intro-returns-and-risks

January 3, 2022

1 Fundamentos de risco e retornos

[ ]:
```
# Bibliiotecas importadas
import numpy as np
import pandas as pd
import yfinance as yf
%matplotlib inline
```

1.1 Retorno Simples ($R_{t,t+1}$)

$$R_{t,t+1} = \frac{P_{t+1} - P_t}{P_t}$$

ou alternativamente

$$R_{t,t+1} = \frac{P_{t+1}}{P_t} - 1$$

Exemplo:

[ ]:
```
empresas = {
    'Banco do Brasil S.A.': 'BBAS3.SA',
    'Banco Bradesco S.A.': 'BBDC3.SA',
    'Itaúsa - Investimentos Itaú SA': 'ITSA4.SA'
}
```

[ ]:
```
tickers = list(empresas.values())
prices = yf.download(tickers=tickers, start='2019-12-31', end='2020-12-31',␣
 ↪progress=False)['Adj Close']
prices.head()
```

[ ]:
```
             BBAS3.SA   BBDC3.SA   ITSA4.SA
Date
2020-01-02  48.002701  27.186684  12.605558
2020-01-03  47.922401  26.762447  12.421086
2020-01-06  47.288906  26.608953  12.394732
2020-01-07  46.932011  26.156134  12.227833
2020-01-08  46.503731  25.757042  12.043357
```

```
[ ]: prices.shape
```

```
[ ]: (248, 3)
```

Cálculo do Retorno Simples para cada ação.

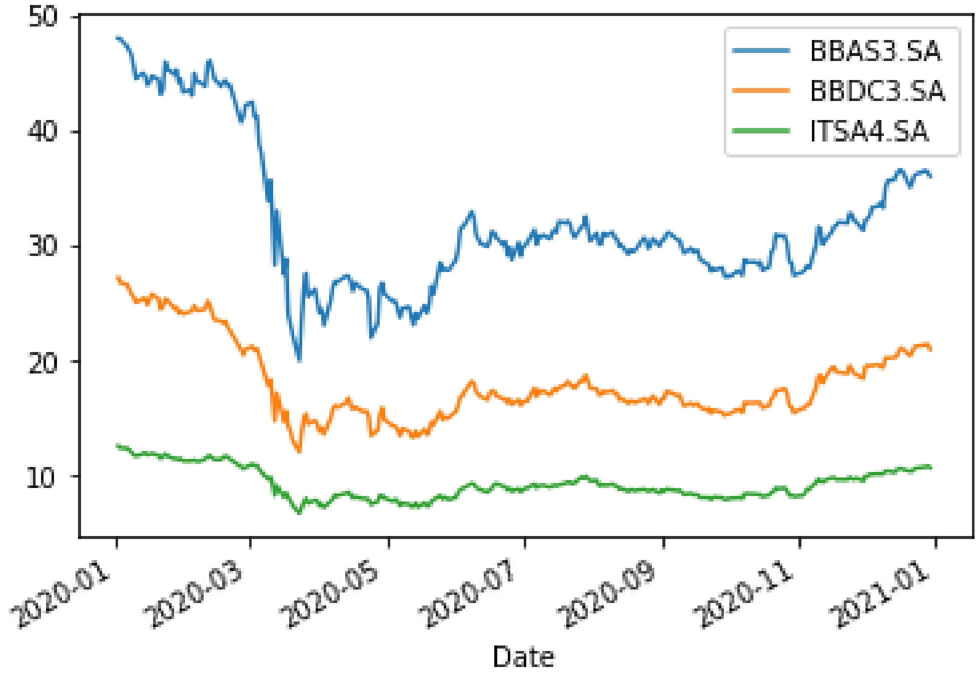
```
[ ]: returns = prices.pct_change()

     returns.head()
```

```
[ ]:             BBAS3.SA  BBDC3.SA  ITSA4.SA
     Date
     2020-01-02       NaN       NaN       NaN
     2020-01-03 -0.001673 -0.015605 -0.014634
     2020-01-06 -0.013219 -0.005735 -0.002122
     2020-01-07 -0.007547 -0.017018 -0.013465
     2020-01-08 -0.009126 -0.015258 -0.015087
```

Visualização Gráfica da série temporal de preços

```
[ ]: prices.plot()
```

```
[ ]: <AxesSubplot:xlabel='Date'>
```



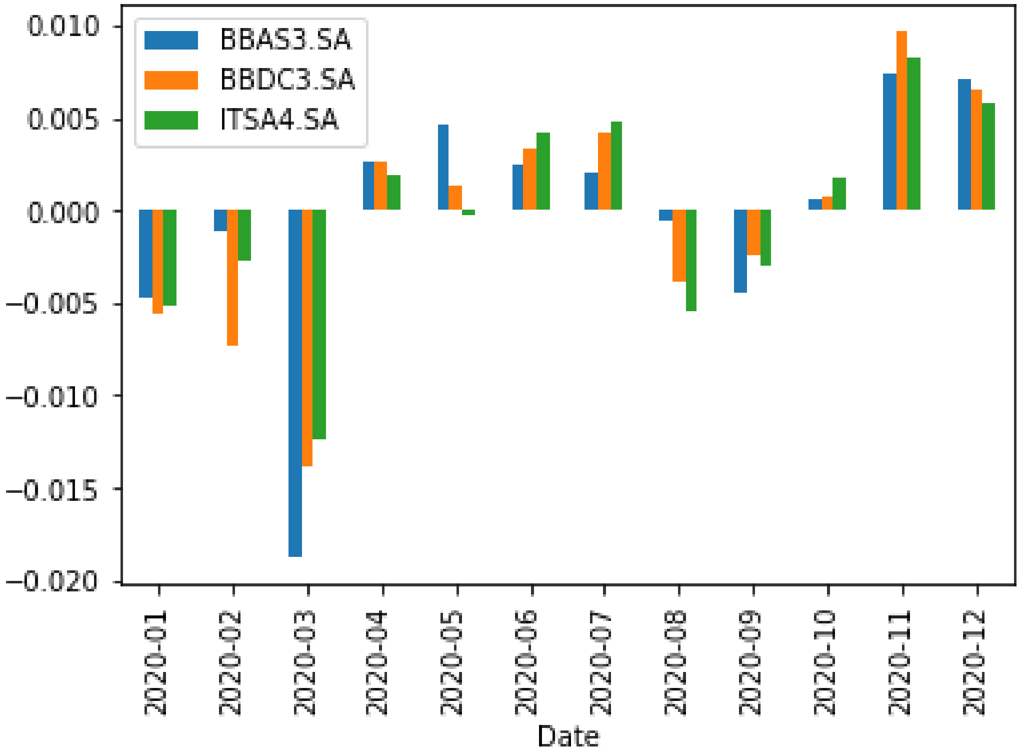
Visualização gráfica dos retornos mensais

```
[ ]: returns_montly = returns.resample('M').mean()

     returns_montly.index = returns_montly.index.to_period('M')

     returns_montly
```

```
[ ]:          BBAS3.SA  BBDC3.SA  ITSA4.SA
     Date
     2020-01 -0.004717 -0.005666 -0.005120
     2020-02 -0.001132 -0.007282 -0.002681
     2020-03 -0.018780 -0.013786 -0.012421
     2020-04  0.002588  0.002647  0.001917
     2020-05  0.004632  0.001372 -0.000284
     2020-06  0.002427  0.003411  0.004214
     2020-07  0.002119  0.004240  0.004851
     2020-08 -0.000538 -0.003868 -0.005518
     2020-09 -0.004387 -0.002387 -0.003065
     2020-10  0.000577  0.000746  0.001757
     2020-11  0.007348  0.009760  0.008255
     2020-12  0.007126  0.006550  0.005812
```

```
[ ]: returns_montly.plot.bar()
```

```
[ ]: <AxesSubplot:xlabel='Date'>
```



Volatilidade dos ativos

Corresponde ao Desvio Padrão dos retornos

```
[ ]: volatility = returns.std()
     volatility
```

```
[ ]: BBAS3.SA    0.039122
     BBDC3.SA    0.035640
     ITSA4.SA    0.027956
     dtype: float64
```

Retorno médio de cada ação

```
[ ]: returns.mean()
```

```
[ ]: BBAS3.SA   -0.000395
     BBDC3.SA   -0.000417
     ITSA4.SA   -0.000269
     dtype: float64
```

Retorno anual

```
[ ]: (returns + 1).prod() - 1
```

```
[ ]: BBAS3.SA   -0.249744
     BBDC3.SA   -0.228378
     ITSA4.SA   -0.150698
     dtype: float64
```

1.2 Segundo Exemplo

```
[ ]: returns = pd.read_csv('data/Portfolios_Formed_on_ME_monthly_EW.csv',
                          header=0, index_col=0, parse_dates=True, na_values=-99.
      ↪99
                        )
     returns.head()
```

```
[ ]:         <= 0  Lo 30  Med 40  Hi 30  Lo 20  Qnt 2  Qnt 3  Qnt 4  Hi 20  Lo 10  \
     192607   NaN  -0.43    1.52   2.68  -0.57   0.59   1.60   1.47   3.33  -1.45
     192608   NaN   3.90    3.04   2.09   3.84   3.59   3.71   1.61   2.33   5.12
     192609   NaN  -1.08   -0.54   0.16  -0.48  -1.40   0.00  -0.50  -0.09   0.93
     192610   NaN  -3.32   -3.52  -3.06  -3.29  -4.10  -2.89  -3.36  -2.95  -4.84
     192611   NaN  -0.46    3.82   3.09  -0.55   2.18   3.41   3.39   3.16  -0.78

             Dec 2  Dec 3  Dec 4  Dec 5  Dec 6  Dec 7  Dec 8  Dec 9  Hi 10
```

```
192607   0.29  -0.15   1.33   1.24   1.98   1.55   1.38   3.38   3.29
192608   2.59   4.03   3.15   2.72   4.72   1.60   1.63   0.98   3.70
192609  -1.87  -2.27  -0.53   0.07  -0.07  -1.64   0.64  -0.86   0.67
192610  -1.77  -3.36  -4.83  -2.98  -2.80  -3.45  -3.27  -3.47  -2.43
192611  -0.32  -0.29   4.65   3.24   3.57   3.82   2.95   3.61   2.70
```

```
[ ]: columns = ['Lo 10', 'Hi 10']

     returns = returns[columns]

     returns.head()
```

```
[ ]:         Lo 10  Hi 10
     192607  -1.45   3.29
     192608   5.12   3.70
     192609   0.93   0.67
     192610  -4.84  -2.43
     192611  -0.78   2.70
```

```
[ ]: returns = returns / 100
     returns.head()
```

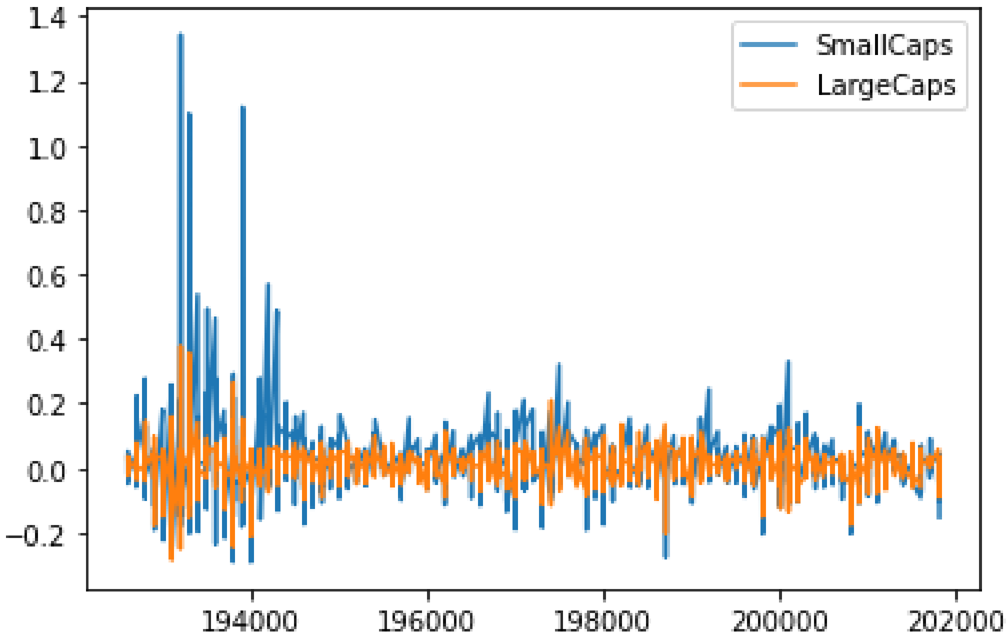
```
[ ]:          Lo 10   Hi 10
     192607 -0.0145  0.0329
     192608  0.0512  0.0370
     192609  0.0093  0.0067
     192610 -0.0484 -0.0243
     192611 -0.0078  0.0270
```

```
[ ]: returns.columns = ['SmallCaps', 'LargeCaps']
     returns.head()
```

```
[ ]:         SmallCaps  LargeCaps
     192607    -0.0145     0.0329
     192608     0.0512     0.0370
     192609     0.0093     0.0067
     192610    -0.0484    -0.0243
     192611    -0.0078     0.0270
```

```
[ ]: returns.plot.line()
```

```
[ ]: <AxesSubplot:>
```



Volatilidade

```
[ ]: volatility = returns.std()
     volatility
```

```
[ ]: SmallCaps    0.106288
     LargeCaps    0.053900
     dtype: float64
```

```
[ ]: annualized_vol = volatility * np.sqrt(12)
     annualized_vol
```

```
[ ]: SmallCaps    0.368193
     LargeCaps    0.186716
     dtype: float64
```

Retorno mensal

```
[ ]: n_months = returns.shape[0]

     return_per_month = ((1 + returns).prod() ** (1 / n_months)) - 1

     return_per_month
```

```
[ ]: SmallCaps    0.012986
     LargeCaps    0.007423
     dtype: float64
```

```
[ ]: annualized_return = (return_per_month + 1) ** 12 - 1

     annualized_return
```

```
[ ]: SmallCaps    0.167463
     LargeCaps    0.092810
     dtype: float64
```

```
[ ]: annualized_return = ((1 + returns).prod() ** (12 / n_months)) - 1

     annualized_return
```

```
[ ]: SmallCaps    0.167463
     LargeCaps    0.092810
     dtype: float64
```

```
[ ]: risk = annualized_return / annualized_vol

     risk
```

```
[ ]: SmallCaps    0.454825
     LargeCaps    0.497063
     dtype: float64
```

```
[ ]: riskfree_rate = 0.03

     excess_return = annualized_return - riskfree_rate

     sharpe_ratio = excess_return / annualized_vol

     sharpe_ratio
```

```
[ ]: SmallCaps    0.373346
     LargeCaps    0.336392
     dtype: float64
```

1.3 Máximo *Drawdown* (Máximo Rebaixamento)

É a perda máxima experimentada a partir de um pico anterior para um vale subseguente.

O *Dradown* é uma medida do risco de queda.

É a perda teórica que você teria sofrido se tivesse investido no pico e vendido na baixa.

Etapas para o cálculo do Máximo *Drawdown*:

1. Converter a série de retornos para o que é chamado de **índice de riqueza**.
2. Calcular os picos a cada ponto de tempo.
3. Plotar os *drawdowns* ao longo do tempo.

Cálcudo dos Rebaixamentos (Drawdowns)

[ ]:
```
# Bibliotecas necessárias
import pandas as pd
```

Fonte: Retornos mensais com base no capital de mercado ou com base no tamanho para *Small Caps* e *Large Caps*.

[ ]:
```
# importação do dataset
me_m = pd.read_csv(
    "data/Portfolios_Formed_on_ME_monthly_EW.csv",
    header=0,
    index_col=0,
    parse_dates=True,
    na_values=-99.99
)

me_m.head()
```

[ ]:
```
        <= 0  Lo 30  Med 40  Hi 30  Lo 20  Qnt 2  Qnt 3  Qnt 4  Hi 20  Lo 10  \
192607   NaN  -0.43    1.52   2.68  -0.57   0.59   1.60   1.47   3.33  -1.45
192608   NaN   3.90    3.04   2.09   3.84   3.59   3.71   1.61   2.33   5.12
192609   NaN  -1.08   -0.54   0.16  -0.48  -1.40   0.00  -0.50  -0.09   0.93
192610   NaN  -3.32   -3.52  -3.06  -3.29  -4.10  -2.89  -3.36  -2.95  -4.84
192611   NaN  -0.46    3.82   3.09  -0.55   2.18   3.41   3.39   3.16  -0.78

        Dec 2  Dec 3  Dec 4  Dec 5  Dec 6  Dec 7  Dec 8  Dec 9  Hi 10
192607   0.29  -0.15   1.33   1.24   1.98   1.55   1.38   3.38   3.29
192608   2.59   4.03   3.15   2.72   4.72   1.60   1.63   0.98   3.70
192609  -1.87  -2.27  -0.53   0.07  -0.07  -1.64   0.64  -0.86   0.67
192610  -1.77  -3.36  -4.83  -2.98  -2.80  -3.45  -3.27  -3.47  -2.43
192611  -0.32  -0.29   4.65   3.24   3.57   3.82   2.95   3.61   2.70
```

Faremos uso apenas dos ativos referentes ao decil superior (Hi 10) e ao decil inferior (Lo 10):

[ ]:
```
# Seleção das colunas de interesse
rets = me_m[['Lo 10', 'Hi 10']]

# Renomear colunas
rets.columns = ['SmallCap', 'LargeCap']

# Converter os retornos
rets = rets / 100
```

[ ]:
```
rets.head()
```

[ ]:
```
        SmallCap  LargeCap
192607   -0.0145    0.0329
```

```
192608    0.0512    0.0370
192609    0.0093    0.0067
192610   -0.0484   -0.0243
192611   -0.0078    0.0270
```

```
[ ]: rets.index
```

```
[ ]: Int64Index([192607, 192608, 192609, 192610, 192611, 192612, 192701, 192702,
                 192703, 192704,
                 …
                 201803, 201804, 201805, 201806, 201807, 201808, 201809, 201810,
                 201811, 201812],
                dtype='int64', length=1110)
```

Vemos que a coluna *index* está no formato ‘int64’ quando deveria estar no formato de ‘data e hora’. Para resolver esse problema, façamos a seguinte conversão:

```
[ ]: rets.index = pd.to_datetime(rets.index, format='%Y%m')
```

```
[ ]: rets.head()
```

```
[ ]:             SmallCap  LargeCap
     1926-07-01   -0.0145    0.0329
     1926-08-01    0.0512    0.0370
     1926-09-01    0.0093    0.0067
     1926-10-01   -0.0484   -0.0243
     1926-11-01   -0.0078    0.0270
```

Vemos que o formato de dados foi convertido para ‘data e hora’. Entretanto, por padrão, foi atribuído o primeiro dia de cada mês. Isso é indesejado na medida em que os retornos referem-se ao mês como um todo. Para esse novo problema, façamos:

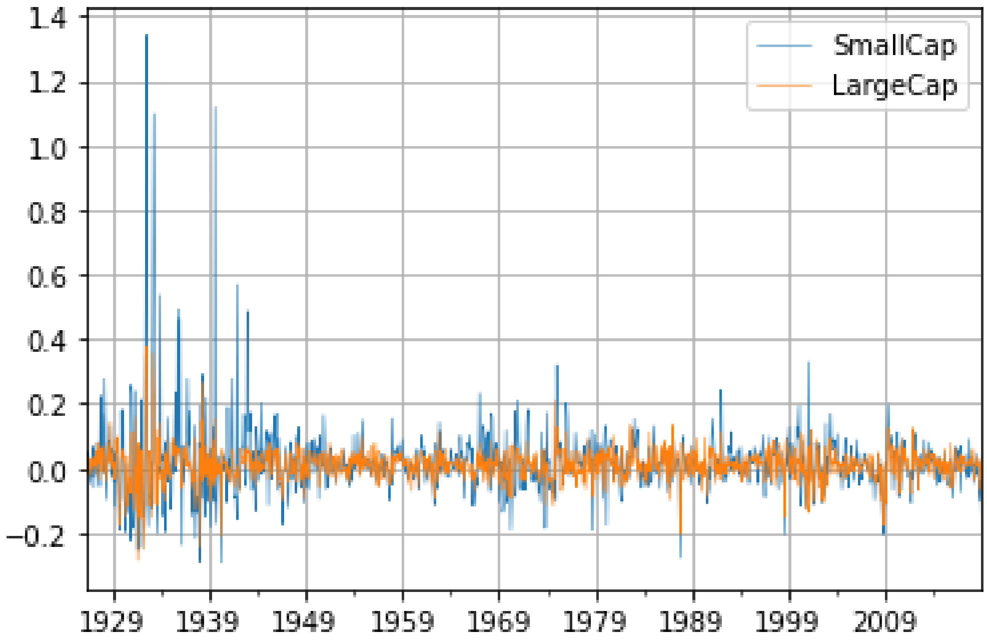
```
[ ]: rets.index = rets.index.to_period('M')
     rets.head()
```

```
[ ]:          SmallCap  LargeCap
     1926-07   -0.0145    0.0329
     1926-08    0.0512    0.0370
     1926-09    0.0093    0.0067
     1926-10   -0.0484   -0.0243
     1926-11   -0.0078    0.0270
```

Visualizamos novamente o gráfico de retornos:

```
[ ]: rets.plot.line(lw=.5, grid=True)
```

```
[ ]: <AxesSubplot:>
```



Podemos ver informações gerais desse dataframe:

[ ]:
```
rets.info()
```

```
<class 'pandas.core.frame.DataFrame'>
PeriodIndex: 1110 entries, 1926-07 to 2018-12
Freq: M
Data columns (total 2 columns):
 #   Column    Non-Null Count  Dtype  
---  ------    --------------  -----  
 0   SmallCap  1110 non-null   float64
 1   LargeCap  1110 non-null   float64
dtypes: float64(2)
memory usage: 26.0 KB
```

Procedimento

1. Calcular o índice de riqueza

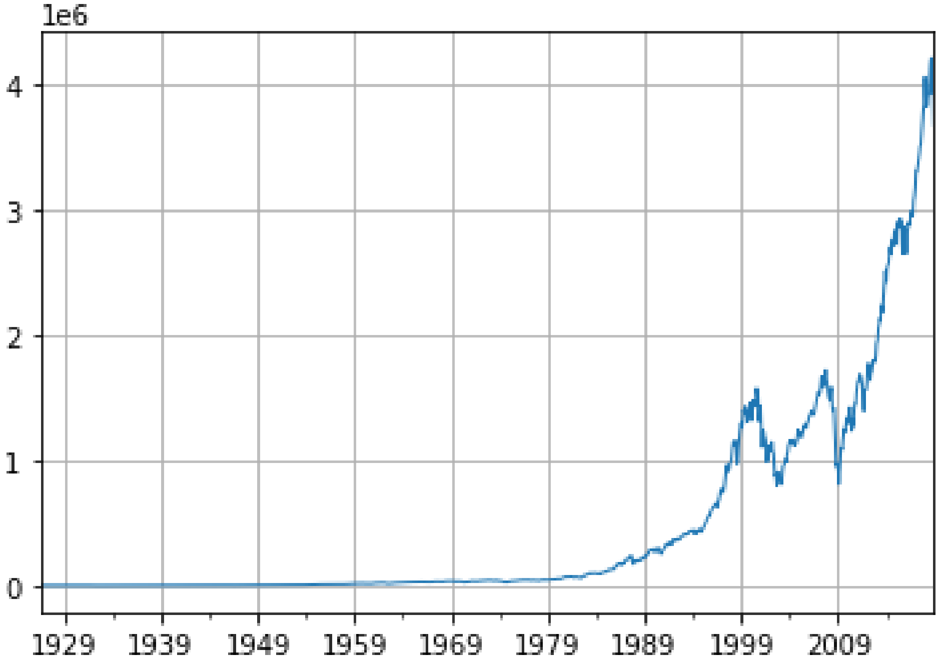
[ ]:
```
wealth_index = 1000. * (1 + rets['LargeCap']).cumprod()

wealth_index.head()
```

[ ]:
```
1926-07    1032.900000
1926-08    1071.117300
1926-09    1078.293786
1926-10    1052.091247
```

```
1926-11    1080.497711
Freq: M, Name: LargeCap, dtype: float64
```

```
[ ]: wealth_index.plot.line(lw=1, grid=True)
```

```
[ ]: <AxesSubplot:>
```

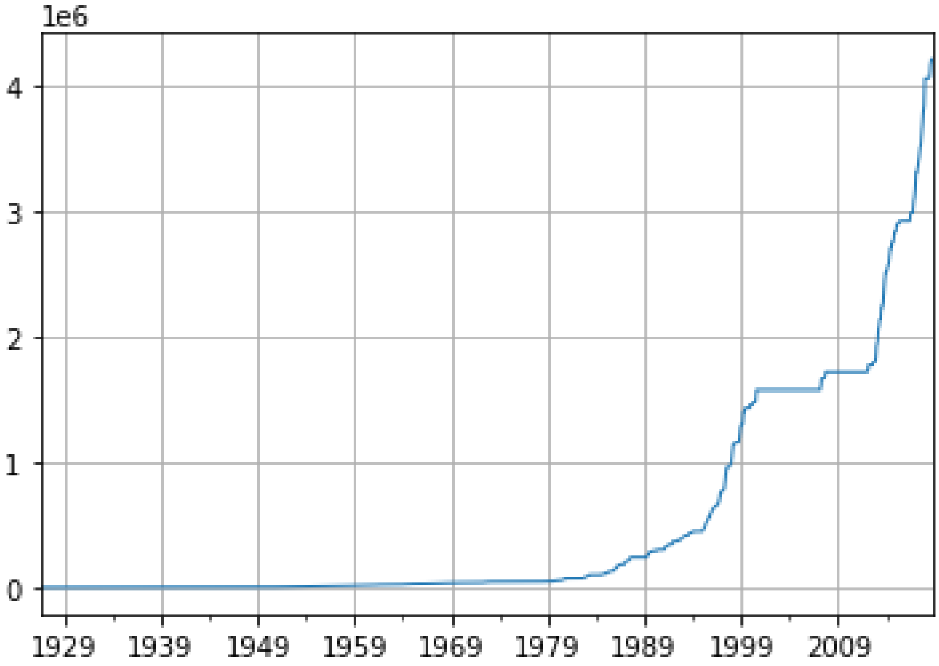


2. Calcular picos anteriores

```
[ ]: previous_peaks = wealth_index.cummax()
```

```
[ ]: previous_peaks.plot(lw=1, grid=True)
```

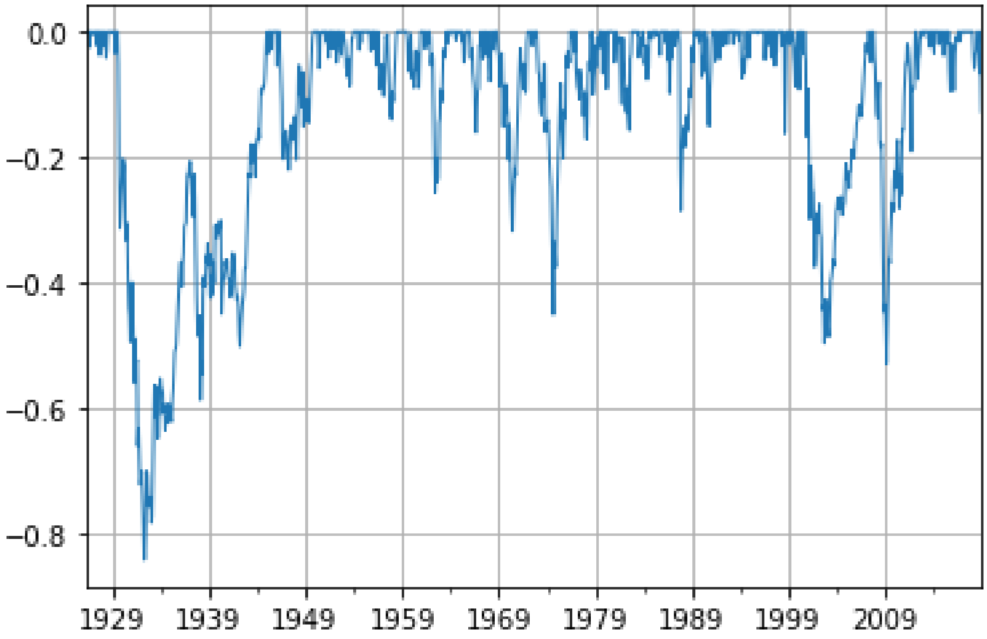
```
[ ]: <AxesSubplot:>
```



3. Calcular o *drawdown* - que é o valor da riqueza como porcentagem do pico anterior

```
[ ]: drawdown = (wealth_index - previous_peaks) / previous_peaks

     drawdown.plot(lw=1, grid=True)
```

```
[ ]: <AxesSubplot:>
```



[ ]:
```
max_drawdown = drawdown.min()

max_drawdown
```

[ ]:
```
-0.8400375277943123
```

O máximo *drawdown* e o momento de sua ocorrência:

[ ]:
```
print(f"O máximo drawdown de todo o período analizado ocorreu no mês {drawdown.
 ↪idxmin().strftime('%m de %Y')} e  foi de {100 * drawdown.min():.2f}%.")
```

```
O máximo drawdown de todo o período analizado ocorreu no mês 05 de 1932 e  foi
de -84.00%.
```

[ ]:
```
dd_value = drawdown["1975":].min()
dd_time = drawdown["1975":].idxmin().strftime('%m de %Y')

print(f"O máximo drawdown desde 1975 ocorreu no mês {dd_time} e  foi de␣
 ↪{dd_value:.2f}%.")
```

```
O máximo drawdown desde 1975 ocorreu no mês 02 de 2009 e  foi de -0.53%.
```

[ ]:
```
drawdown["1975":].idxmin().strftime('%m de %Y')
```

[ ]:
```
'02 de 2009'
```

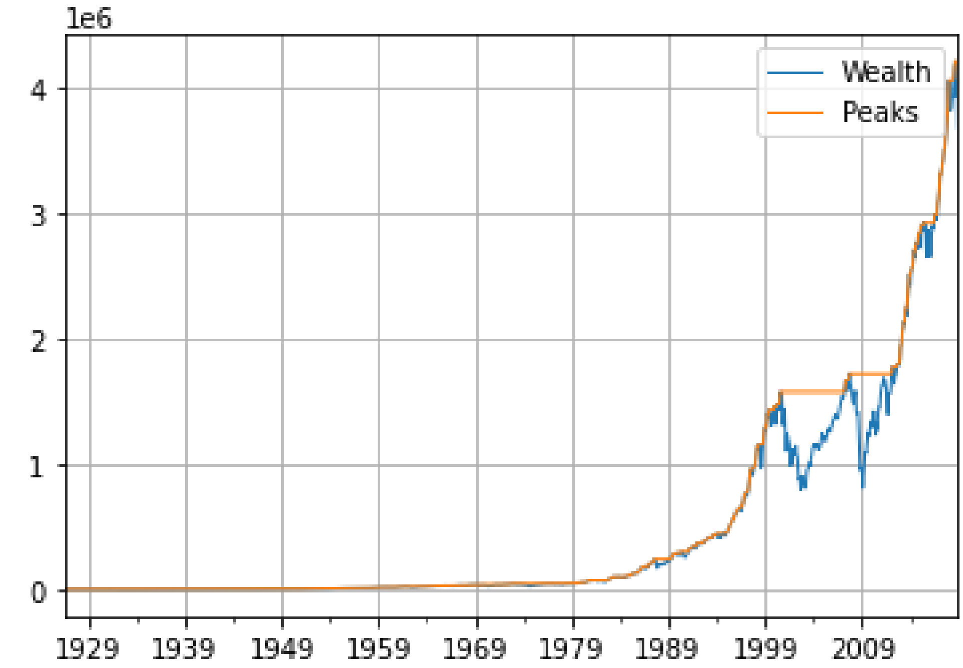
```
[ ]: def drawdown(return_series: pd.Series):
         """
         Takes a times series of asset returns
         Computes and returns a DataFrame that contains:
             - the  wealth index
             - the previous peaks
             - percent drawdowns
         """
         wealth_index = 1000 * (1 + return_series).cumprod()
         previous_peaks = wealth_index.cummax()
         drawdowns = (wealth_index - previous_peaks) / previous_peaks
         return pd.DataFrame({
             'Wealth'    : wealth_index,
             'Peaks'     : previous_peaks,
             'Drawdown' : drawdowns
         })
```

```
[ ]: largecap_drawdown = drawdown(rets['LargeCap'])

     largecap_drawdown.head()
```

```
[ ]:              Wealth        Peaks  Drawdown
     1926-07  1032.900000  1032.900000    0.0000
     1926-08  1071.117300  1071.117300    0.0000
     1926-09  1078.293786  1078.293786    0.0000
     1926-10  1052.091247  1078.293786   -0.0243
     1926-11  1080.497711  1080.497711    0.0000
```

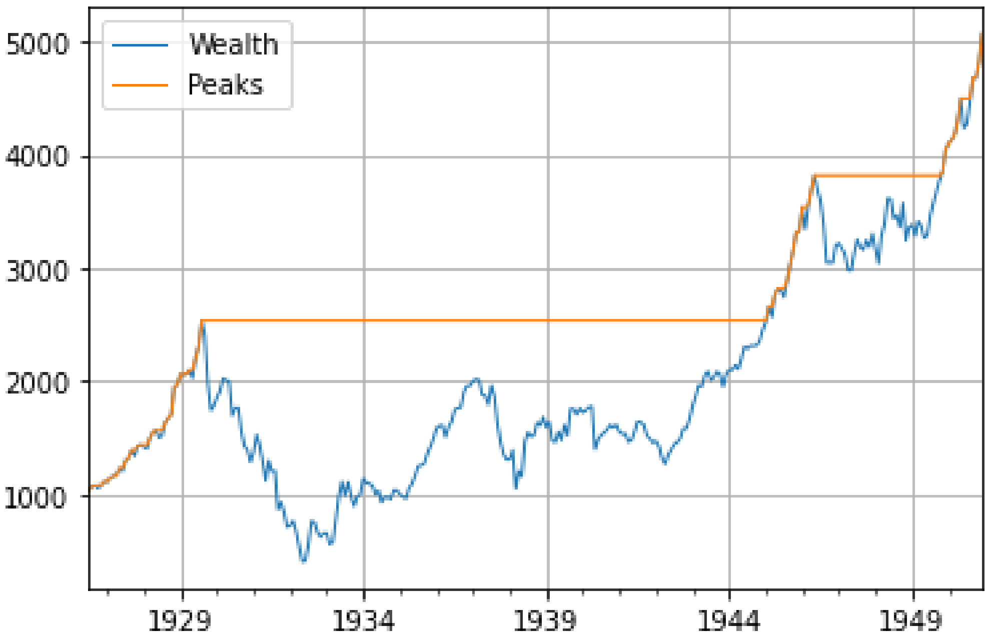
```
[ ]: largecap_drawdown[['Wealth', 'Peaks']].plot(lw=1, grid=True)
```

```
[ ]: <AxesSubplot:>
```



[ ]:
```
largecap_drawdown[:"1950"][['Wealth', 'Peaks']].plot(lw=1, grid=True)
```

[ ]:
```
<AxesSubplot:>
```



```
[ ]: smallcap_drawdown = drawdown(rets['SmallCap'])

     smallcap_drawdown.head()
```

```
[ ]:              Wealth        Peaks  Drawdown
     1926-07   985.500000   985.500000  0.000000
     1926-08  1035.957600  1035.957600  0.000000
     1926-09  1045.592006  1045.592006  0.000000
     1926-10   994.985353  1045.592006 -0.048400
     1926-11   987.224467  1045.592006 -0.055822
```

```
[ ]: smallcap_drawdown_value = smallcap_drawdown['Drawdown'].min()
     smallcap_drawdown_strtime = smallcap_drawdown['Drawdown'].idxmin().strftime('%m␣
      ↪de %Y')

     print(f"Para as SmallCaps, o máximo drawdown de todo o período foi de {100 *␣
      ↪smallcap_drawdown_value:.2f}% e ocorreu eno mês {smallcap_drawdown_strtime}.
      ↪")
```

```
Para as SmallCaps, o máximo drawdown de todo o período foi de -83.30% e ocorreu
eno mês 05 de 1932.
```

```
[ ]: smallcap_drawdown_value_75 = smallcap_drawdown["1975":]['Drawdown'].min()
     smallcap_drawdown_strtime_75 = smallcap_drawdown["1975":]['Drawdown'].idxmin().
      ↪strftime('%m de %Y')

     print(f"Para as SmallCaps, o máximo drawdown desde 1975 foi de {100 *␣
      ↪smallcap_drawdown_value_75:.2f}% e ocorreu eno mês␣
      ↪{smallcap_drawdown_strtime_75}.")
```

```
Para as SmallCaps, o máximo drawdown desde 1975 foi de -63.12% e ocorreu eno mês
02 de 2009.
```

Exemplo Brasileiro

```
[ ]: import yfinance as yf
```

```
[ ]: tickers = ['MGLU3.SA', 'PETR3.SA', 'BBDC3.SA', 'JBSS3.SA']

     prices = yf.download(tickers, '2016-01-01', '2021-12-31', progress=False)
     prices.head()
```

```
[ ]:            Adj Close                                   Close              \
                 BBDC3.SA  JBSS3.SA  MGLU3.SA  PETR3.SA   BBDC3.SA JBSS3.SA
     Date
     2016-01-04  8.173043  9.907081  0.066922  6.635111  10.452175    12.05
     2016-01-05  8.236720  9.413780  0.067593  6.466746  10.524616    11.45
     2016-01-06  8.090937  8.550508  0.061740  6.168281  10.338340    10.40
```

```
2016-01-07  7.831771  9.043809  0.053537  5.992262  10.007181    11.00
2016-01-08  7.815570  9.208241  0.053762  6.015221   9.986484    11.20

                                    High           …       Low            \
            MGLU3.SA PETR3.SA  BBDC3.SA JBSS3.SA  …  MGLU3.SA PETR3.SA
Date                                              …
2016-01-04  0.070117     8.67  10.597057    12.20  …  0.064921     8.31
2016-01-05  0.070820     8.45  10.524616    12.22  …  0.070117     8.35
2016-01-06  0.064687     8.06  10.545313    11.32  …  0.064687     8.06
2016-01-07  0.056093     7.83  10.203807    11.02  …  0.051171     7.65
2016-01-08  0.056328     7.86  10.172761    11.20  …  0.055664     7.68

                 Open                                Volume              \
             BBDC3.SA JBSS3.SA  MGLU3.SA PETR3.SA   BBDC3.SA    JBSS3.SA
Date
2016-01-04  10.452175    12.06  0.066210     8.31  3815169.0   6412800.0
2016-01-05  10.452175    12.14  0.071523     8.73  2707202.0   7522700.0
2016-01-06  10.384909    11.28  0.070820     8.22  2217092.0  15168200.0
2016-01-07  10.095145    10.33  0.062500     7.72  2768659.0  15282900.0
2016-01-08  10.100320    10.72  0.058593     8.00  2940855.0  13947300.0


               MGLU3.SA    PETR3.SA
Date
2016-01-04  138547200.0  16912500.0
2016-01-05   96179200.0   9146500.0
2016-01-06  117324800.0   9908400.0
2016-01-07  226688000.0  10777500.0
2016-01-08   97638400.0  10759800.0

[5 rows x 24 columns]
```

```
[ ]: returns = pd.DataFrame()
     returns[tickers] = prices['Adj Close'].pct_change().dropna()


     returns.head()
```

```
[ ]:            MGLU3.SA  PETR3.SA  BBDC3.SA  JBSS3.SA
     Date
     2016-01-05  0.007791 -0.049793  0.010026 -0.025375
     2016-01-06 -0.017699 -0.091703 -0.086600 -0.046154
     2016-01-07 -0.032032  0.057693 -0.132855 -0.028536
     2016-01-08 -0.002069  0.018182  0.004189  0.003831
     2016-01-11 -0.019172  0.019643  0.000000 -0.035623
```

[ ]:

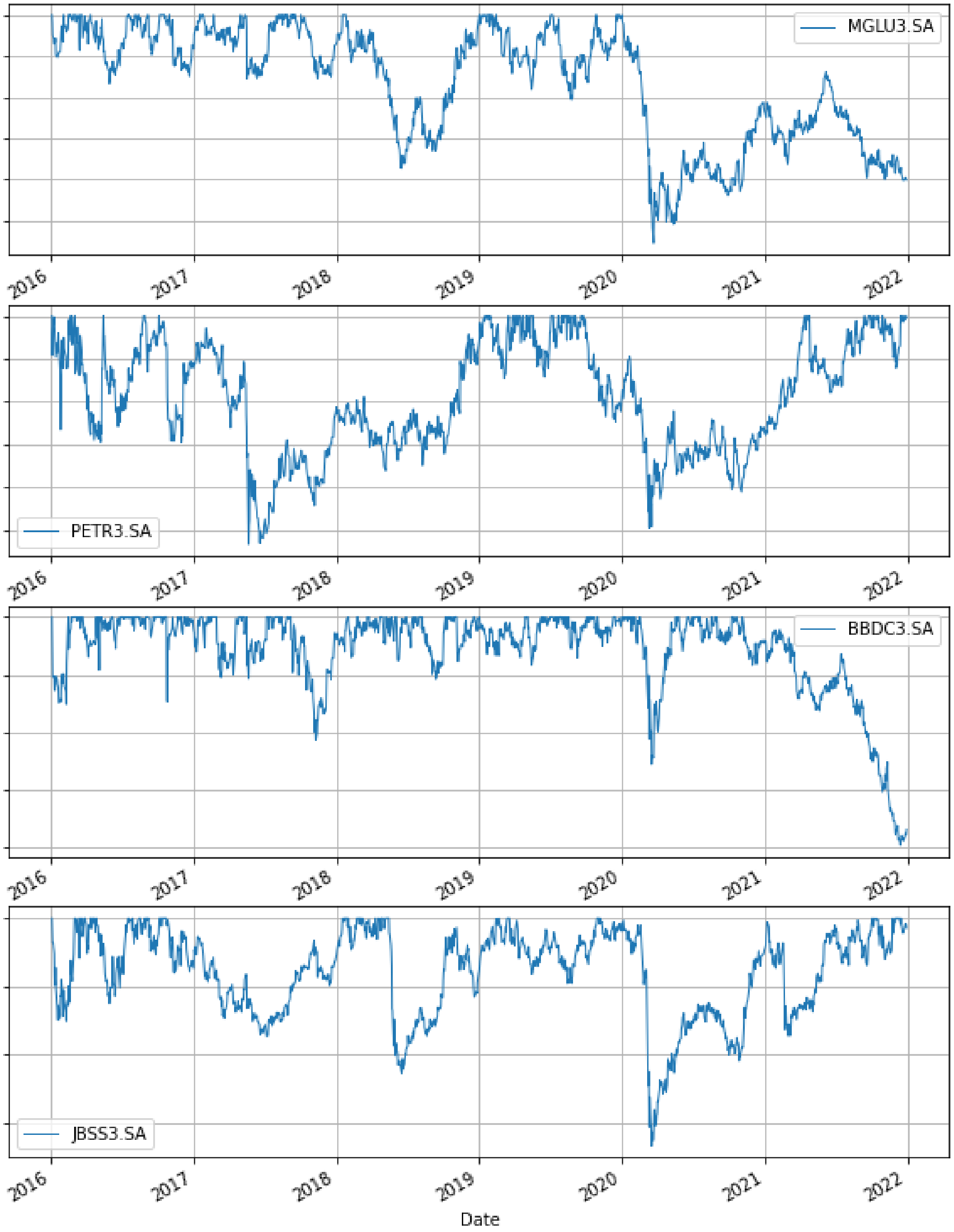
```
df = pd.DataFrame()
for ticker in tickers:
    df[ticker] = drawdown(returns[ticker])['Drawdown']

df.head()
```

[ ]:

```
            MGLU3.SA  PETR3.SA  BBDC3.SA  JBSS3.SA
Date
2016-01-05  0.000000  0.000000  0.000000  0.000000
2016-01-06 -0.017699 -0.091703 -0.086600 -0.046154
2016-01-07 -0.049164 -0.039301 -0.207950 -0.073373
2016-01-08 -0.051131 -0.021834 -0.204632 -0.069823
2016-01-11 -0.069322 -0.002620 -0.204632 -0.102959
```

[ ]:

```
import matplotlib.pyplot as plt

fig, axes = plt.subplots(nrows=4, ncols=1, figsize=(10,14))

for i in range(4):
    ticker = df.columns[i]
    df[ticker].plot(ax=axes[i], legend=ticker, grid=True, lw=1)
```


MGLU3.SA
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
PETR3.SA
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
BBDC3.SA
0.0
−0.2
−0.4
−0.6
−0.8
JBSS3.SA
0.0
−0.2
−0.4
−0.6
2016
2017
2018
2019
2020
2021
2022
Date

[ ]: