Incentive Mechanism Design for Heterogeneous Crowdsourcing Using All-Pay Contests

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Abstract—Many crowdsourcing scenarios are heterogeneous in the sense that, not only the workers' *types* (e.g., abilities or costs) are different, but the *beliefs* (probabilistic knowledge) about their respective types are also different. In this paper, we design an incentive mechanism for such scenarios using an *asymmetric all-pay contest* (or auction) model. Our design objective is an optimal mechanism, i.e., one that maximizes the crowdsourcing revenue minus cost. To achieve this, we furnish the contest with a *prize tuple* which is an array of reward functions each for a potential winner. We prove and characterize the unique equilibrium of this contest, and solve the optimal prize tuple.

In addition, this study discovers a counter-intuitive property called *strategy autonomy* (SA), which means that heterogeneous workers behave independently of one another as if they were in a homogeneous setting. In game-theoretical terms, it says that an asymmetric auction admits a symmetric equilibrium. Not only theoretically interesting, but SA also has important practical implications on mechanism complexity, energy efficiency, crowdsourcing revenue, and system scalability.

By scrutinizing seven mechanisms, our extensive performance evaluation demonstrates the superior performance of our mechanism as well as offers insights into the SA property.

Index Terms—Crowdsourcing, mobile crowd sensing, participatory sensing, all-pay auction, asymmetric auction, strategy autonomy

1 Introduction

Crowdsourcing offers a distributed and cost-effective approach to problem solving and data gathering by soliciting user contributions from a large group of undefined people. Recently, due to the burgeoning smartphone industry and the soaring demand for sensing data, a new mobile computing and sensing paradigm called mobile crowdsensing [1] emerged, which collects data through crowdsourcing and has created significant momentum in both industry and academia. For example, IBM Almaden Research Center launched a citizen science project Creek Watch [2] to enable iPhone users to report water-related information in order to monitor water levels and the vicinity conditions. NoiseTube [3] deals with another environmental issue, noise monitoring, via the microphone sensor on each user's smartphone. ContriSense:Bus [4], on the other hand, addresses public transport problems by allowing bus commuters to send bus arrival times and crowdedness levels via smartphones. In the communications domain, WiFi-Scout [5] measures WiFi signal quality and connection speed by crowdsourcing to smartphone users' surfing experience. The motivation of this study originates from crowdsensing yet the model and results are applicable to the general context of crowdsourcing.

Incentives are key to the success of crowdsourcing applications as it heavily depends on the level of user participation. While some applications are endowed with strong *intrinsic* motivation such as self-fulfillment, skill enhancement, and fun, most applications have to rely on *extrinsic* incentives such as financial reward. In this paper, we design an incentive mechanism with arbitrarily divisible (e.g., financial) reward using an auction-based framework. We choose auctions because they are effective, sophisticated incentive mechanisms that have been well adopted in both theory [6]–[9] and practice [10], [11].

In particular, we use an *all-pay contest* [12] model to design our incentive mechanism. All-pay contests are *isomorphic* to all-pay auctions: given an equilibrium in one model, one can construct one and only one equilibrium in the other model (to draw an analogy, bidders tendering bids in an auction resembles contestants exerting effort in a contest). All-pay auctions or contests are distinct from other mainstream auctions such as first- and second-price auctions and [6], [7], [9], in that all the bidders must pay for their respective bids regardless of who wins the auction, while in the mainstream auctions only winners will need to pay. This seems to be rather peculiar, but when applied to the context of crowdsourcing, it becomes a natural model if we let each bid represent each user's *actual* contribution effort (instead of an *indication* of one's "willingness to contribute"). In that case, a bid once submitted is irrevocable since effort has been sunk, exactly mirroring "all-pay". In addition, as [13] points out, all-pay auctions have two important advantages: they (i) simplify the typical two-stage "bid-contribute" process (e.g. [6], [7], [9]) into a single "bid-cum-contribute" stage, and (ii) eliminate the risk of task non-fulfillment.

This paper themes around *heterogeneous crowdsourcing*, where not only agent (worker) *types* (e.g., abilities or costs)

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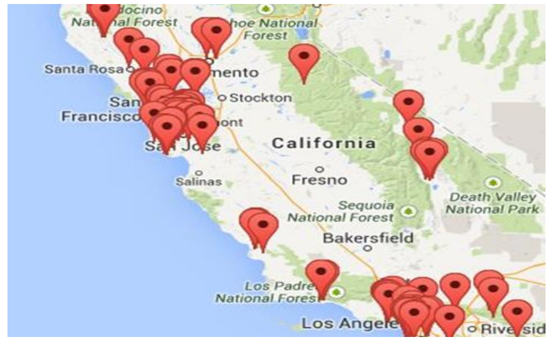
are different, but the *beliefs* (probabilistic knowledge) about their respective types are also different. Here "belief", which is in the form of a probability distribution, also implies that we assume an *incomplete-information* setting where agents do not exactly know each other's types except for their own, as is usually the case in practice. In the vast literature on crowdsourcing and auctions, the majority (e.g., [8], [14]–[19])[1] deals with the *homogeneous* case where all the agents are assumed to be *ex post* or *ex ante* identical. That is, their types are either exactly the same (ex post), or statistically the same (ex ante) (i.e., follow the same, single belief or probability distribution). This typically leads to a symmetric equilibrium and can indeed offer some insights into applications in which players are largely homogeneous. However, a heterogeneous model could provide a better understanding of many other real scenarios. For example, see two crowdsourcing applications illustrated in Fig. 1. In (a), participants of a citizen science project such as Creek Watch [2] often form a community with common interest, and therefore may have some (uncertain) knowledge about each other's types via social contacts or community activities. In (b), a crowdsensing application such as WiFi-Scout [5] often comes with an associated website to publicize user performance or contribution rankings [20], via which users can develop a probabilistic knowledge of other users' types. In both cases, users roughly know who are "stronger" and who are "weaker", bespeaking a heterogeneous setting. In addition, the "everyone contributing" behavior, as in many mobile crowdsourcing applications which directly solicit contributions from all users, falls under the all-pay model. Therefore, we employ *asymmetric all-pay auctions* as our mathematical framework to tackle our design problem.

However, asymmetric auctions are much less understood and applied because they are generally much more challenging to analyze than their symmetric siblings. As a result, the relatively much smaller literature on asymmetric auctions is often limited to two-player cases [21]–[23] or complete-information settings [12], [24] in order to trade for tractability. In fact, many related questions such as characterizing equilibria remain open even after decades. In 2003, a significant progress was made by [25] toward understanding asymmetric *first-price* auctions with more than two players and incomplete information, but the solution is approximate and only applies to *weakly* asymmetric agents[2]. Other studies along the similar line resort to numerical simulations [26]. In this paper, we deal with an arbitrary number of agents and incomplete information, yet we obtain precise solutions analytically, which also apply to any (weak or strong) asymmetry.

In summary, this paper makes the following contributions:

1) To the best of our knowledge, this is the first work that addresses heterogeneous crowdsourcing using an all-pay contest model. Furthermore, the model is rather general and realistic in the sense that it accommodates an arbitrary number n of agents with incomplete information.
2) We design an incentive mechanism for heterogeneous crowdsourcing, and achieve optimality in terms of maximizing the crowdsourcing profit (or the crowdsourcer's utility).
3) Our mechanism reveals a new, counter-intuitive property called *strategy autonomy* (SA), which means that a heterogeneous crowdsourcing or asymmetric auction model behaves like a homogeneous or symmetric one. This has practical implications on mechanism complexity, energy efficiency, crowdsourcing revenue, and system scalability.
4) While most prior work focuses on workers (bidders) only, this work also gives an example of how to deal with different types of crowdsourcers (auctioneers) or the same crowdsourcer with changing types. This is elaborated in Section 3.
5) The performance of our mechanism in terms of crowdsourcing profit and system scalability is demonstrated to be superior by our extensive evaluation that involves seven mechanisms. Our evaluation also sheds light on the SA property, as well as draws intuition from various rationales.



(a) Creek Watch [2]: Waterway conditions reported by iPhone users.

HIGHEST CONTRIBUTION POWER		HIGHEST ENDORSEMENT POWER	
Rank	Users	CP Score	EP Score
1	Rookie Ac	392.23	115.15
2	Jason Pan	179.41	143.80
3	Ting Ting	163.52	123.12
4	Zhang Xinwan	150.32	100.00
5	Jasmine	148.32	114.52

(b) WiFi-Scout [20]: user ranking based on contribution performance.

Fig. 1. Two mobile crowd sensing applications that illustrate heterogeneous crowdsourcing as well as the all-pay behavior.

The rest of this paper is organized as follows. Section 2 reviews the literature and Section 3 presents our contest model. Section 4 analyzes the model and provides the main results. Section 5 provides a detailed performance evaluation for seven mechanisms, and Section 6 concludes.

1. Studies [6], [9] follow a different line and do not have the concept of (Bayesian) "belief"; they are reviewed in Section 2.3. The work [7] is reviewed in Section 2.2.

2. Weak asymmetry means that the difference between any two beliefs (distributions) is small, or formally, $F_i(v) = F(v) + \epsilon H_i(v)$ where $F_i(v)$ is the belief of type v_i, $|H_i(v)| \leq 1$ and $|\epsilon| \ll 1$.

2 Related Work

A preliminary version of this work appeared in [27].

2.1 Symmetric auctions

The vast majority of prior work on auctions or auction-based incentive mechanisms adopts symmetric auctions as a handy framework. As [28] concurs, almost all auction theory concerns symmetric auctions.

A crowdsourcing site that offers a range of contests was studied in [14], where each user chooses one contest to participate based on the skill requirement and the offered reward. Each contest is modeled as a symmetric all-pay auction. The work [15], [16] investigates whether a single or multiple prizes can maximize contest revenue, defined as the sum of all the bids [15] or that of the highest k bids [16]. Both studies assume a symmetric setting. Under similar assumptions, and with a focus on revenue composition, [17] finds that the highest bid is at least half of the total bids in the revenue. Also using a symmetric auction model, [18], [19] compare the revenue in terms of the highest bid and that of the total bids, as well as investigate the cases when bidders value the reward according to additive and multiplicative rules.

Most recently, [8] proposed an all-pay auction based incentive mechanism for participatory sensing and crowd sensing. The model is general in the sense that it assumes risk-averse (subsuming risk-neutral) agents, stochastic (subsuming deterministic) population, as well as incomplete information. The idea was subsequently redeveloped in [13] with substantial extensions, which in particular points out two important merits of all-pay auctions—simplicity and risk-elimination—which we mentioned in Section 1. However, both works still assume a symmetric model.

2.2 Asymmetric auctions

The regime of asymmetric auctions is relatively much smaller and less understood due to its analytical complexity. As a result, most work deals with two-player cases or complete-information settings. Amann and Leininger [22] presented their seminal work on a two-player asymmetric auction, by characterizing the equilibrium bidding strategies. Maskin and Riley [29], [30] extended it by proving the monotonicity and uniqueness of the equilibrium. In the n-player case, Fibich and Gavious [25] provided an approximate characterization of equilibria in the weakly asymmetric case, using a perturbation approach. Another work [31] examines risk aversion and offers some exploratory yet inconclusive results.

With complete information, Siegel [12] obtained closed-form player payoffs. He is also probably the first to coin the term "all-pay contests". Under a similar complete-information model, Xiao [24] studied the problem of allocating $k < n$ prizes to n players and proposed an algorithm to construct the equilibrium. Franke et al. [32] aimed to maximize revenue through discriminating players by associating differentiated weights, assuming complete information about players.

In contrast, our model accommodates an arbitrary number n of agents with incomplete information (i.e., a Bayesian game setting), as well as asymmetric types and beliefs. Furthermore, we furnish the contest with a prize tuple which is a sequence of reward functions. These set this work apart from prior art.

Perhaps the closest model to ours is in [7] which also assumes an asymmetric Bayesian game setting. However, it falls into the category of second-price auctions while we take on the all-pay flavor. Secondly, the objective of [7] is to minimize cost while keeping revenue constant, whereas our objective is to maximize revenue subtracted by cost, or profit, which is presumably more flexible. Third, the prize tuple and SA property are unique to our work only.

2.3 Other incentive mechanisms

Algorithmic mechanism design [33] represents another large field in parallel with the classic economic mechanism design arena where this work sits in. Algorithmic mechanism design stresses computational efficiency and focuses on polynomial-time implementable mechanisms, and therefore frequently use methods such as greedy heuristics and approximations. Work [6], [9] belongs to this category. On the other hand, economic mechanism design focuses on outcomes and equilibrium analysis, rather than procedures that lead to the outcomes (the procedures are often straightforward in many cases including ours).

While auctions are commonly used as an incentive mechanism design framework, there are also non-auction based incentive work in the literature. For instance, [34] designs a contract-based incentive mechanism for a distributed computing scenario using contract theory.[3] On the other hand, [35], [36] take a resource-allocation approach to incentivize user participation by allocating *service quota* to users based on their contribution levels and service demands. Moreover, [37] takes a socio-economic approach to link participants into a social network overlaid by economic incentives in order to stimulate trustworthy contributions.

3 Our All-Pay Contest Model

A *principal* (crowdsourcer) is conducting an *all-pay contest* based crowdsourcing campaign in order to solicit some "effort" from n agents (workers). Here, "effort" is a general term that can be interpreted as, depending on the application, quantity (total or per unit time [7]), quality of contributions [38], or a compound measure of both quality and quantity [39]. By "all-pay contest", we mean that agents directly submit their efforts to the principal as their respective "bids", and the agent who submits the highest effort will win a prize, which can only be determined after the contest (crowdsourcing campaign).

Each agent i is characterized by his type (e.g., ability or unit cost) v_i which is private information, i.e., an agent does not know any other agent's type except for

3. Note that the term usage of "asymmetric" in [34] is slightly different from this paper. In the "data acquisition" scenario of [34], the term means that the client (crowdsourcer) does not know any user's type which is known to each user himself; whereas in the "distributed computing" scenario of [34], the term means that the knowledge about each user's type is individually different. The latter usage is consistent with the standard usage as in "asymmetric auctions" and also coincides with our paper.

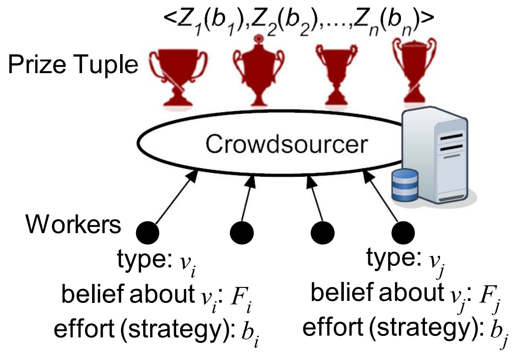


Fig. 2. The asymmetric all-pay contest model for heterogeneous crowdsourcing. Both the worker types and the beliefs about their respective types are different (i.e., $v_i \neq v_j, F_i \neq F_j$). Workers contribute their efforts to the crowdsourcer directly (without an extra "bidding" process to select who to contribute) in order to compete for a reward. The reward is provisioned as a prize tuple, or sequence of reward functions, of which the workers are pre-informed before the contest (the crowdsourcing campaign). At the end of the contest, the worker, say i^*, who has contributed the highest effort b_{i^*}, will be the only winner to receive a reward $Z_{i^*}(b_{i^*})$.

his own. Information is incomplete, i.e., while v_i is private information, each agent has some probabilistic knowledge (belief) about other agents' types. Specifically, it is common knowledge (to both agents and the principal) that each v_i is independently drawn from a nonnegative support $[\underline{v}, \bar{v}]$ according to a probability distribution $F_i(\cdot)$. In our setting, agents are *heterogeneous*, in the sense that not only their types v_i are different, but the beliefs about their respective types are also different, i.e., $F_i \neq F_j$ in general for all $i \neq j$.[4]

We assume that v_i is continuous, $F_i(\cdot)$ is differentiable, and its corresponding p.d.f. $f_i(\cdot)$ is continuous and positive over the open interval $(\underline{v}, \bar{v})$.

The principal is profit seeking, i.e., its objective is to maximize its revenue subtracted by cost, where the revenue is the total solicited agent effort and the cost is the prize it needs to pay out as reward. To achieve this objective of *optimal mechanism design*, we furnish the contest with a *prize tuple* $\mathbf{Z}$, which is an array of reward functions catering to every potential winner, i.e., $\mathbf{Z} := \langle Z_1(b_1), Z_2(b_2), ..., Z_n(b_n)\rangle$ where b_i is agent i's effort, $Z_i(\cdot)$ is a function of b_i and, in general, $Z_i \neq Z_j$ for all $i \neq j$. All the agents are pre-informed of this prize tuple before the crowdsourcing contest starts.[5] Our model is depicted in Fig. 2.

A prize of face value Z is valued by an agent of type v to be of real worth $V(v, Z)$ where $V(\cdot,\cdot)$ is a value function. For example, if $V(v, Z) = vZ$, then a prize Z_i bears a value of $v_i Z_i$ to agent i, which captures the case that an agent of higher type (e.g. ability) can exploit a reward to a better extent;[6] if $V(v, Z) = v$, it means that the reward is fixed (and normalized) and an agent values it at v, which is exactly the case in classic auctions. Therefore, the value function $V(\cdot,\cdot)$ can be treated as a generalization of classic auctions. We assume that $V(v, Z)$ is differentiable with respect to v.

Each agent needs to pay for his cost incurred from participating, as per a payment function $p(b, v)$. That is, an agent i who submits effort b_i has to pay a cost of $p(b_i, v_i)$. For example, if $p(b, v) = b$ and $V(v, Z) = v$, we have a standard all-pay auction; if $p(b, v) = 0$ and $V(v, Z(b)) = v - b$, we have a standard first-price auction. Now, we can formulate the expected utility of an agent i, as

$$u_i := q_i V(v_i, Z_i(b_i)) - p(b_i, v_i) \tag{1}$$

where q_i is the probability that agent i wins the contest.

In this paper, we introduce a new modelling variable called the *principal's type*, λ, in order to characterize different crowdsourcers or the same crowdsourcer of changing types. For example, if λ is the principal's (marginal) valuation of reward, then a prize Z will be of value λZ to the principal; in general, we use the value function $V(\cdot,\cdot)$ introduced above. Therefore, we can formulate the utility of the principal, π, as follows:

$$\pi := \mathbb{E}\big[\sum_{i=1}^{n} b_i - V(\lambda, Z_w(b_w))\big] \tag{2}$$

which is its expected profit as it is profit seeking. Here, $w \in [1..n]$ is the winner's index (which is a random variable), and $\lambda > 0$ is common knowledge. Our objective is to design a contest (auction) mechanism such that π is maximized.

For mathematical convenience, we make the following assumptions on the payment function $p(b, v)$. It is twice continuously differentiable and $p(0, v) = 0$; $p'_b(b, v) > 0$, i.e., the higher effort, the higher payment (cost); $p'_v(b, v) \leq 0$, i.e., the higher type (ability), the lower payment; $p''_{bb}(b, v) > 0$, i.e., striving from higher effort levels is more costly than from lower effort levels, or conversely, the marginal output by adding more effort is decreasing; $p'''_{b^2 v}(b, v) \leq 0$, i.e., lower types are more vulnerable to the effect of decreasing marginal output.[7]

4 Optimal Mechanism Design

We first analyze the asymmetric equilibrium strategy for each agent (Section 4.1; Lemma 2), as a function of any given (arbitrary) prize tuple, and then determine the optimal prize tuple that induces the maximum profit for the principal (Section 4.2; Theorem 1). Following that, Section 4.3 remarks on three important properties including SA.

4. In practice, belief F_i can be formed via social interactions [2] or via publicized information (e.g., contribution performance) [20], as mentioned in Section 1.

5. Agents can be informed, for example, via communication between a cloud service that represents the principal, and software running on each agent's personal device (smartphone, PC, wearable widget, etc.). The software can also calculate the strategy for the agent.

6. More concretely, imagine that the reward is a R&D fund or a contract budget, an agent with better ability may be able to produce more R&D outputs or make more profit from the same contract. For another example, the reward may be money, vouchers, or coupons, which can lead to different utilities when used by different agents. It (V) could also simply be the psychological value one perceives from the reward (e.g., a trophy).

7. We follow the notation convention that, for a function $g(x, y)$, $g'_x := \frac{\partial g}{\partial x}$, $g''_{xy} := \frac{\partial^2 g}{\partial x \partial y}$, and $g'''_{x^2 y} := \frac{\partial^3 g}{\partial x^2 \partial y}$. As a technical pointer, our assumptions on the function $p(\cdot,\cdot)$ are used later in the proof of Theorem 1 and the proof of Proposition 1.

4.1 Asymmetric equilibrium

As our model constitutes a Bayesian game setting, we need the following solution concept which is an extension of Nash equilibrium.

Definition 1 (Bayes-Nash equilibrium)**.** *A pure-strategy Bayes-Nash equilibrium is a strategy profile* $\mathbf{b}^* := (b_1^*, b_2^*, ..., b_n^*)$ *in which*

$$u_i(b_i^*, b_{-i}^*) \geq u_i(b_i, b_{-i}^*), \forall b_i, \forall i.$$

In words, each agent in a Bayes-Nash equilibrium plays a strategy that maximizes his expected payoff given his belief about other agents' types and that other agents play their respective equilibrium strategies.

Lemma 1 (Existence, uniqueness, monotonicity, and common support)**.** *Our asymmetric all-pay contest admits a unique pure-strategy Bayes-Nash equilibrium. The (asymmetric) equilibrium strategies are strictly increasing in type, and share a common support* $[0, \bar{b}]$ *where* $\bar{b}$ *is unknown.*

Proofs: Most of the proofs for this paper are contained in the Appendix available in the IEEE Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TMC.2015.2485978.

Notation convention: Henceforth, we will exclusively deal with the equilibrium state. Hence for brevity, we slightly deviate from the general notational convention by dropping the superscript * from equilibrium variables. For example, we write b_i instead of b_i^* and $v_i(\cdot)$ instead of $v_i^*(\cdot)$.

Lemma 1 tells that an agent's equilibrium strategy b_i is a strictly monotone (increasing) function of v_i, which we denote by $\beta_i(\cdot)$, i.e., $b_i = \beta_i(v_i)$. Hence, its inverse function exists and is also increasing, which we denote by $v_i(\cdot) := \beta_i^{-1}(\cdot)$. Thus, noticing that $b_j = \beta_j(v_j)$, we have

$$\Pr(b_i > b_j) = \Pr(\beta_j^{-1}(b_i) > v_j) = F_j(v_j(b_i)).$$

Furthermore, because of the strict monotonicity and the type continuity, event $b_i = b_j$ is of zero probability and tie-breaking is trivial. So, agent i's winning probability $q_i = \prod_{j \neq i} \Pr(b_i > b_j)$, and thus (1) is rewritten as

$$u_i = V(v_i, Z_i(b_i)) \prod_{j \neq i} F_j(v_j(b_i)) - p(b_i, v_i). \tag{3}$$

Lemma 2. *Given a prize function* $Z_i(b_i)$*, agent* i*'s equilibrium strategy* $b_i(v_i)$ *is determined by*

$$V(v_i, Z_i(b_i)) \prod_{j \neq i} F_j(v_j(b_i)) - p(b_i, v_i) = \int_{\underline{v}}^{v_i} [V'_{v_i}(\tilde{v}_i, Z_i(b_i)) \prod_{j \neq i} F_j(v_j(b_i)) - p'_{v_i}(b_i, \tilde{v}_i)] \, \mathrm{d}\tilde{v}_i. \tag{4}$$

No further reduction can be made to (4) because the derivatives in the above integrand denote partial derivatives.

Remark: Asymmetric auctions, regardless of winner-pay or all-pay, do not have closed-form expressions for equilibrium strategies in general (an approximate solution to first-price auctions can be found in [25]). However, even without the closed form of equilibrium strategies, we are able to solve for the optimal prize tuple using Lemma 2, as shown next.

4.2 Optimal prize tuple

Solving for the optimal prize tuple $\mathbf{Z}$ requires the value function $V(\cdot)$ to be specified, for which we consider a general form of $V(v, Z) = h(v)Z$ where $h(\cdot)$ satisfies $h(0) = 0$ and $h'(v) > 0$. This form further generalizes the form $V = vZ$ which, as mentioned in Section 3, is already a generalization of the standard all-pay auctions.

Corollary 1. *If the value function takes the form* $V(v, Z) = h(v)Z$*, agent* i*'s equilibrium strategy* $b_i(v_i)$ *is determined by*

$$Z_i(b_i) \prod_{j \neq i} F_j(v_j(b_i)) - \hat{p}(b_i, v_i) = -\int_{\underline{v}}^{v_i} \hat{p}'_{v_i}(b_i, \tilde{v}_i) \, \mathrm{d}\tilde{v}_i. \tag{5}$$

where

$$\hat{p}(b, v) := \frac{p(b, v)}{h(v)}.$$

Corollary 1 follows from Lemma 2. Now we state our main result.

Theorem 1. *The optimal prize tuple that maximizes the crowdsourcing profit (or principal's utility) is given by* $\mathbf{Z} = \langle Z_1(b_1), Z_2(b_2), ..., Z_n(b_n) \rangle$ *in which*

$$Z_i(b_i) = \frac{\hat{p}(b_i, v_i(b_i)) - \int_0^{b_i} \hat{p}'_{v_i}(\tilde{b}_i, v_i(\tilde{b}_i)) \, \mathrm{d}v_i(\tilde{b}_i)}{\prod_{j \neq i} F_j(v_j(b_i))}, \quad i = 1, 2, ..., n, \tag{6}$$

where $v_i(b_i)$ *is the inverse function of* $b_i(v_i)$ *which is the corresponding equilibrium effort, determined by*

$$\hat{p}'_{b_i}(b_i, v_i) = \frac{1}{h(\lambda)} + \hat{p}''_{b_i, v_i}(b_i, v_i) \frac{1 - F_i}{f_i}. \tag{7}$$

The resultant maximum crowdsourcing profit is given by

$$\pi = \sum_i \int_{\underline{v}}^{\bar{v}} \Big[b_i(v_i) - h(\lambda)\hat{p}(b_i, v_i) + h(\lambda)\hat{p}'_{v_i}(b_i, v_i) \frac{1 - F_i}{f_i} \Big] \, \mathrm{d}F_i. \tag{8}$$

Theorem 1 specifies our mechanism which we refer to as **OPT** in the sequel. Note that our mechanism is not specified by a (procedure-oriented) algorithm like algorithmic mechanism design works [6], [9], [33] but falls under the classic economic mechanism design genre, as mentioned in Section 2.3.

4.3 Properties: SA, IR, IC

Based on the analytical results above, this section examines three important properties pertaining to **OPT**.

4.3.1 Strategy Autonomy (SA)

This is the most salient property of **OPT**, particularly in the presence of asymmetry. None of the prior work on asymmetric mechanisms possesses this property and it has practical significance.

Definition 2 (**Strategy Autonomy).** *A mechanism is strategy-autonomous if, given the asymmetric common prior (i.e., the different beliefs* $F_i(\cdot)|_{i=1}^n$ *about the* n *agents) under incomplete information, the mechanism induces an equilibrium in which each agent adopts an strategy independent of his knowledge (beliefs) about other agents, i.e.,* $b_i(v_i|\{F_j|_{j=1}^n\}) = b_i(v_i|F_i), \forall i.$

Essentially, SA says that an *asymmetric* mechanism (auction or contest) admits a *symmetric* equilibrium.

Corollary 2. OPT *satisfies strategy autonomy.*

Proof. Immediately follows from Theorem 1, where the equilibrium strategy b_i (7) is independent of any $j \neq i$. □

SA is rather counter-intuitive, and somewhat surprising. This is because in a game theoretical setting, a rational agent will reason about how other agents would act so as to react on it (as a best-response), which naturally depends on his belief about other (heterogeneous) agents. Indeed, Lemma 2 does show that the equilibrium strategy b_i depends on $F_j|_{j\neq i}$. In fact, prior work on asymmetric auctions, regardless of winner-pay or all-pay, with complete or incomplete information, all exclusively admit asymmetric equilibria or even no equilibria; see, e.g., [22], [24], [25] and a comprehensive survey [40]. So the puzzling question is: *why* do agents behave autonomously in the **OPT** mechanism?

The fundamental reason is that the asymmetric belief about agent types is *endogenized* by the optimal prize tuple (6) where each prize $Z_i(b_i)$ contains the winning probability $\prod_{j\neq i} F_j(v_j(b_i))$ which absorbs all the heterogeneity or asymmetry. In a sense, each agent's concern about other agents is now taken care of by the principal who stipulates the prizes.

Not just theoretically interesting, SA also has three important practical implications:

• *Reduces mechanism complexity and energy consumption:* SA remarkably reduces the computational complexity and storage requirement, from $O(n)$ to $O(1)$, for *each* agent. The $O(n)$ can be understood from (4) where each agent's strategy involves $n-1$ beliefs about other $j \neq i$, as is also the case in, e.g., [25], [28], [32]. Such considerable reduction of computational and storage requirements will lead to lower energy consumption as well. Thus in practice, as agent strategies would typically be computed by software residing on each agent's mobile device (e.g., smartphone, smart watch), this merit is an important enabler for those miniature devices to support heterogeneous crowdsourcing applications.

• *Increases crowdsourcing revenue:* SA overcomes an *effort reservation* effect that exists in standard (fixed-prize) asymmetric auctions [41]: when the prize is fixed, any agent only needs to beat the other agents by an infinitesimal *winning margin*; therefore, by illustrating with a two-agent scenario, if the stronger agent believes that the other agent is statistically weaker, he has the incentive to reserve effort in order to reduce his winning margin since a larger margin does not make the winner better off at all. This effect outweighs the strategy adjustment of the weaker agent and results in a reduced total revenue compared to symmetric auctions [41], as will also be demonstrated in Section 5. However, SA insulates agents from such negative mutual influence, allowing an agent to not be concerned with minimizing winning margin but to focus on exerting more effort toward the "self-adjusting" reward. This is supposed to be beneficial to revenue, and will be verified in Section 5.

• *Enhances system scalability*: The prevailing law of diminishing marginal returns (DMR) governs many phenomena in (network) economics. It states that, as new employees (or more generally, resources) are being added, the marginal product of an additional employee will at some point be less than that of the previous employee [42]. Mathematically, DMR corresponds to *concave* nonlinear growth of revenue when resources are being added linearly. However, this submodularity-resembling law of DMR is neutralized by SA and we will see in Section 5 that the principal's profit grows *linearly* as the number of agents increases. This conveys a dramatic enhancement to system *scalability*, and will be demonstrated later too.

4.3.2 Individual Rationality (IR)

Definition 3 (Individual Rationality). *A mechanism is individually rational if any participating agent will expect a surplus no lower than not participating. That is, in equilibrium, the expected utility $u_i(b_i, b_{-i}) \geq u_i(0, b_{-i})$ for all i.*

Proposition 1. OPT *satisfies individual rationality. Furthermore, an agent i reaps* strictly positive *utility if $b_i > 0$.*

4.3.3 Incentive Compatibility (IC)

A mechanism is incentive-compatible or strategy-proof if each agent's dominant strategy is to reveal his true type (in an equivalent *direct-revelation mechanism*). In our all-pay contest mechanism, prize allocation is based on each agent's *observable* actual effort which is a function of his (true) type and cannot be lied about. Therefore, IC is inherently satisfied.

5 Performance Evaluation

To derive a quantitative and intuitive understanding of the analytical results, we evaluate the performance of **OPT** in comparison to other six mechanisms in this section.

We consider three scenarios. In the first scenario, there are two agents of types $v_1, v_2 \in [0,1]$ independently drawn from $F_1(v) = v$ (uniform distribution) and $F_2(v) = \frac{v+1}{2}$, respectively. Hence, $f_2(v) = \frac{1}{2}\delta(v) + \frac{1}{2}$ where $\delta(\cdot)$ is the Dirac delta function.[8] In other words, agent 2 is equally probable to be of type zero or of a type uniformly drawn from (0,1]. Therefore, agent 1 is statistically *stronger* than agent 2. The value function is $V(v, Z) = vZ$ and the payment function is $p(b, v) = b^2$. Hence, $h(v) = v$ and $\hat{p}(b, v) = b^2/v$.

In this first scenario, we compare **OPT** with the following three canonical all-pay auctions:

- **FIX**: Fixed-prize asymmetric all-pay auctions.
- SYM: Fixed-prize symmetric all-pay auctions, particularly,
  - **SYM-1**: both types follow $F_1(v)$;
  - **SYM-2**: both types follow $F_2(v)$.

In the second scenario, there are n agents which allows us to investigate system scalability, namely whether and how SA neutralizes the law of DMR. We compare **OPT-n**

8. Neither our model nor analysis assumes continuity of the p.d.f. at the boundary of the support. Also, our analysis can apply to other differentiable c.d.f.'s as well. For example, using the power-law distribution $F_2(v) = v^\alpha, \alpha > 0$ will obtain similar results, but the expressions are lengthy (due to the inverse effort function $v_2(b_2)$) and hence not reported. The function choice in this section allows for a neater presentation.

to **FIX-n**, which are OPT and FIX each with n symmetric agents (choosing agent 1 for illustration), respectively.

In the third and last scenario, we consider a realistic environmental sensing application and another incentive mechanism, both introduced by [7]. We evaluate the performance of that mechanism (which we refer to as **INFOCOM13**) in parallel with **OPT**.

In summary, this section evaluates the following seven mechanisms: OPT, FIX, SYM-1, SYM-2, OPT-n, FIX-n, and INFOCOM13.

5.1 Theoretical underpinnings

With the introduction of **FIX** and SYM (**SYM-1** and **SYM-2**), we first need to characterize their respective bidding strategies.

Proposition 2 (Equilibrium strategy in **FIX**). *In an asymmetric all-pay contest with incomplete information, two agents, and a fixed prize Z, if the common prior is $F_1(v), F_2(v), v \in [\underline{v}, \bar{v}]$, the value function $V(v,Z) = vZ$, and the payment function $p(b)$ satisfies $p(0) = 0$ and $p'(b) > 0$, then there exists a unique Bayes-Nash equilibrium $\mathbf{b} = (b_1, b_2)$ given by*

$$b_1(v_1) = p^{-1}\Big(Z\int_{k^{-1}(\underline{v})}^{v_1} k(v)F_1'(v)\,\mathrm{d}v\Big), \tag{9}$$

$$b_2(v_2) = b_1(k^{-1}(v_2)), \tag{10}$$

where $b_1(v) = 0$ iff $v_1 = k^{-1}(\underline{v})$, and $k(v)$ is determined by

$$k'(v) = \frac{k(v)F_1'(v)}{vF_2'(k(v))}$$

with boundary condition $k(\bar{v}) = \bar{v}$.

Proposition 3 (Equilibrium strategy in **SYM**). *In a symmetric all-pay contest with incomplete information, n agents, and a fixed prize Z, if the common prior is $F(v), v \in [\underline{v}, \bar{v}]$, the value function is $V(v,Z) = vZ$, and the payment function $p(b)$ satisfies $p(0) = 0$ and $p'(b) > 0$, then there exists a unique Bayesian Nash (symmetric) equilibrium given by*

$$b(v) = p^{-1}\Big(vZF^{n-1}(v) - Z\int_{\underline{v}}^{v} F^{n-1}(t)\,\mathrm{d}t\Big). \tag{11}$$

5.2 Computing strategy, prize, and profit

5.2.1 Bidding strategy (Agent effort b)

OPT: Using Theorem 1, we apply (7) with $\hat{p}(b_i, v_i) = b_i^2/v_i$ and $F_1 = v_1$ to obtain

$$\frac{2b_1}{v_1} = \frac{1}{\lambda} - \frac{2b_1}{v_1^2}(1 - v_1),$$

which gives the optimal equilibrium strategy for agent 1:

$$b_1(v_1) = \frac{v_1^2}{2\lambda}, \qquad v_1(b_1) = \sqrt{2\lambda b_1}. \tag{12}$$

Similarly, applying (7) with $F_2 = \frac{v_2+1}{2}$ yields for agent 2:

$$b_2(v_2) = \frac{v_2^2}{2\lambda}, \qquad v_2(b_2) = \sqrt{2\lambda b_2} \tag{13}$$

which is the same (i.e., symmetric) as agent 1. This is because the two type distributions happen to have identical *hazard rate* [43], $\frac{f(v)}{1-F(v)}$, which is used (inversely) in (7). Of course, this should not be generalized to all distributions; indeed, we shall see later that the optimal prizes for the two agents (18)(19) as well as their individual contributions to the principal's profit (20)(21) are different.

FIX: Instantiating Proposition 2 with $F_1(v) = v$ and $F_2(v) = \frac{v+1}{2}$ yields

$$k'(v) = \frac{2k(v)}{v} \Rightarrow k(v) = v^2, \qquad k^{-1}(v) = \sqrt{v}.$$

$$\text{Therefore} \quad b_1^{fix}(v_1) = \Big(Z\int_0^{v_1} v^2\,\mathrm{d}v\Big)^{\frac{1}{2}} = \frac{v_1^{3/2}}{\sqrt{3}}\sqrt{Z}, \tag{14}$$

$$b_2^{fix}(v_2) = \frac{v_2^{3/4}}{\sqrt{3}}\sqrt{Z}. \tag{15}$$

SYM: For **SYM-1**, applying Proposition 3 with $F(v) = v$ gives

$$b_1^{sym}(v) = \frac{v}{\sqrt{2}}\sqrt{Z}. \tag{16}$$

For **SYM-2**, applying Proposition 3 with $F(v) = \frac{v+1}{2}$ gives

$$b_2^{sym}(v) = \frac{v}{2}\sqrt{Z}. \tag{17}$$

5.2.2 Prize

The optimal prize tuple in **OPT** can be computed using Theorem 1, particularly (6). The prize for agent 1 is

$$Z_1(b_1) = \frac{\frac{v_1^3}{4\lambda^2} + \int_0^{v_1}\frac{\tilde{v}_1^2}{4\lambda^2}\,\mathrm{d}\tilde{v}_1}{\frac{v_1+1}{2}}\Big|_{v_1=\sqrt{2\lambda b_1}} = \frac{2(\sqrt{2\lambda b_1})^3}{3\lambda^2(\sqrt{2\lambda b_1}+1)}, \tag{18}$$

and similarly for agent 2 is

$$Z_2(b_2) = \frac{2}{3\lambda}b_2. \tag{19}$$

The prizes in **FIX** and **SYM** are normalized as 1 as used by all the standard auctions. However, in Section 5.4, we go one step further by optimizing them and then comparing with **OPT** again.

5.2.3 Principal's profit

OPT: Using Theorem 1, particularly (8), we calculate each agent's individual contribution π_1 and π_2:

$$\pi_1 = \int_0^1 \big[\frac{v^2}{2\lambda} - \frac{v^3}{4\lambda} - \frac{v^2}{4\lambda}(1-v)\big]\,\mathrm{d}v = \frac{1}{12\lambda}, \tag{20}$$

$$\pi_2 = \int_{0^+}^1 \big[\frac{v^2}{2\lambda} - \frac{v^3}{4\lambda} - \frac{v^2}{4\lambda}(1-v)\big]\frac{\mathrm{d}v}{2} = \frac{1}{24\lambda}, \tag{21}$$

$$\therefore\ \pi = \pi_1 + \pi_2 = \frac{1}{8\lambda}. \tag{22}$$

When calculating π_2, we can integrate from 0^+ onward because, although there is a probability atom of 0.5 at $v_2 = 0$, the corresponding effort and payment are both zero, and hence it does not contribute to the profit.

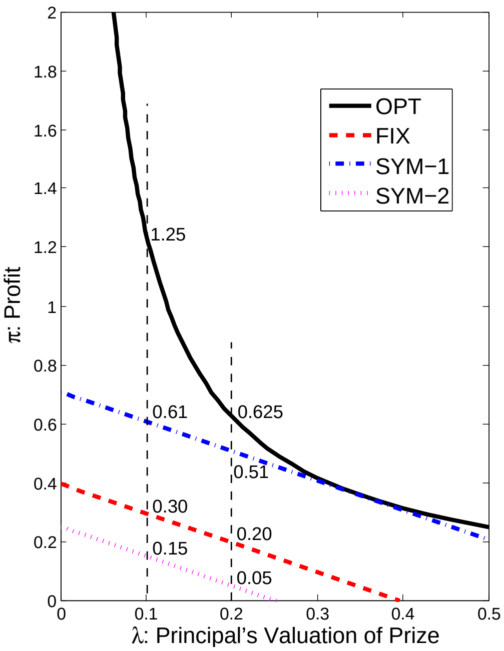


Fig. 3. Profit comparison of four mechanisms.

FIX: The profit in this case is $\pi^{fix} = r_1^{fix} + r_2^{fix} - \lambda Z$ where r_1^{fix} and r_2^{fix} are the revenue contributed by agent 1 and 2, respectively, and

$$r_1^{fix} = \int_0^1 b_1^{fix}(v_1)\,\mathrm{d}F_1(v_1) = \frac{2\sqrt{Z}}{5\sqrt{3}},$$
$$r_2^{fix} = \int_{0+}^1 b_2^{fix}(v_2)\,\mathrm{d}F_2(v_2) = \frac{2\sqrt{Z}}{7\sqrt{3}}.$$
$$\therefore\ \pi^{fix} = \frac{24\sqrt{Z}}{35\sqrt{3}} - \lambda Z. \tag{23}$$

Similar to **OPT** above, the probability atom at $v_2 = 0$ is nullified by $b_2^{fix}(0) = 0$.

SYM: The profits of **SYM-1** and **SYM-2** are, respectively,

$$\pi_1^{sym} = 2\int_0^1 b_1^{sym}(v_1)\,\mathrm{d}F_1(v_1) - \lambda Z = \frac{\sqrt{Z}}{\sqrt{2}} - \lambda Z, \tag{24}$$
$$\pi_2^{sym} = 2\int_0^1 b_2^{sym}(v_2)\,\mathrm{d}F_2(v_2) - \lambda Z = \frac{\sqrt{Z}}{4} - \lambda Z. \tag{25}$$

5.3 Result Set 1-A: Profit ranking

In view of the ultimate objective of a principal, we first compare the profit of the above four mechanisms in Fig. 3, based on formulae (22)–(25), where the prize Z in standard auctions is normalized. The plot clearly shows the profit ranking as **SYM-2** $\prec$ **FIX** $\prec$ **SYM-1** $\prec$ **OPT**, where $\prec$ denotes "is inferior to". In particular, OPT garners the highest profit compared to all the other mechanisms over all possible λ: from eight specific profit values marked in Fig. 3 at $\lambda = 0.1$ and 0.3, we see that OPT significantly outperforms the other three mechanisms by about 105%, 315% and 730%, respectively (at $\lambda = 0.1$). Furthermore, if λ is sufficiently high, FIX, SYM-1, and SYM-2 even run into deficit (negative profit), at $\lambda > 0.396$, $\lambda > 0.707$, and $\lambda > 0.25$, respectively. Reversely, as λ becomes smaller (i.e., the principal values the prize less), OPT reaps exponential profit growth whereas the other mechanisms only have linear profit increase.

5.3.1 *Rationale behind* **SYM-2** $\prec$ **FIX** $\prec$ **SYM-1**

The ranking of these three mechanisms is fairly intuitive if we notice their population compositions: SYM-1 and SYM-2 are composed of two strong and two weak agents, respectively, and FIX is a mixture. However, taking a closer look at Fig. 3, one would notice that FIX is even *lower than half* of SYM-1, which is not intuitive and there is no straightforward answer.

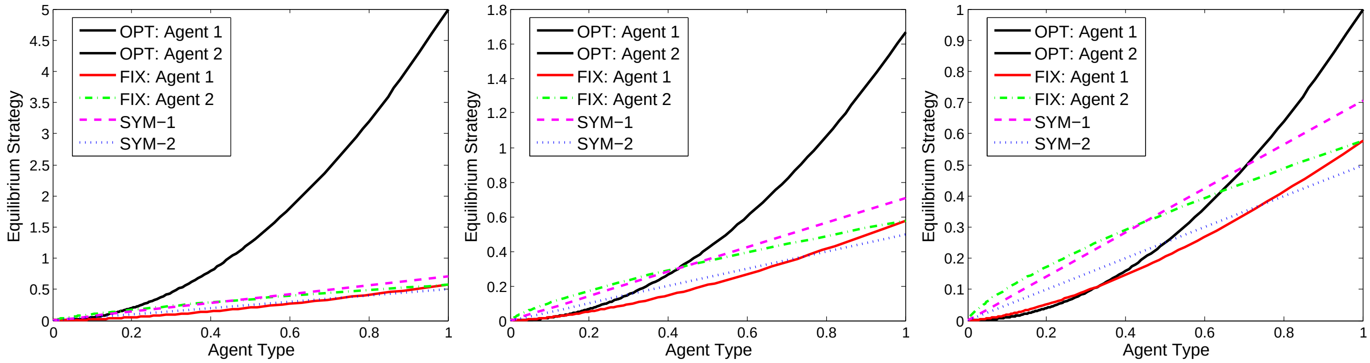
This is explained by the *effort reservation* effect in asymmetric auctions, where a stronger agent shades his bid when facing a weaker agent, as described earlier in Section 4.3. To verify this, we plot formulae (12)–(17) in Fig. 4, where we indeed see that agent 1 in FIX bids significantly lower than in SYM-1. Although agent 2, on the other hand, exerts higher effort than in SYM-2,[9] such small effort increase is outweighed by the effort reduction of agent 1. This is because, mathematically, the p.d.f. of the stronger type (agent 1) concentrates on the higher region of the common support $[\underline{v}, \bar{v}]$ and thus has a larger impact on the revenue (as a result of integration). Intuitively, this tells that "stronger agents matter more", and allows us to draw the insight that, when designing an incentive mechanism, it is *more productive* to focus on incentivizing *stronger* agents as they will constitute the main contributors to the revenue. This rule of thumb is also concurred by *discriminatory* auction design [44].

5.3.2 *Rationale behind* **SYM-1** $\prec$ **OPT**

Unlike the other three mechanisms, the ranking of SYM-1 $\prec$ OPT is rather perplexing, because SYM-1 is composed of two strong agents while OPT contains a weak agent. To understand why, first we examine agent strategies by look at Fig. 4. It shows that agents in OPT exert significantly higher effort than SYM-1 (and other mechanisms as well) especially at higher types (recall that stronger agents matter more), which seems to explain the ranking. But, how is this achieved? To find the answer, we plot formulae (18)(19) in Fig. 5 to examine the optimal prize tuple of OPT. We see that OPT gives slightly higher reward to agent 2 if he exerts the same amount of effort as agent 1. This motivates the weaker agent to strive harder insofar as he becomes a *competitive rival* to the stronger agent, which in turn "threatens" the stronger agent *not* to reserve effort. Essentially, the asymmetric contest recuperates from the fierceness of competition level of symmetric contests by virtue of the prize tuple which endogenizes the agent asymmetry. Moreover, Fig. 5 also shows that the prize for any agent is increasing with respect to effort, which also motivates agents to exert higher effort. Consequently, SYM-1 $\prec$ OPT.

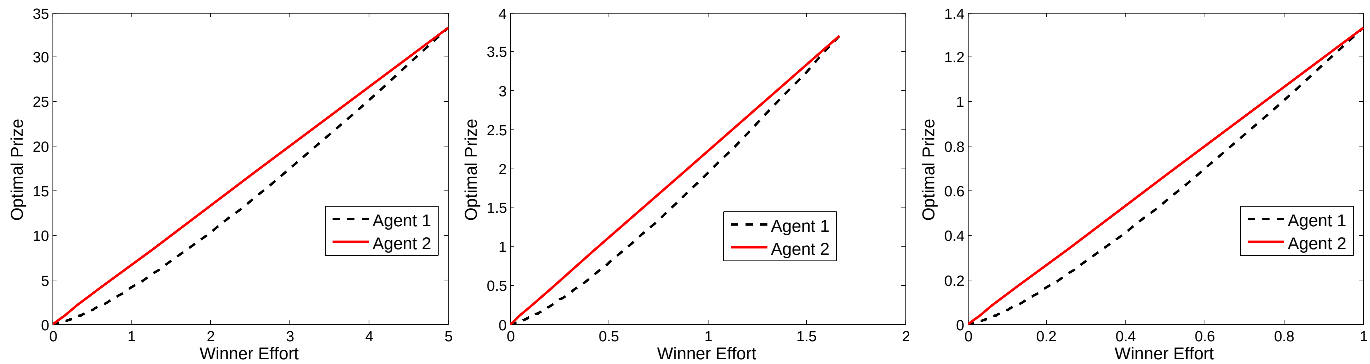
As a side note, Fig. 4 also indicates that agent strategies in all the mechanisms are monotone increasing, which conforms to Lemma 1.

9. The reason why agent 2 works harder in FIX than in SYM-2 is because he can deduce that the stronger agent will reserve effort and hence he (agent 2) sees a better chance to win by striving above his (usual) effort level as in the symmetric case (SYM-2).



(a) $\lambda = 0.1$. (b) $\lambda = 0.3$. (c) $\lambda = 0.5$.

Fig. 4. Equilibrium strategy (agent effort). For **OPT**, the same line-spec is used for both agents as they adopt the same strategy.



(a) $\lambda = 0.1$; the maximum effort is 5. (b) $\lambda = 0.3$; the maximum effort is $\frac{5}{3}$. (c) $\lambda = 0.5$; the maximum effort is 1.

Fig. 5. Optimal prize tuple of **OPT**, as functions of winner efforts. The range of X axis is determined by the maximum effort. Also note that the ranges of Y axes in the three plots are considerably different.

5.4 Result Set 1-B: Profit ranking with optimized standard auctions

In Section 5.3, the prize Z is normalized as per standard auctions. In this section, we go one step further by optimizing the prizes in standard auctions, and then compare their profits with our mechanism **OPT**.

FIX: To solve $\max_Z \pi^{fix}$, we apply the first order condition (FOC) to (23) to obtain

$$\sqrt{Z^*} = \frac{12}{35\sqrt{3}\lambda} \quad \text{and} \quad \pi^{fix^*} = (\frac{24}{35\sqrt{3}})^2 \Big/ (4\lambda) = \frac{48}{1225\lambda}.$$

SYM: The profit is maximized again by using FOC:

$$\sqrt{Z_1^*} = \frac{1}{2\sqrt{2}\lambda} \qquad \text{and} \qquad \pi_1^{sym^*} = \frac{1}{8\lambda},$$
$$\sqrt{Z_2^*} = \frac{1}{8\lambda} \qquad \text{and} \qquad \pi_2^{sym^*} = \frac{1}{64\lambda}.$$

Now, we plot the above optimized profits, $\pi^{fix^*}, \pi_1^{sym^*}, \pi_2^{sym^*}$, against π of OPT (22) in Fig. 7. First, we observe that OPT is still a clear winner over FIX and SYM-2 after optimization. Second, the optimized SYM-1 parallels OPT, but note that this is still a remarkable result because SYM-1 has the strongest population composition (two strong agents) while OPT has a weak agent. Finally, all the mechanisms (especially FIX, SYM-1 and SYM-2) are now deficit free over the entire range of λ, thanks to the prize optimization.

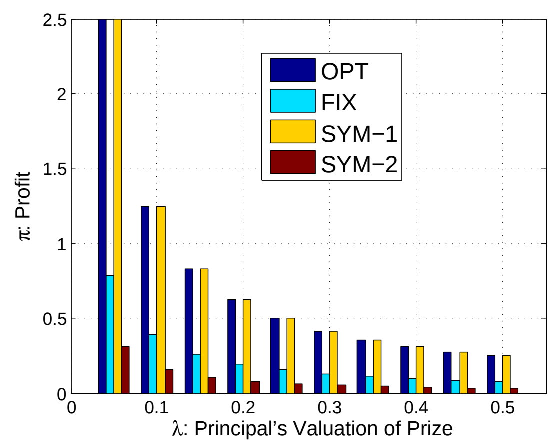
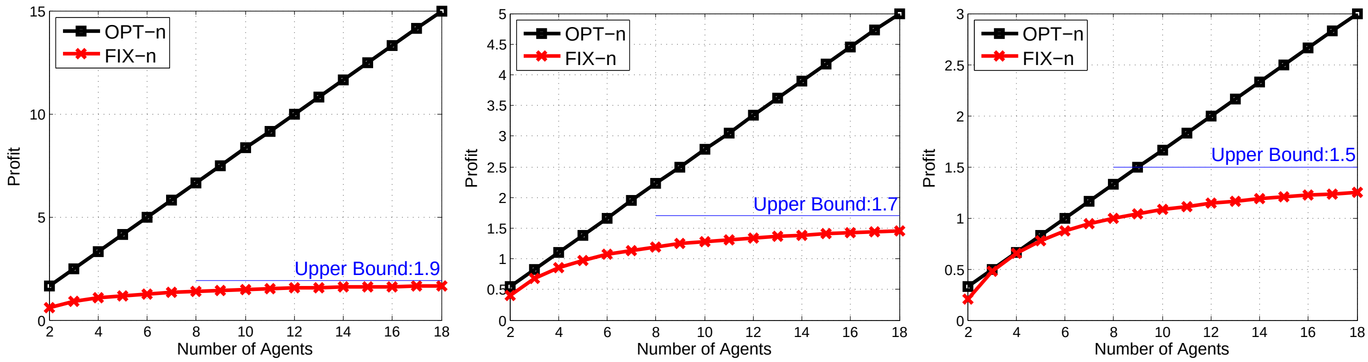


Fig. 7. Profit comparison with optimized canonical auctions.

5.5 Result Set 2-A: SA neutralizing DMR

In the second scenario, we investigate how SA neutralizes the law of DMR, by comparing **OPT-n** and **FIX-n**. In **OPT-n**, since the n agents are homogeneous, the prize tuple collapses into a single prize function, which can be derived still from Theorem 1 as

$$Z^{opt\text{-}n}(b) = \frac{(2\lambda b)^{2-\frac{n}{2}}}{3\lambda^2}.$$



(a) $\lambda = 0.1$. (b) $\lambda = 0.3$. (c) $\lambda = 0.5$.

Fig. 6. Strategy autonomy (SA) enhances scalability by neutralizing the law of diminishing marginal returns (DMR).

Accordingly, the equilibrium agent strategy changes to

$$b = \frac{v^2}{2\lambda}, \tag{26}$$

and the resultant profit becomes

$$\pi^{opt\text{-}n} = \frac{n}{12\lambda}. \tag{27}$$

In **FIX-n**, the equilibrium agent strategy is calculated using Proposition 3, as

$$b^{fix\text{-}n}(v) = \sqrt{\frac{n-1}{n} v^{\frac{n}{2}} \sqrt{Z}}, \tag{28}$$

and the resultant profit is

$$\pi^{fix\text{-}n} = n \int_0^1 b^{fix\text{-}n}(v)\, \mathrm{d}F_1 - \lambda Z = \frac{2\sqrt{n(n-1)Z}}{n+2} - \lambda Z \tag{29}$$

We plot $\pi^{opt\text{-}n}$ and $\pi^{fix\text{-}n}$ in Fig. 6 with respect to n for different λ values, where FIX-n adopts a normalized prize. As we can see, FIX-n as a standard auction is indeed governed by the law of DMR, exhibiting concave profit growth as n increases. Moreover, it saturates at the upper bound $\lim_{n\to\infty} \pi^{fix\text{-}n} = 2 - \lambda$, which is depicted in Fig. 6 as well.

In contrast, OPT-n is not confined by the DMR law and its profit grows *linearly* as n increases. It even exceeds the *upper bound* of FIX-n, when n is not too small as is common in reality, and continuously generates constantly larger profit as n increases. This manifests a very healthy *scalability* of OPT-based crowdsourcing systems.

To understand why, note that SA in symmetric cases (as of OPT-n and FIX-n) translates to the property that agent strategy in equilibrium is independent of the number of agents. This is evidenced by (26) where b does not depend on n, whereas the equilibrium strategy in FIX-n (28) does. Therefore the revenue—the sum of all the bids—of OPT-n is a linear function of n; specifically, revenue $r = n \int_0^1 \frac{v^2}{2\lambda}\, \mathrm{d}F(v) = \frac{n}{6\lambda}$. The cost (prize) of OPT-n is also linear: $\lambda \mathbb{E}[Z] = \lambda \int_0^1 \frac{v^{4-n}}{3\lambda^2}\, \mathrm{d}v^n = \frac{n}{12\lambda}$. As a result, the profit is a linear function of n, as $\frac{n}{6\lambda} - \frac{n}{12\lambda} = \frac{n}{12\lambda}$ which coincides with (27).

5.6 Result Set 2-B: SA neutralizing DMR with optimized FIX-n

Applying FOC to (29) yields the optimized prize and profit for FIX-n:

$$\sqrt{Z_n^*} = \frac{\sqrt{n(n-1)}}{(n+2)\lambda} \quad \text{and} \quad \pi^{fix\text{-}n^*} = \frac{n(n-1)}{(n+2)^2\lambda}.$$

The results are plotted in Fig. 8, with a larger range of n up to 30 in order to match with the new upper bound of FIX-n. We see that FIX-n can now approach a higher upper bound as compared to Fig. 6, as a result of optimization. Specifically, $\lim_{n\to\infty} \pi^{fix\text{-}n^*} = 1/\lambda$, which values to 10, 3.3 and 2 for different λ. Also by comparing to Fig. 6, we see that FIX-n and OPT-n generate profits much closer to each other when n is small. However, the prominent observation is that OPT-n still outperforms FIX-n (as long as n is not too small) even though it is optimized, and in contrast to FIX-n, is not restricted by DMR when it scales up.

5.7 Result Set 3: Environmental sensing

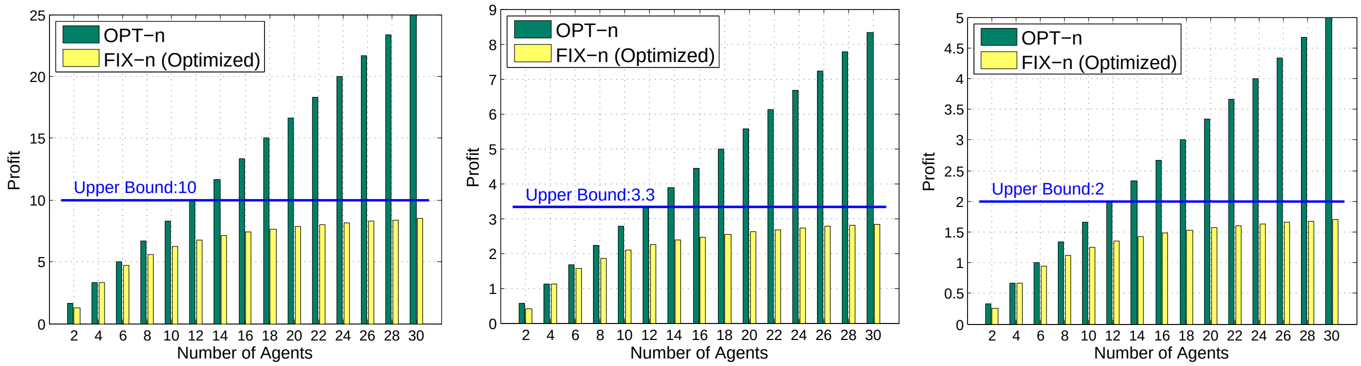
An environmental sensing application outlined in [7] (Sec. IV) exploits smartphone microphone sensors or wearable/handheld sensors from the crowd to monitor air pollution or EMF radiation in a certain area. There are n agents whose unit costs (types) c_i follow heterogeneous belief F_i^{inf} ($F_i^{inf} \neq F_j^{inf}$ if $i \neq j$).[10] Let $\delta_i(c_i) = c_i + \frac{F_i^{inf}(c_i)}{f_i^{inf}(c_i)}$, the optimal participation level (sampling rate) $\mathbf{x}^*$ according to [7] is the solution to

$$\arg\min_{\mathbf{x}} \sum_i x_i \delta_i(c_i) \text{ subject to } \sum_i x_i/\sigma_i^2 = 1/\epsilon, \tag{30}$$

where σ_i^2 is the variance of user i's measurements and indicates his data quality, ϵ is the mean square error (MSE) and is the system-targeted QoS. When $\delta_i(c_i)$ is linear in c_i, the problem (30) is a linear programming one and the solution can be obtained as $x_{i^*} = \sigma_i^2/\epsilon$ and $x_j = 0$ for all $j \neq i^*$, where $i^* = \arg\min_i \delta_i \sigma_i^2$.

Consider two agents with their unit costs c_i drawn from $[0, 1]$ as per $F_1^{inf}(c_1) = c_1$ and $F_2^{inf}(c_2) = c_2^2$, respectively.

10. We retain the same notation in [7] so that readers can do easy cross-reference; whenever there is a notation clash, e.g. $F()$, we add a superscript 'inf' for differentiation.



(a) $\lambda = 0.1$.   (b) $\lambda = 0.3$.   (c) $\lambda = 0.5$.

Fig. 8. SA neutralizes DMR: **OPT-n** versus optimized **FIX-n**. Note that both the upper bounds and the Y-axes are rather different from Fig. 6.

Hence we have $\delta_1 = 2c_1$ and $\delta_2 = 3c_2/2$ which are linear. Suppose $\sigma_1^2 = \sigma_2^2 = 2.5$, $c_1 = 0.5$ and $c_2 = 0.75$ (the costs will be truthfully reported because the mechanism is incentive-compatible). Then we have $i^* = \arg\min_i \delta_i \sigma_i^2 = 1$, $x_1 = \sigma_1^2/\epsilon$ and $x_2 = 0$. User 1 will receive remuneration of, according to [7], $p_1^{inf} = \frac{\sigma_1^2}{\epsilon}\frac{\sigma_2^2 c_2}{\sigma_1^2} = \sigma_2^2 c_2/\epsilon = 1.875/\epsilon$.

The objective of [7] is to minimize total remuneration subject to a QoS constraint $\sum_i x_i/\sigma_i^2 = 1/\epsilon$. Since x_i denotes participation level and $1/\sigma_i^2$ denotes data quality, x_i/σ_i^2 can be treated as user contribution (bid) and $\sum_i x_i/\sigma_i^2$ as system revenue. Hence the profit is

$$\pi^{inf} = \sum_i x_i/\sigma_i^2 - \lambda \sum_i p_i^{inf} = (1 - 1.875\lambda)/\epsilon.$$

In our case, the type v_i represents user ability which relates to unit cost c_i in an opposite way. Suppose $v_i = 1 - c_i$. It can thus be derived that $F_1(v_1) = v_1$ and $F_2(v_2) = 1-(1-v_2)^2$ (and $f_2(v_2) = 2(1-v_2)$). Therefore, $\frac{1-F_1(v_1)}{f_1(v_1)} = 1 - v_1$ and $\frac{1-F_2(v_2)}{f_2(v_2)} = (1-v_2)/2$. Using Theorem 1, we first calculate agent bidding strategy to be $b_1 = v_1^2/\lambda$ and $b_2 = v_2^2/[\lambda(1+v_2)]$, and then the profit from each agent to be $\pi_1 = 1/(12\lambda)$ and $\pi_2 = \frac{2}{\lambda}\int_0^1 \frac{v^2(1-v)}{1+v} - \frac{v^3(1-v)}{(1+v)^2} - \frac{v^2(1-v)^2}{2(1+v)^2}\,\mathrm{d}v = \frac{1}{\lambda}\int_0^1 \frac{v^2(1-v)}{1+v}\,\mathrm{d}v = \frac{\ln 4}{\lambda} - \frac{4}{3\lambda}$. Therefore, the total profit $\pi = \pi_1 + \pi_2 = (\ln 4 - \frac{5}{4})/\lambda$.

We plot the profits from the above two mechanisms in Fig. 9. The horizonal line of $\pi = 0$ clearly indicates that the profit obtained in [7] can be negative, when the crowdsourcer values his payout too high, i.e., λ exceeds some threshold (in this case 0.53). On the other hand, profit of OPT is always positive.

Note that the above evaluation which pertains to a particular scenario should not be overgeneralized. Due to the difference between the model of [7] and that of this study, there is no precise one-to-one mapping between the parameters or functions used in these two studies. However, as mentioned in Section 2.2, [7] appears to be the closest to our work and hence is chosen here for evaluation as a possible numerical illustration.

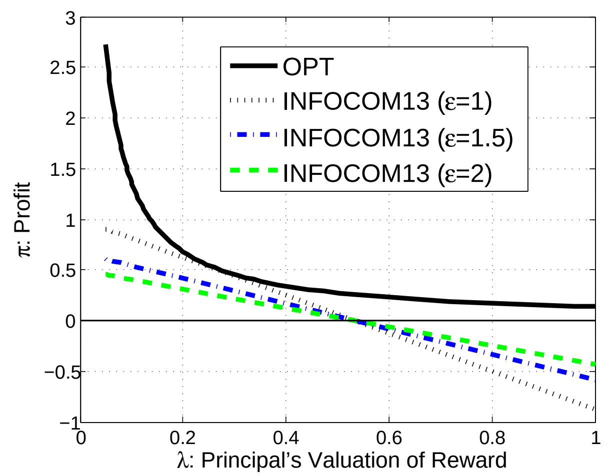


Fig. 9. Profit results in an environmental sensing application setting [7] (INFOCOM13).

6 Conclusion

This paper addresses the problem of incentive mechanism design for heterogeneous crowdsourcing, by casting it as an asymmetric all-pay contest. For the first time, this model accommodates an arbitrary number of heterogeneous workers with incomplete information, and is instrumented with a prize tuple for the objective of maximizing the crowdsourcer's utility. We solve for this model and demonstrate that the resultant mechanism induces maximal effort from self-interested agents while minimizing the cost to the crowdsourcer, and significantly outperforms traditional mechanisms that employ a single, fixed prize in both symmetric and asymmetric cases.

Our asymmetric auction based mechanism also yields a counter-intuitive property called strategy autonomy (SA). It captures an equilibrium behavior that agents with heterogeneous knowledge behave independently of each other as if they were in a homogeneous setting, or in other words, an asymmetric auction admits a symmetric equilibrium. SA could be an interesting enrichment to the mechanism design theory, and also has several desirable practical implications.

One possible future direction is to explore an incentive mechanism with multiple winners.

APPENDIX

Available online at the IEEE Computer Society Digital Library (http://www.computer.org/csdl).

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