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DU_NE

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2025-01-17

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Contest (1)	
template.cpp	17 lines
#include <bits/stdc++.h> using namespace std; #define rep(i, a, b) for(int i = a; i<(b); ++i) #define all(x) begin(x), end(x) #define sz(x) (int)(x).size() typedef long long ll; typedef pair<int, int> pii; typedef vector<int> vi; int main() { cin.tie(0)->sync_with_stdio(0); cin.exceptions(cin.failbit); #ifdef ONPC cerr << endl << "finished in " << clock() * 1.0 / CLOCKS_PER_SEC << " sec" << endl; #endif } .bashrc3 lines alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \ -fsanitize=undefined,address' xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps = < .vimrc6 lines set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul sy on im jk <esc> im kj <esc> no ; : " Select region and then type :Hash to hash your selection. " Useful for verifying that there aren't mistypes. ca Hash w !cpp -dD -P -fpreprocessed \ tr -d '[:space:]' \ \ md5sum \ cut -c-6 hash.sh3 lines # Hashes a file, ignoring all whitespace and comments. Use for # verifying that code was correctly typed. cpp -dD -P -fpreprocessed tr -d '[:space:]' md5sum cut -c-6 stress.sh10 lines #!/bin/bash ["\$#" -ne 3] && echo "Usage: \$0 test_file brute_file mycode_file" && exit 1 g++ -O2 \$1 -o test && g++ -O2 \$2 -o brute && g++ -O2 \$3 -o mycode for i in {1..10000}; do ./test > tests.txt ./brute < tests.txt > correct.txt ./mycode < tests.txt > myans.txt	

```
diff -q correct.txt myans.txt >/dev/null || { echo -e "\e[31mTest $i: WA\e[0m"; cat tests.txt; break; }  
echo -e "\e[32mTest $i: AC\e[0m"  
done
```

interactiveStress.py19 lines

```
import subprocess, random  
def generate_permutation(n): return random.sample(range(1, n + 1), n)  
def handle_queries(hidden, n, max_q=6666):  
process = subprocess.Popen(["./solve"], stdin=subprocess.  
PIPE, stdout=subprocess.PIPE, text=True)  
process.stdin.write(f"{n}\n"); process.stdin.flush()  
for _ in range(max_q):  
query = process.stdout.readline().strip().split()  
if query[0] == "1":  
print("Correct!" if list(map(int, query[1:])) ==  
hidden else "Wrong!")  
break  
matches = sum(p == h for p, h in zip(map(int, query  
[1:]), hidden))  
process.stdin.write(f"{matches}\n"); process.stdin.  
flush()  
else: print("Query limit exceeded!")  
process.terminate()  
  
n = 1000  
hidden_permutation = generate_permutation(n)  
print("Hidden permutation:", hidden_permutation)  
handle_queries(hidden_permutation, n)
```

makefile10 lines

```
# runs by make run file=filename, use *tab*  
CC = g++  
CFLAGS = -fsanitize=address -std=c++17 -Wall -Wextra -Wshadow -  
DONPC -O2  
all:  
%: %.cpp  
$(CC) $(CFLAGS) -o "$@" "$<  
run: $(file)  
./$(file)  
clean:  
find . -type f -executable -delete
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$ax + by = e \Rightarrow \begin{matrix} x = \frac{ed - bf}{ad - bc} \\ cx + dy = f \Rightarrow y = \frac{af - ec}{ad - bc} \end{matrix}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where A'_i is A with the i 'th column replaced by b .

2.2 Recurrences

If $a_n = c_1a_{n-1} + \cdots + c_ka_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k - c_1x^{k-1} - \cdots - c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1r_1^n + \cdots + d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1n + d_2)r^n$.

2.3 Trigonometry

$$\begin{aligned}\sin(v+w) &= \sin v \cos w + \cos v \sin w \\ \cos(v+w) &= \cos v \cos w - \sin v \sin w\end{aligned}$$

$$\begin{aligned}\tan(v+w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}\end{aligned}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$\begin{aligned}a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi)\end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b+c} \right)^2 \right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$

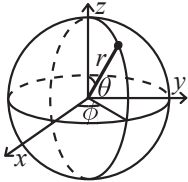
2.4.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned}x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \text{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \text{atan2}(y, x)\end{aligned}$$

2.5 Derivatives/Integrals

$$\begin{aligned}\frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1+x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \text{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1)\end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$\begin{aligned}1 + 2 + 3 + \cdots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \cdots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \cdots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \cdots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}\end{aligned}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \cdots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x . It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y ,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $U(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Data structures (3)

OrderStatisticTree.h

Description: ...
Time: $\mathcal{O}(\log N)$
`<ext/pb.ds/assoc.container.hpp>`, `<ext/pb.ds/tree.policy.hpp>` d41d8c, 14 lines
`using namespace __gnu_pbds;`

`#define ordered_set tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>`
`#define ordered_pair_set tree<pair<int, int>, null_type, less<pair<int, int>>, rb_tree_tag, tree_order_statistics_node_update>`
`ordered_set os;`
`// Example using ordered_set`
`os.insert(5);os.insert(1);os.insert(10);os.insert(3);`
`cout << "2nd smallest element: " << *os.find_by_order(2) << endl;` `// Output: 5`
`cout << "Elements less than 6: " << os.order_of_key(6) << endl;`
`// Output: 3`
`// Example using ordered_pair_set`
`ordered_pair_set ops;`
`ops.insert({1, 100});ops.insert({2, 200});ops.insert({1, 150});`
`ops.insert({3, 250});`
`cout << "1st smallest pair: (" << ops.find_by_order(0)->first << ", " << ops.find_by_order(0)->second << ")" << endl;`
`// Output: (1, 100)`
`cout << "Pairs less than (2, 150): " << ops.order_of_key({2, 150}) << endl;` `// Output: 2`

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).
`<bits/extc++.h>` d41d8c, 6 lines
`struct chash {`
`const uint64_t C = uint64_t(4e18 * acos(0)) | 71;`
`ll operator()(ll x) const { return __builtin_bswap64(x * C); }`
`};`

`__gnu_pbds::gp_hash_table<ll, int, chash> h;`

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$ d41d8c, 113 lines
`struct Segtree {`
`// 0 base indexing`
`int n;`
`vector<ll> tree;`
`ll merge(ll x, ll y) {`

`return x + y;`
`}`
`void build(vector<ll> &a, int node, int l, int r) {`
`if(l == r) {`
`tree[node] = a[l];`
`return;`
`}`
`int mid = l + ((r - l) >> 1);`
`build(a, (node << 1)+1, l, mid);`
`build(a, (node << 1)+2, mid+1, r);`
`tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);`
`}`
`void update(int i, ll value, int node, int l, int r) {`
`if(l == i && r == i) {`
`tree[node] = value;`
`return;`
`}`
`int mid = l + ((r-l) >> 1);`
`if(i <= mid)update(i, value, (node << 1)+1, l, mid);`
`else update(i, value, (node << 1)+2, mid+1, r);`
`tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);`
`}`
`void update(int i, int value) {`
`update(i, value, 0, 0, n-1);`
`}`
`ll query(int i, int j, int node, int l, int r) {`
`if(l > j || r < i) return 0;`
`if(l >= i && r <= j)return tree[node];`
`int mid = l + ((r - l) >> 1);`
`return merge(query(i, j, (node << 1)+1, l, mid), query(i, j, (node << 1)+2, mid+1, r));`
`}`
`ll query(int i, int j) {`
`return query(i, j, 0, 0, n-1);`
`}`
`void init(vector<ll> &a, int _n) {`
`n = _n;`
`int size = 1;`
`while(size < n) size = size << 1;`
`tree.resize((size << 1)-1);`
`build(a, 0, 0, n-1);`
`}`
`} st;`
`struct Segtree {`
`// 0 base indexing`
`int n;`
`vector<ll> tree, lazy;`

`ll merge(ll x, ll y) {`
`return x + y;`
`}`
`void push(int node, int l, int r) {`
`int a = (node << 1)+1, b = (node << 1)+2;`
`int mid = l + ((r-l) >> 1);`
`tree[a]+=(mid-l+1)*lazy[node], tree[b]+=(r-(mid+1)+1)*lazy[node];`
`lazy[a]+=lazy[node], lazy[b]+=lazy[node];`
`lazy[node] = 0;`
`}`
`void build(vector<ll> &a, int node, int l, int r) {`
`if(l == r) {`
`tree[node] = a[l];`
`return;`
`}`
`int mid = l + ((r-l) >> 1);`
`build(a, (node << 1)+1, l, mid);`
`build(a, (node << 1)+2, mid+1, r);``tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);`
`}`
`void build(vector<ll> &a) {`
`build(a, 0, 0, n-1);`
`}`
`void update(int i, int j, ll value, int node, int l, int r) {`
`{`
`if(l > j || r < i)return;`
`if(l >= i && r <= j) {`
`lazy[node]+=value;`
`tree[node]+=(r-l+1)*value;`
`return;`
`}`
`if(lazy[node])push(node, l, r);`
`int mid = l + ((r-l) >> 1);`
`update(i, j, value, (node << 1)+1, l, mid);`
`update(i, j, value, (node << 1)+2, mid+1, r);`
`tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);`
`}`
`void update(int i, int j, ll value) {`
`update(i, j, value, 0, 0, n-1);`
`}`
`ll query(int i, int j, int node, int l, int r) {`
`if(l > j || r < i)`
`return 0;`
`if(l >= i && r <= j)`
`return tree[node];`

`if(lazy[node]) push(node, l, r);`
`int mid = l + ((r-l) >> 1);`
`return merge(query(i, j, (node << 1)+1, l, mid), query(i, j, (node << 1)+2, mid+1, r));`
`}`
`ll query(int i, int j) {`
`return query(i, j, 0, 0, n-1);`
`}`
`void init(vector<ll> &a, int _n) {`
`n = _n;`
`int size = 1;`
`while(size < n) size = size << 1;`
`tree.resize((size << 1)-1);`
`lazy.assign((size << 1)-1, 0);`
`build(a, 0, 0, n-1);`
`}`
`} st;`

LazySegmentTree.h

Description: Segment tree with lazy propagation
Usage: update(l, 0, n - 1, ql, qr, val), query(l, 0, n - 1, ql, qr)
Time: $\mathcal{O}(\log N)$ d41d8c, 66 lines
`struct Segtree {`
`// 0 base indexing`
`int n;`
`vector<ll> tree, lazy;`

`ll merge(ll x, ll y) {`
`return x + y;`
`}`
`void push(int node, int l, int r) {`
`int a = (node << 1)+1, b = (node << 1)+2;`
`int mid = l + ((r-l) >> 1);`
`tree[a]+=(mid-l+1)*lazy[node], tree[b]+=(r-(mid+1)+1)*lazy[node];`
`lazy[a]+=lazy[node], lazy[b]+=lazy[node];`
`lazy[node] = 0;`

```

}
void build(vector<ll> &a, int node, int l, int r) {
    if(l == r) {
        tree[node] = a[l];
        return;
    }
    int mid = l + ((r-l) >> 1);
    build(a, (node << 1)+1, l, mid);
    build(a, (node << 1)+2, mid+1, r);
    tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);
}
void build(vector<ll> &a) {
    build(a, 0, 0, n-1);
}
void update(int i, int j, ll value, int node, int l, int r)
{
    if(l > j || r < i) return;
    if(l >= i && r <= j) {
        lazy[node] += value;
        tree[node] += (r-l+1)*value;
        return;
    }
    if(lazy[node]) push(node, l, r);
    int mid = l + ((r-l) >> 1);
    update(i, j, value, (node << 1)+1, l, mid);
    update(i, j, value, (node << 1)+2, mid+1, r);
    tree[node] = merge(tree[(node << 1)+1], tree[(node << 1)+2]);
}
void update(int i, int j, ll value) {
    update(i, j, value, 0, 0, n-1);
}
ll query(int i, int j, int node, int l, int r) {
    if(l > j || r < i)
        return 0;
    if(l >= i && r <= j)
        return tree[node];

    if(lazy[node]) push(node, l, r);
    int mid = l + ((r-l) >> 1);
    return merge(query(i, j, (node << 1)+1, l, mid), query(
        i, j, (node << 1)+2, mid+1, r));
}
ll query(int i, int j) {
    return query(i, j, 0, 0, n-1);
}
void init(vector<ll> &a, int _n) {
    n = _n;
    int size = 1;
    while(size < n) size = size << 1;
    tree.resize((size << 1)-1);
    lazy.assign((size << 1)-1, 0);
    build(a, 0, 0, n-1);
}
} st;

```

PersistentSegtree.h

Description: Persistent Segment Tree

d41d8c, 76 lines

```

struct persistentSegtree {
    // 0 base indexing
    ll data;
    persistentSegtree *left, *right;

    ll merge(ll x, ll y) {
        return x + y;
    }
    void build(vector<ll> &a, int l, int r) {

```

```

        if(l == r) {
            data = a[l];
            return;
        }
        int mid = l + ((r - l) >> 1);
        left = new persistentSegtree();
        right = new persistentSegtree();
        left->build(a, l, mid);
        right->build(a, mid+1, r);
        data = merge(left->data, right->data);
    }
    persistentSegtree* update(int i, ll value, int l, int r) {
        if(l > i || r < i) return this;
        if(l == i && r == i) {
            persistentSegtree *rslt = new persistentSegtree();
            rslt->data = value;
            return rslt;
        }
        int mid = l + ((r-l) >> 1);
        persistentSegtree *rslt = new persistentSegtree();

        rslt->left = left->update(i, value, l, mid);
        rslt->right = right->update(i, value, mid+1, r);
        rslt->data = merge(rslt->left->data, rslt->right->data);
    }

    return rslt;
}
ll query(int i, int j, int l, int r) {
    if(l > j || r < i) return 0;
    if(l >= i && r <= j) return data;
    int mid = l + ((r - l) >> 1);
    return merge(left->query(i, j, l, mid), right->query(i,
        j, mid+1, r));
}
} *roots[N];
int main() { // Idea from Mahmudul Yeamim
    int tt = 1;
    while(tt--) {
        int n, q, k = 0;
        cin >> n >> q;
        vector<ll> a(n);
        for(int i = 0; i < n; i++) {
            cin >> a[i];
        }
        roots[0] = new persistentSegtree();
        roots[k++] -> build(a, 0, n-1);
        while(q--) {
            int type;
            cin >> type;
            if(type == 1) {
                int _k, i;
                ll x;
                cin >> _k >> i >> x;
                --_k;
                roots[_k] = roots[_k] -> update(--i, x, 0, n-1);
            } else if(type == 2) {
                int _k, i, j;
                cin >> _k >> i >> j;
                cout << roots[--_k] -> query(--i, --j, 0, n-1) <<
                    "\n";
            } else {
                int _k;
                cin >> _k;
                roots[k++] = roots[--_k];
            }
        }
    }
    return 0;
}

```

```

}

```

UnionFind.h

Description: Disjoint-set data structure.

Time: $\mathcal{O}(\alpha(N))$

d41d8c, 17 lines

```

void make_set(int v) {
    parent[v] = v;
    Size[v] = 1;
}
int find_set(int v) {
    if (v == parent[v]) return v;
    return parent[v] = find_set(parent[v]);
}
void union_sets(int a, int b) {
    a = find_set(a);
    b = find_set(b);
    if (a != b) {
        if(Size[a] < Size[b]) swap(a, b);
        parent[b] = a;
        Size[a] += Size[b];
    }
}

```

UnionFindRollback.h

Description: 2D prefix with update

d41d8c, 34 lines

2DPrefix.h

Description: 2D prefix with update

Usage: SubMatrix<int> m(matrix);

m.sum(0, 0, 2, 2); // top left 4 elements

Time: $\mathcal{O}(N^2 + Q)$

d41d8c, 34 lines

```

void update(vector<vector<ll>>& grid, int x1, int y1, int x2,
    int y2, int val) {
    grid[x1][y1] += val;
    if (x2 + 1 < n) grid[x2 + 1][y1] += val;
    if (y2 + 1 < m) grid[x1][y2 + 1] += val;
    if (x2 + 1 < n && y2 + 1 < m) grid[x2 + 1][y2 + 1] += val;
}
vector<vector<ll>> calculate(vector<vector<ll>> &grid) {
    vector<vector<ll>> ans(n, vector<ll>(m, 0));
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            ans[i][j] = grid[i][j];
            if(i > 0) ans[i][j] += ans[i - 1][j];
            if(j > 0) ans[i][j] += ans[i][j - 1];
            if(i > 0 && j > 0) ans[i][j] += ans[i - 1][j - 1];
        }
    }
    return ans;
}
template<class T> struct SubMatrix {
    vector<vector<T>> p;
    SubMatrix(const vector<vector<T>>& v) {
        int R = v.size(), C = v[0].size();
        p.assign(R + 1, vector<T>(C + 1, 0));

        for (int r = 0; r < R; ++r) {
            for (int c = 0; c < C; ++c) {
                p[r + 1][c + 1] = v[r][c] + p[r][c + 1] + p[r + 1][c] - p[r][c];
            }
        }
    }
    T sum(int u, int l, int d, int r) {
        return p[d + 1][r + 1] - p[u][r + 1] - p[d + 1][l] + p[u][l];
    }
}

```

```

    }
};

Matrix.h
Description: Basic operations on square matrices.
Usage: Matrix<int, 3, 3> A;
A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};
vector<int> vec = {1,2,3};
vec = (A^N) * vec;

```

d41d8c, 34 lines

```

template<class T, int N, int M> struct Matrix {
    typedef Matrix Mx;
    array<array<T, M>, N> d{};
    // Matrix multiplication
    template<int P>
    Matrix<T, N, P> operator*(const Matrix<T, M, P>& m) const {
        Matrix<T, N, P> a;
        for (int i = 0; i < N; i++)
            for (int j = 0; j < P; j++)
                for (int k = 0; k < M; k++)
                    a.d[i][j] += d[i][k] * m.d[k][j];
        return a;
    }
    // Matrix-vector multiplication
    vector<T> operator*(const vector<T>& vec) const {
        vector<T> ret(N, 0);
        for (int i = 0; i < N; i++)
            for (int j = 0; j < M; j++)
                ret[i] += d[i][j] * vec[j];
        return ret;
    }
    // Matrix exponentiation
    Matrix<T, N, N> operator^(ll p) const {
        static_assert(N == M); assert(p >= 0);
        Matrix<T, N, N> a, b(*this);
        for (int i = 0; i < N; i++) a.d[i][i] = 1; // Identity matrix
        while (p) {
            if (p & 1) a = a * b;
            b = b * b;
            p >>= 1;
        }
        return a;
    }
};

```

CHT.h

Description: Container where you can add lines of the form $kx+m$, and query minimum values at points x . Useful for dynamic programming (“convex hull trick”).
Time: $\mathcal{O}(\log N)$

d41d8c, 38 lines

```

struct Line {
    // m = slope, c = intercept
    ll m, c;
    Line(ll a, ll b) : m(a), c(b) {}
};

struct CHT {
    // SayeefMahmud
    vector<Line> lines;

    bool bad(Line l1, Line l2, Line l3) {
        __int128 a = (__int128)(l2.c - l1.c) * (l2.m - l3.m);
        __int128 b = (__int128)(l3.c - l2.c) * (l1.m - l2.m);
        return a >= b;
    }

    void add(Line line) {
        lines.push_back(line);
        int sz = lines.size();
    }
};

```

```

        while (sz >= 3 && bad(lines[sz - 3], lines[sz - 2],
            lines[sz - 1])) {
            lines.erase(lines.end() - 2);
            sz--;
        }
    }
    ll query(ll x) {
        int l = 0, r = lines.size() - 1;
        ll ans = LLONG_MAX;
        while (l <= r) {
            int mid1 = l + (r - l) / 3;
            int mid2 = r - (r - l) / 3;
            ans = min(ans, min(lines[mid1].m * x + lines[mid1].c,
                lines[mid2].m * x + lines[mid2].c));
            if (lines[mid1].m * x + lines[mid1].c <= lines[mid2].m * x + lines[mid2].c) {
                r = mid2 - 1;
            } else {
                l = mid1 + 1;
            }
        }
        return ans;
    }
};

```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

FenwickTree.h

Description: Computes partial sums $a[0] + a[1] + \dots + a[pos - 1]$, and updates single elements $a[i]$, taking the difference between the old and new value.

d41d8c, 26 lines

```

struct FenwickTree {
    // 0 base indexing
    vector<int> bit;
    int n;
    FenwickTree(int n) {
        this->n = n;
        bit.assign(n, 0);
    }
    FenwickTree(vector<int> const &a) : FenwickTree(a.size()) {
        for (size_t i = 0; i < a.size(); i++)
            add(i, a[i]);
    }
    int sum(int r) {
        int ret = 0;
        for (; r >= 0; r = (r & (r + 1)) - 1)
            ret += bit[r];
        return ret;
    }
    int sum(int l, int r) {
        return sum(r) - sum(l - 1);
    }
    void add(int idx, int delta) {
        for (; idx < n; idx = idx | (idx + 1))
            bit[idx] += delta;
    }
};

```

FenwickTree2d.h

Description: Computes sums $a[i,j]$ for all $i < I, j < J$, and increases single elements $a[i,j]$. Requires that the elements to be updated are known in advance (call `fakeUpdate()` before `init()`).

d41d8c, 36 lines

```

struct FenwickTree2D {

```

```

    // 0 base indexing
    vector<vector<int>> bit;
    int n, m;
    FenwickTree2D(int n, int m) {
        this->n = n;
        this->m = m;
        bit.assign(n, vector<int>(m, 0));
    }
    FenwickTree2D(vector<vector<int>>& matrix) : FenwickTree2D(
        matrix.size(), matrix[0].size()) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                add(i, j, matrix[i][j]);
            }
        }
    }
    int sum(int x, int y) {
        int ret = 0;
        for (int i = x; i >= 0; i = (i & (i + 1)) - 1) {
            for (int j = y; j >= 0; j = (j & (j + 1)) - 1) {
                ret += bit[i][j];
            }
        }
        return ret;
    }
    int sum(int x1, int y1, int x2, int y2) {
        return sum(x2, y2) - sum(x2, y1 - 1) - sum(x1 - 1, y2)
            + sum(x1 - 1, y1 - 1);
    }
    void add(int x, int y, int delta) {
        for (int i = x; i < n; i = i | (i + 1)) {
            for (int j = y; j < m; j = j | (j + 1)) {
                bit[i][j] += delta;
            }
        }
    }
};

```

RMQ.h

Description: Range Minimum Queries on an array. Returns $\min(V[a], V[a + 1], \dots, V[b - 1])$ in constant time.

Usage: RMQ rmq(values);
 rmq.query(inclusive, exclusive);

Time: $\mathcal{O}(|V| \log |V| + Q)$

d41d8c, 26 lines

```

struct RMQ {
    // 0-base indexing
    int n, logN;
    vector<vector<int>> st;
    vector<int> lg;

    void init(const vector<int>& array) {
        n = array.size();
        logN = ceil(log2(n));
        st.resize(logN, vector<int>(n));
        lg.resize(n + 1);
        lg[1] = 0;
        for (int i = 2; i <= n; i++)
            lg[i] = lg[i / 2] + 1;
        copy(array.begin(), array.end(), st[0].begin());
        for (int i = 1; i < logN; i++) {
            for (int j = 0; j + (1 << i) <= n; j++) {
                st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
            }
        }
    }
    int query(int L, int R) {
        int i = lg[R - L + 1];
    }
};

```

```
        return min(st[i][L], st[i][R - (1 << i) + 1]);
    }
} ST;

MoQueries.h
Description: ...
// 0-base indexing
void add(int x) {
    if(!freq[x]) distinct++;
    freq[x]++;
}

void remove(int x) {
    freq[x]--;
    if(!freq[x]) distinct--;
}

void adjust(int &curr_l, int &curr_r, int L, int R) {
    while(curr_l > L) {
        curr_l--;
        add(a[curr_l]);
    }
    while(curr_r < R) {
        curr_r++;
        add(a[curr_r]);
    }
    while(curr_l < L) {
        remove(a[curr_l]);
        curr_l++;
    }
    while(curr_r > R) {
        remove(a[curr_r]);
        curr_r--;
    }
}

void solve(vector<array<int, 3>> &queries) {
    // const int BLOCK_SIZE = sqrt(queries.size()) + 1;
    const int BLOCK_SIZE = 555;
    sort(queries.begin(), queries.end(), [&](const array<int, 3>& a, const array<int, 3>& b) {
        int blockA = a[0] / BLOCK_SIZE;
        int blockB = b[0] / BLOCK_SIZE;
        if (blockA != blockB)
            return blockA < blockB;
        return a[1] < b[1];
    });
    auto [L, R, id] = queries[0];
    int curr_l = L, curr_r = L;
    distinct = 1;
    freq[a[curr_l]]++;
    vector<int> ans(queries.size());
    for(auto [L, R, id] : queries) {
        adjust(curr_l, curr_r, L, R);
        ans[id] = distinct;
    }
    for(auto x : ans) cout << x << "\n";
}
```

Numerical (4)

4.1 Polynomials and recurrences

```
Polynomial.h
struct Poly {
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val += x) += a[i];
    }
}
```

```
        return val;
    }

    void diff() {
        rep(i, 1, sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    }

    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
        a.pop_back();
    }
};

PolyRoots.h
Description: Finds the real roots to a polynomial.
Usage: polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0
Time: O(n^2 log(1/epsilon))
"Polynomial.h"
```

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = polyRoots(der, xmin, xmax);
    dr.push_back(xmin-1);
    dr.push_back(xmax+1);
    sort(all(dr));
    rep(i, 0, sz(dr)-1) {
        double l = dr[i], h = dr[i+1];
        bool sign = p(l) > 0;
        if (sign ^ (p(h) > 0)) {
            rep(it, 0, 60) { // while (h - l > 1e-8)
                double m = (l + h) / 2, f = p(m);
                if ((f <= 0) ^ sign) l = m;
                else h = m;
            }
            ret.push_back((l + h) / 2);
        }
    }
    return ret;
}
```

```
PolyInterpolate.h
Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}. For numerical precision, pick x[k] = c * cos(k/(n-1) * pi), k = 0 ... n-1.
Time: O(n^2)
```

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k, 0, n-1) rep(i, k+1, n)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0; temp[0] = 1;
    rep(k, 0, n) rep(i, 0, n) {
        res[i] += y[k] * temp[i];
        swap(last, temp[i]);
        temp[i] -= last * x[k];
    }
    return res;
}
```

```
BerlekampMassey.h
Description: Recovers any n-order linear recurrence relation from the first 2n terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size <= n.
Usage: berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}
```

```
Time: O(N^2)
"../number-theory/ModPow.h"
vector<ll> berlekampMassey(vector<ll> s) {
    int n = sz(s), L = 0, m = 0;
    vector<ll> C(n), B(n), T;
    C[0] = B[0] = 1;
    ll b = 1;
    rep(i, 0, n) { ++m;
        ll d = s[i] % mod;
        rep(j, 1, L+1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C; ll coef = d * modpow(b, mod-2) % mod;
        rep(j, m, n) C[j] = (C[j] - coef * B[j - m]) % mod;
        if (2 * L > i) continue;
        L = i + 1 - L; B = T; b = d; m = 0;
    }
    C.resize(L + 1); C.erase(C.begin());
    for (ll& x : C) x = (mod - x) % mod;
    return C;
}
```

```
LinearRecurrence.h
Description: Generates the k'th term of an n-order linear recurrence S[i] = sum_j S[i-j-1]tr[j], given S[0...n-1] and tr[0...n-1]. Faster than matrix multiplication. Useful together with Berlekamp-Massey.
Usage: linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number
Time: O(n^2 log k)
```

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);
    auto combine = [&](Poly a, Poly b) {
        Poly res(n * 2 + 1);
        rep(i, 0, n+1) rep(j, 0, n+1)
            res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
        for (int i = 2 * n; i > n; --i) rep(j, 0, n)
            res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
        res.resize(n + 1);
        return res;
    };
    Poly pol(n + 1), e(pol);
    pol[0] = e[1] = 1;
    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    }
    ll res = 0;
    rep(i, 0, n) res = (res + pol[i + 1] * S[i]) % mod;
    return res;
}
```

4.2 Optimization

```
GoldenSectionSearch.h
Description: Finds the argument minimizing the function f in the interval [a, b] assuming f is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See Ternary-Search.h in the Various chapter for a discrete version.
Usage: double func(double x) { return 4+x+.3*x*x; }
double xmin = gss(-1000,1000,func);
Time: O(log((b-a)/epsilon))
```

```
double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5)-1)/2, eps = 1e-7;
    double x1 = b - r*(b-a), x2 = a + r*(b-a);
    double f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
```



```
        b = x2; x2 = x1; f2 = f1;
        x1 = b - r*(b-a); f1 = f(x1);
    } else {
        a = x1; x1 = x2; f1 = f2;
        x2 = a + r*(b-a); f2 = f(x2);
    }
    return a;
}
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions.

d41d8c, 14 lines

typedef array<double, 2> P;

```
template<class F> pair<double, P> hillClimb(P start, F f) {
    pair<double, P> cur(f(start), start);
    for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
        rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {
            P p = cur.second;
            p[0] += dx*jmp;
            p[1] += dy*jmp;
            cur = min(cur, make_pair(f(p), p));
        }
    }
    return cur;
}
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4 , although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

d41d8c, 7 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
    double h = (b - a) / 2 / n, v = f(a) + f(b);
    rep(i,1,n*2)
        v += f(a + i*h) * (i&1 ? 4 : 2);
    return v * h / 3;
}
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.

Usage: double sphereVolume = quad(-1, 1, [](double x) { return quad(-1, 1, [&](double y) { return quad(-1, 1, [&](double z) { return x*x + y*y + z*z < 1; }}});});});

d41d8c, 15 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
    d c = (a + b) / 2;
    d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
    if (abs(T - S) <= 15 * eps || b - a < 1e-10)
        return T + (T - S) / 15;
    return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}

template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
    return rec(f, a, b, eps, S(a, b));
}
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b, x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}};

vd b = {1,1,-4}, c = {-1,-1}, x;

T val = LPSolver(A, b, c).solve(x);

Time: $\mathcal{O}(NM * \text{\#pivots})$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

```
typedef double T; // long double, Rational, double + modP>...
typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j
struct LPSolver {
    int m, n;
    vi N, B;
    vvd D;
    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m+1][n] = 1;
    }
    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            rep(j,0,n+2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }
    bool simplex(int phase) {
        int x = m + phase - 1;
        for (;;) {
            int s = -1;
            rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;
            rep(i,0,m) {
                if (D[i][s] <= eps) continue;
                if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                    < MP(D[r][n+1] / D[r][s], B[r])) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }
    T solve(vd &x) {
        int r = 0;
        rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
        if (D[r][n+1] < -eps) {
            pivot(r, n);
            if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
            rep(i,0,m) if (B[i] == -1) {
                int s = 0;
                rep(j,1,n+1) ltj(D[i]);
                pivot(i, s);
            }
        }
    }
}
```

```
bool ok = simplex(1); x = vd(n);
rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
return ok ? D[m][n+1] : inf;
}
};
```

4.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.

Time: $\mathcal{O}(N^3)$

d41d8c, 15 lines

```
double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
        }
    }
    return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.

Time: $\mathcal{O}(N^3)$

d41d8c, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a); ll ans = 1;
    rep(i,0,n) {
        rep(j,i+1,n) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i];
                if (t) rep(k,i,n)
                    a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;
            }
        }
        ans = ans * a[i][i] % mod;
        if (!ans) return 0;
    }
    return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.

Time: $\mathcal{O}(n^2m)$

d41d8c, 36 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);
    vi col(m); iota(all(col), 0);
    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m)
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
```



```
        rep(j,i,n) if (fabs(b[j]) > eps) return -1;
        break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
        double fac = A[j][i] * bv;
        b[j] -= fac * b[i];
        rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
}
x.assign(m, 0);
for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
}
return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h
Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

```
"SolveLinear.h"
d41d8c, 7 lines

rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i];
}
fail:; }
```

SolveLinearBinary.h
Description: Solves $Ax = b$ over \mathbb{F}_2 . If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b .
Time: $\mathcal{O}(n^2m)$

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0);
    rep(i,0,n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
        if (br == n) {
            rep(j,i,n) if(b[j]) return -1;
            break;
        }
        int bc = (int)A[br]._Find_next(i-1);
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) if (A[j][i] != A[j][bc]) {
            A[j].flip(i); A[j].flip(bc);
        }
        rep(j,i+1,n) if (A[j][i]) {
            b[j] ^= b[i];
            A[j] ^= A[i];
        }
        rank++;
    }
    x = bs();
    for (int i = rank; i--;) {
        if (!b[i]) continue;
    }
}
```

```
        x[col[i]] = 1;
        rep(j,0,i) b[j] ^= A[j][i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h
Description: Invert matrix A . Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of $A \pmod{p}$, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

```
d41d8c, 32 lines

int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;
    rep(i,0,n) {
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i;
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i];
        rep(j,i+1,n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
        }
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    }
    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        double v = A[j][i];
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    }
    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
    return n;
}
```

MatrixExpo.h
Description: Matrix Exponentiation

```
d41d8c, 33 lines

using row = vector<int>;
using matrix = vector<row>;
matrix unit_mat(int n) {
    matrix I(n, row(n));
    for (int i = 0; i < n; ++i){
        I[i][i] = 1;
    }
    return I;
}

matrix mat_mul(matrix a, matrix b) {
    int m = a.size(), n = a[0].size();
    int p = b.size(), q = b[0].size();
    // assert(n==p);
    matrix res(m, row(q));
    for (int i = 0; i < m; ++i){
        for (int j = 0; j < q; ++j){
            for (int k = 0; k < n; ++k){
                res[i][j] = (res[i][j] + a[i][k]*b[k][j]) % mod;
            }
        }
    }
}
```

```
    }
    return res;
}

matrix mat_exp(matrix a, int p) {
    int m = a.size(), n = a[0].size(); // assert(m==n);
    matrix res = unit_mat(m);
    while (p) {
        if (p&1) res = mat_mul(a, res);
        a = mat_mul(a, a);
        p >>= 1;
    }
    return res;
}
```

Gauss.h
Description: Gauss

```
d41d8c, 60 lines

ll bigMod (ll a, ll e, ll mod) {
    if (e == -1) e = mod - 2;
    ll ret = 1;
    while (e) {
        if (e & 1) ret = ret * a % mod;
        a = a * a % mod, e >>= 1;
    }
    return ret;
}

pair <int, ld> gaussJordan (int n, int m, ld eq[N][N], ld res[N][N]) {
    ld det = 1;
    vector <int> pos(m, -1);
    for (int i = 0, j = 0; i < n and j < m; ++j) {
        int piv = i;
        for (int k = i; k < n; ++k) if (fabs(eq[k][j]) > fabs(eq[piv][j])) piv = k;
        if (fabs(eq[piv][j]) < EPS) continue; pos[j] = i;
        for (int k = j; k <= m; ++k) swap(eq[piv][k], eq[i][k]);
        if (piv ^ i) det = -det; det *= eq[i][j];
        for (int k = 0; k < n; ++k) if (k ^ i) {
            ld x = eq[k][j] / eq[i][j];
            for (int l = j; l <= m; ++l) eq[k][l] -= x * eq[i][l];
        } ++i;
    }
    int free_var = 0;
    for (int i = 0; i < m; ++i) {
        pos[i] == -1 ? ++free_var, res[i] = det = 0 : res[i] = eq[
            pos[i]][m] / eq[pos[i]][i];
    }
    for (int i = 0; i < n; ++i) {
        ld cur = -eq[i][m];
        for (int j = 0; j < m; ++j) cur += eq[i][j] * res[j];
        if (fabs(cur) > EPS) return make_pair(-1, det);
    }
    return make_pair(free_var, det);
}

pair <int, int> gaussJordanModulo (int n, int m, int eq[N][N],
    int res[N], int mod) {
    int det = 1;
    vector <int> pos(m, -1);
    const ll mod_sq = (ll) mod * mod;
    for (int i = 0, j = 0; i < n and j < m; ++j) {
        int piv = i;
        for (int k = i; k < n; ++k) if (eq[k][j] > eq[piv][j]) piv = k;
        if (!eq[piv][j]) continue; pos[j] = i;
        for (int k = j; k <= m; ++k) swap(eq[piv][k], eq[i][k]);
        if (piv ^ i) det = det ? MOD - det : 0; det = (ll) det * eq[i][j] % MOD;
        for (int k = 0; k < n; ++k) if (k ^ i and eq[k][j]) {
            ll x = eq[k][j] * bigMod(eq[i][j], -1, mod) % mod;
        }
    }
}
```

```
        for (int l = j; l <= m; ++l) if (eq[i][l]) eq[k][l] = (eq
            [k][l] + mod_sq - x * eq[i][l]) % mod;
    } ++i;
}
int free_var = 0;
for (int i = 0; i < m; ++i) {
    pos[i] == -1 ? ++free_var, res[i] = det = 0 : res[i] = eq[
        pos[i]][m] * bigMod(eq[pos[i]][i], -1, mod) % mod;
}
for (int i = 0; i < n; ++i) {
    ll cur = -eq[i][m];
    for (int j = 0; j < m; ++j) cur += (ll) eq[i][j] * res[j],
        cur %= mod;
    if (cur) return make_pair(-1, det);
}
return make_pair(free_var, det);
}
```

Tridiagonal.h
Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & q_{n-2} & d_{n-1} & \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, \, 1 \leq i \leq n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, \dots, -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i , or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}(N)$

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
    int n = sz(b); vi tr(n);
    rep(i, 0, n-1) {
        if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
            b[i+1] -= b[i] * diag[i+1] / super[i];
            if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
            diag[i+1] = sub[i]; tr[i+1] = 1;
        } else {
            diag[i+1] -= super[i]*sub[i]/diag[i];
            b[i+1] -= b[i]*sub[i]/diag[i];
        }
    }
    for (int i = n; i--;) {
        if (tr[i]) {
            swap(b[i], b[i-1]);
            diag[i-1] = diag[i];
            b[i] /= super[i-1];
        } else {
            b[i] /= diag[i];
            if (i) b[i-1] -= b[i]*super[i-1];
        }
    }
    return b;
}
```

```
Xorbasis.h
Description: Xor basis
d41d8c, 13 lines

int basis[d] = {0};
int sz = 0;
void insertVector(int mask) {
    for (int i = 0; i < d; i++) {
        if ((mask & (1 << i)) == 0) continue;
        if (!basis[i]) {
            basis[i] = mask;
            ++sz;
            return;
        }
        mask ^= basis[i];
    }
}
```

4.4 Fourier transforms

```
FastFourierTransform.h
Description: Returns coefficient of multiplication of two polynomials.
d41d8c, 46 lines

const double PI = acos(-1);
struct base {
    double a, b;
    base(double a = 0, double b = 0) : a(a), b(b) {}
    const base operator + (const base &c) const
    { return base(a + c.a, b + c.b); }
    const base operator - (const base &c) const
    { return base(a - c.a, b - c.b); }
    const base operator * (const base &c) const
    { return base(a * c.a - b * c.b, a * c.b + b * c.a); }
};

void fft(vector<base> &p, bool inv = 0) {
    int n = p.size(), i = 0;
    for(int j = 1; j < n - 1; ++j) {
        for(int k = n >> 1; k > (i ^= k); k >>= 1);
        if(j < i) swap(p[i], p[j]);
    }
    for(int l = 1, m; (m = 1 << l) <= n; l <== l) {
        double ang = 2 * PI / m;
        base wn = base(cos(ang), (inv ? 1. : -1.) * sin(ang)), w;
        for(int i = 0, j, k; i < n; i += m) {
            for(w = base(1, 0), j = i, k = i + 1; j < k; ++j, w = w *
                wn) {
                base t = w * p[j + 1];
                p[j + 1] = p[j] - t;
                p[j] = p[j] + t;
            }
        }
        if(inv) for(int i = 0; i < n; ++i) p[i].a /= n, p[i].b /= n;
    }
}

vector<long long> multiply(vector<ll> &a, vector<ll> &b) {
    int n = a.size(), m = b.size(), t = n + m - 1, sz = 1;
    while(sz < t) sz <== 1;
    vector<base> x(sz), y(sz), z(sz);
    for(int i = 0; i < sz; ++i) {
        x[i] = i < (int)a.size() ? base(a[i], 0) : base(0, 0);
        y[i] = i < (int)b.size() ? base(b[i], 0) : base(0, 0);
    }
    fft(x), fft(y);
    for(int i = 0; i < sz; ++i) z[i] = x[i] * y[i];
    fft(z, 1);
    vector<long long> ret(sz);
    for(int i = 0; i < sz; ++i) ret[i] = (long long) round(z[i].a
        );
    while((int)ret.size() > 1 && ret.back() == 0) ret.pop_back();
    return ret;
}
```

```
FastFourierTransformMod.h
Description: Higher precision FFT, can be used for convolutions modulo
arbitrary integers as long as N log_2 N * mod < 8.6 * 10^14 (in practice 10^16 or
higher). Inputs must be in [0, mod).
Time: O(N log N), where N = |A| + |B| (twice as slow as NTT or FFT)
"FastFourierTransform.h"
d41d8c, 22 lines

typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
    if (a.empty() || b.empty()) return {};
    vl res(sz(a) + sz(b) - 1);
    int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
    vector<C> L(n), R(n), outs(n), outl(n);
    rep(i, 0, sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
    rep(i, 0, sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
    fft(L), fft(R);
    rep(i, 0, n) {
        int j = -i & (n - 1);
        outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
    }
    fft(outl), fft(outs);
    rep(i, 0, sz(res)) {
        ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
        ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
    }
    return res;
}
```

NumberTheoreticTransform.h
Description: `ntt(a)` computes $f(k) = \sum_x a[x]g^{xk}$ for all k , where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a . For arbitrary modulo, see `FFTMod`. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x - i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n , `reverse(start+1, end)`, NTT back. Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$

```
"../number-theory/ModPow.h"
d41d8c, 35 lines

const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vl rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)};
        rep(i, k, 2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
    }
    vi rev(n);
    rep(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i, 0, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j, 0, k) {
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        }
    }
    vl conv(const vl &a, const vl &b) {
        if (a.empty() || b.empty()) return {};
        int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
            n = 1 << B;
        int inv = modpow(n, mod - 2);
        vl L(a), R(b), out(n);
        L.resize(n), R.resize(n);
```

```
    ntt(L), ntt(R);
    rep(i,0,n)
        out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
    ntt(out);
    return {out.begin(), out.begin() + s};
}
```

FastSubsetTransform.h
Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where \oplus is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

```
void FST(vi& a, bool inv) {
    for (int n = sz(a), step = 1; step < n; step *= 2) {
        for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
            int &u = a[j], &v = a[j + step]; tie(u, v) =
                inv ? pii(v - u, u) : pii(v, u + v); // AND
                inv ? pii(v, u - v) : pii(u + v, u); // OR
                pii(u + v, u - v); // XOR
        }
    }
    if (inv) for (int& x : a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
    FST(a, 0); FST(b, 0);
    rep(i,0,sz(a)) a[i] *= b[i];
    FST(a, 1); return a;
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.

```
"euclid.h" d41d8c, 18 lines
const ll mod = 17; // change to something else
struct Mod {
    ll x;
    Mod(ll xx) : x(xx) {}
    Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
    Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
    Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
    Mod operator/(Mod b) { return *this * invert(b); }
    Mod invert(Mod a) {
        ll x, y, g = euclid(a.x, mod, x, y);
        assert(g == 1); return Mod((x + mod) % mod);
    }
    Mod operator^(ll e) {
        if (!e) return Mod(1);
        Mod r = *this ^ (e / 2); r = r * r;
        return e&1 ? *this * r : r;
    }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes $\text{LIM} \leq \text{mod}$ and that mod is a prime.

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

```
d41d8c, 11 lines
int bigPow(ll base, ll power, const int mod) {
    int ans = 1 % mod;
    base %= mod;
    if (base < 0) base += mod;
    while (power) {
        if (power & 1) ans = (ll) ans * base % mod;
        base = (ll) base * base % mod;
        power >>= 1;
    }
    return ans;
}
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,l,m) can be used to calculate the order of a .

```
"ModPow.h" d41d8c, 11 lines
ll modLog(ll a, ll b, ll m) {
    ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
    unordered_map<ll, ll> A;
    while (j <= n && (e = f = e * a % m) != b % m)
        A[e * b % m] = j++;
    if (e == b % m) return j;
    if (__gcd(m, e) == __gcd(m, b))
        rep(i,2,n+2) if (A.count(e = e * f % m))
            return n * i - A[e];
    return -1;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
 $\text{modsum}(\text{to}, c, k, m) = \sum_{i=0}^{\text{to}-1} (ki + c) \% m$. divsum is similar but for floored division.

```
"ModPow.h" d41d8c, 14 lines
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }
ull divsum(ull to, ull c, ull k, ull m) {
    ull res = k / m * sumsq(to) + c / m * to;
    k %= m; c %= m;
    if (!k) return res;
    ull to2 = (to * k + c) / m;
    return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}
ll modsum(ull to, ll c, ll k, ll m) {
    c = ((c % m) + m) % m;
    k = ((k % m) + m) % m;
    return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \pmod c$ (or $a^b \pmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow

```
d41d8c, 11 lines
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
    ll ret = a * b - M * ull(1.L / M * a * b);
    return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = modmul(b, b, mod), e /= 2)
        if (e & 1) ans = modmul(ans, b, mod);
    return ans;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

```
"ModPow.h" d41d8c, 24 lines
ll sqrt(ll a, ll p) {
    a %= p; if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p-1)/2, p) == 1); // else no solution
    if (p % 4 == 3) return modpow(a, (p+1)/4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
    ll s = p - 1, n = 2;
    int r = 0, m;
    while (s % 2 == 0)
        ++r, s /= 2;
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    ll x = modpow(a, (s + 1) / 2, p);
    ll b = modpow(a, s, p), g = modpow(n, s, p);
    for (;;) r = m) {
        ll t = b;
        for (m = 0; m < r && t != 1; ++m)
            t = t * t % p;
        if (m == 0) return x;
        ll gs = modpow(g, 1LL << (r - m - 1), p);
        g = gs * gs % p;
        x = x * gs % p;
        b = b * g % p;
    }
}
```

5.2 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.
Time: $\text{LIM} = 1\text{e}9 \approx 1.5\text{s}$

```
d41d8c, 20 lines
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int)round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S+1); pr.reserve((int)(LIM/log(LIM)*1.1));
    vector<pii> cp;
    for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
        cp.push_back({i, i * i / 2});
        for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
    }
    for (int L = 1; L <= R; L += S) {
        array<bool, S> block{};
        for (auto &[p, idx] : cp)
            for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
        rep(i,0,min(S, R - L))
            if (!block[i]) pr.push_back((L + i) * 2 + 1);
    }
    for (int i : pr) isPrime[i] = 1;
    return pr;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \pmod c$.

```
"ModMulLL.h" d41d8c, 12 lines
bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ull a : A) { // ^ count trailing zeroes
        ull p = modpow(a%n, d, n), i = s;
```

```
while (p != 1 && p != n - 1 && a % n && i--)
    p = modmul(p, p, n);
if (p != n-1 && i != s) return 0;
}
return 1;
}
```

Factor.h
Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

```
"ModMullL.h", "MillerRabin.h"
d41d8c, 18 lines

ull pollard(ull n) {
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    auto f = [&](ull x) { return modmul(x, x, n) + i; };
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}

vector<ull> factor(ull n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), all(r));
    return l;
}
```

5.3 Divisibility

euclid.h
Description: Finds two integers x and y , such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in `__gcd` instead. If a and b are coprime, then x is the inverse of $a \pmod{b}$.

```
ll euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
}
```

5.3.1 Chinese Remainder Theorem

Let $m = m_1 \cdot m_2 \cdots m_k$, where m_i are pairwise coprime. In addition to m_i , we are also given a system of congruences

$$\begin{cases} a &\equiv a_1 & (\text{mod } m_1) \\ a &\equiv a_2 & (\text{mod } m_2) \\ &\vdots \\ a &\equiv a_k & (\text{mod } m_k) \end{cases}$$

where a_i are some given constants. CRT will give the unique solution modulo m .

CRT.h
Description: Chinese Remainder Theorem.
`crt(a, m, b, n)` computes x such that $x \equiv a \pmod{m}$, $x \equiv b \pmod{n}$. If $|a| < m$ and $|b| < n$, x will obey $0 \leq x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
Time: $\log(n)$

```
"euclid.h"
d41d8c, 7 lines

ll crt(ll a, ll m, ll b, ll n){
```

```
if (n > m) swap(a, b), swap(m, n);
ll x, y, g = euclid(m, n, x, y);
assert((a - b) % g == 0); // else no solution
x = (b - a) % n * x % n / g * m + a;
return x < 0 ? x + m*n/g : x;
}
```

CRT2.h
Description: Chinese Remainder Theorem.
Time: $\mathcal{O}(n)$ here n is the number of congruences.

```
d41d8c, 17 lines

struct Congruence {
    ll a, m;
};

ll CRT(vector<Congruence> const &congruences) {
    ll M = 1;
    for (auto const &congruence : congruences) {
        M *= congruence.m;
    }
    ll solution = 0;
    for (auto const &congruence : congruences) {
        ll a_i = congruence.a;
        ll M_i = M / congruence.m;
        ll N_i = mod_inv(M_i, congruence.m);
        solution = (solution + a_i * M_i % M * N_i) % M;
    }
    return solution;
}
```

5.3.2 Bézout’s identity

For $a \neq, b \neq 0$, then $d = \gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

Diophantine.h
Description: Provides any solution of $ax + by = c$
Time: $\mathcal{O}(\log(n))$

```
"euclid.h"
d41d8c, 8 lines

bool find_any_solution(int a, int b, int c, int &x0, int &y0,
    int &g) {
    g = euclid(abs(a), abs(b), x0, y0);
    if (c % g) return false;
    x0 *= c / g, y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
```

phiFunction.h
Description: Euler’s ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n . $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p - 1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1}p_2^{k_2} \dots p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} \dots (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$.
 $\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k, n)=1} k = n\phi(n)/2, n > 1$.
Euler’s thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat’s little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p} \forall a$.
Time: $\mathcal{O}(\log \log n)$ and $\mathcal{O}(\sqrt{n})$ for the second version.

```
d41d8c, 21 lines
```

```
const int LIM = 5000000;
int phis[LIM];

void calculatePhi() {
    rep(i, 0, LIM) phis[i] = i & 1 ? i : i / 2;
    for (int i = 3; i < LIM; i += 2)
        if (phis[i] == i)
            for (int j = i; j < LIM; j += i)
                phis[j] -= phis[j] / i;
}

int phi(int n) {
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}
```

5.4 Fractions

ContinuedFractions.h
Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. (p_k/q_k alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞ ; if x is the root of a degree 2 polynomial the a ’s eventually become cyclic.
Time: $\mathcal{O}(\log N)$

```
d41d8c, 21 lines

typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
    ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
    for (;;) {
        ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
            a = (ll)floor(y), b = min(a, lim),
            NP = b*P + LP, NQ = b*Q + LQ;
        if (a > b) {
            // If b > a/2, we have a semi-convergent that gives us a
            // better approximation; if b = a/2, we *may* have one.
            // Return {P, Q} here for a more canonical approximation.
            return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
                make_pair(NP, NQ) : make_pair(P, Q);
        }
        if (abs(y = 1/(y - (d)a)) > 3*N) {
            return {NP, NQ};
        }
        LP = P; P = NP;
        LQ = Q; Q = NQ;
    }
}
```

FracBinarySearch.h
Description: Given f and N , finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.
Usage: `fracBS({}(Frac f) { return f.p>=3*f.q; }, 10);` // {1,3}
Time: $\mathcal{O}(\log(N))$

```
d41d8c, 25 lines

struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
    bool dir = 1, A = 1, B = 1;
    Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
    if (f(lo)) return lo;
    assert(f(hi));
```

```
while (A || B) {
  ll adv = 0, step = 1; // move hi if dir, else lo
  for (int si = 0; step; (step *= 2) >>= si) {
    adv += step;
    Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
    if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
      adv -= step; si = 2;
    }
  }
  hi.p += lo.p * adv;
  hi.q += lo.q * adv;
  dir = !dir;
  swap(lo, hi);
  A = B; B = !!adv;
}
return dir ? hi : lo;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$

with $m > n > 0, k > 0, m \perp n$, and either m or n even.

5.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.7 Fibonacchi

Fibonacci numbers are defined by

$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$. Again, $F_n = \frac{\phi^n - \hat{\phi}^n}{\sqrt{5}} \approx \frac{\phi^n}{\sqrt{5}}$,

where $\phi = \frac{1+\sqrt{5}}{2}$ and $\hat{\phi} = \frac{1-\sqrt{5}}{2}$. Some important properties of Fibonacci numbers:

$F_{n-1}F_{n+1} - F_n^2 = (-1)^n$

$F_{n+k} = F_{k-1}F_n + F_kF_{n+1}$

$F_{2n} = F_n(F_{n-1} + F_{n+1})$

$F_{2n+1} = F_n^2 + F_{n+1}^2$

$n \mid m \Leftrightarrow F_n \mid F_m$

$\gcd(F_m, F_n) = F_{\gcd(m,n)}$

Fibonacchi.h

Description: nthFibonacci

Time: $\mathcal{O}(\log n)$ d41d8c, 8 lines

```
ll f(ll n) {
  if (n == 0 || n == 1) return dp[n] = 1;
  if (dp[n]) return dp[n];
  ll k = n/2;
  if (n % 2 == 0) return dp[n] = (f(k)*f(k) + f(k-1)*f(k-1)) % M;
  return dp[n] = (f(k)*f(k+1) + f(k-1) * f(k)) % M;
}
(n == 0 ? 0 : f(n-1));
```

5.8 Estimates

$\sum_{d \mid n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.9 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d \mid n} f(d) \Leftrightarrow f(n) = \sum_{d \mid n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d \mid n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n \mid d} f(d) \Leftrightarrow f(n) = \sum_{n \mid d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.

Time: $\mathcal{O}(n)$ d41d8c, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for (int x:v) r = r * ++i + __builtin_popcount(use & ~(1<<x)),
    use |= 1 << x; // (note: minus, not ~!)
  return r;
}
```

6.1.2 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^\infty g_S(n) \frac{x^n}{n!} = \exp \left(\sum_{n \in S} \frac{x^n}{n} \right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

6.1.4 Burnside’s lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts “configurations” (of some sort) of length n , we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k \mid n} f(k) \phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2e5$	$\sim 2e8$

6.2.2 Lucas’ Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \dots + n_1 p + n_0$ and $m = m_k p^k + \dots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$.

6.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \dots k_n!}$. d41d8c, 5 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i, 1, sz(v)) rep(j, 0, v[i]) c = c * ++m / (j+1);
  return c;
}
```


6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
 $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^\infty f(i) = \int_m^\infty f(x)dx - \sum_{k=1}^\infty \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k) x^k = x(x+1) \dots (x+n-1)$$

$$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$$

$$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j :s s.t. $\pi(j) \geq j$, k j :s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

BellmanFord FloydWarshall

6.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i : $n_1 n_2 \dots n_k n^{k-2}$
with degrees d_i : $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Fundamentals

BellmanFord.h
Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get `dist = inf`; nodes reachable through negative-weight cycles get `dist = -inf`. Assumes $V^2 \max |w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

```
void BellmanFord(int st, int n) {
    vector<ll> dist(n+1, INF);
    vector<int> parent(n+1, -1);
    dist[st] = 0;
    for (int i = 0; i < n-1; i++) {
        bool any = false;
        for (auto [u, v, cost] : edges)
            if (dist[u] < INF)
                if (dist[v] > dist[u] + cost) {
                    dist[v] = dist[u] + cost;
                    parent[v] = u;
                    any = true;
                }
        if (!any) break;
    }
    if (dist[n] == INF)
        cout << "-1\n";
    else {
        vector<int> path;
        for (int cur = n; cur != -1; cur = parent[cur])
            path.push_back(cur);
        reverse(path.begin(), path.end());
        for (int u : path)
            cout << u << ' ';
    }
}
```

void BellmanFord(int s, int n) {

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```
vector<ll> dist(n+1, 0); // No need to init INF here because
                        // there can be a negative cycle where you can't reach
                        // from node 1
                        // and the Graph is not necessarily
                        // connected
                        // Our concern is about to find
                        // negative cycle not shortest
                        // distance

vector<int> parent(n+1, -1);
dist[s] = 0;
int flag;
for (int i = 0; i < n; i++) {
    flag = -1;
    for (auto [u, v, cost] : edges) {
        if (dist[u] + cost < dist[v]) {
            dist[v] = dist[u] + cost;
            parent[v] = u;
            flag = v;
        }
    }
}
if (flag == -1)
    cout << "NO\n";
else {
    int y = flag;
    for (int i = 0; i < n; ++i)
        y = parent[y];

    vector<int> path;
    for (int cur = y; cur = parent[cur]) {
        path.push_back(cur);
        if (cur == y && path.size() > 1)
            break;
    }
    reverse(path.begin(), path.end());
    cout << "YES\n";
    for (int u : path)
        cout << u << ' ';
}

}

FloydWarshall.h
Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix  $m$ , where  $m[i][j] = \text{inf}$  if  $i$  and  $j$  are not adjacent. As output,  $m[i][j]$  is set to the shortest distance between  $i$  and  $j$ , inf if no path, or -inf if the path goes through a negative-weight cycle.
Time:  $\mathcal{O}(N^3)$ 
```

```
void init() {
    for(int i = 0; i < n; i++) {
        for(int j = 0; j < n; j++) {
            d[i][j] = INF;
        }
        d[i][i] = 0;
    }
}

void floydWarshall() {
    for (int k = 0; k < n; ++k) {
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) {
                if (d[i][k] < INF && d[k][j] < INF) {
                    d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
                }
            }
        }
    }
}
```


Dijkstra.h

Description: Dijkstra

d41d8c, 22 lines

```
vector<ll> dijkstra(int s, int n, vector<vector<pair<int, ll>>>
    &adj) {
    vector<ll> dist(n+1, INF);
    dist[s] = 0;
    priority_queue<pair<ll, int>, vector<pair<ll, int>>,
        greater<pair<ll, int>>> pq;
    pq.push({0, s});
    bool vis[n+1];
    memset(vis, false, sizeof(vis));
    while(!pq.empty()) {
        auto [d, u] = pq.top();
        pq.pop();
        if(vis[u]) continue;
        vis[u] = true;
        for(auto [v, wt] : adj[u]) {
            ll _d = d + wt;
            if(_d < dist[v]) {
                dist[v] = _d;
                pq.push({_d, v});
            }
        }
    }
    return dist;
}
```

7.2 Network flow

MinCostMaxFlow.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}(FE \log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi.

d41d8c, 76 lines

```
const int N = 500;
vector<int> adj[N+1];
int capacity[N+1][N+1];

int bfs(int s, int d, int n, vector<int> &parent) {
    parent.assign(n+1, -1);
    parent[s] = 0;
    queue<pair<int, int>> q;
    q.push({s, INT_MAX});
    while(!q.empty()) {
        int u = q.front().first;
        int f = q.front().second;
        q.pop();
        for(auto v : adj[u]) {
            if(parent[v] == -1 && capacity[u][v]) {
                parent[v] = u;
                int n_f = min(f, capacity[u][v]);
                if(v == d) return n_f;
                q.push({v, n_f});
            }
        }
    }
    return 0;
}

int max_flow(int s, int d, int n) {
    int mx_flow = 0;
    vector<int> parent;
    int flow;
    while(flow = bfs(s, d, n, parent)) {
        mx_flow+=flow;
        int now = d;
        while(now != s) {
            int prev = parent[now];
            capacity[prev][now] -= flow;
```

```
        capacity[now][prev] += flow;
        now = prev;
    }
    return mx_flow;
}
bool visited[N+1];
void dfs(int u) {
    visited[u] = true;
    for(auto v : adj[u]) if(!visited[v] && capacity[u][v]) dfs(v);
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, m;
        cin >> n >> m;
        for(int i = 0; i < m; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            adj[v].push_back(u);
            capacity[u][v] += 1;
            capacity[v][u] += 1;
        }
        cout << max_flow(1, n, n) << "\n";
        dfs(1);
        for(int u = 1; u <= n; u++) {
            if(visited[u]) {
                for(auto v : adj[u]) {
                    if(!visited[v]) {
                        cout << u << " " << v << "\n";
                    }
                }
            }
        }
    }
    return 0;
}
```

7.3 Matching

7.4 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.

Usage: scc(graph, [&](vi& v) { ... }) visits all components in reverse topological order. comp[i] holds the component index of a node (a component only has edges to components with lower index). ncomps will contain the number of components.

Time: $\mathcal{O}(E + V)$

d41d8c, 29 lines

```
struct SCC {
    // 1-base indexing
    int n;
    vector<vector<int>> adj, radj;
    vector<int> todo, comps, id;
    vector<bool> vis;
    void init(int _n) {
        n = _n;
        adj.resize(n+1), radj.resize(n+1), id.assign(n+1, -1),
            vis.resize(n+1);
    }
}
```

```
void build(int x, int y) { adj[x].push_back(y), radj[y].
    push_back(x); }
void dfs(int x) {
    vis[x] = 1;
    for(auto y : adj[x]) if (!vis[y]) dfs(y);
    todo.push_back(x);
}
void dfs2(int x, int v) {
    id[x] = v;
    for(auto y : radj[x]) if (id[y] == -1) dfs2(y, v);
}
void gen() {
    for(int i = 1; i <= n; i++) if (!vis[i]) dfs(i);
    reverse(todo.begin(), todo.end());
    for(auto x : todo) if (id[x] == -1) {
        dfs2(x, x);
        comps.push_back(x);
    }
}
} scc;
```

ArticulationPoint.h

Description: Finding articulation points in a graph.

d41d8c, 22 lines

```
vector<int> adj[N];
int t = 0;
vector<int> tin(N, -1), low(N), ap;
void dfs(int u, int p) {
    tin[u] = low[u] = t++;
    int is_ap = 0, child = 0;
    for (int v : adj[u]) {
        if (v != p) {
            if (tin[v] != -1) {
                low[u] = min(low[u], tin[v]);
            } else {
                child++;
                dfs(v, u);
                if (tin[u] <= low[v]) is_ap = 1;
                low[u] = min(low[u], low[v]);
            }
        }
    }
    if ((p != -1 or child > 1) and is_ap)
        ap.push_back(u);
}
dfs(0, -1);
```

Bridge.h

Description: Finds all the bridges in a graph.

d41d8c, 19 lines

```
void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    bool parent_skipped = false;
    for (int to : adj[v]) {
        if (to == p && !parent_skipped) {
            parent_skipped = true;
            continue;
        }
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
            dfs(to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] > tin[v])
                IS_BRIDGE(v, to);
        }
    }
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)&&(a||c)&&(d||b)&&...$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: TwoSat ts(number of boolean variables);

ts.either(0, ~3); // Var 0 is true or var 3 is false

ts.setValue(2); // Var 2 is true

ts.atMostOne({0,~1,2}); // ≤ 1 of vars 0, ~1 and 2 are true

ts.solve(); // Returns true iff it is solvable

ts.values[0..N-1] holds the assigned values to the vars

Time: $\mathcal{O}(N + E)$, where N is the number of boolean variables, and E is the number of clauses.

d41d8c, 80 lines

```
struct _2SAT {
    // 0-base indexing
    int n;
    vector<vector<int>>> adj, radj;
    vector<int> todo, comps, id;
    vector<bool> vis, assignment;
    void init(int _n) {
        n = _n;
        adj.resize(n), radj.resize(n), id.assign(n, -1), vis.
            resize(n);
        assignment.assign(n/2, false);
    }
    void build(int x, int y) { adj[x].push_back(y), radj[y].
        push_back(x); }
    void dfs1(int x) {
        vis[x] = 1;
        for(auto y : adj[x]) if (!vis[y]) dfs1(y);
        todo.push_back(x);
    }
    void dfs2(int x, int v) {
        id[x] = v;
        for(auto y : radj[x]) if (id[y] == -1) dfs2(y, v);
    }
    bool solve_2SAT() {
        for(int i = 0; i < n; i++) if (!vis[i]) dfs1(i);
        reverse(todo.begin(), todo.end());
        int j = 0;
        for(auto x : todo) if (id[x] == -1) {
            dfs2(x, j++);
            // comps.push_back(x);
        }
        for (int i = 0; i < n; i += 2) {
            if (id[i] == id[i + 1]) {
                return false;
            }
            assignment[i / 2] = id[i] > id[i + 1];
        }
        return true;
    }
    void add_disjunction(int a, bool na, int b, bool nb) {
        // na and nb signify whether a and b are to be negated
        a = 2 * a ^ na;
        b = 2 * b ^ nb;
        int neg_a = a ^ 1;
        int neg_b = b ^ 1;
        build(neg_a, b);
        build(neg_b, a);
    }
} _2sat;

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
```

```
// cin >> tt;
while(tt--) {
    int n, m;
    cin >> n >> m;
    _2sat.init(m*2);
    for(int i = 0; i < n; i++) {
        int a, b;
        char _na, _nb;
        cin >> _na >> a >> _nb >> b;
        bool na, nb;
        --a, --b;
        if(_na == '+') na = false;
        else na = true;
        if(_nb == '+') nb = false;
        else nb = true;
        _2sat.add_disjunction(a, na, b, nb);
    }
    bool possible = _2sat.solve_2SAT();
    if(possible) {
        for(int i = 0; i < m; i++) {
            if(_2sat.assignment[i]) cout << "+ ";
            else cout << "- ";
        }
    } else cout << "IMPOSSIBLE";
}
return 0;
}
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

Time: $\mathcal{O}(V + E)$

<bits/stdc++.h>, <bits/stdc++.h>

d41d8c, 114 lines

```
/*
Problem Link: https://cses.fi/problemset/task/1691/
Idea: Euler Circuit in undirected graph Hierholzer Algorithm
*/
using namespace std;
const int N = 100000;
vector<pair<int, int>> adj[N+1];
int degree[N+1];
bool visited[2*N+1]; // total edge size
vector<int> euler_path;

void dfs(int u) {
    while(!adj[u].empty()) {
        auto [v, idx] = adj[u].back();
        adj[u].pop_back();
        if(visited[idx]) continue;
        visited[idx] = true;
        dfs(v);
    }
    euler_path.push_back(u);
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, m;
        cin >> n >> m;
        for(int i = 0; i < m; i++) {
            int u, v;
            cin >> u >> v;
```

```
adj[u].push_back({v, i});
adj[v].push_back({u, i});
degree[u]++, degree[v]++;
}
/*
Undirected Graphs:
Euler Circuit: All vertices must have even degree.
Euler Path: Exactly zero or two vertices can have odd
degree.
*/
for(int i = 1; i <= n; i++) {
    if(degree[i] % 2 != 0) {
        cout << "IMPOSSIBLE\n";
        return 0;
    }
}
dfs(1);
if(euler_path.size() != m+1) {
    cout << "IMPOSSIBLE\n";
    return 0;
}
for(auto x : euler_path) {cout << x << " ";}
}
return 0;
}
/*
Problem Link: https://cses.fi/problemset/task/1693/
Idea: Euler Path in Directed graph Hierholzer Algorithm
*/
using namespace std;
const int N = 100000;
vector<int> adj[N+1];
int in[N+1], out[N+1];
vector<int> euler_path;

void dfs(int u) {
    while(!adj[u].empty()) {
        int v = adj[u].back();
        adj[u].pop_back();
        dfs(v);
    }
    euler_path.push_back(u);
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, m;
        cin >> n >> m;
        for(int i = 0; i < m; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            out[u]++, in[v]++;
        }
    }
    /*
Directed Graphs:
Euler Circuit: All vertices must have equal in-degree
and out-degree.
Euler Path: Exactly two vertices can have a difference
of one between their in-degree and out-degree.
*/
for(int i = 1; i <= n; i++) {
    if((i == 1 && out[1]-in[1] != 1) ||
        (i == n && in[n]-out[n] != 1) ||
        (i > 1 && i < n && out[i] != in[i])) {
```

```

        cout << "IMPOSSIBLE\n";
        return 0;
    }
}
dfs(1);
reverse(euler_path.begin(), euler_path.end());
if(euler_path.size() - 1 != m || euler_path.back() != n) {
    cout << "IMPOSSIBLE\n";
    return 0;
}
for(auto x : euler_path) {cout << x << " ";}
}
return 0;
}
}

```

7.5 Coloring

7.6 Heuristics

7.7 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.

Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

d41d8c, 39 lines

```

const int N = 2e5 + 1;
const int LOG = 18; // LOG = ceil(log2(N))
vector<int> adj[N+1];
int up[N+5][LOG], depth[N+5]; // up[v][j] is the 2^j-th
    Ancestor of node v

void ancestor(int u) {
    for(auto v : adj[u]) {
        depth[v] = depth[u] + 1;
        up[v][0] = u;
        for(int j = 1; j < LOG; j++) up[v][j] = up[up[v][j-1]][j-1];
        ancestor(v);
    }
}

int get_lca(int a, int b) {
    if(depth[a] < depth[b]) swap(a, b);
    int k = depth[a] - depth[b];
    for(int i = LOG-1; i >= 0; i--)
        if(k & (1 << i))
            a = up[a][i];

    if(a == b)
        return a;

    for(int i = LOG-1; i >= 0; i--) {
        if(up[a][i] != up[b][i]) {
            a = up[a][i];
            b = up[b][i];
        }
    }
    return up[a][0];
}

/*
int getKthAncestor(int a, int k) {
    for(int i = LOG - 1; i >= 0; i--)
        if(k & (1 << i))
            a = up[a][i];
    return a;
}
*/

```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

Time: $\mathcal{O}(N \log N + Q)$

../data-structures/RMQ.h d41d8c, 20 lines

```

struct LCA {
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;

    LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1), ret)) {}
    void dfs(vector<vi>& C, int v, int par) {
        time[v] = T++;
        for (int y : C[v]) if (y != par) {
            path.push_back(v), ret.push_back(time[v]);
            dfs(C, y, v);
        }
    }
    int lca(int a, int b) {
        if (a == b) return a;
        tie(a, b) = minmax(time[a], time[b]);
        return path[rmq.query(a, b)];
    }
    //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
};

```

DsuOnTree.h

Description: Dsu on tree

<bits/stdc++.h> d41d8c, 88 lines

```

using namespace std;
const int N = 2e5 + 1;
int color[N+1];
vector<int> adj[N+1];

int idx = 0, euler[N+1], pos[N+1], sz[N+1], H_C[N+1];

void dfs(int u, int p) {
    pos[u] = idx;
    euler[idx++] = u;
    H_C[u] = -1, sz[u] = 1;
    for(auto v: adj[u]) {
        if(v == p) continue;
        dfs(v, u);
        sz[u] += sz[v];
        if(H_C[u] == -1 || sz[v] > sz[H_C[u]]) {
            H_C[u] = v;
        }
    }
}

int freq[N+1], cur_distinct = 0, distinct[N+1];
void add(int u) {
    freq[color[u]]++;
    if(freq[color[u]] == 1) cur_distinct++;
}

void remove(int u) {
    freq[color[u]]--;
    if(freq[color[u]] == 0) cur_distinct--;
}

void dsu(int u, int p, int keep) {
    for(auto v : adj[u]) {
        if(v == p || v == H_C[u]) continue;
        dsu(v, u, 0);
    }
    if(H_C[u] != -1) {

```

```

        dsu(H_C[u], u, 1);
    }

    for(auto v : adj[u]) {
        if(v == p || v == H_C[u]) continue;
        for(int i = pos[v]; i < pos[v] + sz[v]; i++) {
            add(euler[i]);
        }
    }
    add(u);
    distinct[u] = cur_distinct;

    if(!keep) {
        for(int i = pos[u]; i < pos[u] + sz[u]; i++) {
            remove(euler[i]);
        }
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n;
        cin >> n;
        map<int, int> compress;
        int id = 1;
        for(int i = 1; i <= n; i++) {
            cin >> color[i];
            if(compress[color[i]]) color[i] = compress[color[i]];
            else {
                compress[color[i]] = id++;
                color[i] = compress[color[i]];
            }
        }
        for(int i = 0; i < n-1; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            adj[v].push_back(u);
        }
        dfs(1, -1);
        dsu(1, -1, 1);
        for(int i = 1; i <= n; i++) cout << distinct[i] << " ";
        return 0;
    }
}

```

HLD.h

Description: Heavy Light Decomposition

<bits/stdc++.h> d41d8c, 136 lines

```

/*
Problem Link: https://cses.fi/problemset/task/2134
*/
using namespace std;
const int N = 2e5 + 1;
int values[N+1], subtree[N+1], parent[N+1], depth[N+1];
int heavy[N+1], head[N+1], id[N+1];
vector<int> adj[N+1];

// 0 Base indexing
struct Segtree {
    int size;
    vector<int> tree;

```

```

int merge(int x, int y) {
    return max(x, y);
}
void build(vector<int> &a, int node, int l, int r) {
    if(l == r) {
        tree[node] = a[l];
        return;
    }
    int mid = l + (r - l)/2;
    build(a, node*2+1, l, mid);
    build(a, node*2+2, mid+1, r);
    tree[node] = merge(tree[node*2+1], tree[node*2+2]);
}
void update(int i, int value, int node, int l, int r) {
    if(l == i && r == i) {
        tree[node] = value;
        return;
    }
    int mid = l + (r-l)/2;
    if(i <= mid) update(i, value, node*2+1, l, mid);
    else update(i, value, node*2+2, mid+1, r);
    tree[node] = merge(tree[node*2+1], tree[node*2+2]);
}
void update(int i, int value) {
    update(i, value, 0, 0, size-1);
}
int query(int i, int j, int node, int l, int r) {
    if(l > j || r < i) return INT_MIN;
    if(l >= i && r <= j) return tree[node];
    int mid = l + (r - l)/2;
    return merge(query(i, j, node*2+1, l, mid), query(i, j,
        node*2+2, mid+1, r));
}
int query(int i, int j) {
    return query(i, j, 0, 0, size-1);
}
int sz(int n) {
    int size = 1;
    while(size < n) size = size << 1;
    return 2*size-1;
}
void init(vector<int> &a, int n) {
    size = 1;
    while(size < n) size = size << 1;
    tree.resize(2*size-1);
    build(a, 0, 0, size-1);
}
} st;

void dfs(int u, int p) {
    subtree[u] = 1;
    int mx = 0;
    for(auto v : adj[u]) {
        if(v == p) continue;
        parent[v] = u;
        depth[v] = depth[u]+1;
        dfs(v, u);
        subtree[v] += subtree[u];
        if(subtree[v] > mx) {
            mx = subtree[v];
            heavy[u] = v;
        }
    }
}
int idx = 0;
void HLD(int u, int h) {
    head[u] = h;
    id[u] = idx++;
    if(heavy[u]) HLD(heavy[u], h);
}

```

```

for(auto v : adj[u]) {
    if(v != parent[u] && v != heavy[u]) {
        HLD(v, v);
    }
}
int path(int x, int y) {
    int ans = 0;
    while(head[x] != head[y]) {
        if(depth[head[x]] > depth[head[y]]) swap(x, y);
        ans = max(ans, st.query(id[head[y]], id[y]));
        y = parent[head[y]];
    }
    if(depth[x] > depth[y]) swap(x, y);
    ans = max(ans, st.query(id[x], id[y]));
    return ans;
}
int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n, q;
        cin >> n >> q;
        for(int i = 0; i < n; i++) cin >> values[i];
        for(int i = 0; i < n-1; i++) {
            int u, v;
            cin >> u >> v;
            adj[u].push_back(v);
            adj[v].push_back(u);
        }
        dfs(1, -1);
        HLD(1, 1);
        vector<int> a(n);
        for(int i = 0; i < n; i++) a[id[i+1]] = values[i];
        st.init(a, n);
        while(q--) {
            int type;
            cin >> type;
            if(type == 1) {
                int s, x;
                cin >> s >> x;
                st.update(id[s], x);
            } else {
                int a, b;
                cin >> a >> b;
                cout << path(a, b) << " ";
            }
        }
    }
    return 0;
}

```

CentroidDecomp.h

Description: Centroid decompose, Finding 1 to K length Path Source : <https://www.codechef.com/problems/PRIMEDST>

d41d8c, 93 lines

```

const int N = 50001;
vector<int> adj[N];
int n, k;
int subtree[N], cnt[N], mx_depth, all_cnt[N];
bool visited[N];
// ll ans;

vector<bool> is_prime(N, true);
set<int> primes;
// O(Nlog(log(N)))

```

```

void sieve() {
    is_prime[0] = is_prime[1] = false;
    for (int i = 2; i * i <= N; i++) {
        if (is_prime[i]) {
            for (int j = i * i; j <= N; j += i)
                is_prime[j] = false;
        }
    }
}
int getSubtree(int u, int p) {
    subtree[u] = 1;
    for(auto v : adj[u]) {
        if(!visited[v] && v != p) {
            getSubtree(v, u);
            subtree[u] += subtree[v];
        }
    }
    return subtree[u];
}
int getCentroid(int u, int p, int desired) {
    for(auto v : adj[u])
        if(!visited[v] && v != p && subtree[v] > desired)
            return getCentroid(v, u, desired);
    return u;
}
void compute(int u, int p, bool filling, int depth) {
    if(depth > k) return;
    mx_depth = max(mx_depth, depth);
    if(filling) {
        cnt[depth]++;
        all_cnt[depth]++;
    } else {
        // ans += cnt[k - depth] * 1LL;
        for(int i = 1; i <= mx_depth; i++) {
            if(cnt[i] all_cnt[i + depth] += cnt[i];
        }
    }
    for(auto v : adj[u]) if(!visited[v] && v != p) compute(v, u,
        filling, depth+1);
}
void centroidDecomposition(int u) {
    int centroid = getCentroid(u, -1, getSubtree(u, -1) >> 1);
    visited[centroid] = true;
    mx_depth = 0;
    for(auto v : adj[centroid]) {
        if(!visited[v]) {
            compute(v, centroid, false, 1);
            compute(v, centroid, true, 1);
        }
    }
    for(int i = 1; i <= mx_depth; i++) cnt[i] = 0;
    for(auto v : adj[centroid]) if(!visited[v])
        centroidDecomposition(v);
}
int main() {
    int tt;
    sieve();
    tt = 1;
    // cin >> tt;
    while(tt--) {
        cin >> n;
        for(int i = 2; i <= n-1; i++) {
            if(is_prime[i]) {
                primes.insert(i);
            }
        }
        for(int i = 0; i < n-1; i++) {
            int u, v;
            cin >> u >> v;

```

```
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    // ans = 0;
    cnt[0] = 1;
    k = *primes.rbegin();
    centroidDecomposition(1);
    ll p_path = 0;
    for(auto x : primes) {
        p_path+=all_cnt[x];
    }
    ll total = n*1LL*(n-1)/2;
    cout << fixed << setprecision(6) << (p_path*1.0)/(total
        *1.0) << "\n";
}
return 0;
}
```

DPOntree.h

Description: DPonTree d41d8c, 64 lines

```
const int N = 100000;
int n, mod;
vector<int> adj[N];
// up[i] = total ways to paint all the ancestors of node i
// if the parent of node i is painted black.
vector<ll> up(N, 1);
// down[i] = total ways to paint the subtree of node i
// if the node i is painted black or white.
ll down[N];

void dfs1(int u, int parent) {
    down[u] = 1;
    for(auto v : adj[u]) {
        if(v == parent)continue;
        dfs1(v, u);
        down[u] = (down[u] * down[v]) % mod;
    }
    down[u] = (down[u] + 1) % mod;
}

void dfs2(int u, int parent) {
    int pref = 1;
    for(auto v : adj[u]) {
        if(v == parent)continue;
        up[v] = pref % mod;
        pref = pref*down[v] % mod;
    }
    reverse(adj[u].begin(), adj[u].end());
    int suff = 1;
    for(auto v : adj[u]) {
        if(v == parent)continue;
        up[v] = up[v]*suff % mod;
        suff = suff*down[v] % mod;
    }
    for(auto v : adj[u]) {
        if(v == parent)continue;
        up[v] = up[u] * up[v] % mod;
        up[v] = (up[v] + 1) % mod;
        dfs2(v, u);
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
```

```
    cin >> n >> mod;
    for(int i = 0; i < n-1; i++) {
        int u, v;
        cin >> u >> v;
        --v, --u;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    dfs1(0, -1);
    dfs2(0, -1);
    for(int i = 0; i < n; i++) {
        cout << up[i]*(down[i] - 1 + mod) % mod << "\n";
    }
}
return 0;
}
```

7.8 Math

7.8.1 Number of Spanning Trees

Create an $N \times N$ matrix mat , and for each edge $a \rightarrow b \in G$, do $mat[a][b]--, mat[b][b]++$ (and $mat[b][a]--, mat[a][a]++$ if G is undirected). Remove the i th row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.8.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \dots \geq d_n$ exists iff $d_1 + \dots + d_n$ is even and for every $k = 1 \dots n$,

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.) d41d8c, 172 lines

```
using ftype = ll;
const double eps = 1e-9;
const double PI = acos((double)-1.0);
int sign(double x) { return (x > eps) - (x < -eps);}

struct P {
    ftype x, y;
    P() {}
    P(ftype x, ftype y): x(x), y(y) {}
    void read() {
        cin >> x >> y;
    }
    P& operator+=(const P &t) {
        x += t.x;
        y += t.y;
        return *this;
    }
    P& operator-=(const P &t) {
        x -= t.x;
        y -= t.y;
        return *this;
    }
};
```

```
}
P& operator*=(ftype t) {
    x *= t;
    y *= t;
    return *this;
}
P& operator/=(ftype t) {
    x /= t;
    y /= t;
    return *this;
}
P operator+(const P &t) const {return P(*this) += t;}
P operator-(const P &t) const {return P(*this) -= t;}
P operator*(ftype t) const {return P(*this) *= t;}
P operator/(ftype t) const {return P(*this) /= t;}
bool operator == (P a) const { return sign(a.x - x) == 0 &&
    sign(a.y - y) == 0; }
bool operator != (P a) const { return !(*this == a); }
bool operator < (P a) const { return sign(a.x - x) == 0 ? y
    < a.y : x < a.x; }
bool operator > (P a) const { return sign(a.x - x) == 0 ? y
    > a.y : x > a.x; }
P perp() const {
    return P(y, -x); // Or P(y, -x) depending on the
        desired direction.
}

};

P operator*(ftype a, P b) {return b * a;}
inline ftype dot(P a, P b) {return a.x * b.x + a.y * b.y;}
inline ftype cross(P a, P b) {return a.x * b.y - a.y * b.x;}
ftype norm(P a) {return dot(a, a);}
double abs(P a) {return sqrt(norm(a));}
double proj(P a, P b) {return dot(a, b) / abs(b);}
double angle(P a, P b) {return acos(dot(a, b) / abs(a) / abs(b)
    );}
P intersect(P a1, P d1, P a2, P d2) {return a1 + cross(a2 - a1,
    d2) / cross(d1, d2) * d1;}

bool LineSegmentIntersection(P p1, P p2, P p3, P p4) {
    // Check if they are parallel
    if(cross(p1-p2, p3-p4) == 0) {
        // If they are not collinear
        if(cross(p2-p1, p3-p1) != 0) {
            return false;
        }
        // Check if they are collinear and do not intersect
        for(int it = 0; it < 2; it++) {
            if(max(p1.x, p2.x) < min(p3.x, p4.x) ||
                max(p1.y, p2.y) < min(p3.y, p4.y)) {
                return false;
            }
            swap(p1, p3), swap(p2, p4);
        }
        return true;
    }
    // Check one segment totally on the left or right side of
        other segment
    for(int it = 0; it < 2; it++) {
        ll sign1 = cross(p2-p1, p3-p1);
        ll sign2 = cross(p2-p1, p4-p1);
        if((sign1 < 0 && sign2 < 0) || (sign1 > 0 && sign2 > 0)
            ) {
            return false;
        }
        swap(p1, p3), swap(p2, p4);
    }
    // For all other case return true
    return true;
}
```

```

}

// here return value is area*2
ftype PolygonArea(vector<P> &polygon, int n) {
    ll area = 0;
    for(int i = 0; i < n; i++) {
        int j = (i+1) % n;
        area+=cross(polygon[i], polygon[j]);
    }
    return abs(area);
}

string PointInPolygon(vector<P> &polygon, int n, P &p) {
    int cnt = 0;
    for(int i = 0; i < n; i++) {
        int j = (i+1) % n;
        if(LineSegmentIntersection(polygon[i], polygon[j], p, p
        )) {
            return "BOUNDARY";
        }
        /*
        Imagine a vertically infinite line from point p to
        positive infinity.
        Check if a line from the polygon is totally on the left
        or right side of the infinite line and makes a
        positive cross product or positive triangle.
        Here, "right" means to the right or equal.
        */
        if((polygon[i].x >= p.x && polygon[j].x < p.x && cross(
            polygon[i]-p, polygon[j]-p) > 0) ||
            (polygon[i].x < p.x && polygon[j].x >= p.x && cross(
                polygon[j]-p, polygon[i]-p) > 0))
            cnt++;
    }
    if(cnt & 1) return "INSIDE";
    return "OUTSIDE";
}

void ConvexHull(vector<P> &points, int n) {
    vector<P> hull;
    sort(points.begin(), points.end());
    for(int rep = 0; rep < 2; rep++) {
        const int h = (int)hull.size();
        for(auto C : points) {
            while((int)hull.size() - h >= 2) {
                P A = hull[(int)hull.size()-2];
                P B = hull[(int)hull.size()-1];
                if(cross(B-A, C-A) <= 0) {
                    break;
                }
                hull.pop_back();
            }
            hull.push_back(C);
        }
        hull.pop_back();
        reverse(points.begin(), points.end());
    }
    cout << hull.size() << "\n";
    for(auto p : hull) {
        cout << p.x << " " << p.y << "\n";
    }
}

bool circleInter(P a, P b, double r1, double r2, pair<P, P>*
    out) {
    P vec = b - a;
    double d2 = norm(vec);
    double d = sqrt(d2);
    if (d > r1 + r2 || d < fabs(r1 - r2)) {
        return false;
    }

```

```

    }
    double p = (d2 + r1 * r1 - r2 * r2) / (2 * d);
    double h2 = r1 * r1 - p * p;
    if (h2 < 0) h2 = 0;
    P mid = a + vec * (p / d);
    P per = vec.perp() * (sqrt(h2) / d);
    *out = {mid + per, mid - per};
    return true;
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    int tt;
    tt = 1;
    // cin >> tt;
    while(tt--) {
        int n;
        cin >> n;
        vector<P> points;
        for(int i = 0; i < n; i++) {
            P p;
            p.read();
            points.push_back(p);
        }
        ConvexHull(points, n);
    }
    return 0;
}

```

8.2 Circles

8.3 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.

Time: $\mathcal{O}(n \log n)$

d41d8c, 64 lines

```

#define pii pair<ll, ll>
#define ff first
#define ss second

bool comparex(pii a, pii b) { return a.first < b.first; }
bool comparey(pii a, pii b) { return a.second < b.second; }
ll dist(pii x, pii y) { return (x.ff - y.ff) * (x.ff - y.ff) +
    (x.ss - y.ss) * (x.ss - y.ss); }

pair<pii, pii> closestAmongThree(pii a, pii b, pii c) {
    ll d1 = dist(a, b);
    ll d2 = dist(b, c);
    ll d3 = dist(a, c);
    ll mn = min({ d1, d2, d3 });
    if (mn == d1) return { a, b };
    else if (mn == d2) return { b, c };
    else return { a, c };
}

pair<pii, pii> closest(vector<pii>& points, ll st, ll en) {
    if (st + 1 == en) return { points[st], points[en] };
    if (st + 2 == en) return closestAmongThree(points[st],
        points[st + 1], points[en]);

    ll mid = st + (en - st) / 2;

    pair<pii, pii> left = closest(points, st, mid);
    pair<pii, pii> right = closest(points, mid + 1, en);
    ll left_d = dist(left.ff, left.ss);
    ll right_d = dist(right.ff, right.ss);
    ll d = min(left_d, right_d);
    pair<pii, pii> ans = (d == left_d) ? left : right;

    vector<pii> middle;
    for (int i = st; i <= en; i++)

```

```

        if (abs(points[i].ff - points[mid].ff) < d)
            middle.push_back(points[i]);
        sort(middle.begin(), middle.end(), comparey);

        for (int i = 0; i < (int)middle.size(); i++) {
            for (int j = i + 1; j < (int)middle.size() and (middle[
                j].ss - middle[i].ss) * (middle[j].ss - middle[i].
                ss) < d; j++) {
                ll dst = dist(middle[i], middle[j]);
                if (dst < d) {
                    ans = { middle[i], middle[j] };
                    d = dst;
                }
            }
        }
        middle.clear();

        return ans;
    }

int main() {
    int tt;
    tt = 1;
    while (tt--) {
        int n;
        cin >> n;
        vector<pii> points(n);
        for (int i = 0; i < n; i++) {
            cin >> points[i].first >> points[i].second;
        }
        sort(points.begin(), points.end(), comparex);
        pair<pii, pii> ans = closest(points, 0, n - 1);
        cout << dist(ans.ff, ans.ss) << '\n';
    }
    return 0;
}

```

SweepLine.h

Description: Returns any intersecting segments, or -1, -1 if none exist.

Time: $\mathcal{O}(N \log N)$

Strings (9)

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

Time: $\mathcal{O}(n)$

d41d8c, 15 lines

```

vi pi(const string& s) {
    vi p(sz(s));
    rep(i, 1, sz(s)) {
        int g = p[i-1];
        while (g && s[i] != s[g]) g = p[g-1];
        p[i] = g + (s[i] == s[g]);
    }
    return p;
}

vi match(const string& s, const string& pat) {
    vi p = pi(pat + '\0' + s), res;
    rep(i, sz(p)-sz(s), sz(p))
        if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat));
    return res;
}

```


Zfunc.h

Description: $z[i]$ computes the length of the longest common prefix of $s[i]$ and s , except $z[0] = 0$. (abacaba -> 0010301)

Time: $\mathcal{O}(n)$

d41d8c, 12 lines

```
vi Z(const string& S) {
    vi z(sz(S));
    int l = -1, r = -1;
    rep(i, 1, sz(S)) {
        z[i] = i >= r ? 0 : min(r - i, z[i - l]);
        while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
            z[i]++;
        if (i + z[i] > r)
            l = i, r = i + z[i];
    }
    return z;
}
```

Manacher.h

Description: For each position in a string, computes $p[0][i] = \text{half length of longest even palindrome around pos } i$, $p[1][i] = \text{longest odd (half rounded down)}$.

Time: $\mathcal{O}(N)$

d41d8c, 13 lines

```
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi, 2> p = {vi(n+1), vi(n)};
    rep(z, 0, 2) for (int i=0, l=0, r=0; i < n; i++) {
        int t = r-i+!z;
        if (i < r) p[z][i] = min(t, p[z][l+t]);
        int L = i-p[z][i], R = i+p[z][i]-!z;
        while (L >= 1 && R+1 < n && s[L-1] == s[R+1])
            p[z][i]++, L--, R++;
        if (R > r) l=L, r=R;
    }
    return p;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: rotate(v.begin(), v.begin()+minRotation(v), v.end());

Time: $\mathcal{O}(N)$

d41d8c, 8 lines

```
int minRotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b, 0, N) rep(k, 0, N) {
        if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
        if (s[a+k] > s[b+k]) {a = b; break;}
    }
    return a;
}
```

SuffixArray.h

Description: Suffix Array

d41d8c, 44 lines

```
void count_sort(vector<pli> &b, int bits) {
    int mask = (1 << bits) - 1;
    rep(it, 0, 2) {
        int shift = it * bits;
        vi q(1 << bits), w(sz(q) + 1);
        rep(i, 0, sz(b)) q[(b[i].first >> shift) & mask]++;
        partial_sum(q.begin(), q.end(), w.begin() + 1);
        vector<pli> res(sz(b));
        rep(i, 0, sz(b)) res[w[(b[i].first >> shift) & mask]++] = b[i];
        swap(b, res);
    }
}

struct SuffixArray {
    vi a; string s;
```

```
SuffixArray(const string &str) : s(str + '\0') {
    int N = sz(s), q = 8;
    while ((1 << q) < N) q++;
    vector<pli> b(N);
    a.resize(N);
    rep(i, 0, N) b[i] = {s[i], i};
    for (int moc = 0;; moc++) {
        count_sort(b, q);
        rep(i, 0, N) a[b[i].second] = (i && b[i].first == b[i - 1].first) ? a[b[i - 1].second] : i;
        if ((1 << moc) >= N) break;
        rep(i, 0, N) {
            b[i] = {(1ll)a[i] << q, i + (1 << moc) < N ? a[i + (1 << moc)] : 0};
            b[i].second = i;
        }
    }
    rep(i, 0, N) a[i] = b[i].second;
}

vi lcp() {
    int n = sz(a), h = 0;
    vi inv(n), res(n);
    rep(i, 0, n) inv[a[i]] = i;
    rep(i, 0, n) if (inv[i]) {
        int p0 = a[inv[i] - 1];
        while (s[i + h] == s[p0 + h]) h++;
        res[inv[i]] = h;
        if (h) h--;
    }
    return res;
}
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices $[l, r]$ into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining $[l, r]$ substrings. The root is 0 (has $l = -1, r = 0$), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

Time: $\mathcal{O}(26N)$

d41d8c, 47 lines

```
struct SuffixTree {
    enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
    int toi(char c) { return c - 'a'; }
    string a; // v = cur node, q = cur position
    int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;
    void ukkadd(int i, int c) { suff:
        if (r[v] <= q) {
            if (t[v][c] == -1) { t[v][c] = m; l[m] = i;
                p[m++] = v; v = s[v]; q = r[v]; goto suff; }
            v = t[v][c]; q = l[v];
        }
        if (q == -1 || c == toi(a[q])) q++; else {
            l[m+1] = i; p[m+1] = m; l[m] = l[v]; r[m] = q;
            p[m] = p[v]; t[m][c] = m+1; t[m][toi(a[q])] = v;
            l[v] = q; p[v] = m; t[p[m]][toi(a[l[m]])] = m;
            v = s[p[m]]; q = l[m];
            while (q < r[m]) { v = t[v][toi(a[q])]; q = r[v] - l[v]; }
            if (q == r[m]) s[m] = v; else s[m] = m+2;
            q = r[v] - (q - r[m]); m += 2; goto suff;
        }
    }

    SuffixTree(string a) : a(a) {
        fill(r, r+N, sz(a));
        memset(s, 0, sizeof s);
        memset(t, -1, sizeof t);
        fill(t[1], t[1]+ALPHA, 0);
        s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
```

```
        rep(i, 0, sz(a)) ukkadd(i, toi(a[i]));
    }
    // example: find longest common substring (uses ALPHA = 28)
    pii best;
    int lcs(int node, int i1, int i2, int olen) {
        if (l[node] <= i1 && i1 < r[node]) return 1;
        if (l[node] <= i2 && i2 < r[node]) return 2;
        int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
        rep(c, 0, ALPHA) if (t[node][c] != -1)
            mask |= lcs(t[node][c], i1, i2, len);
        if (mask == 3)
            best = max(best, {len, r[node] - len});
        return mask;
    }
    static pii LCS(string s, string t) {
        SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
        st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
        return st.best;
    }
};
```

Hashing.h

Description: Self-explanatory methods for string hashing. (Arithmetic mod $2^{64} - 1$. 2x slower than mod 2^{64} and more code, but works on evil test data (e.g. Thue-Morse, where ABBA... and BAAB... of length 2^{10} hash the same mod 2^{64}). "typedef ull H;" instead if you think test data is random, or work mod $10^9 + 7$ if the Birthday paradox is not a problem.)

d41d8c, 36 lines

```
typedef uint64_t ull;
struct H {
    ull x; H(ull x=0) : x(x) {}
    H operator+(H o) { return x + o.x + (x + o.x < x); }
    H operator-(H o) { return *this + ~o.x; }
    H operator*(H o) { auto m = (__uint128_t)x * o.x;
        return H((ull)m) + (ull)(m >> 64); }
    ull get() const { return x + !~x; }
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (1ll)1e11+3; // (order ~ 3e9; random also ok)
struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
        pw[0] = 1;
        rep(i, 0, sz(str))
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    }
    H hashInterval(int a, int b) { // hash [a, b]
        return ha[b] - ha[a] * pw[b - a];
    }
};
vector<H> getHashes(string& str, int length) {
    if (sz(str) < length) return {};
    H h = 0, pw = 1;
    rep(i, 0, length)
        h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h};
    rep(i, length, sz(str)) {
        ret.push_back(h = h * C + str[i] - pw * str[i-length]);
    }
    return ret;
}

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

AhoCorasick.h

Description: Aho Corasick

d41d8c, 56 lines

```
struct AC {
```

```
int N, P;
const int A = 26;
vector <vector <int>> next;
vector <int> link, out_link;
vector <vector <int>> out;
AC(): N(0), P(0) {node();}
int node() {
    next.emplace_back(A, 0);
    link.emplace_back(0);
    out_link.emplace_back(0);
    out.emplace_back(0);
    return N++;
}
inline int get (char c) {
    return c - 'a';
}
int add_pattern (const string T) {
    int u = 0;
    for (auto c : T) {
        if (!next[u][get(c)]) next[u][get(c)] = node();
        u = next[u][get(c)];
    }
    out[u].push_back(P);
    return P++;
}
void compute() {
    queue <int> q;
    for (q.push(0); !q.empty(); ) {
        int u = q.front(); q.pop();
        for (int c = 0; c < A; ++c) {
            int v = next[u][c];
            if (!v) next[u][c] = next[link[u]][c];
            else {
                link[v] = u ? next[link[u]][c] : 0;
                out_link[v] = out[link[v]].empty() ? out_link[link[v]] : link[v];
                q.push(v);
            }
        }
    }
}
int advance (int u, char c) {
    while (u && !next[u][get(c)]) u = link[u];
    u = next[u][get(c)];
    return u;
}
void match (const string S) {
    int u = 0;
    for (auto c : S) {
        u = advance(u, c);
        for (int v = u; v; v = out_link[v]) {
            for (auto p : out[v]) cout << "match " << p << endl;
        }
    }
}
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

set<pii>::iterator addInterval(set<pii>& is, int L, int R) {

```
if (L == R) return is.end();
auto it = is.lower_bound({L, R}), before = it;
while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
}
if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
}
return is.insert(before, {L,R});
}
```

```
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second;
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

template<class T>

```
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
    vi S(sz(I)), R;
    iota(all(S), 0);
    sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
    T cur = G.first;
    int at = 0;
    while (cur < G.second) { // (A)
        pair<T, int> mx = make_pair(cur, -1);
        while (at < sz(I) && I[S[at]].first <= cur) {
            mx = max(mx, make_pair(I[S[at]].second, S[at]));
            at++;
        }
        if (mx.second == -1) return {};
        cur = mx.first;
        R.push_back(mx.second);
    }
    return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});
Time: $\mathcal{O}(k \log \frac{n}{k})$

template<class F, class G, class T>

```
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
    if (p == q) return;
    if (from == to) {
        g(i, to, p);
        i = to; p = q;
    } else {
        int mid = (from + to) >> 1;
        rec(from, mid, f, g, i, p, f(mid));
        rec(mid+1, to, f, g, i, p, q);
    }
}
```

```
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
    if (to <= from) return;
    int i = from; auto p = f(i), q = f(to-1);
    rec(from, to-1, f, g, i, p, q);
    g(i, to, q);
}
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a,b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f , change it to $>$, also at (B).
Usage: int ind = ternSearch(0,n-1,[&](int i){return a[i];});
Time: $\mathcal{O}(\log(b-a))$

template<class F>

```
int ternSearch(int a, int b, F f) {
    assert(a <= b);
    while (b - a >= 5) {
        int mid = (a + b) / 2;
        if (f(mid) < f(mid+1)) a = mid; // (A)
        else b = mid+1;
    }
    rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
    return a;
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

template<class I> vi lis(const vector<I>& S) {

```
    if (S.empty()) return {};
    vi prev(sz(S));
    typedef pair<I, int> p;
    vector<p> res;
    rep(i,0,sz(S)) {
        // change 0 -> i for longest non-decreasing subsequence
        auto it = lower_bound(all(res), p{S[i], 0});
        if (it == res.end()) res.emplace_back(), it = res.end()-1;
        *it = {S[i], i};
        prev[i] = it == res.begin() ? 0 : (it-1)->second;
    }
    int L = sz(res), cur = res.back().second;
    vi ans(L);
    while (L--) ans[L] = cur, cur = prev[cur];
    return ans;
}
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t , computes the maximum $S \leq t$ such that S is the sum of some subset of the weights.
Time: $\mathcal{O}(N \max(w_i))$

int knapsack(vi w, int t) {

```
    int a = 0, b = 0, x;
    while (b < sz(w) && a + w[b] <= t) a += w[b++];
    if (b == sz(w)) return a;
    int m = *max_element(all(w));
    vi u, v(2*m, -1);
    v[a+m-t] = b;
    rep(i,b,sz(w)) {
        u = v;
        rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
        for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
```

```
        v[x-w[j]] = max(v[x-w[j]], j);
    }
    for (a = t; v[a+m-t] < 0; a--);
    return a;
}
```

10.3 Dynamic programming

KnuthDP.h
Description: When doing DP on intervals: $a[i][j] = \min_{i < k < j} (a[i][k] + a[k][j]) + f(i, j)$, where the (minimal) optimal k increases with both i and j , one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j - 1]$ and $p[i + 1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b, c) \leq f(a, d)$ and $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^2)$

DivideAndConquerDP.h
Description: Given $a[i] = \min_{lo(i) \leq k < hi(i)} (f(i, k))$ where the (minimal) optimal k increases with i , computes $a[i]$ for $i = L..R - 1$.
Time: $\mathcal{O}((N + (hi - lo)) \log N)$

```
struct DP { // Modify at will:
    int lo(int ind) { return 0; }
    int hi(int ind) { return ind; }
    ll f(int ind, int k) { return dp[ind][k]; }
    void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

    void rec(int L, int R, int LO, int HI) {
        if (L >= R) return;
        int mid = (L + R) >> 1;
        pair<ll, int> best (LLONG_MAX, LO);
        rep(k, max(LO, lo(mid)), min(HI, hi(mid)))
            best = min(best, make_pair(f(mid, k), k));
        store(mid, best.second, best.first);
        rec(L, mid, LO, best.second+1);
        rec(mid+1, R, best.second, HI);
    }
    void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });`
converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.5 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x;) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).

- `c = x&-x, r = x+c; ((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b, 0, K) rep(i, 0, (1 << K))`
if `(i & 1 << b) D[i] += D[i^(1 << b)];`
computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastMod.h
Description: Compute $a \% b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod b$ in the range $[0, 2b)$.

```
typedef unsigned long long ull;
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m(-1ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t)m * a) >> 64) * b;
    }
};
```

10.6 Miscellaneous

SOSDP.h
Description: SOS DP

submaskiterate.h
Description: Submask iterate

nCrNotP.h
Description: Finds nCr modulo a number that is not necessarily prime. Its good when m is small and not fixed.
Time: $\mathcal{O}(m \log m)$

```
ll f(ll n) {
    if (!n) return 1;
    return modpow(F[pe], n / pe, pe) * (F[n % pe] * f(n / p) % pe) % pe;
}
ll ncr(ll n, ll r) {
    ll c;
    if ((c = lg(n, p) - lg(r, p) - lg(n - r, p)) >= e)
        return 0;
    for (int i = 1; i <= pe; i++)
        F[i] = F[i - 1] * (i % p == 0 ? 1 : i) % pe;
    return (f(n) * modpow(p, c, pe) % pe) *
        modpow(f(r) * f(n - r), pe - (pe / p) - 1, pe) % pe;
}
ll ncr(ll n, ll r, ll m) {
    ll a0 = 0, m0 = 1;
    for (p = 2; m != 1; p++) {
        e = 0, pe = 1;
        while (m % p == 0)
            m /= p, e++, pe *= p;
        if (e) {
            a0 = crt(a0, m0, ncr(n, r), pe);
            m0 = m0 * pe;
        }
    }
    return a0;
}
```