R_0 for compartmental models with homogeneous mixing (HIV)

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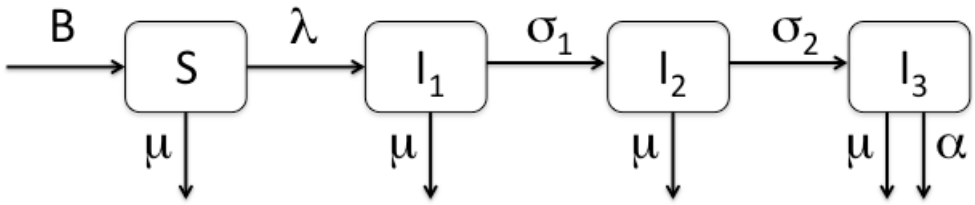
1. Model

This is a simple flow diagram for HIV infection:



S is the number of susceptible people in the population

I_1 is the number of people who are in the early stage of infection

I_2 is the number of people who are in the chronic stage of infection

I_3 is the number of people who are in the late stage of infection

$N=S+ I_1+ I_2+ I_3$ is the total population size

2. Questions

For each question select one of the answers a), b), c), or d).

Assuming that the flow diagram follows the same rules as those used in your lectures:

1. Which of these statements is false?

a. HIV cannot be cured

b. Everyone infected with HIV will die of HIV induced mortality

c. The total population size can vary during the epidemic

d. Infected individuals cannot be re-infected

b. as $\mu>0$, some individuals infected with HIV will die of "natural" mortality

2. Which of these statements is true?

a. α is the per capita death rate due to HIV infection

b. B is the rate of entry into the susceptible population

c. μ is the probability of dying due to natural mortality

d. σ_1 is the probability that individuals in the early stage of infection enter the chronic stage of infection

b. B is the rate of entry in the population. (a. is false because it is the death rate due to late HIV stage; c and d are false because μ and σ_1 are rates, not probabilities)

3. If primary infection lasts on average 3 months, and there is no natural mortality ($\mu=0$), what does this tell us about σ_1?

a. $\sigma_1 = 4$ years^{-1}

b. $\sigma_1 = 0.25$ years^{-1}

c. $\sigma_1 = 4$ years

d. $\sigma_1 = 0.25$ years

a. an average duration of 3 months corresponds to a yearly rate (noted years^{-1}) of 4

4. β_1 is the transmission rate for people in the early stage of infection, β_2 is the transmission rate for people in the chronic stage of infection, and β_3 is the transmission rate for people in the late stage of infection. What is the force of infection?

a. $\lambda = \beta_1 \frac{I_1 N}{S} + \beta_2 \frac{I_2 N}{S} + \beta_3 \frac{I_3 N}{S}$

b. $\lambda = \beta_1 \frac{I_1}{N} + \beta_2 \frac{I_2}{N} + \beta_3 \frac{I_3}{N}$

c. $\lambda = \beta_1 \frac{I_1 S}{N} + \beta_2 \frac{I_2 S}{N} + \beta_3 \frac{I_3 S}{N}$

d. $\lambda = \beta_1 \frac{I_1}{I_1+I_2+I_3} + \beta_2 \frac{I_2}{I_1+I_2+I_3} + \beta_3 \frac{I_3}{I_1+I_2+I_3}$

b. the force of infection is the rate at which susceptible individuals get infected. In a density-dependent model such as this one, it can been seen as the sum of the rates of acquiring HIV from individuals of each of the infectious compartments; each of these rates depending on a specific transmission rate and a proportion of individuals that belongs to the specific compartment.

5. What is the mean duration of chronic infection?

a. $\frac{\sigma_2}{\mu+\sigma_2}$

b. $\frac{\mu}{\mu+\sigma_2}$

c. $\frac{1}{\sigma_2}$

d. $\frac{1}{\mu+\sigma_2}$

d. this duration is the inverse of the "loss" rate which is = $\mu + \sigma_2$

6. Which of these equations describes the dynamics of the late stage of infection?

a. $\frac{dI_3}{dt} = \sigma_2 I_2 - \mu I_3 - \alpha I_3$

b. $\frac{dI_3}{dt} = \mu I_3 + \alpha I_3 - \sigma_2 I_2$

c. $\frac{dI_3}{dt} = \sigma_1 I_1 + \sigma_2 I_2 - \mu I_3 - \alpha I_3$

d. $\frac{dI_3}{dt} = \mu I_3 + \alpha I_3 - \sigma_1 I_1 - \sigma_2 I_2$

a.

$\frac{dI_3}{dt}$ represent the instantaneous change in the number of individuals in the I_3 compartment.

I_3: "In": I_2 progressing at a rate σ_2 ; "out": background (or "natural") and late HIV-specific mortality rates

7. What is the probability of entering the late stage of infection once infected?

a. 1

b. $\frac{\sigma_2}{\mu+\sigma_2}$

c. $\frac{\sigma_1}{\mu+\sigma_1}\frac{\sigma_2}{\mu+\sigma_2}$

d. $\frac{\sigma_1}{\mu+\sigma_1} + \frac{\sigma_2}{\mu+\sigma_2}$

c. As both events needs to happen, p(I_1 → I_2) AND p(I_2 → I_3), the probabilities of surviving each stage are multiplied

8. What is the R_0 of HIV in this example?

a. $\frac{\beta_1}{\mu+\sigma_1}+\frac{\beta_2}{\mu+\sigma_2}+\frac{\beta_3}{\mu+\alpha}$

b. $\frac{\beta_1}{\mu+\sigma_1}+\frac{1}{\sigma_1}\frac{\beta_2}{\mu+\sigma_2}+\frac{1}{\sigma_1}\frac{1}{\sigma_2}\frac{\beta_3}{\mu+\alpha}$

c. $\frac{\beta_1}{\mu+\sigma_1}+\frac{\sigma_1}{\mu+\sigma_1}\frac{\beta_2}{\mu+\sigma_2}+\left(\frac{\sigma_1}{\mu+\sigma_1}+\frac{\sigma_2}{\mu+\sigma_2}\right)\frac{\beta_3}{\mu+\alpha}$

d. $\frac{\beta_1}{\mu+\sigma_1}+\frac{\sigma_1}{\mu+\sigma_1}\frac{\beta_2}{\mu+\sigma_2}+\frac{\sigma_1}{\mu+\sigma_1}\frac{\sigma_2}{\mu+\sigma_2}\frac{\beta_3}{\mu+\alpha}$

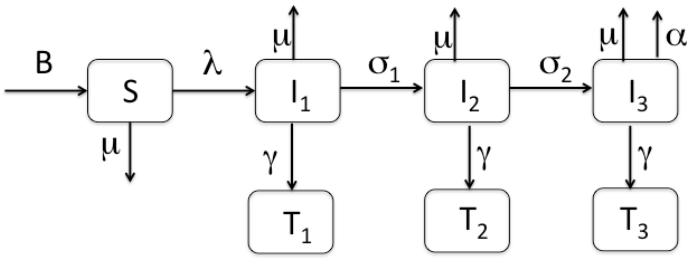
d. $R_0 = \beta D$. Here, this can be seen as:

$R_0 = R_0(\text{while } I_1) + R_0(\text{while } I_2) + R_0(\text{while } I_3)$, and we need to take into account the fact that not all infected individuals will progress into I_2 and I_3 (c.f. previous question)

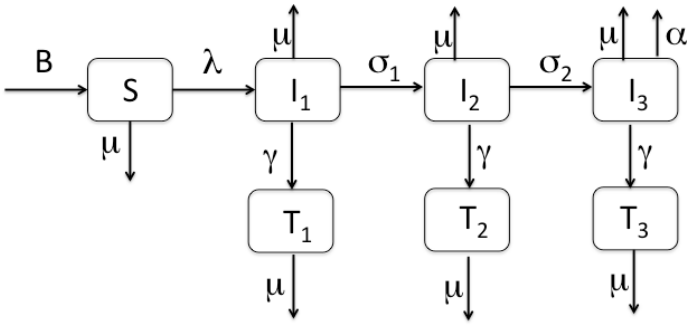
9. Suppose a 'treatment as prevention' regime is introduced. A test can detect whether individuals have HIV with 100% accuracy and all people testing positive for HIV are given antiretrovirals immediately. Treated individuals remain on treatment for life, do not progress to the next stage of infection and do not die of HIV induced mortality.

If γ is the testing rate, which of these flow diagrams represents the new regime?

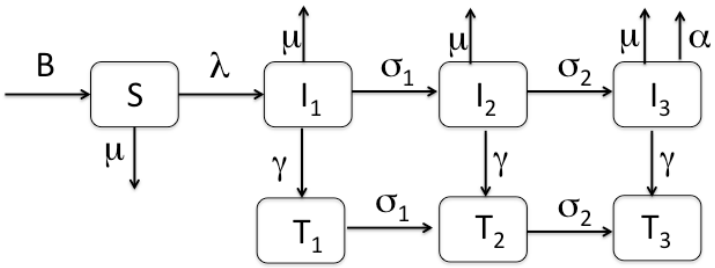
a.



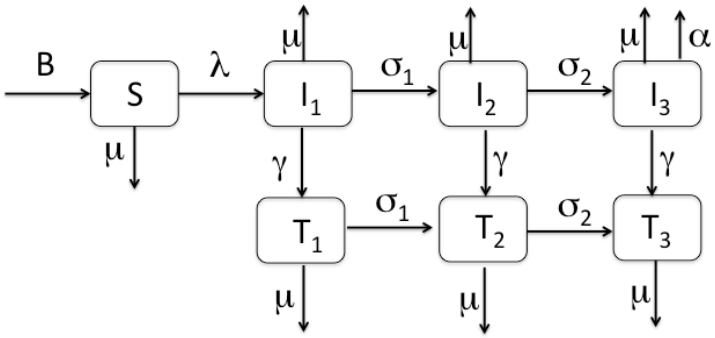
b.



c.



d.



b. Treated individuals should still die "naturally", and they don't progress to a different stage

10. If $\gamma=2$ years^{-1}, what is the average duration between infection and treatment?

a. 2 years

b. 6 months

c. 2 months

d. 20 years

b. A rate of 2 years^{-1} is equivalent to a 6 months average duration

11. Assuming treated individuals cannot transmit the virus, what is the force of infection for this new regime?

a. $\lambda = \frac{\beta_1}{\alpha}\frac{I_1}{N} + \frac{\beta_2}{\alpha}\frac{I_2}{N} + \frac{\beta_3}{\alpha}\frac{I_3}{N}$

b. $\lambda = \beta_1\frac{I_1}{N} + \beta_2\frac{I_2}{N} + \beta_3\frac{I_3}{N}$

c. $\lambda = \beta_1\frac{I_1+T_1}{N} + \beta_2\frac{I_2+T_2}{N} + \beta_3\frac{I_3+T_3}{N}$

d. $\lambda = \beta_1\frac{I_1-T_1}{N} + \beta_2\frac{I_2-T_2}{N} + \beta_3\frac{I_3-T_3}{N}$

b. Only I_1, I_2 and I_3 can transmit the infection, like in the previous model

12. What is R_0 under this model of treatment as prevention?

a. $\frac{\beta_1}{\mu+\sigma_1} + \frac{\sigma_1}{\mu+\sigma_1}\frac{\beta_2}{\mu+\sigma_2} + \left(\frac{\sigma_1}{\mu+\sigma_1} + \frac{\sigma_2}{\mu+\sigma_2}\right)\frac{\beta_3}{\mu+\alpha}$

b. $\frac{\beta_1}{\mu+\sigma_1+\gamma} + \frac{\beta_2}{\mu+\sigma_2+\gamma} + \frac{\beta_3}{\mu+\alpha+\gamma}$

c. $\frac{\beta_1}{\mu+\sigma_1+\gamma} + \frac{\sigma_1}{\mu+\sigma_1+\gamma}\frac{\beta_2}{\mu+\sigma_2+\gamma} + \left(\frac{\sigma_1}{\mu+\sigma_1+\gamma} + \frac{\sigma_2}{\mu+\sigma_2+\gamma}\right)\frac{\beta_3}{\mu+\alpha+\gamma}$

d. $\frac{\beta_1}{\mu+\sigma_1+\gamma} + \frac{\sigma_1}{\mu+\sigma_1+\gamma}\frac{\beta_2}{\mu+\sigma_2+\gamma} + \frac{\sigma_1}{\mu+\sigma_1+\gamma}\frac{\sigma_2}{\mu+\sigma_2+\gamma}\frac{\beta_3}{\mu+\alpha+\gamma}$

d. It can be seen as: $R_0 = R_0(while\ I_1) + R_0(while\ I_2) + R_0(while\ I_3)$, and we need to take into account that not all infected individuals will progress into I_2 and I_3, as some of them will die from "natural" or HIV death (c.f. previous questions), and some other will be put into treatment along the way.

13. If a new cheaper test is introduced that only detects virus in half of infected individuals, regardless of their stage of infection, at what rate will infected individuals now progress onto treatment?

a. $\gamma/2$ year^{-1}

b. 2γ year^{-1}

c. $\gamma/2$

d. 2γ

c. if only half of infected individuals are detected, then the rate of treatment initiation is divided by two. As the rate γ was already defined as a yearly rate, the answer "a." is not accurate.

14. As a policy maker, you have to make the most of your limited resources. You have 2 options that will reduce the cost of treatment as prevention by the same amount. Option 1: Introduce the new cheaper test that only detects virus in half of infected people. Option 2: Reduce the rate of testing from 2 year^{-1} to 1 year^{-1}. Which option should you use to minimise the number of new infections?

a. Option 1

b. Option 2

c. Both options will give the same outcome

d. You don't have enough information to decide

c. Both options will decrease the number of infectious individuals by the same amount:

At every instant, twice as many tests are run under Option 1 compared to under Option 2, but only half of the tests of option 1 that should have been positive will lead to treatment initiation, whereas in option 2, all tested HIV+ individuals will initiate treatment. At the end (of this instant!), the number of individuals initiating treatment will be the same under both options. In this case, the number of infectious individuals and the number of new infections will be the same under both options.

15. From a broader public health perspective, which option should you choose?

Option 2. It is best to avoid telling people they are HIV- when they are actually HIV+ (for example, it's best not to tell someone that his partner(s) is(are) not "at risk" when this person is actually HIV+)