Proofs for *TruTeam* (ICC'15)

Qing Liu*, Tie Luo†, Ruiming Tang*, Stéphane Bressan§
*School of Computing, National University of Singapore
†Institute for Infocomm Research, A*STAR, Singapore
§National University of Singapore and CNRS, Image & Pervasive Access Lab (IPAL UMI), Singapore
{liuqing,tangruiming,steph}@nus.edu.sg, luot@i2r.a-star.edu.sg

A. Optimal Mechanism

Lemma -A.1. *OPT is not Computationally Efficient.*

Proof. The complexity of OPT is $O(2^n)$ (where n is the number of workers) since it considers every possible team (namely, every possible subset of W). □

Lemma -A.2. *OPT is Individually Rational.*

Proof. A rational worker bids the task with a value no less than her cost to perform this task, i.e., $b_i \geq c_i$. Therefore, the utility for each selected worker is $u_i = p_i - c_i \geq p_i - b_i = 0$. □

Lemma -A.3. *OPT is Profitable.*

Proof. This is obvious, since if the the payment to the selected team is larger than the value of the task, this task will not be performed. When performing the task, the utility of the requester is $U_R = v - \sum_{w_i \in S} p_i = v - \sum_{w_i \in S} b_i \geq 0$. □

Lemma -A.4. *OPT is not Truthful.*

Proof. OPT pays each selected worker with exactly the value of her bid, therefore she has incentive to bid a higher value than her true cost.

□

B. Greedy Mechanism

Lemma -B.1. *GREEDY is Computationally Efficient.*

Proof. Every time, selecting the worker with minimal value of $\frac{b_i}{\Delta_i(S)}$ takes $O(n)$ time. We have to consider at most n workers. Therefore, the complexity of this mechanism is $O(n^2)$. □

Lemma -B.2. *GREEDY is Individually Rational.*

Proof. The same with OPT □

Lemma -B.3. *GREEDY is Profitable.*

Proof. Every time we select a work, we check if the remaining value of the task can cover the payment of this worker. If not, the worker will not be selected. This process guarantees that the value of this task can cover the payment to the whole team. □

Lemma -B.4. *GREEDY is Not Truthful.*

Proof. The same with OPT □

C. VCG based Mechanism

Lemma -C.1. *VCG is not Computationally Efficient.*

Proof. Running time of VCG is $O(n2^n)$, where n is number of workers. There are $O(2^n)$ teams to consider, therefore it takes $O(2^n)$ to select the cheapest team. To decide the payment of each selected worker w_i, it takes again $O(2^n)$ time to find the cheapest team that excludes w_i. The total time complexity is $O(n2^n)$. □

Lemma -C.2. *VCG is Individually Rational.*

Proof. If worker w_i is selected, then

$$\begin{aligned} u_i &= p_i - c_i \\ &= \min_T \sum_{w_j \in T \wedge w_i \notin T} c_j - \sum_{w_j \in S \wedge j \neq i} c_j - c_i \\ &= \min_T \sum_{w_j \in T \wedge w_i \notin T} c_j - \sum_{w_j \in S} c_j \\ &\geq 0 \end{aligned} \tag{1}$$

Note that $b_i = c_i$ because this is a truthful mechanism (we will prove it later). The last inequality holds because team S is overall the cheapest team. □

Lemma -C.3. *VCG is Profitable.*

Proof. This is obvious since the task is abandoned if it is not profitable. □

Lemma -C.4. *VCG is Truthful.*

Proof.

- Assume that worker w_i is selected by bidding her true cost and she is also selected by bidding untruthfully. We want to compare her utilities when she bids truthfully and untruthfully, respectively. u_i, u_i', p_i, p_i' are the corresponding utilities, payments of w_i, when she bids truthfully and untruthfully, respectively. S' is the team of selected workers when w_i bids untruthfully.

$$\begin{aligned} u_i' &= p_i' - c_i \\ &= \min_T \sum_{w_j \in T \wedge w_i \notin T} c_j - \sum_{w_j \in S' \wedge j \neq i} c_j - c_i \\ &= \min_T \sum_{w_j \in T \wedge w_i \notin T} c_j - \sum_{w_j \in S'} c_j \end{aligned} \tag{2}$$

$u_i' \leq u_i$, since $\sum_{w_j \in S'} c_j \geq \sum_{w_j \in S} c_j$.

- When w_i is not selected when she bids truthfully (i.e., $u_i = 0$) and she is selected by underbidding.

$$\begin{aligned} u_i' &= p_i' - c_i \\ &= \min_T \sum_{w_j \in T \wedge w_i \notin T} c_j - \sum_{w_j \in S'} c_j \\ &= \sum_{w_j \in S} c_j - \sum_{w_j \in S'} c_j \leq 0 \end{aligned} \tag{3}$$

Therefore. a worker gets her maximum utility by bidding her true cost. □

D. Efficient and Truthful Mechanism (TruTeam)

Lemma -D.1. *TruTeam is computationally efficient, with a time complexity of $O(n^2 l)$.*

Proof. Selecting the worker who has the minimal value $\frac{b_i}{\Delta_i(S)}$ takes $O(n)$ time. Deciding the payment for the selected worker takes $O(nl)$. Since there are n workers, time complexity of this mechanism is $O(n^2 l)$. □

Lemma -D.2. *TruTeam is individually rational.*

Proof. According to the payment rule, we can see that for any selected worker w_i.

$$\begin{aligned} b_i &\leq \frac{b_{j_1}}{\Delta_{j_1}(S)} \times \Delta_i(S) \\ &\leq max_{j \in \{j_1, j_2, ..., j_k\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\} \\ &= p_i \end{aligned} \tag{4}$$

We assume a worker will not bid bellow her true cost, i.e. $b_i \geq c_i$ (In fact, we will show in Lemma -D.4 that $b_i = c_i$). Therefore, w_i's utility is $u_i = p_i - c_i \geq p_i - b_i \geq 0$. □

Lemma -D.3. *TruTeam is profitable.*

Proof. Every time when a worker is considered, we check if the remaining value of the task can cover the payment to this worker. If not, the worker will not be selected. This process guarantees that the value of this task is more than the total payment to the whole team. □

Lemma -D.4. *TruTeam is truthful.*

Proof. In the following detailed proof, b_i' is an untruthful bid, p_i' and u_i' are the corresponding payment and utility of worker w_i when she bid b_i'.

- If w_i is selected by biding her true cost c_i:
  - If w_i overbids, i.e., $b_i' > c_i$
    - If w_i is not selected by overbidding, her utility drops to 0.
    - If w_i is still selected by overbidding. Suppose $\{w_1, w_2, ..., w_{i-1}, w_i, w_{i+1}, ..., w_q\}$ is the original selected team when w_i bids c_i, $\{w_1, w_2, ..., w_{i-1}, w_{j_1}, w_{j_2} ..., w_{j_x}, w_{j_y}, ..., w_{j_q}\}$ is the selected team if w_i did not participate. Then $p_i = \max_{j \in \{j_1, ..., j_q\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\}$. Let $\{w_1, w_2, ..., w_{i-1}, w_{j_1}, w_{j_2}, ..., w_{j_x}, w_i, ..., w_{q'}\}$ be the selected team when w_i overbids. $\{w_1, w_2, ..., w_{i-1}, w_{j_1}, w_{j_2} ..., w_{j_x}, w_{j_y}, ..., w_{j_q}\}$ is still the selected team if w_i did not participate. So, $p_i' = \max_{j \in \{j_y, ..., j_q\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\}$. We have $p_i \geq p_i'$ since $\{j_y, ..., j_q\} \in \{j_1, ..., j_q\}$.
  - If w_i underbids, w_i will be selected for sure. Suppose $\{w_1, w_2, ..., w_{i-x}, ..., w_i, w_{i+1}, ..., w_q\}$ is the original selected team when w_i bids c_i, $\{w_1, ..., w_{i-x}, ... w_{i-1}, w_{j_1}, w_{j_2}, ..., w_{j_q}\}$ is the selected team if w_i did not participate. Then $p_i = \max_{j \in \{j_1, ..., j_q\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\}$. Let $\{w_1, , ..., w_{i-x}, w_i, ..., w_{q'}\}$ be the selected team when w_i underbids, $\{w_1, ..., w_{i-x}, ... w_{i-1}, w_{j_1}, w_{j_2}, ..., w_{j_q}\}$ is still the selected team if w_i did not participate. So, $p_i' = \max_{j \in \{i-x+1, ..., i-1, j_1, ..., j_q\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\}$. Obviously, $p_i \leq p_i'$. Worker w_i is the i^{th} selected worker when bidding c_i, because $\frac{b_j}{\Delta_j(T)} \leq \frac{c_i}{\Delta_i(T)}$ ($j \in \{1, 2, ..., i-1\}$). Therefore, $p_i' = \max_{j \in \{i-x, ..., i-1, j_1, ..., j_q\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\} \leq \max\{c_i, max_{j \in \{j_1, ..., j_q\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\}\} = \max\{c_i, p_i\} = p_i$. Therefore, $p_i' = p_i$
- If w_i is not selected by biding her true cost c_i:
  - If w_i overbids, w_i is still not selected for the task.
  - If w_i underbids, i.e., $b_i' < c_i$
    - If w_i is still not selected, her utility keep unchanged.
    - If w_i is luckily selected by underbidding. Suppose $\{w_1, w_2, ..., w_{j_0}, w_{j_1}, ..., w_{j_q}\}$ is the selected team when w_i bids c_i. Originally, w_i is not selected since $\frac{b_j}{\Delta_j(T)} < \frac{c_i}{\Delta_i(T)}$ ($j \in \{1, 2, ..., j_q\}$). Suppose $\{w_1, w_2, ..., w_{j_0}, w_i, ..., w_{q'}\}$ is the selected team if w_i underbids. Set $\{w_1, w_2, ..., w_{j_0}, w_{j_1}, ..., w_{j_q}\}$ determines the payment to worker w_i, as $p_i' = \max_{j \in \{j_1, ..., j_q\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\} \leq \max_{j \in \{1, 2 ..., j_q\}} \{\frac{b_j}{\Delta_j(T)} \times \Delta_i(T)\} < c_i$. The utility of worker w_i is $u_i' = p_i' - c_i < 0$.

To conclude, w_i has no incentive to bid a value other than her true cost. □

Theorem 1. *TruTeam is computationally efficient, individually rational, profitable and truthful [1].*

References

[1] Q. Liu, T. Luo, R. Tang, and S. Bressan, "An efficient and truthful pricing mechanism for team formation in crowdsourcing markets," in *IEEE ICC*, 2015.