R0 for compartmental models with homogeneous mixing (HIV)

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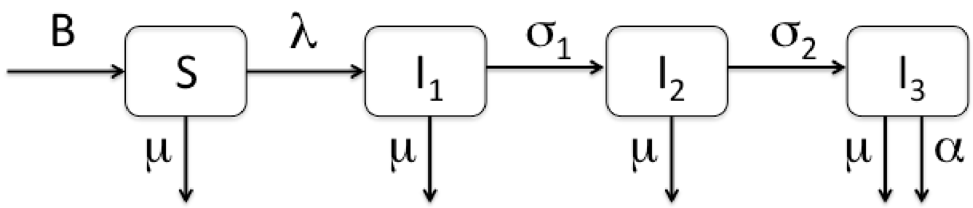
1. Model

This is a simple flow diagram for HIV infection:



S is the number of susceptible people in the population

I_1 is the number of people who are in the early stage of infection

I_2 is the number of people who are in the chronic stage of infection

I_3 is the number of people who are in the late stage of infection

$N=S+ I_1+ I_2+ I_3$ is the total population size

2. Questions

For each question select one of the answers a), b), c), or d).

Assuming that the flow diagram follows the same rules as those used in your lectures:

1. Which of these statements is false?

a. HIV cannot be cured

b. Everyone infected with HIV will die of HIV induced mortality

c. The total population size can vary during the epidemic

d. Infected individuals cannot be re-infected

2. Which of these statements is true?

a. α is the per capita death rate due to HIV infection

b. B is the rate of entry into the susceptible population

c. μ is the probability of dying due to natural mortality

d. σ_1 is the probability that individuals in the early stage of infection enter the chronic stage of infection

3. If primary infection lasts on average 3 months, and there is no natural mortality (μ=0), what does this tell us about σ_1?

a. σ_1 = 4 years^{-1}

b. σ_1 = 0.25 years^{-1}

c. σ_1 = 4 years

d. σ_1 = 0.25 years

4. β_1 is the transmission rate for people in the early stage of infection, β_2 is the transmission rate for people in the chronic stage of infection, and β_3 is the transmission rate for people in the late stage of infection. What is the force of infection?

a. $\lambda = \beta_1 \frac{I_1 N}{S} + \beta_2 \frac{I_2 N}{S} + \beta_3 \frac{I_3 N}{S}$

b. $\lambda = \beta_1 \frac{I_1}{N} + \beta_2 \frac{I_2}{N} + \beta_3 \frac{I_3}{N}$

c. $\lambda = \beta_1 \frac{I_1 S}{N} + \beta_2 \frac{I_2 S}{N} + \beta_3 \frac{I_3 S}{N}$

d. $\lambda = \beta_1 \frac{I_1}{I_1+I_2+I_3} + \beta_2 \frac{I_2}{I_1+I_2+I_3} + \beta_3 \frac{I_3}{I_1+I_2+I_3}$

5. What is the mean duration of chronic infection?

a. $\frac{\sigma_2}{\mu+\sigma_2}$

b. $\frac{\mu}{\mu+\sigma_2}$

c. $\frac{1}{\sigma_2}$

d. $\frac{1}{\mu+\sigma_2}$

6. Which of these equations describes the dynamics of the late stage of infection?

a. $\frac{dI_3}{dt} = \sigma_2 I_2 - \mu I_3 - \alpha I_3$

b. $\frac{dI_3}{dt} = \mu I_3 + \alpha I_3 - \sigma_2 I_2$

c. $\frac{dI_3}{dt} = \sigma_1 I_1 + \sigma_2 I_2 - \mu I_3 - \alpha I_3$

d. $\frac{dI_3}{dt} = \mu I_3 + \alpha I_3 - \sigma_1 I_1 - \sigma_2 I_2$

7. What is the probability of entering the late stage of infection once infected?

a. 1

b. $\frac{\sigma_2}{\mu+\sigma_2}$

c. $\frac{\sigma_1}{\mu+\sigma_1}\frac{\sigma_2}{\mu+\sigma_2}$

d. $\frac{\sigma_1}{\mu+\sigma_1} + \frac{\sigma_2}{\mu+\sigma_2}$

8. What is the R_0 of HIV in this example?

a. $\frac{\beta_1}{\mu+\sigma_1}+\frac{\beta_2}{\mu+\sigma_2}+\frac{\beta_3}{\mu+\alpha}$

b. $\frac{\beta_1}{\mu+\sigma_1}+\frac{1}{\sigma_1}\frac{\beta_2}{\mu+\sigma_2}+\frac{1}{\sigma_1}\frac{1}{\sigma_2}\frac{\beta_3}{\mu+\alpha}$

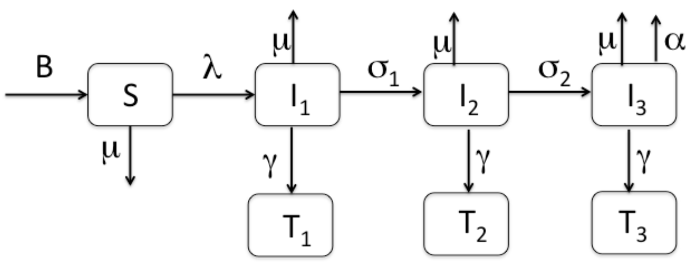
c. $\frac{\beta_1}{\mu+\sigma_1}+\frac{\sigma_1}{\mu+\sigma_1}\frac{\beta_2}{\mu+\sigma_2}+\left(\frac{\sigma_1}{\mu+\sigma_1}+\frac{\sigma_2}{\mu+\sigma_2}\right)\frac{\beta_3}{\mu+\alpha}$

d. $\frac{\beta_1}{\mu+\sigma_1}+\frac{\sigma_1}{\mu+\sigma_1}\frac{\beta_2}{\mu+\sigma_2}+\frac{\sigma_1}{\mu+\sigma_1}\frac{\sigma_2}{\mu+\sigma_2}\frac{\beta_3}{\mu+\alpha}$

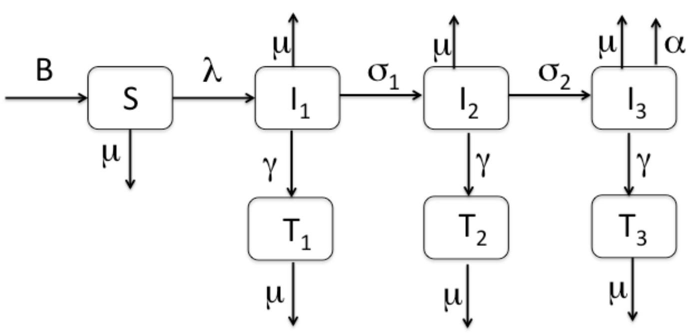
9. Suppose a 'treatment as prevention' regime is introduced. A test can detect whether individuals have HIV with 100% accuracy and all people testing positive for HIV are given antiretrovirals immediately. Treated individuals remain on treatment for life, do not progress to the next stage of infection and do not die of HIV induced mortality.

If γ is the testing rate, which of these flow diagrams represents the new regime?

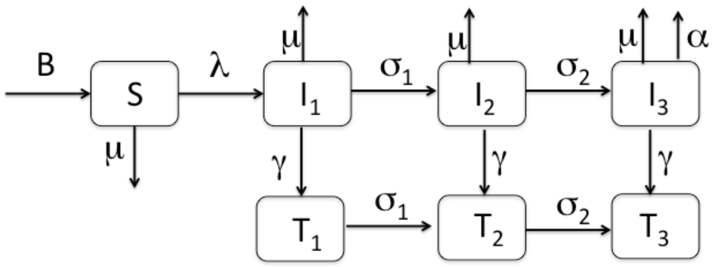
a.



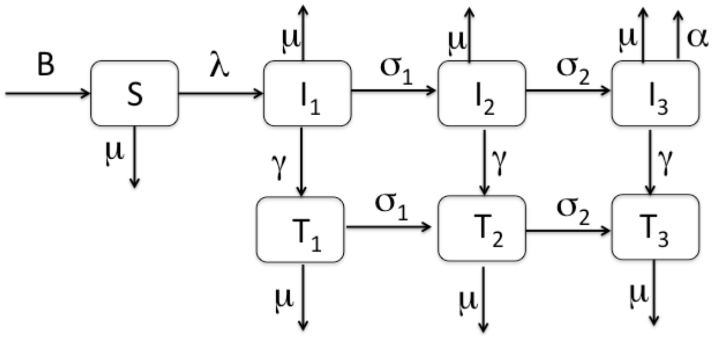
b.



c.



d.



10. If γ=2 years^{-1}, what is the average duration between infection and treatment?

a. 2 years

b. 6 months

c. 2 months

d. 20 years

11. Assuming treated individuals cannot transmit the virus, what is the force of infection for this new regime?

a. $\lambda = \frac{\beta_1}{\alpha}\frac{I_1}{N} + \frac{\beta_2}{\alpha}\frac{I_2}{N} + \frac{\beta_3}{\alpha}\frac{I_3}{N}$

b. $\lambda = \beta_1\frac{I_1}{N} + \beta_2\frac{I_2}{N} + \beta_3\frac{I_3}{N}$

c. $\lambda = \beta_1\frac{I_1+T_1}{N} + \beta_2\frac{I_2+T_2}{N} + \beta_3\frac{I_3+T_3}{N}$

d. $\lambda = \beta_1\frac{I_1-T_1}{N} + \beta_2\frac{I_2-T_2}{N} + \beta_3\frac{I_3-T_3}{N}$

12. What is R_0 under this model of treatment as prevention?

a. $\frac{\beta_1}{\mu+\sigma_1} + \frac{\sigma_1}{\mu+\sigma_1}\frac{\beta_2}{\mu+\sigma_2} + \left(\frac{\sigma_1}{\mu+\sigma_1} + \frac{\sigma_2}{\mu+\sigma_2}\right)\frac{\beta_3}{\mu+\alpha}$

b. $\frac{\beta_1}{\mu+\sigma_1+\gamma} + \frac{\beta_2}{\mu+\sigma_2+\gamma} + \frac{\beta_3}{\mu+\alpha+\gamma}$

c. $\frac{\beta_1}{\mu+\sigma_1+\gamma} + \frac{\sigma_1}{\mu+\sigma_1+\gamma}\frac{\beta_2}{\mu+\sigma_2+\gamma} + \left(\frac{\sigma_1}{\mu+\sigma_1+\gamma} + \frac{\sigma_2}{\mu+\sigma_2+\gamma}\right)\frac{\beta_3}{\mu+\alpha+\gamma}$

d. $\frac{\beta_1}{\mu+\sigma_1+\gamma} + \frac{\sigma_1}{\mu+\sigma_1+\gamma}\frac{\beta_2}{\mu+\sigma_2+\gamma} + \frac{\sigma_1}{\mu+\sigma_1+\gamma}\frac{\sigma_2}{\mu+\sigma_2+\gamma}\frac{\beta_3}{\mu+\alpha+\gamma}$

13. If a new cheaper test is introduced that only detects virus in half of infected individuals, regardless of their stage of infection, at what rate will infected individuals now progress onto treatment?

a. $\gamma/2$ year^{-1}

b. 2γ year^{-1}

c. $\gamma/2$

d. 2γ

14. As a policy maker, you have to make the most of your limited resources. You have 2 options that will reduce the cost of treatment as prevention by the same amount. Option 1: Introduce the new cheaper test that only detects virus in half of infected people. Option 2: Reduce the rate of testing from 2 year^{-1} to 1 year^{-1}. Which option should you use to minimise the number of new infections?

a. Option 1

b. Option 2

c. Both options will give the same outcome

d. You don't have enough information to decide

15. From a broader public health perspective, which option should you choose?
