

HOMEWORK
DEPENDENT RANDOM VARIABLES
(MACKEY - CHAPTER 8)

Necessary reading for this assignment:

- *Information Theory, Inference, and Learning Algorithms* (MacKay): *Information Theory, Inference, and Learning Algorithms* (MacKay):
 - Chapter 8.1: *More about entropy*

Note: The exercises are labeled according to their level of difficulty: [Easy], [Medium] or [Hard]. This labeling, however, is subjective: different people may disagree on the perceived level of difficulty of any given exercise. Don't be discouraged when facing a hard exercise, you may find a solution that is simpler than the one the instructor had in mind!

Review questions.

1. The entropy $H(X) = -\sum_x p(x) \log p(x)$ can be interpreted as the uncertainty one has about the random variable X . With that in mind, for each of the items below, give its name, its mathematical formula and explain its meaning in terms of uncertainty.
 - (a) $H(X, Y)$.
 - (b) $H(X | Y)$.
 - (c) $I(X; Y)$.
 - (d) $I(X; Y | Z)$.
2. State the following "laws" of information theory.
 - (a) The chain rule for entropy $H(X_1, X_2, \dots, X_n)$.
 - (b) The chain rule for mutual information $I(X_1, X_2, \dots, X_n; Y)$.
 - (c) The data-processing inequality (DPI), and explain what it intuitively means.

Exercises.

3. (MacKay 8.1) [Medium]
4. (MacKay 8.2) [Medium]
5. (MacKay 8.6) [Easy]
6. (MacKay 8.7) [Medium]
7. (MacKay 8.9) [Hard]
8. (MacKay 8.10) [Medium]