HOMEWORK 09
DECISION THEORY

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REVIEW QUESTIONS

1. Answer formally the following questions. In a decision theory problem:
   a) What do the set of states of the world model?
   b) What do the set of actions model?
   c) What does the utility function model?
   d) What is the quantity we are trying to maximize?

Resposta

a) Conjunto de estados de todos os estados possíveis do domínio da realidade.

b) Conjunto de ações que o "jogador/entidade" pode escolher.

c) Mensura o benefício de uma certa ação dado um estado. Em outras palavras é uma função que mapeia uma tupla de ações e estados para um número real.

d) Queremos maximizar a utilizade em relação sobre uma ação.

$$E[U \mid a] = \sum_{a} p(x \mid a) u(a, x)$$

$p(x \mid a)$ é a probabilidade de ocorrer um estado x dado que pegamos a ação a.

EXERCISES

2. [Medium] (The bicycle shop.) Zed and Adrian and run a small bicycle shop called "Z to A Bicycles". They must order bicycles for the coming season. Orders for the bicycles must be placed in quantities multiple of twenty (20). The cost per bicycle is $70 if they order 20, $67 if they order 40, $65 if they order 60, and $64 if they order 80. The bicycles will be sold for $100 each. Any bicycles left over at the end of the season can be sold (for certain) at $45 each. If Zed and Adrian run out of bicycles during the season, then they will suffer a loss of "goodwill"among their customers. They estimate this goodwill loss to be $5 per customer who was unable to buy a bicycle. Zed and Adrian estimate that the demand for bicycles this season will be 10, 30, 50, or 70 bicycles with probabilities of 0.2, 0.4, 0.3, and 0.1 respectively. Decide Zed and Adrian's best possible action in this scenario, that is, determine the amount of bicycles they must order for the coming season in order to maximize their expected profit.

Resposta

Temos 4 ações para Zed e Adrian:

- comprar 20 biclicletas
- comprar 40 biclicletas
- comprar 60 biclicletas
- comprar 80 biclicletas

Temos 4 estados possíveis:

- a demanda é 10 bicicletas
- a demanda é 30 bicicletas
- a demanda é 50 bicicletas
- a demanda é 70 bicicletas

Vamos calcular cada utilidade esperada.

$$E[U \mid a] = \sum_{x \in X} p(x \mid a) U(x, a)$$

$$p(demanda = 10 \mid a) = 0.2$$
$$p(demanda = 30 \mid a) = 0.4$$
$$p(demanda = 50 \mid a) = 0.3$$
$$p(demanda = 70 \mid a) = 0.1$$

Calculando para cada x.

$$E[U \mid comprar = 20] = 400$$
$$E[U \mid comprar = 40] = 740$$
$$E[U \mid comprar = 60] = 720$$
$$E[U \mid comprar = 80] = 460$$

A melhor opção é comprar=40 pois é o que possui o maior payoff.

3. One of the challenges of decision theory is figuring out exactly what the utility function is. The utility of money, for example, is notoriously nonlinear for most people. In fact, the behaviour of many people cannot be captured by a coherent utility function, as illustrated by the Allais paradox, which runs as follows.

Which of these choices do you find most attractive?

a. £1 million guaranteed.

b. 89% chance of £1 million;
   10% chance of £2.5 million;
   1% chance of nothing

Now consider these choices:

c. 89% chance of nothing;
   11% chance of £1 million.

d. 90% chance of nothing;
   10% chance of £2.5 million

Many people prefer A to B, and, at the same time, D to C. Prove that these preferences are inconsistent with any utility function U(x) for money.

(Hint: Model the problem as a decision theory problem (i.e., identify the states of the world, the possible actions, the probabilities involved) in which the utility function U(x) is free. Then impose the restrictions that action A is preferred over B and that action D is preferred over C to derive a contradiction on the existence of U(x).)

Resposta

Preferência de A pra B

$$u(1) > 0.01 \cdot u(0) + 0.89 \cdot u(1) + 0.10 \cdot (2.5)$$

Preferencia de D para C

$$0.89 \cdot u(0) + 0.11 \cdot u(1) < 0.9 \cdot u(0) + 0.1 \cdot u(2.5)$$
$$0.11 \cdot u(1) < 0.01 \cdot u(0) + 0.1 \cdot u(2.5)$$
$$u(1) < 0.89 \cdot u(1) + 0.1 \cdot u(2.5) + 0.01 \cdot u(0)$$

Contradizendo a inequação anterior.