l_i];
        l_i++;
    }else if(l_i == l_sz) {
        v[i] = R[r_i];
        r_i++;
    }else if(L[l_i] <= R[r_i]) {
        v[i] = L[l_i];
        l_i++;
    }else {
        v[i] = R[r_i];
        r_i++;
    }
}
}

```

// 0-base indexing

```

void mergeSort(vector<int> &v, int l, int r) {
    if(l == r)return;
    int mid = l + (r - l)/2;
    mergeSort(v, l, mid);
    mergeSort(v, mid+1, r);
    merge(v, l, r, mid);
}

```

```

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    cin >> tt;
    while(tt--) {
        int n;
        cin >> n;
        vector<int> v(n), _v(n);
        set<int> st, _st;
        for(int i = 0; i < n; i++) {
            cin >> v[i];
            st.insert(v[i]);
        }
        for(int i = 0; i < n; i++) {
            cin >> _v[i];
            _st.insert(_v[i]);
        }
        if(st != _st) {
            cout << "No\n";
            continue;
        }
        mergeSort(v, 0, n-1);
        int count_l = inversion_count;
        inversion_count = 0;
        mergeSort(_v, 0, n-1);
        if((count_l % 2 == 0 && inversion_count % 2 == 0) || (
            count_l % 2 != 0 && inversion_count % 2 != 0)) {
            cout << "Yes\n";
        }else cout << "No\n";
        inversion_count = 0;
    }
}

```

```

    return 0;
}

```

XORHash.h

Description: ...

Time: $\mathcal{O}(\log N)$

<bits/stdc++.h>

d41d8c, 47 lines

```

// https://codeforces.com/contest/2014/problem/H
using namespace std;
#define ll long

```

```

mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    count());

```

```

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    cin >> tt;
    while(tt--) {
        int n, q;
        cin >> n >> q;
        vector<ll> v(n);
        map<int, int> compress;
        int id = 0;
        for(int i = 0; i < n; i++) {
            cin >> v[i];
            if(compress.find(v[i]) != compress.end()) {
                v[i] = compress[v[i]];
            } else {
                compress[v[i]] = id++;
                v[i] = compress[v[i]];
            }
        }
        vector<ll> rv(n);
        for(int i = 0; i < n; i++) {
            rv[i] = rng();
        }
        for(int i = 0; i < n; i++) {
            v[i] = rv[v[i]];
        }
        vector<ll> pfx(n+1, 0);
        for(int i = 1; i <= n; i++) {
            pfx[i] = (pfx[i-1] ^ v[i-1]);
        }
        while(q--) {
            int l, r;
            cin >> l >> r;
            if(pfx[l-1] == pfx[r]) {
                cout << "YES\n";
            }else cout << "NO\n";
        }
    }
    return 0;
}

```

Numerical (4)

4.1 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.

Time: $\mathcal{O}(N^3)$

d41d8c, 15 lines

```

double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;

```



```

rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
if (i != b) swap(a[i], a[b]), res *= -1;
res *= a[i][i];
if (res == 0) return 0;
rep(j,i+1,n) {
    double v = a[j][i] / a[i][i];
    if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
}
}
return res;
}

```

Number theory (5)

5.1 Modular arithmetic

ModPow.h

d41d8c, 11 lines

```

int bigPow(ll base, ll power, const int mod) {
    int ans = 1 % mod;
    base %= mod;
    if (base < 0) base += mod;
    while (power) {
        if (power & 1) ans = (ll) ans * base % mod;
        base = (ll) base * base % mod;
        power >>= 1;
    }
    return ans;
}

```

MatrixExpo.h

<bits/stdc++.h> d41d8c, 52 lines

// <https://codeforces.com/gym/102644/problem/C>

```

using namespace std;
#define ll long long
const int M = 1e9 + 7;

```

```

struct Matrix {
    int a[2][2] = {{0, 0}, {0, 0}};
    Matrix operator *(const Matrix& other) {
        Matrix product;
        for (int i = 0; i < 2; i++) {
            for (int j = 0; j < 2; j++) {
                for (int k = 0; k < 2; k++) {
                    product.a[i][j] = (product.a[i][k] + (ll) a[i][k] * other.a[j][k]) % M;
                }
            }
        }
        return product;
    }
};

```

```

Matrix expo_power(Matrix a, ll k) {
    Matrix product;
    for (int i = 0; i < 2; i++) {
        product.a[i][i] = 1;
    }
    while (k > 0) {
        if (k % 2) {
            product = product * a;
        }
        a = a * a;
        k /= 2;
    }
    return product;
}
int main() {

```

```

ios::sync_with_stdio(false);
cin.tie(0);
int tt;
tt = 1;
// cin >> tt;
while(tt--) {
    ll k;
    cin >> k;
    Matrix M;
    M.a[0][0] = 1;
    M.a[0][1] = 1;
    M.a[1][0] = 1;
    M.a[1][1] = 0;
    cout << expo_power(M, k).a[1][0] << "\n";
}
return 0;
}

```

SumProductCountOfDivisors.h

<bits/stdc++.h> d41d8c, 57 lines

```

/*
Problem Link: https://cses.fi/problemset/task/2182/
*/

```

```

using namespace std;
const int M = 1e9 + 7;
#define ll long long

```

```

int bigPow(ll base, ll power, const int mod) {
    int ans = 1 % mod;
    base %= mod;
    if (base < 0) base += mod;
    while (power) {
        if (power & 1) ans = (ll) ans * base % mod;
        base = (ll) base * base % mod;
        power >>= 1;
    }
    return ans;
}

```

```

}
// S_n = a(1-r^n)/(1-r)
int geometricSeriesSum(int r, int n) {
    int nu = bigPow(r, n, M) - 1; // Numerator
    int de = r - 1; // Denominator
    de = bigPow(de, M-2, M);
    return nu*llL*de % M;
}

```

```

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n;
        cin >> n;
        ll cnt = 1, sum = 1, prod = 1, num1 = 1, num2 = 1, pw = 1;
        bool ok = true;
        for(int i = 0; i < n; i++) {
            int x, k;
            cin >> x >> k;
            cnt = cnt * (k + 1) % M;
            sum = sum * geometricSeriesSum(x, k+1) % M;
            num1 = num1 * bigPow(x, k, M) % M;
            num2 = num2 * bigPow(x, k/2, M) % M;
            if(k % 2 != 0 && ok) {
                pw = (pw * (k+1)/2) % (M-1);
                ok = false;
            } else {
                pw = (pw * (k+1)) % (M-1);
            }
        }
    }
}

```

```

    }
}
// Product of divisors = (Num)^(d(Num)/2)
if(!ok) prod = bigPow(num1, pw, M);
else prod = bigPow(num2, pw, M);
cout << cnt << " " << sum << " " << prod << "\n";
}
return 0;
}

```

Sieve.h

<bits/stdc++.h> d41d8c, 105 lines

```

using namespace std;
#define ll long long
const int N = 100000;
vector<bool> is_prime(N+1, true);

```

```

// O(Nlog(N))
void divisors() {
    vector<vector<int>> d(N+1);
    for(int i = 1; i <= N; i++) {
        for(int j = i; j <= N; j+=i) {
            d[j].push_back(i);
        }
    }
}
// O(sqrt(N))
vector<ll> divisor(ll a) {
    vector<ll> divisors;
    for (ll i = 1; i*i <= a; ++i) {
        if(a % i == 0) {
            if(a / i == i) divisors.push_back(i);
            else {
                divisors.push_back(i);
                divisors.push_back(a/i);
            }
        }
    }
    return divisors;
}
// O(Nlog(log(N)))
void sieve() {
    is_prime[0] = is_prime[1] = false;
    for (int i = 2; i * i <= N; i++) {
        if (is_prime[i]) {
            for (int j = i * i; j <= N; j += i)
                is_prime[j] = false;
        }
    }
}
// O(sqrt(N))
vector<ll> prime_factorization(ll n) {
    vector<ll> factorization;
    while (n % 2 == 0) {
        factorization.push_back(2);
        n /= 2;
    }
    for (ll d = 3; d * d <= n; d += 2) {
        while (n % d == 0) {
            factorization.push_back(d);
            n /= d;
        }
    }
    if (n > 1) factorization.push_back(n);
    return factorization;
}
// O(sqrt(N))
int phi(int n) {
    int result = n;

```



```
for (int i = 2; i * i <= n; i++) {
    if (n % i == 0) {
        while (n % i == 0)
            n /= i;
        result -= result / i;
    }
}
if (n > 1)
    result -= result / n;
return result;
}
// O(Nloglog(N))
void phi_1_to_n(int n) {
    vector<int> phi(n + 1);
    for (int i = 0; i <= n; i++)
        phi[i] = i;

    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}
// O(Nloglog(N))
void phi_1_to_n_(int n) {
    vector<int> phi(n + 1);
    phi[0] = 0;
    phi[1] = 1;
    for (int i = 2; i <= n; i++)
        phi[i] = i - 1;

    for (int i = 2; i <= n; i++)
        for (int j = 2 * i; j <= n; j += i)
            phi[j] -= phi[i];
}
int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n;
        cin >> n;

        return 0;
    }
}
```

5.2 Primality

5.3 Divisibility

5.3.1 Chinese Remainder Theorem

Let $m = m_1 \cdot m_2 \cdots m_k$, where m_i are pairwise coprime. In addition to m_i , we are also given a system of congruences

$$\begin{cases} a &\equiv a_1 \pmod{m_1} \\ a &\equiv a_2 \pmod{m_2} \\ &\vdots \\ a &\equiv a_k \pmod{m_k} \end{cases}$$

where a_i are some given constants. CRT will give the unique solution modulo m .

5.3.2 Bézout’s identity

For $a \neq 0, b \neq 0$, then $d = \gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

5.4 Fractions

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0, k > 0, m \perp n$, and either m or n even.

5.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.7 Fibonacci

Fibonacci numbers are defined by

$$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}. \text{ Again, } F_n = \frac{\phi^n - \hat{\phi}^n}{\sqrt{5}} \approx \frac{\phi^n}{\sqrt{5}},$$

where $\phi = \frac{1+\sqrt{5}}{2}$ and $\hat{\phi} = \frac{1-\sqrt{5}}{2}$. Some important properties of Fibonacci numbers:

$$\begin{aligned} F_{n-1}F_{n+1} - F_n^2 &= (-1)^n & F_{n+k} &= F_{k-1}F_n + F_kF_{n+1} \\ F_{2n} &= F_n(F_{n-1} + F_{n+1}) & F_{2n+1} &= F_n^2 + F_{n+1}^2 \\ n|m &\Leftrightarrow F_n|F_m & \gcd(F_m, F_n) &= F_{\gcd(m,n)} \end{aligned}$$

Fibonacci.h

```
Description: nthFibonacci
Time: O(log n)
d41d8c, 8 lines

11 f(11 n) {
    if (n == 0 || n == 1) return dp[n] = 1;
    if (dp[n]) return dp[n];
    11 k = n/2;
    if (n % 2 == 0) return dp[n] = (f(k)*f(k) + f(k-1)*f(k-1)) % M;
    return dp[n] = (f(k)*f(k+1) + f(k-1) * f(k)) % M;
}
(n == 0 ? 0 : f(n-1));
```

5.8 Estimates

$$\sum_{d|n} d = O(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.9 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g(n/d)$$

Other useful formulas/forms:

$$\sum_{d|n} \mu(d) = [n = 1] \text{ (very useful)}$$

$$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n)g(d)$$

$$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m)g(\lfloor \frac{n}{m} \rfloor)$$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.
Time: $O(n)$

```
int permToInt(vi& v) {
    int use = 0, i = 0, r = 0;
    for (int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
}
```

6.1.2 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^\infty g_S(n) \frac{x^n}{n!} = \exp \left(\sum_{n \in S} \frac{x^n}{n} \right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n)=(n-1)(D(n-1)+D(n-2))=nD(n-1)+(-1)^n=\left\lfloor\frac{n!}{e}\right\rfloor$$

6.1.4 Burnside’s lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x=x$).

If $f(n)$ counts “configurations” (of some sort) of length n , we can ignore rotational symmetry using $G=\mathbb{Z}_n$ to get

$$g(n)=\frac{1}{n}\sum_{k=0}^{n-1}f(\gcd(n,k))=\frac{1}{n}\sum_{k|n}f(k)\phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0)=1,\;p(n)=\sum_{k\in\mathbb{Z}\setminus\{0\}}(-1)^{k+1}p(n-k(3k-1)/2)$$

$$p(n)\sim 0.145/n\cdot\exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2\text{e}5$	$\sim 2\text{e}8$

6.2.2 Lucas’ Theorem

Let n,m be non-negative integers and p a prime. Write $n=n_kp^k+\dots+n_1p+n_0$ and $m=m_kp^k+\dots+m_1p+m_0$. Then $\binom{n}{m}\equiv\prod_{i=0}^k\binom{n_i}{m_i}\pmod p$.

6.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1+\dots+k_n}{k_1,k_2,\dots,k_n}=\frac{(\sum k_i)!}{k_1!k_2!\dots k_n!}$.

d41d8c, 5 lines

```
11 multinomial(vi& v) {
11   c = 1, m = v.empty() ? 1 : v[0];
11   rep(i,1,sz(v)) rep(j,0,v[i]) c = c * ++m / (j+1);
11   return c;
}
```

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t)=\frac{t}{e^t-1}$ (FFT-able).
 $B[0,\dots]=[1,-\frac{1}{2},\frac{1}{6},0,-\frac{1}{30},0,\frac{1}{42},\dots]$

Sums of powers:

$$\sum_{i=1}^n n^m=\frac{1}{m+1}\sum_{k=0}^m\binom{m+1}{k}B_k\cdot(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned}\sum_{i=m}^\infty f(i) &= \int_m^\infty f(x)dx - \sum_{k=1}^\infty \frac{B_k}{k!}f^{(k-1)}(m) \\ &\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))\end{aligned}$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$\begin{aligned}c(n,k) &= c(n-1,k-1) + (n-1)c(n-1,k),\; c(0,0)=1 \\ \sum_{k=0}^n c(n,k)x^k &= x(x+1)\dots(x+n-1)\end{aligned}$$

$$\begin{aligned}c(8,k) &= 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1 \\ c(n,2) &= 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots\end{aligned}$$

6.3.3 Eulerian numbers

Number of permutations $\pi\in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j)>\pi(j+1)$, $k+1$ j :s s.t. $\pi(j)\geq j$, k j :s s.t. $\pi(j)> j$.

$$E(n,k)=(n-k)E(n-1,k-1)+(k+1)E(n-1,k)$$

$$E(n,0)=E(n,n-1)=1$$

$$E(n,k)=\sum_{j=0}^k(-1)^j\binom{n+1}{j}(k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k)=S(n-1,k-1)+kS(n-1,k)$$

$$S(n,1)=S(n,n)=1$$

$$S(n,k)=\frac{1}{k!}\sum_{j=0}^k(-1)^{k-j}\binom{k}{j}j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n)=1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ For p prime,

$$B(p^m+n)\equiv mB(n)+B(n+1)\pmod p$$

6.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i : $n_1n_2\cdots n_kn^{k-2}$
with degrees d_i : $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

6.3.7 Catalan numbers

$$C_n=\frac{1}{n+1}\binom{2n}{n}=\binom{2n}{n}-\binom{2n}{n+1}=\frac{(2n)!}{(n+1)!n!}$$

$$C_0=1,\;C_{n+1}=\frac{2(2n+1)}{n+2}C_n,\;C_{n+1}=\sum C_iC_{n-i}$$

$$C_n=1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an $n\times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2\max|w_i|<\sim 2^{63}$.
Time: $\mathcal{O}(VE)$

d41d8c, 64 lines

```
void BellmanFord(int st, int n) {
    vector<ll> dist(n+1, INF);
    vector<int> parent(n+1, -1);
    dist[st] = 0;
    for (int i = 0; i < n-1; i++) {
        bool any = false;
        for (auto[u, v, cost] : edges)
            if (dist[u] < INF)
                if (dist[v] > dist[u] + cost) {
                    dist[v] = dist[u] + cost;
                    parent[v] = u;
                    any = true;
                }
        if (!any) break;
    }
    if (dist[n] == INF)
        cout << "-1\n";
    else {
        vector<int> path;
        for (int cur = n; cur != -1; cur = parent[cur])
            path.push_back(cur);
        reverse(path.begin(), path.end());
        for (int u : path)
            cout << u << ' ';
    }
}

void BellmanFord(int s, int n) {
    vector<ll> dist(n+1, 0); // No need to init INF here because
                           // there can be a negative cycle where you can't reach
                           // from node 1
                           // and the Graph is not necessarily
                           // connected
                           // Our concern is about to find
                           // negative cycle not shortest
                           // distance
}
```

```

vector<int> parent(n+1, -1);
dist[s] = 0;
int flag;
for (int i = 0; i < n; i++) {
    flag = -1;
    for (auto [u, v, cost] : edges) {
        if (dist[u] + cost < dist[v]) {
            dist[v] = dist[u] + cost;
            parent[v] = u;
            flag = v;
        }
    }
}
if (flag == -1)
    cout << "NO\n";
else {
    int y = flag;
    for (int i = 0; i < n; ++i)
        y = parent[y];

    vector<int> path;
    for (int cur = y; cur = parent[cur]) {
        path.push_back(cur);
        if (cur == y && path.size() > 1)
            break;
    }
    reverse(path.begin(), path.end());
    cout << "YES\n";
    for (int u : path)
        cout << u << ' ';
}
}

```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m , where $m[i][j] = \text{inf}$ if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j , inf if no path, or $-\text{inf}$ if the path goes through a negative-weight cycle.

Time: $\mathcal{O}(N^3)$ d41d8c, 19 lines

```

void init() {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            d[i][j] = INF;
        }
        d[i][i] = 0;
    }
}

void floydWarshall() {
    for (int k = 0; k < n; ++k) {
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) {
                if (d[i][k] < INF && d[k][j] < INF) {
                    d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
                }
            }
        }
    }
}

```

Dijkstra.h

Description: Dijkstra

d41d8c, 22 lines

```

vector<ll> dijkstra(int s, int n, vector<vector<pair<int, ll>>>
    &adj) {
    vector<ll> dist(n+1, INF);
    dist[s] = 0;

```

```

priority_queue<pair<ll, int>, vector<pair<ll, int>>,
    greater<pair<ll, int>>> pq;
pq.push({0, s});
bool vis[n+1];
memset(vis, false, sizeof(vis));
while (!pq.empty()) {
    auto [d, u] = pq.top();
    pq.pop();
    if (vis[u]) continue;
    vis[u] = true;
    for (auto [v, wt] : adj[u]) {
        ll _d = d + wt;
        if (_d < dist[v]) {
            dist[v] = _d;
            pq.push({_d, v});
        }
    }
}
return dist;
}

```

Cycle.h

Description: Heavy Light Decomposition

d41d8c, 130 lines

```

/*
Problem Name: Round Trip II (Directed)
Problem Link: https://cses.fi/problemset/task/1678/
*/
using namespace std;
const int N = 100000;
vector<int> adj[N+1], parent(N+1);
vector<int> color(N+1);
int st, en;

```

```

bool dfs(int u) {
    color[u] = 1;
    for (auto v : adj[u]) {
        if (!color[v]) {
            parent[v] = u;
            if (dfs(v)) return true;
        } else if (color[v] == 1) {
            st = v, en = u;
            return true;
        }
    }
    color[u] = 2;
    return false;
}

void checkCycle(int n) {
    st = en = -1;
    for (int i = 1; i <= n; i++) {
        if (!color[i] && dfs(i)) {
            break;
        }
    }
    if (st == -1) cout << "IMPOSSIBLE\n";
    else {
        vector<int> path;
        path.push_back(st);
        for (int i = en; i != st; i = parent[i]) path.push_back(i);
        path.push_back(st);
        reverse(path.begin(), path.end());
        cout << path.size() << "\n";
        for (auto i : path) cout << i << " ";
        cout << "\n";
    }
}

int main() {

```

```

ios::sync_with_stdio(false);
cin.tie(0);
int tt;
tt = 1;
while (tt--) {
    int n, m;
    cin >> n >> m;
    for (int i = 0; i < m; i++) {
        int u, v;
        cin >> u >> v;
        adj[u].push_back(v);
    }
    checkCycle(n);
}
return 0;
}

/*
Problem Name: Round Trip (Undirected)
Problem Link: https://cses.fi/problemset/task/1669/
*/
using namespace std;
const int N = 100000;
vector<int> adj[N + 1];
bool vis[N + 1], cycle_found;
int parent[N + 1], en, st;

void dfs(int u, int p) {
    vis[u] = true;
    for (auto v : adj[u]) {
        if (p == v)
            continue;
        if (!vis[v]) {
            parent[v] = u;
            dfs(v, u);
        }
        else {
            if (!cycle_found)
                en = u, st = v;
            cycle_found = true;
            return;
        }
    }
}

void checkCycle(int n) {
    for (int i = 1; i <= n; i++) {
        if (!vis[i]) {
            parent[i] = -1;
            dfs(i, -1);
        }
    }
    if (!cycle_found) {
        cout << "IMPOSSIBLE\n";
        return;
    }
    vector<int> path;
    path.push_back(st);
    path.push_back(en);
    int j = en;
    while (parent[j] != st) {
        path.push_back(parent[j]);
        j = parent[j];
    }
    path.push_back(st);
    cout << path.size() << "\n";
    for (int i = path.size() - 1; i >= 0; i--)
        cout << path[i] << " ";
}

int main() {

```

```
ios::sync_with_stdio(false);
cin.tie(0);
int tt;
tt = 1;
while (tt--) {
    int n, m;
    cin >> n >> m;
    for (int i = 0; i < m; i++) {
        int u, v;
        cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    checkCycle(n);
}
return 0;
}
```

7.2 Network flow

MinCostMaxFlow.h
Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: $\mathcal{O}(FE \log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi.

```
const int N = 500;
vector<int> adj[N+1];
int capacity[N+1][N+1];

int bfs(int s, int d, int n, vector<int> &parent) {
    parent.assign(n+1, -1);
    parent[s] = 0;
    queue<pair<int, int>> q;
    q.push({s, INT_MAX});
    while(!q.empty()) {
        int u = q.front().first;
        int f = q.front().second;
        q.pop();
        for(auto v : adj[u]) {
            if(parent[v] == -1 && capacity[u][v]) {
                parent[v] = u;
                int n_f = min(f, capacity[u][v]);
                if(v == d) return n_f;
                q.push({v, n_f});
            }
        }
    }
    return 0;
}

int max_flow(int s, int d, int n) {
    int mx_flow = 0;
    vector<int> parent;
    int flow;
    while(flow = bfs(s, d, n, parent)) {
        mx_flow+=flow;
        int now = d;
        while(now != s) {
            int prev = parent[now];
            capacity[prev][now] -= flow;
            capacity[now][prev] += flow;
            now = prev;
        }
    }
    return mx_flow;
}

bool visited[N+1];
void dfs(int u) {
    visited[u] = true;
```

```
for(auto v : adj[u]) if(!visited[v] && capacity[u][v]) dfs(v);
};

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, m;
        cin >> n >> m;
        for(int i = 0; i < m; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            adj[v].push_back(u);
            capacity[u][v] += 1;
            capacity[v][u] += 1;
        }
        cout << max_flow(1, n, n) << "\n";
        dfs(1);
        for(int u = 1; u <= n; u++) {
            if(visited[u]) {
                for(auto v : adj[u]) {
                    if(!visited[v]) {
                        cout << u << " " << v << "\n";
                    }
                }
            }
        }
    }
    return 0;
}
```

7.3 Matching

7.4 DFS algorithms

SCC.h
Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: scc(graph, [&](vi& v) { ... }) visits all components in reverse topological order. comp[i] holds the component index of a node (a component only has edges to components with lower index). ncomps will contain the number of components.
Time: $\mathcal{O}(E+V)$

```
struct SCC {
    // 1-base indexing
    int n;
    vector<vector<int>> adj, radj;
    vector<int> todo, comps, id;
    vector<bool> vis;
    void init(int _n) {
        n = _n;
        adj.resize(n+1), radj.resize(n+1), id.assign(n+1, -1),
            vis.resize(n+1);
    }
    void build(int x, int y) { adj[x].push_back(y), radj[y].push_back(x); }
    void dfs(int x) {
        vis[x] = 1;
        for(auto y : adj[x]) if (!vis[y]) dfs(y);
        todo.push_back(x);
    }
    void dfs2(int x, int v) {
        id[x] = v;
        for(auto y : radj[x]) if (id[y] == -1) dfs2(y, v);
    }
}
```

```
}
void gen() {
    for(int i = 1; i <= n; i++) if (!vis[i]) dfs(i);
    reverse(todo.begin(), todo.end());
    for(auto x : todo) if (id[x] == -1) {
        dfs2(x, x);
        comps.push_back(x);
    }
}
} scc;
```

ArticulationPoint.h
Description: Finding articulation points in a graph.

```
vector<int> adj[N];
int t = 0;
vector<int> tin(N, -1), low(N), ap;
void dfs(int u, int p) {
    tin[u] = low[u] = t++;
    int is_ap = 0, child = 0;
    for (int v : adj[u]) {
        if (v != p) {
            if (tin[v] != -1) {
                low[u] = min(low[u], tin[v]);
            } else {
                child++;
                dfs(v, u);
                if (tin[u] <= low[v]) is_ap = 1;
                low[u] = min(low[u], low[v]);
            }
        }
    }
    if ((p != -1 or child > 1) and is_ap)
        ap.push_back(u);
}
dfs(0, -1);
```

Bridge.h
Description: Finds all the bridges in a graph.

```
void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    bool parent_skipped = false;
    for (int to : adj[v]) {
        if (to == p && !parent_skipped) {
            parent_skipped = true;
            continue;
        }
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
            dfs(to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] > tin[v])
                IS_BRIDGE(v, to);
        }
    }
}
```

2sat.h
Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type (a||b)&&(!a||c)&&(d||b)&&... becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: TwoSat ts(number of boolean variables);
 ts.either(0, ~3); // Var 0 is true or var 3 is false
 ts.setValue(2); // Var 2 is true
 ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
 ts.solve(); // Returns true iff it is solvable
 ts.values[0..N-1] holds the assigned values to the vars
Time: $O(N + E)$, where N is the number of boolean variables, and E is the number of clauses.

d41d8c, 80 lines

```
struct _2SAT {
    // 0-base indexing
    int n;
    vector<vector<int>>> adj, radj;
    vector<int> todo, comps, id;
    vector<bool> vis, assignment;
    void init(int _n) {
        n = _n;
        adj.resize(n), radj.resize(n), id.assign(n, -1), vis.
            resize(n);
        assignment.assign(n/2, false);
    }
    void build(int x, int y) { adj[x].push_back(y), radj[y].
        push_back(x); }
    void dfs1(int x) {
        vis[x] = 1;
        for(auto y : adj[x]) if (!vis[y]) dfs1(y);
        todo.push_back(x);
    }
    void dfs2(int x, int v) {
        id[x] = v;
        for(auto y : radj[x]) if (id[y] == -1) dfs2(y, v);
    }
    bool solve_2SAT() {
        for(int i = 0; i < n; i++) if (!vis[i]) dfs1(i);
        reverse(todo.begin(), todo.end());
        int j = 0;
        for(auto x : todo) if (id[x] == -1) {
            dfs2(x, j++);
            // comps.push_back(x);
        }
        for (int i = 0; i < n; i += 2) {
            if (id[i] == id[i + 1]) {
                return false;
            }
            assignment[i / 2] = id[i] > id[i + 1];
        }
        return true;
    }
    void add_disjunction(int a, bool na, int b, bool nb) {
        // na and nb signify whether a and b are to be negated
        a = 2 * a ^ na;
        b = 2 * b ^ nb;
        int neg_a = a ^ 1;
        int neg_b = b ^ 1;
        build(neg_a, b);
        build(neg_b, a);
    }
} _2sat;

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, m;
        cin >> n >> m;
        _2sat.init(m*2);
```

```
        for(int i = 0; i < n; i++) {
            int a, b;
            char _na, _nb;
            cin >> _na >> a >> _nb >> b;
            bool na, nb;
            --a, --b;
            if(_na == '+') na = false;
            else na = true;
            if(_nb == '+') nb = false;
            else nb = true;
            _2sat.add_disjunction(a, na, b, nb);
        }
        bool possible = _2sat.solve_2SAT();
        if(possible) {
            for(int i = 0; i < m; i++) {
                if(_2sat.assignment[i]) cout << "+ ";
                else cout << "- ";
            }
            cout << "IMPOSSIBLE";
        }
        return 0;
    }
}
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

Time: $O(V + E)$

<bits/stdc++.h>, <bits/stdc++.h> d41d8c, 114 lines

```
/*
Problem Link: https://cses.fi/problemset/task/1691/
Idea: Euler Circuit in undirected graph Hierholzer Algorithm
*/
using namespace std;
const int N = 100000;
vector<pair<int, int>> adj[N+1];
int degree[N+1];
bool visited[2*N+1]; // total edge size
vector<int> euler_path;
```

```
void dfs(int u) {
    while(!adj[u].empty()) {
        auto [v, idx] = adj[u].back();
        adj[u].pop_back();
        if(visited[idx]) continue;
        visited[idx] = true;
        dfs(v);
    }
    euler_path.push_back(u);
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, m;
        cin >> n >> m;
        for(int i = 0; i < m; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back({v, i});
            adj[v].push_back({u, i});
            degree[u]++, degree[v]++;
        }
    }
    /*
```

Undirected Graphs:
 Euler Circuit: All vertices must have even degree.
 Euler Path: Exactly zero or two vertices can have odd degree.

```
*/
for(int i = 1; i <= n; i++) {
    if(degree[i] % 2 != 0) {
        cout << "IMPOSSIBLE\n";
        return 0;
    }
}
dfs(1);
if(euler_path.size() != m+1) {
    cout << "IMPOSSIBLE\n";
    return 0;
}
for(auto x : euler_path) {cout << x << " ";}
return 0;
}
/*
Problem Link: https://cses.fi/problemset/task/1693/
Idea: Euler Path in Directed graph Hierholzer Algorithm
*/
using namespace std;
const int N = 100000;
vector<int> adj[N+1];
int in[N+1], out[N+1];
vector<int> euler_path;

void dfs(int u) {
    while(!adj[u].empty()) {
        int v = adj[u].back();
        adj[u].pop_back();
        dfs(v);
    }
    euler_path.push_back(u);
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, m;
        cin >> n >> m;
        for(int i = 0; i < m; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            out[u]++, in[v]++;
        }
        /*
        Directed Graphs:
        Euler Circuit: All vertices must have equal in-degree
        and out-degree.
        Euler Path: Exactly two vertices can have a difference
        of one between their in-degree and out-degree.
        */
        for(int i = 1; i <= n; i++) {
            if((i == 1 && out[1]-in[1] != 1) ||
               (i == n && in[n]-out[n] != 1) ||
               (i > 1 && i < n && out[i] != in[i])) {
                cout << "IMPOSSIBLE\n";
                return 0;
            }
        }
        dfs(1);
```

```

reverse(euler_path.begin(), euler_path.end());
if(euler_path.size() - 1 != m || euler_path.back() !=
n) {
    cout << "IMPOSSIBLE\n";
    return 0;
}
for(auto x : euler_path) {cout << x << " ";}
return 0;
}

```

7.5 Coloring

7.6 Heuristics

7.7 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.

Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

d41d8c, 39 lines

```

const int N = 2e5 + 1;
const int LOG = 18; // LOG = ceil(log2(N))
vector<int> adj[N+1];
int up[N+5][LOG], depth[N+5]; // up[v][j] is the 2^j-th
    Ancestor of node v

void ancestor(int u) {
    for(auto v : adj[u]) {
        depth[v] = depth[u] + 1;
        up[v][0] = u;
        for(int j = 1; j < LOG; j++) up[v][j] = up[up[v][j-1]][j-1];
        ancestor(v);
    }
}

int get_lca(int a, int b) {
    if(depth[a] < depth[b]) swap(a, b);
    int k = depth[a] - depth[b];
    for(int i = LOG-1; i >= 0; i--)
        if(k & (1 << i))
            a = up[a][i];

    if(a == b)
        return a;

    for(int i = LOG-1; i >= 0; i--) {
        if(up[a][i] != up[b][i]) {
            a = up[a][i];
            b = up[b][i];
        }
    }
    return up[a][0];
}

/*
int getKthAncestor(int a, int k) {
    for(int i = LOG - 1; i >= 0; i--)
        if(k & (1 << i))
            a = up[a][i];
    return a;
}
*/

```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h"

d41d8c, 20 lines

```

struct LCA {
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;

    LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1), ret)) {}
    void dfs(vector<vi>& C, int v, int par) {
        time[v] = T++;
        for (int y : C[v]) if (y != par) {
            path.push_back(v), ret.push_back(time[v]);
            dfs(C, y, v);
        }
    }
    int lca(int a, int b) {
        if (a == b) return a;
        tie(a, b) = minmax(time[a], time[b]);
        return path[rmq.query(a, b)];
    }
    //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
};

```

DsuOnTree.h

Description: Dsu on tree

<bits/stdc++.h>

d41d8c, 88 lines

```

using namespace std;
const int N = 2e5 + 1;
int color[N+1];
vector<int> adj[N+1];

int idx = 0, euler[N+1], pos[N+1], sz[N+1], H_C[N+1];

void dfs(int u, int p) {
    pos[u] = idx;
    euler[idx++] = u;
    H_C[u] = -1, sz[u] = 1;
    for(auto v: adj[u]) {
        if(v == p) continue;
        dfs(v, u);
        sz[u] += sz[v];
        if(H_C[u] == -1 || sz[v] > sz[H_C[u]]) {
            H_C[u] = v;
        }
    }
}

int freq[N+1], cur_distinct = 0, distinct[N+1];
void add(int u) {
    freq[color[u]]++;
    if(freq[color[u]] == 1) cur_distinct++;
}

void remove(int u) {
    freq[color[u]]--;
    if(freq[color[u]] == 0) cur_distinct--;
}

void dsu(int u, int p, int keep) {
    for(auto v : adj[u]) {
        if(v == p || v == H_C[u]) continue;
        dsu(v, u, 0);
    }
    if(H_C[u] != -1) {
        dsu(H_C[u], u, 1);
    }

    for(auto v : adj[u]) {
        if(v == p || v == H_C[u]) continue;
        for(int i = pos[v]; i < pos[v] + sz[v]; i++) {

```

```

            add(euler[i]);
        }
    }
    add(u);
    distinct[u] = cur_distinct;

    if(!keep) {
        for(int i = pos[u]; i < pos[u] + sz[u]; i++) {
            remove(euler[i]);
        }
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n;
        cin >> n;
        map<int, int> compress;
        int id = 1;
        for(int i = 1; i <= n; i++) {
            cin >> color[i];
            if(compress[color[i]]) color[i] = compress[color[i]];
            else {
                compress[color[i]] = id++;
                color[i] = compress[color[i]];
            }
        }
        for(int i = 0; i < n-1; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            adj[v].push_back(u);
        }
        dfs(1, -1);
        dsu(1, -1, 1);
        for(int i = 1; i <= n; i++) cout << distinct[i] << " ";

        return 0;
    }
}

```

HLD.h

Description: Heavy Light Decomposition

<bits/stdc++.h>

d41d8c, 136 lines

```

/*
Problem Link: https://cses.fi/problemset/task/2134
*/
using namespace std;
const int N = 2e5 + 1;
int values[N+1], subtree[N+1], parent[N+1], depth[N+1];
int heavy[N+1], head[N+1], id[N+1];
vector<int> adj[N+1];

// 0 Base indexing
struct Segtree {
    int size;
    vector<int> tree;

    int merge(int x, int y) {
        return max(x, y);
    }
    void build(vector<int> &a, int node, int l, int r) {
        if(l == r) {
            tree[node] = a[l];

```



```

        return;
    }
    int mid = l + (r - 1)/2;
    build(a, node*2+1, l, mid);
    build(a, node*2+2, mid+1, r);
    tree[node] = merge(tree[node*2+1], tree[node*2+2]);
}
void update(int i, int value, int node, int l, int r) {
    if(l == i && r == i) {
        tree[node] = value;
        return;
    }
    int mid = l + (r-1)/2;
    if(i <= mid) update(i, value, node*2+1, l, mid);
    else update(i, value, node*2+2, mid+1, r);
    tree[node] = merge(tree[node*2+1], tree[node*2+2]);
}
void update(int i, int value) {
    update(i, value, 0, 0, size-1);
}
int query(int i, int j, int node, int l, int r) {
    if(l > j || r < i) return INT_MIN;
    if(l >= i && r <= j) return tree[node];
    int mid = l + (r - 1)/2;
    return merge(query(i, j, node*2+1, l, mid), query(i, j,
        node*2+2, mid+1, r));
}
int query(int i, int j) {
    return query(i, j, 0, 0, size-1);
}
int sz(int n) {
    int size = 1;
    while(size < n) size = size << 1;
    return 2*size-1;
}
void init(vector<int> &a, int n) {
    size = 1;
    while(size < n) size = size << 1;
    tree.resize(2*size-1);
    build(a, 0, 0, size-1);
}
} st;

void dfs(int u, int p) {
    subtree[u] = 1;
    int mx = 0;
    for(auto v : adj[u]) {
        if(v == p) continue;
        parent[v] = u;
        depth[v] = depth[u]+1;
        dfs(v, u);
        subtree[v] += subtree[u];
        if(subtree[v] > mx) {
            mx = subtree[v];
            heavy[u] = v;
        }
    }
}
int idx = 0;
void HLD(int u, int h) {
    head[u] = h;
    id[u] = idx++;
    if(heavy[u]) HLD(heavy[u], h);
    for(auto v : adj[u]) {
        if(v != parent[u] && v != heavy[u]) {
            HLD(v, v);
        }
    }
}
}
int idx = 0;
void HLD(int u, int h) {
    head[u] = h;
    id[u] = idx++;
    if(heavy[u]) HLD(heavy[u], h);
    for(auto v : adj[u]) {
        if(v != parent[u] && v != heavy[u]) {
            HLD(v, v);
        }
    }
}
}

```

```

int path(int x, int y) {
    int ans = 0;
    while(head[x] != head[y]) {
        if(depth[head[x]] > depth[head[y]]) swap(x, y);
        ans = max(ans, st.query(id[head[y]], id[y]));
        y = parent[head[y]];
    }
    if(depth[x] > depth[y]) swap(x, y);
    ans = max(ans, st.query(id[x], id[y]));
    return ans;
}
int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, q;
        cin >> n >> q;
        for(int i = 0; i < n; i++) cin >> values[i];
        for(int i = 0; i < n-1; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            adj[v].push_back(u);
        }
        dfs(1, -1);
        HLD(1, 1);
        vector<int> a(n);
        for(int i = 0; i < n; i++) a[id[i+1]] = values[i];
        st.init(a, n);
        while(q--) {
            int type;
            cin >> type;
            if(type == 1) {
                int s, x;
                cin >> s >> x;
                st.update(id[s], x);
            } else {
                int a, b;
                cin >> a >> b;
                cout << path(a, b) << " ";
            }
        }
        return 0;
    }
}

```

CentroidDecomp.h

Description: Centroid decompose, Finding 1 to K length Path Source : <https://www.codechef.com/problems/PRIMEDST>

d41d8c, 93 lines

```

const int N = 50001;
vector<int> adj[N];
int n, k;
int subtree[N], cnt[N], mx_depth, all_cnt[N];
bool visited[N];
// ll ans;

vector<bool> is_prime(N, true);
set<int> primes;
// O(Nlog(log(N)))
void sieve() {
    is_prime[0] = is_prime[1] = false;
    for (int i = 2; i * i <= N; i++) {
        if (is_prime[i]) {
            for (int j = i * i; j <= N; j += i)
                is_prime[j] = false;
        }
    }
}

```

```

    }
}
int getSubtree(int u, int p) {
    subtree[u] = 1;
    for(auto v : adj[u]) {
        if(!visited[v] && v != p) {
            getSubtree(v, u);
            subtree[u] += subtree[v];
        }
    }
    return subtree[u];
}
int getCentroid(int u, int p, int desired) {
    for(auto v : adj[u])
        if(!visited[v] && v != p && subtree[v] > desired)
            return getCentroid(v, u, desired);
    return u;
}
void compute(int u, int p, bool filling, int depth) {
    if(depth > k) return;
    mx_depth = max(mx_depth, depth);
    if(filling) {
        cnt[depth]++;
        all_cnt[depth]++;
    } else {
        // ans += cnt[k - depth] * 1LL;
        for(int i = 1; i <= mx_depth; i++) {
            if(cnt[i] all_cnt[i + depth] += cnt[i];
        }
    }
    for(auto v : adj[u]) if(!visited[v] && v != p) compute(v, u,
        filling, depth+1);
}
void centroidDecomposition(int u) {
    int centroid = getCentroid(u, -1, getSubtree(u, -1) >> 1);
    visited[centroid] = true;
    mx_depth = 0;
    for(auto v : adj[centroid]) {
        if(!visited[v]) {
            compute(v, centroid, false, 1);
            compute(v, centroid, true, 1);
        }
    }
    for(int i = 1; i <= mx_depth; i++) cnt[i] = 0;
    for(auto v : adj[centroid]) if(!visited[v])
        centroidDecomposition(v);
}
int main() {
    int tt;
    sieve();
    tt = 1;
    // cin >> tt;
    while(tt--) {
        cin >> n;
        for(int i = 2; i <= n-1; i++) {
            if(is_prime[i]) {
                primes.insert(i);
            }
        }
        for(int i = 0; i < n-1; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            adj[v].push_back(u);
        }
        // ans = 0;
        cnt[0] = 1;
        k = *primes.rbegin();
    }
}

```



```

centroidDecomposition(1);
ll p_path = 0;
for(auto x : primes) {
    p_path+=all_cnt[x];
}
ll total = n*1LL*(n-1)/2;
cout << fixed << setprecision(6) << (p_path*1.0)/(total
    *1.0) << "\n";
}
return 0;
}

```

DPOntree.h

Description: DPonTree

d41d8c, 64 lines

```

const int N = 100000;
int n, mod;
vector<int> adj[N];
// up[i] = total ways to paint all the ancestors of node i
// if the parent of node i is painted black.
vector<ll> up(N, 1);
// down[i] = total ways to paint the subtree of node i
// if the node i is painted black or white.
ll down[N];

```

```

void dfs1(int u, int parent) {
    down[u] = 1;
    for(auto v : adj[u]) {
        if(v == parent)continue;
        dfs1(v, u);
        down[u] = (down[u] * down[v]) % mod;
    }
    down[u] = (down[u] + 1) % mod;
}

```

```

void dfs2(int u, int parent) {
    int pref = 1;
    for(auto v : adj[u]) {
        if(v == parent)continue;
        up[v] = pref % mod;
        pref = pref*down[v] % mod;
    }
    reverse(adj[u].begin(), adj[u].end());
    int suff = 1;
    for(auto v : adj[u]) {
        if(v == parent)continue;
        up[v] = up[v]*suff % mod;
        suff = suff*down[v] % mod;
    }
    for(auto v : adj[u]) {
        if(v == parent)continue;
        up[v] = up[u] * up[v] % mod;
        up[v] = (up[v] + 1) % mod;
        dfs2(v, u);
    }
}

```

```

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        cin >> n >> mod;
        for(int i = 0; i < n-1; i++) {
            int u, v;
            cin >> u >> v;
            --v, --u;
            adj[u].push_back(v);

```

```

        adj[v].push_back(u);
    }
    dfs1(0, -1);
    dfs2(0, -1);
    for(int i = 0; i < n; i++) {
        cout << up[i]*(down[i] - 1 + mod) % mod << "\n";
    }
}
return 0;
}

```

7.8 Math

7.8.1 Number of Spanning Trees

Create an $N \times N$ matrix mat , and for each edge $a \rightarrow b \in G$, do $\text{mat}[a][b]--$, $\text{mat}[b][b]++$ (and $\text{mat}[b][a]--$, $\text{mat}[a][a]++$ if G is undirected). Remove the i th row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.8.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \dots \geq d_n$ exists iff $d_1 + \dots + d_n$ is even and for every $k = 1 \dots n$,

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

d41d8c, 172 lines

```

using ftype = ll;
const double eps = 1e-9;
const double PI = acos((double)-1.0);
int sign(double x) { return (x > eps) - (x < -eps);}

```

```

struct P {
    ftype x, y;
    P() {}
    P(ftype x, ftype y): x(x), y(y) {}
    void read() {
        cin >> x >> y;
    }
    P& operator+=(const P &t) {
        x += t.x;
        y += t.y;
        return *this;
    }
    P& operator-=(const P &t) {
        x -= t.x;
        y -= t.y;
        return *this;
    }
    P& operator*=(ftype t) {
        x *= t;
        y *= t;
        return *this;
    }
}

```

```

P& operator/=(ftype t) {
    x /= t;
    y /= t;
    return *this;
}
P operator+(const P &t) const {return P(*this) += t;}
P operator-(const P &t) const {return P(*this) -= t;}
P operator*(ftype t) const {return P(*this) *= t;}
P operator/(ftype t) const {return P(*this) /= t;}
bool operator == (P a) const { return sign(a.x - x) == 0 &&
    sign(a.y - y) == 0; }
bool operator != (P a) const { return !(*this == a); }
bool operator < (P a) const { return sign(a.x - x) == 0 ? y
    < a.y : x < a.x; }
bool operator > (P a) const { return sign(a.x - x) == 0 ? y
    > a.y : x > a.x; }
P perp() const {
    return P(y, -x); // Or P(y, -x) depending on the
        desired direction.
}
};

P operator*(ftype a, P b) {return b * a;}
inline ftype dot(P a, P b) {return a.x * b.x + a.y * b.y;}
inline ftype cross(P a, P b) {return a.x * b.y - a.y * b.x;}
ftype norm(P a) {return dot(a, a);}
double abs(P a) {return sqrt(norm(a));}
double proj(P a, P b) {return dot(a, b) / abs(b);}
double angle(P a, P b) {return acos(dot(a, b) / abs(a) / abs(b)
    );}
P intersect(P a1, P d1, P a2, P d2) {return a1 + cross(a2 - a1,
    d2) / cross(d1, d2) * d1;}

bool LineSegmentIntersection(P p1, P p2, P p3, P p4) {
    // Check if they are parallel
    if(cross(p1-p2, p3-p4) == 0) {
        // If they are not collinear
        if(cross(p2-p1, p3-p1) != 0) {
            return false;
        }
        // Check if they are collinear and do not intersect
        for(int it = 0; it < 2; it++) {
            if(max(p1.x, p2.x) < min(p3.x, p4.x) ||
                max(p1.y, p2.y) < min(p3.y, p4.y)) {
                return false;
            }
            swap(p1, p3), swap(p2, p4);
        }
        return true;
    }
    // Check one segment totally on the left or right side of
        other segment
    for(int it = 0; it < 2; it++) {
        ll sign1 = cross(p2-p1, p3-p1);
        ll sign2 = cross(p2-p1, p4-p1);
        if((sign1 < 0 && sign2 < 0) || (sign1 > 0 && sign2 > 0)
            ) {
            return false;
        }
        swap(p1, p3), swap(p2, p4);
    }
    // For all other case return true
    return true;
}

```

```

// here return value is area*2
ftype PolygonArea(vector<P> &polygon, int n) {
    ll area = 0;
    for(int i = 0; i < n; i++) {

```

```

    int j = (i+1) % n;
    area+=cross(polygon[i], polygon[j]);
}
return abs(area);
}

string PointInPolygon(vector<P> &polygon, int n, P &p) {
    int cnt = 0;
    for(int i = 0; i < n; i++) {
        int j = (i+1) % n;
        if(LineSegmentIntersection(polygon[i], polygon[j], p, p
        )) {
            return "BOUNDARY";
        }
        /*
        Imagine a vertically infinite line from point p to
        positive infinity.
        Check if a line from the polygon is totally on the left
        or right side of the infinite line and makes a
        positive cross product or positive triangle.
        Here, "right" means to the right or equal.
        */
        if((polygon[i].x >= p.x && polygon[j].x < p.x && cross(
            polygon[i]-p, polygon[j]-p) > 0) ||
            (polygon[i].x < p.x && polygon[j].x >= p.x && cross(
                polygon[j]-p, polygon[i]-p) > 0))
            cnt++;
    }
    if(cnt & 1) return "INSIDE";
    return "OUTSIDE";
}

void ConvexHull(vector<P> &points, int n) {
    vector<P> hull;
    sort(points.begin(), points.end());
    for(int rep = 0; rep < 2; rep++) {
        const int h = (int)hull.size();
        for(auto C : points) {
            while((int)hull.size() - h >= 2) {
                P A = hull[(int)hull.size()-2];
                P B = hull[(int)hull.size()-1];
                if(cross(B-A, C-A) <= 0) {
                    break;
                }
                hull.pop_back();
            }
            hull.push_back(C);
        }
        hull.pop_back();
        reverse(points.begin(), points.end());
    }
    cout << hull.size() << "\n";
    for(auto p : hull) {
        cout << p.x << " " << p.y << "\n";
    }
}

bool circleInter(P a, P b, double r1, double r2, pair<P, P>*
    out) {
    P vec = b - a;
    double d2 = norm(vec);
    double d = sqrt(d2);
    if (d > r1 + r2 || d < fabs(r1 - r2)) {
        return false;
    }
    double p = (d2 + r1 * r1 - r2 * r2) / (2 * d);
    double h2 = r1 * r1 - p * p;
    if (h2 < 0) h2 = 0;
    P mid = a + vec * (p / d);
    P per = vec.perp() * (sqrt(h2) / d);

```

```

    *out = {mid + per, mid - per};
    return true;
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n;
        cin >> n;
        vector<P> points;
        for(int i = 0; i < n; i++) {
            P p;
            p.read();
            points.push_back(p);
        }
        ConvexHull(points, n);
    }
    return 0;
}

```

8.2 Circles

8.3 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.

Time: $\mathcal{O}(n \log n)$

d41d8c, 64 lines

```

#define pii pair<ll, ll>
#define ff first
#define ss second

bool comparex(pii a, pii b) { return a.first < b.first; }
bool comparey(pii a, pii b) { return a.second < b.second; }
ll dist(pii x, pii y) { return (x.ff - y.ff) * (x.ff - y.ff) +
    (x.ss - y.ss) * (x.ss - y.ss); }

pair<pii, pii> closestAmongThree(pii a, pii b, pii c) {
    ll d1 = dist(a, b);
    ll d2 = dist(b, c);
    ll d3 = dist(a, c);
    ll mn = min({ d1, d2, d3 });
    if (mn == d1) return { a, b };
    else if (mn == d2) return { b, c };
    else return { a, c };
}

pair<pii, pii> closest(vector<pii> &points, ll st, ll en) {
    if (st + 1 == en) return { points[st], points[en] };
    if (st + 2 == en) return closestAmongThree(points[st],
        points[st + 1], points[en]);

    ll mid = st + (en - st) / 2;

    pair<pii, pii> left = closest(points, st, mid);
    pair<pii, pii> right = closest(points, mid + 1, en);
    ll left_d = dist(left.ff, left.ss);
    ll right_d = dist(right.ff, right.ss);
    ll d = min(left_d, right_d);
    pair<pii, pii> ans = (d == left_d) ? left : right;

    vector<pii> middle;
    for (int i = st; i <= en; i++)
        if (abs(points[i].ff - points[mid].ff) < d)
            middle.push_back(points[i]);
    sort(middle.begin(), middle.end(), comparey);

    for (int i = 0; i < (int)middle.size(); i++) {

```

```

        for (int j = i + 1; j < (int)middle.size() and (middle[
            j].ss - middle[i].ss) * (middle[j].ss - middle[i].
            ss) < d; j++) {
            ll dst = dist(middle[i], middle[j]);
            if (dst < d) {
                ans = { middle[i], middle[j] };
                d = dst;
            }
        }
        middle.clear();

        return ans;
    }

int main() {
    int tt;
    tt = 1;
    while (tt--) {
        int n;
        cin >> n;
        vector<pii> points(n);
        for (int i = 0; i < n; i++) {
            cin >> points[i].first >> points[i].second;
        }
        sort(points.begin(), points.end(), comparex);
        pair<pii, pii> ans = closest(points, 0, n - 1);
        cout << dist(ans.ff, ans.ss) << '\n';
    }

    return 0;
}

```

SweepLine.h

Description: Returns any intersecting segments, or -1, -1 if none exist.

Time: $\mathcal{O}(N \log N)$

Strings (9)

KMP.h

Description: $\text{pi}[x]$ computes the length of the longest prefix of s that ends at x , other than $s[0...x]$ itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

Time: $\mathcal{O}(n)$

d41d8c, 13 lines

```

vector<int> prefix_function(string s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}

```

Zfunc.h

Description: $z[i]$ computes the length of the longest common prefix of $s[i:]$ and s , except $z[0] = 0$. (abacaba -> 0010301)

Time: $\mathcal{O}(n)$

d41d8c, 18 lines

```

vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for (int i = 1; i < n; i++) {

```

```

        if(i < r) {
            z[i] = min(r - i, z[i - 1]);
        }
        while(i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if(i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    return z;
}

```

Manacher.h

Description: For each position in a string, computes $p[0][i]$ = half length of longest even palindrome around pos i , $p[1][i]$ = longest odd (half rounded down).

Time: $\mathcal{O}(N)$

d41d8c, 31 lines

```

vector<int> manacher(string t) {
    string s;
    for(auto c: t) {
        s += string("#") + c;
    }
    s+="#";
    int n = s.size();
    s = "$" + s + "^";
    vector<int> p(n + 2);
    int l = 1, r = 1;
    for(int i = 1; i <= n; i++) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while(s[i - p[i]] == s[i + p[i]]) {
            p[i]++;
        }
        if(i + p[i] > r) {
            l = i - p[i], r = i + p[i];
        }
    }
    return vector<int>(begin(p) + 1, end(p) - 1);
}

// 0-base indexing
bool is_palindrome(int l, int r, vector<int> &pal) {
    l++, r++;
    int range = (r - l) + 1;
    l = (l << 1) - 1;
    r = (r << 1) - 1;
    int mid = (l + r) >> 1;
    return pal[mid] >= range;
}

```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: rotate(v.begin(), v.begin() + minRotation(v), v.end());

Time: $\mathcal{O}(N)$

d41d8c, 24 lines

```

int least_rotation(const string &s) {
    int n = s.length();
    vector<int> f(2 * n, -1);
    int k = 0;

    for(int j = 1; j < 2 * n; ++j) {
        int i = f[j - k - 1];
        while(i != -1 && s[j % n] != s[(k + i + 1) % n]) {
            if(s[j % n] < s[(k + i + 1) % n]) {
                k = j - i - 1;
            }
            i = f[i];
        }
    }
}

```

```

    }
    if(i == -1 && s[j % n] != s[(k + i + 1) % n]) {
        if(s[j % n] < s[(k + i + 1) % n]) {
            k = j;
        }
        f[j - k] = -1;
    } else {
        f[j - k] = i + 1;
    }
}

return k;
}

```

SuffixArray.h

Description: Suffix Array

<bits/stdc++.h> d41d8c, 109 lines

```

/*
Problem Name: Finding Patterns
Problem Link: https://cses.fi/problemset/task/2102/
Idea: Suffix Array
Complexity:
Resource:
*/
using namespace std;

```

```

void radix_sort(vector<int> &p, vector<int> &c) {
    int n = p.size();

    vector<int> cnt(n);
    for(auto x: c) {
        cnt[x]++;
    }
}

```

```

vector<int> p_new(n);
vector<int> pos(n);
pos[0] = 0;
for(int i = 1; i < n; i++) {
    pos[i] = pos[i - 1] + cnt[i - 1];
}

```

```

for(auto x: p) {
    int i = c[x];
    p_new[pos[i]] = x;
    pos[i]++;
}
p = p_new;
}

```

```

void SA() {
    string s;
    cin >> s;
    s += "$";
    int n = s.size();
    vector<int> p(n), c(n);
}

```

```

// k = 0
vector<pair<char, int>> a(n);
for(int i = 0; i < n; i++) a[i] = {s[i], i};
sort(a.begin(), a.end());
for(int i = 0; i < n; i++) p[i] = a[i].second;
c[p[0]] = 0;
for(int i = 1; i < n; i++) {
    if(a[i].first == a[i - 1].first) {
        c[p[i]] = c[p[i - 1]];
    } else {
        c[p[i]] = c[p[i - 1]] + 1;
    }
}
}

```

int k = 0;

```

while((l << k) < n) {
    // k -> k + 1
    for(int i = 0; i < n; i++) {
        p[i] = (p[i] - (l << k) + n) % n;
    }
    radix_sort(p, c);
    vector<int> c_new(n);
    c_new[p[0]] = 0;
    for(int i = 1; i < n; i++) {
        pair<int, int> prev = {c[p[i - 1]], c[(p[i - 1] + (l << k)) % n]};
        pair<int, int> now = {c[p[i]], c[(p[i] + (l << k)) % n]};
        if(prev == now) {
            c_new[p[i]] = c_new[p[i - 1]];
        } else {
            c_new[p[i]] = c_new[p[i - 1]] + 1;
        }
    }
    c = c_new;
    k++;
}

```

```

int q;
cin >> q;
while(q--) {
    string t;
    cin >> t;
    int lo = 0, hi = n - 1;
    string ans = "NO\n";
}

```

```

while(lo <= hi) {
    int mid = lo + (hi - lo) / 2;
    string sub = s.substr(p[mid], min((int)t.size(), n - p[mid]));

    if(sub.compare(0, t.size(), t) == 0) {
        ans = "YES\n";
        break;
    } else if(t > sub) {
        lo = mid + 1;
    } else {
        hi = mid - 1;
    }
}
cout << ans;
}
}

```

```

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        SA();
    }
    return 0;
}

```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices $[l, r]$ into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining $[l, r]$ substrings. The root is 0 (has $l = -1, r = 0$), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

Time: $\mathcal{O}(26N)$

d41d8c, 47 lines

```

struct SuffixTree {

```

```
enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
int toi(char c) { return c - 'a'; }
string a; // v = cur node, q = cur position
int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;
void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
        if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
            p[m++]=v; v=s[v]; q=r[v]; goto suff; }
        v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
        l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
        p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
        l[v]=q; p[v]=m; t[p[m]][toi(a[l[m])]]=m;
        v=s[p[m]]; q=l[m];
        while (q<r[m]) { v=t[v][toi(a[q])]; q+=r[v]-l[v]; }
        if (q==r[m]) s[m]=v; else s[m]=m+2;
        q=r[v]-(q-r[m]); m+=2; goto suff;
    }
}
}
SuffixTree(string a) : a(a) {
    fill(r, r+N, sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1], t[1]+ALPHA, 0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i, 0, sz(a)) ukkadd(i, toi(a[i]));
}
// example: find longest common substring (uses ALPHA = 28)
pii best;
int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c, 0, ALPHA) if (t[node][c] != -1)
        mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
        best = max(best, {len, r[node] - len});
    return mask;
}
static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
}
};
```

Hashing.h

Description: Self-explanatory methods for string hashing. (Arithmetic mod $2^{64} - 1$. 2x slower than mod 2^{64} and more code, but works on evil test data (e.g. Thue-Morse, where ABBA... and BAAB... of length 2^{10} hash the same mod 2^{64}). "typedef ull H;" instead if you think test data is random, or work mod $10^9 + 7$ if the Birthday paradox is not a problem.)

d41d8c, 36 lines

```
struct Hashing {
    // 0-base indexing
    int n;
    FenwickTree ft1, ft2;
    Hashing() : n(0), ft1(0), ft2(0) {}
    void build_hash(string &s, int size) {
        init();
        n = size;
        ft1 = FenwickTree(n);
        ft2 = FenwickTree(n);
        for(int i = 0; i < n; i++) {
            int hash_value = p_ip1[i][0]*1LL*s[i] % M1;
            ft1.add(i, hash_value);
            hash_value = p_ip2[i][0]*1LL*s[i] % M2;
            ft2.add(i, hash_value);
        }
    }
};
```

```

    }
}
void update(int i, char c) {
    int hash_value = p_ip1[i][0]*1LL*c % M1;
    ft1.add(i, (-ft1.sum(i, i) + hash_value + M1) % M1);
    hash_value = p_ip2[i][0]*1LL*c % M2;
    ft2.add(i, (-ft2.sum(i, i) + hash_value + M2) % M2);
}
array<int, 2> get_hash(int l, int r) {
    array<int, 2> ans;
    ans[0] = ((ft1.sum(l, r) + M1) % M1)*1LL*p_ip1[l][1] % M1;
    ans[1] = ((ft2.sum(l, r) + M2) % M2)*1LL*p_ip2[l][1] % M2;
    return ans;
}
array<int, 2> get_hash() {return get_hash(0, n-1);}
} h, rh;

bool check_palindrome(int i, int j, int n) {
    // 0-base indexing
    return h.get_hash(i, j) == rh.get_hash(n - j - 1, n - i - 1);
}
}
```

AhoCorasick.h

Description: Aho Corasick

d41d8c, 56 lines

```
struct AC {
    int N, P;
    const int A = 26;
    vector <vector <int>> next;
    vector <int> link, out_link;
    vector <vector <int>> out;
    AC() : N(0), P(0) {node();}
    int node() {
        next.emplace_back(A, 0);
        link.emplace_back(0);
        out_link.emplace_back(0);
        out.emplace_back(0);
        return N++;
    }
    inline int get(char c) {
        return c - 'a';
    }
    int add_pattern(const string T) {
        int u = 0;
        for (auto c : T) {
            if (!next[u][get(c)]) next[u][get(c)] = node();
            u = next[u][get(c)];
        }
        out[u].push_back(P);
        return P++;
    }
    void compute() {
        queue <int> q;
        for (q.push(0); !q.empty();) {
            int u = q.front(); q.pop();
            for (int c = 0; c < A; ++c) {
                int v = next[u][c];
                if (!v) next[u][c] = next[link[u]][c];
                else {
                    link[v] = u ? next[link[u]][c] : 0;
                    out_link[v] = out[link[v]].empty() ? out_link[link[v]] : link[v];
                    q.push(v);
                }
            }
        }
    }
};
```

```

    }
    int advance(int u, char c) {
        while (u && !next[u][get(c)]) u = link[u];
        u = next[u][get(c)];
        return u;
    }
    void match(const string S) {
        int u = 0;
        for (auto c : S) {
            u = advance(u, c);
            for (int v = u; v; v = out_link[v]) {
                for (auto p : out[v]) cout << "match " << p << endl;
            }
        }
    }
};
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).

Time: $\mathcal{O}(\log N)$

d41d8c, 23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it);
    }
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    }
    return is.insert(before, {L, R});
}
```

```
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second;
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty).

Time: $\mathcal{O}(N \log N)$

d41d8c, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
    vi S(sz(I)), R;
    iota(all(S), 0);
    sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
    T cur = G.first;
    int at = 0;
    while (cur < G.second) { // (A)
        pair<T, int> mx = make_pair(cur, -1);
        while (at < sz(I) && I[S[at]].first <= cur) {
```

```
mx = max(mx, make_pair(I[S[at]].second, S[at]));
at++;
}
if (mx.second == -1) return {};
cur = mx.first;
R.push_back(mx.second);
}
return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.

Usage: constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});
Time: $\mathcal{O}(k \log \frac{n}{k})$

d41d8c, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
    if (p == q) return;
    if (from == to) {
        g(i, to, p);
        i = to; p = q;
    } else {
        int mid = (from + to) >> 1;
        rec(from, mid, f, g, i, p, f(mid));
        rec(mid+1, to, f, g, i, p, q);
    }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
    if (to <= from) return;
    int i = from; auto p = f(i), q = f(to-1);
    rec(from, to-1, f, g, i, p, q);
    g(i, to, q);
}
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f , change it to $>$, also at (B).

Usage: int ind = ternSearch(0,n-1,[&](int i){return a[i];});
Time: $\mathcal{O}(\log(b-a))$

d41d8c, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
    assert(a <= b);
    while (b - a >= 5) {
        int mid = (a + b) / 2;
        if (f(mid) < f(mid+1)) a = mid; // (A)
        else b = mid+1;
    }
    rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
    return a;
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

d41d8c, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
    if (S.empty()) return {};
    vi prev(sz(S));
    typedef pair<I, int> p;
    vector<p> res;
    rep(i,0,sz(S)) {
```

```
// change 0 -> i for longest non-decreasing subsequence
auto it = lower_bound(all(res), p{S[i], 0});
if (it == res.end()) res.emplace_back(), it = res.end()-1;
*it = {S[i], i};
prev[i] = it == res.begin() ? 0 : (it-1)->second;
}
int L = sz(res), cur = res.back().second;
vi ans(L);
while (L-->) ans[L] = cur, cur = prev[cur];
return ans;
}
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t , computes the maximum $S \leq t$ such that S is the sum of some subset of the weights.

Time: $\mathcal{O}(N \max(w_i))$

d41d8c, 16 lines

```
int knapsack(vi w, int t) {
    int a = 0, b = 0, x;
    while (b < sz(w) && a + w[b] <= t) a += w[b++];
    if (b == sz(w)) return a;
    int m = *max_element(all(w));
    vi u, v(2*m, -1);
    v[a+m-t] = b;
    rep(i,b,sz(w)) {
        u = v;
        rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
        for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
            v[x-w[j]] = max(v[x-w[j]], j);
    }
    for (a = t; v[a+m-t] < 0; a--);
    return a;
}
```

10.3 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i < k < j} (a[i][k] + a[k][j]) + f(i, j)$, where the (minimal) optimal k increases with both i and j , one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b, c) \leq f(a, d)$ and $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

Time: $\mathcal{O}(N^2)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \leq k < hi(i)} (f(i, k))$ where the (minimal) optimal k increases with i , computes $a[i]$ for $i = L..R-1$.

Time: $\mathcal{O}((N + (hi-lo)) \log N)$

d41d8c, 18 lines

```
struct DP { // Modify at will:
    int lo(int ind) { return 0; }
    int hi(int ind) { return ind; }
    ll f(int ind, int k) { return dp[ind][k]; }
    void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

    void rec(int L, int R, int LO, int HI) {
        if (L >= R) return;
        int mid = (L + R) >> 1;
        pair<ll, int> best(LLONG_MAX, LO);
        rep(k, max(LO, lo(mid)), min(HI, hi(mid)))
            best = min(best, make_pair(f(mid, k), k));
        store(mid, best.second, best.first);
        rec(L, mid, LO, best.second+1);
        rec(mid+1, R, best.second, HI);
    }
    void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
```

```
};
```

10.4 Debugging tricks

- signal(SIGSEGV, [](int) { _Exit(0); }); converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). _GLIBCXX_DEBUG failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).

- feenableexcept(29); kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.5 Optimization tricks

__builtin_ia32_ldmxcsr(40896); disables denormals (which make floats 20x slower near their minimum value).

10.5.1 Bit hacks

- $x \& -x$ is the least bit in x .
- for (int x = m; x;) { --x &= m; ... } loops over all subset masks of m (except m itself).
- $c = x \& -x, r = x + c; (((r^x) >> 2) / c) \mid r$ is the next number after x with the same number of bits set.
- rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)]; computes all sums of subsets.

10.5.2 Pragmas

- #pragma GCC optimize ("Ofast") will make GCC auto-vectorize loops and optimizes floating points better.
- #pragma GCC target ("avx2") can double performance of vectorized code, but causes crashes on old machines.
- #pragma GCC optimize ("trapv") kills the program on integer overflows (but is really slow).

FastMod.h

Description: Compute $a \% b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod b$ in the range $[0, 2b)$.

d41d8c, 8 lines

```
typedef unsigned long ull;
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m(-1ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b;
    }
};
```

10.6 Miscellaneous

SOSDP.h

Description: SOS DP

d41d8c, 10 lines

```
vector<vector<int>>> dp(1 << n, vector<int>(n));
vector<int> sos(1 << n);
for (int mask = 0; mask < (1 << n); mask++) {
```

```
dp[mask][n-1] = a[mask];
for (int x = 0; x < n; x++) {
    dp[mask][x] = dp[mask][x-1];
    if (mask & (1 << x)) { dp[mask][x] += dp[mask - (1 << x)][x-1]; }
}
sos[mask] = dp[mask][n-1];
}
```

submaskiterate.h

Description: Submask iterate

d41d8c, 3 lines

```
for (int m=0; m<(1<<n); ++m)
    for (int s=m; s; s=(s-1)&m)
        ... s and m ...
```

nCrNotP.h

Description: Finds nCr modulo a number that is not necessarily prime. Its good when m is small and not fixed.
Time: $\mathcal{O}(m \log m)$

../number-theory/CRT.h", "../number-theory/ModPow.h"

d41d8c, 32 lines

```
int F[1000002] = {1}, p, e, pe;
ll lg(ll n, int p) {
    ll r = 0;
    while (n) n /= p, r += n;
    return r;
}
ll f(ll n) {
    if (!n) return 1;
    return modpow(F[pe], n / pe, pe) * (F[n % pe] * f(n / p) % pe) % pe;
}
ll ncr(ll n, ll r) {
    ll c;
    if ((c = lg(n, p) - lg(r, p) - lg(n-r, p)) >= e)
        return 0;
    for (int i = 1; i <= pe; i++)
        F[i] = F[i-1] * (i % p == 0 ? 1 : i) % pe;
    return (f(n) * modpow(p, c, pe) % pe) *
        modpow(f(r) * f(n-r), pe - (pe / p) - 1, pe) % pe;
}
ll ncr(ll n, ll r, ll m) {
    ll a0 = 0, m0 = 1;
    for (p = 2; m != 1; p++) {
        e = 0, pe = 1;
        while (m % p == 0)
            m /= p, e++, pe *= p;
        if (e) {
            a0 = crt(a0, m0, ncr(n, r), pe);
            m0 = m0 * pe;
        }
    }
    return a0;
}
```