**Information Theory** **DCC/ICEx/UFMG**
**Prof. Mário S. Alvim** **2017/02**

# HOMEWORK

Discrete Probability - Part 2/2
(Based on slide-set)

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**Necessary reading for this assignment:**

- *Slide-set of Lecture 01 - Discrete Probability*:
  - *Bayes' Theorem*
  - *Expected Value and Variance*

**Note:** The exercises are labeled according to their level of difficulty: `[Easy]`, `[Medium]` or `[Hard]`. This labeling, however, is subjective: different people may disagree on the perceived level of difficulty of any given exercise. Don't be discouraged when facing a hard exercise, you may find a solution that is simpler than the one the instructor had in mind!

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**Review questions.**

1. (Rosen Review Question 7-6) `[Easy]`
   (a) Define the expected value of a random variable $X$.
   (b) What is the expected value of the random variable $X$ that assigns to a roll of two dice the larger number that appears on the two dice?
2. (Rosen Review Question 7-8) `[Easy]`
   (a) What is meant by a "*Bernoulli trial*"?
   (b) What is the probability of $k$ successes in $n$ independent Bernoulli trials?
   (c) What is the expected value of the number of successes in $n$ independent Bernoulli trials?
3. (Rosen Review Question 7-9) `[Easy]`
   (a) What does the linearity of expectations of random variables mean?
4. (Rosen Review Question 7-11) `[Easy]` State Bayes' theorem and use it to find $p(F \mid E)$ if $p(E \mid F) = 1/3$, $p(E \mid \overline{F}) = 1/4$, and $p(F) = 2/3$, where $E$ and $F$ are events from a sample space $S$.
5. (Rosen Review Question 7-13) `[Easy]`
   (a) What is the variance of a random variable?
6. (Rosen Review Question 7-14) `[Easy]`
   (a) What is the variance of the sum of $n$ independent random variables?

**Exercises.**

7. (Rosen 7.3-3) `[Medium]` Suppose that Frida selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains two white balls and three blue balls, and the second box contains four white balls and one blue ball. What is the probability that Frida picked a ball from the first box if she has selected a blue ball?

8. (Rosen 7.3-9) `[Medium]` Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that 3% of the patients not infected with HIV test positive. What is the probability that

   (a) a patient testing positive for HIV with this test is infected with it?

   (b) a patient testing positive for HIV with this test is not infected with it?

   (c) a patient testing negative for HIV with this test is infected with it?

   (d) a patient testing negative for HIV with this test is not infected with it?

9. (Rosen 7.4-3) `[Easy]` What is the expected number of times a 6 appears when a fair die is rolled 10 times?

10. (Rosen 7.4-7) `[Easy]` The final exam of a discrete mathematics course consists of 50 true/false questions, each worth two points, and 25 multiple-choice questions, each worth four points. The probability that Linda answers a true/false question correctly is 0.9, and the probability that she answers a multiple-choice question correctly is 0.8. What is her expected score on the final?

11. (Rosen 7.4-19) `[Hard]` Let $X$ be the number appearing on the first die when two fair dice are rolled and let $Y$ be the sum of the numbers appearing on the two dice. Show that $E(X)E(Y) \neq E(XY)$.

12. (Rosen 7.4-27) What is the variance of the number of heads that come up when a fair coin is flipped 10 times?