Altruistic DISH: Unsafe-Pair formation, MCC-free Condition, NP-Hardness

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Abstract

This technical report provides the technical proofs for three propositions. The first is the conditions for forming an unsafe pair (UP) in an undirected graph. The second is the necessary and sufficient conditions for full cooperation coverage to achieve the void of multi-channel coordination (MCC) problems. The last is the NP-hardness of determining the minimum number and locations of altruistic nodes to achieve full cooperation coverage.

1 Forming Unsafe Pairs

Proposition 1. *In an undirected graph where each vertex represents a peer and each edge represents the relationship between two neighboring peers, denote by d_i the degree of an arbitrary vertex i. If PSM is not used, two adjacent vertices i and j form an UP if and only if:*

(a) $d_i \geq 2$, $d_j \geq 2$, and $d_i = d_j = 2$ does not hold, or

(b) $d_i = d_j = 2$, and i and j are not on the same three-cycle (i.e., triangle).

If PSM is used (peers sleep when idle), the above condition remains unchanged for the channel conflict problem, but changes to the following for the deaf terminal problem:

$d_i \geq 1$, $d_j \geq 1$, and $d_i = d_j = 1$ does not hold.

The above quotes Proposition 1 from [1].

Proof. First consider the case without PSM.

Sufficiency: If condition (a) or (b) is satisfied, i and j form two independent communicable pairs, say p_i (i, i') and p_j (j, j'), as illustrated in Fig. 1. Suppose p_i switches to a data channel ch_i when p_j is communicating on data channel ch_j, then this channel usage of ch_i is unknown to j. After p_j switches back to the control channel and if j initiates another communication while p_i is still communicating on ch_i, then (i) a channel conflict problem is created if j choose to use channel ch_i, or (ii) a deaf terminal problem is created if j initiates this communication with i.

Necessity: Equivalently, we prove that if neither of the conditions (a) and (b) is satisfied, i.e., d_i (or d_j) is 1 or i and j are on the same three-cycle, i and j

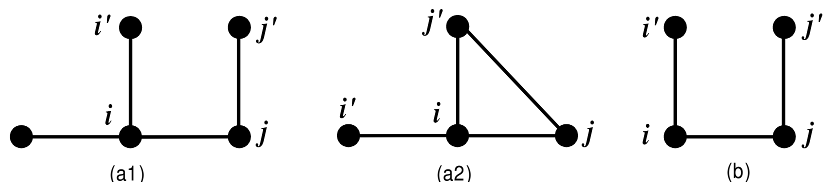

Figure 1: Edges represent neighboring relationships. Corresponding to Proposition 1, subfigures (a1) and (a2) illustrate condition (a), subfigure (b) illustrates condition (b).

does *not* form an UP. Since there is only one communicable pair, channel conflict problems will not be possible. Furthermore, whenever the (one) communicable pair performs a control channel handshake, the third node (if there is) will always be informed since we have assumed a node will always listen to the control channel when idle (if without PSM). Therefore, deaf terminal problems are also not possible.

Clearly, in the case with PSM, since there are at least three nodes, a sleeping node, say i, will miss the communication between j and a third node. Hence a deaf terminal problem will be created when i wakes up and initiates communication with j. Note that in this paper, deaf terminals are defined w.r.t. multi-channel; a sleeping receiver is not called a deaf terminal. □

2 MCC-free Condition

Proposition 2. *Consider a network using altruistic DISH. In order to achieve free of MCC problems, full cooperation coverage is*

1. *necessary for a multi-hop network, and*
2. *necessary and sufficient for a single-hop network.*

The above quotes Proposition 2 from [1].

Proof. Necessity: Since it is always possible for an UP to create MCC problems, an UP has to become a CUP to avoid these problems in a network using altruistic DISH. In other words, full cooperation coverage is a necessary condition for the network, irrespective of single-hop or multi-hop, to be free of MCC problems.

Sufficiency: In a single-hop network, one altruist achieves full cooperation coverage. Due to CSMA, each time only one control channel handshake can be accomplished. Therefore, every MCC problem created by such handshakes will be identified and prevented by the altruist. In case that there are more than one altruist, there is a marginal chance of collision between cooperative messages (in fact the chances are very low because of CCAP). However the proposition still holds because such collision still indicates an MCC problem as explained in Section II-A.

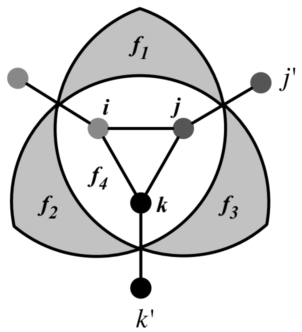

Figure 2: An illustration of Theorem 2. Edges represent neighboring relationships, and arcs represent radio ranges of i, j and k.

Remark: Full cooperation coverage is not a sufficient condition for multi-hop networks to be free of MCC problems, because concurrent and geographically distributed transmissions may overlap at altruists and hence not all MCC problems may be identified. □

3 NP-hardness of altruistic nodes placement

Theorem 1. *Consider a network with a given topology formed by peers on a finite plane. The problem of determining the minimum number and the locations of altruists to achieve full cooperation coverage, is NP-hard.*

The above quotes Proposition 2 from [1].

Proof. **Step 1: Identify UPs**

This step is to obtain a set U of all the UPs in the network by identifying UPs according to Proposition 1. As an example, see a six-node network shown in Fig. 2 and we only consider the case without PSM for conciseness. There are three UPs, and $U = \{(i,j),(j,k),(i,k)\}$.

Step 2: Construct Orphanage Set

This step is to construct $\mathcal{H} = \{H_i | i = 1, 2, ...p\}$ which is a set of all the orphanages in a network. To define orphanage, we first define *face*.

Definition 1. *A* face *is a region bounded by the (circular) radio boundaries of the peers who form UPs (there is no boundary inside a face). We say that* a face covers an UP*, if an altruist on any point of this face covers this UP.*

For example, in Fig. 2, i, j and k are all the peers that form UPs, f_1, f_2, f_3 and f_4 are all the faces, where, e.g., f_1 covers UP (i,j). Note that $f_1 \cup f_4$ is not a face.

Definition 2. *An* orphanage *is the maximum set of UPs covered by a face. Rigorously, an orphanage H is a set of UPs ($H \subseteq U$) covered by a face f_H, and $\forall u \in U \setminus H$, u is not covered by f_H.*

For example, in Fig. 2, $H_1 = \{(i, j)\}$ and $H_4 = \{(i, j), (j, k), (i, k)\}$ are two orphanages covered by faces f_1 and f_4, respectively. But $H'_4 = \{(i, j), (i, k)\}$ is not an orphanage. There are totally four orphanages in Fig. 2.

By definition, there is a one-to-one mapping between each orphanage and its covering face. Thus, finding all the orphanages in a network is equivalent to finding all the faces that covers at least one UP. This problem is the same as the target coverage problem [2] in sensor networks, and is shown by [3] that the number of such faces is bounded by $|U|(|U| - 1) + 2$ and these faces can be found in time $O(|U|^3)$ by simply finding all the intersecting points of the circles (e.g., there are six such points in Fig. 2).

Step 3: Formulate Problem

With U and $\mathcal{H}$, two problems can be posed:

1. Decision problem: given U, $\mathcal{H}$ and an integer k, determine whether a subset $\mathcal{C} = \{H_i | i = 1, 2, ...q\} \subseteq \mathcal{H}$ exists such that $\bigcup_{i=1}^{q} H_i = U$ and $q \leq k$.
2. Optimization problem: given U and $\mathcal{H}$, minimize $k = |\mathcal{C}|$ over all possible $\mathcal{C} = \{H_i | i = 1, 2, ...q\} \subseteq \mathcal{H}$, subject to $\bigcup_{i=1}^{q} H_i = U$.

Since each orphanage $H_i \in \mathcal{H}$ corresponds to a unique face containing an altruist, q ($q \leq p$) is the minimum number of altruists that achieve full cooperation coverage.

The above two problems are the variants of the *set cover problem* defined by Karp [4]; the decision problem is NP-complete and the optimization problem is NP-hard. □

References

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