Information Theory **DCC/ICEx/UFMG**
Prof. Mário S. Alvim **2017/02**

HOMEWORK

Discrete Probability - Part 1/2
(Based on slide-set)

Necessary reading for this assignment:

- *Slide-set of Lecture 01 - Discrete Probability*:
  - *An Introduction to Discrete Probability*
  - *Probability Theory*

Note: The exercises are labeled according to their level of difficulty: `[Easy]`, `[Medium]` or `[Hard]`. This labeling, however, is subjective: different people may disagree on the perceived level of difficulty of any given exercise. Don't be discouraged when facing a hard exercise, you may find a solution that is simpler than the one the instructor had in mind!

Review questions.

1. (Rosen Review Question 7-1) `[Easy]`
   (a) Define the probability of an event when all outcomes are equally likely.
   (b) What is the probability that you select the six winning numbers in a lottery if the six different winning numbers are selected from the first 50 positive integers?
2. (Rosen Review Question 7-2) `[Easy]`
   (a) What conditions should be met by the probabilities assigned to the outcomes from a finite sample space?
   (b) What probabilities should be assigned to the outcome of heads and the outcome of tails if heads comes up three times as often as tails?
3. (Rosen Review Question 7-3) `[Easy]`
   (a) Define the conditional probability of an event E given an event F.
   (b) Suppose E is the event that when a die is rolled it comes up an even number, and F is the event that when a die is rolled it comes up 1, 2, or 3. What is the probability of F given E?
4. (Rosen Review Question 7-4) `[Easy]`
   (a) When are two events E and F independent?
   (b) Suppose E is the event that an even number appears when a fair die is rolled, and F is the event that a 5 or 6 comes up. Are E and F independent?
5. (Rosen Review Question 7-5) `[Easy]`
   (a) What is a random variable?
   (b) What are the possible values assigned by the random variable X that assigns to a roll of two dice the larger number that appears on the two dice? .

Exercises.

6. (Rosen 7.1-5) `[Easy]` What is the probability that the sum of the numbers on two dice is even when they are rolled?

7. (Rosen 7.1-21) `[Easy]` What is the probability that a fair die never comes up an even number when it is rolled six times?

8. (Rosen 7.1-37) `[Medium]` Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?

9. (Rosen 7.2-1) `[Easy]` What probability should be assigned to the outcome of heads when a biased coin is tossed, if heads is three times as likely to come up as tails? What probability should be assigned to the outcome of tails?

10. (Rosen 7.2-11) `[Hard]` Suppose that E and F are events such that $p(E) = 0.7$ and $p(F) = 0.5$. Show that $p(E \cup F) \geq 0.7$ and $p(E \cap F) \geq 0.2$.

11. (Rosen 7.2-17) `[Hard]` If E and F are independent events, prove or disprove that $\overline{E}$ and F are necessarily independent events.

12. (Rosen 7.2-25) `[Medium]` What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)

13. (Rosen 7.2-27(b)) `[Medium]` Let E and F be the events that a family of 4 children has children of both sexes and has at most one boy, respectively. Are E and F independent?