1. (a)

$$
\begin{aligned}
&V\left(\sum_{i=1}^{n} X_i\right) \\
&= E\left(\left(\sum_{i=1}^{n} X_i\right)^2\right) - E\left(\sum_{i=1}^{n} X_i\right)^2 \\
&= V\left(X_1 + \sum_{i=2}^{n} X_i\right) \\
&= E\left(\left(X_1 + \sum_{i=2}^{n} X_i\right)^2\right) - E\left(X_1 + \sum_{i=2}^{n} X_i\right)^2 \\
&= E\left({X_1}^2 + 2X_1 \sum_{i=2}^{n} X_i + \left(\sum_{i=2}^{n} X_i\right)^2\right) \\
&\quad - \left(E\left(X_1\right) + E\left(\sum_{i=2}^{n} X_i\right)\right)^2 \\
&= E({X_1}^2) + \underbrace{E\left(2X_1 \sum_{i=2}^{n} X_i\right)}_{2E(X_1)E\left(\sum_{i=2}^{n} X_i\right)} + E\left(\left(\sum_{i=2}^{n} X_i\right)^2\right) \\
&\quad - \left(E(X_1)^2 + 2E\left(X_1\right) E\left(\sum_{i=2}^{n} X_i\right) + \left(E\left(\sum_{i=2}^{n} X_i\right)\right)^2\right) \\
&= E({X_1}^2) + 2E\left(X_1\right) E\left(\sum_{i=2}^{n} X_i\right) + E\left(\left(\sum_{i=2}^{n} X_i\right)^2\right) \\
&\quad - E(X_1)^2 - 2E\left(X_1\right) E\left(\sum_{i=2}^{n} X_i\right) - \left(E\left(\sum_{i=2}^{n} X_i\right)\right)^2 \\
&= E({X_1}^2) + E\left(\left(\sum_{i=2}^{n} X_i\right)^2\right) \\
&\quad - E(X_1)^2 - \left(E\left(\sum_{i=2}^{n} X_i\right)\right)^2 \\
&= \underbrace{E({X_1}^2) - E(X_1)^2}_{V(X_1)} \\
&\quad + \underbrace{E\left(\left(\sum_{i=2}^{n} X_i\right)^2\right) - \left(E\left(\sum_{i=2}^{n} X_i\right)\right)^2}_{V\left(\sum_{i=2}^{n} X_i\right)}
\end{aligned}
$$

Se refizermos a operação para as somas restantes $\sum_{i=2}^{n} X_i$, teremos:

$$V\left(\sum_{i=1}^{n} X_i\right) = V(X_1) + V(X_2) + \ldots V(X_n) = \sum_{i=1}^{n} V(X_i)$$