Information Theory **DCC/ICEx/UFMG**
Prof. Mário S. Alvim **2017/02**

HOMEWORK

Kolmogorov Complexity and Universal Probability
(Based on slide-set)

Necessary reading for this assignment:

- *Slide-set of Lecture 08 - Kolmogorov Complexity and Universal Probability*

Note: The exercises are labeled according to their level of difficulty: `[Easy]`, `[Medium]` or `[Hard]`. This labeling, however, is subjective: different people may disagree on the perceived level of difficulty of any given exercise. Don't be discouraged when facing a hard exercise, you may find a solution that is simpler than the one the instructor had in mind!

Review questions.

1. Answer formally the following questions:

   (a) Define the Kolmogorov Complexity of a string

   (b) When is a string considered truly random? Give an example of a truly random binary string and an example of a non-random string that looks random.

   (c) What is the universal probability of a string? How is it related to the string's Kolmogorov complexity?

Problems.

2. (Cover & Thomas 14.1) `[Medium]` Let $x, y \in \{0,1\}^*$ be two binary sequences. Argue that the Kolmogorov complexity $K(xy)$ of the concatenation of x and y satisfies $K(xy) \leq K(x) + K(y) + c$.

3. (Cover & Thomas 14.2) `[Medium]` Let n_1 and n_2 be two binary numbers.

   (a) Argue that the complexity $K(n_1 + n_2)$ of the sum of n_1 and n_2 satisfies $K(n_1 + n_2) \leq K(n_1) + K(n_2) + c$.

   (b) Give an example of binary numbers n_1 and n_2 that are complex, but such that $n_1 + n_2$ is simple.

4. (Cover & Thomas 14.5 - *Monkeys on a computer*) `[Medium]` Suppose that a random program is typed into a computer. Give a rough estimate of the probability that the computer prints the following sequence:

   a) 0^n followed by any arbitrary sequence.

   b) $\pi_1 \pi_2 \ldots \pi_n$ followed by any arbitrary sequence, where π_i is the i-th bit in the expansion of π.