

1. De princípio, definimos:

- H : HIV positivo
- \overline{H} : HIV negativo
- T : teste indica HIV positivo
- \overline{T} : teste indica HIV negativo

Pelo enunciado, temos:

$$\begin{aligned}p(H) &= 0.08 \\p(\overline{H}) &= 1 - p(H) = 1 - 0.08 = 0.92 \\p(T|H) &= 0.98 \\p(\overline{T}|H) &= 1 - p(T|H) = 0.02 \\p(T|\overline{H}) &= 0.03 \\p(\overline{T}|\overline{H}) &= 1 - p(T|\overline{H}) = 0.97\end{aligned}$$

(a) Utilizando o Teorema de Bayes.

$$\begin{aligned}p(H|T) &= \frac{p(T|H)p(H)}{p(T|H)p(H) + p(T|\overline{H})p(\overline{H})} \\&= \frac{0.98 \cdot 0.08}{0.98 \cdot 0.08 + 0.03 \cdot 0.92} \approx 0.739\end{aligned}$$

(b)

$$p(\overline{H}|T) = 1 - p(H|T) \approx 1 - 0.739 = 0.261$$

(c) Utilizando o Teorema de Bayes.

$$\begin{aligned}p(H|\overline{T}) &= \frac{p(\overline{T}|H)p(H)}{p(\overline{T}|H)p(H) + p(\overline{T}|\overline{H})p(\overline{H})} \\&= \frac{0.02 \cdot 0.08}{0.02 \cdot 0.08 + 0.97 \cdot 0.92} \approx 0.002\end{aligned}$$

(d)

$$p(\overline{H}|\overline{T}) = 1 - p(H|\overline{T}) \approx 1 - 0.002 = 0.998$$