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Abstract

The paper deals with the dynamic analysis of the circular vertical ground supported liquid storage tank with the aim to determine dynamic properties of a system (structure – liquid). Tanks are used to store a variety of liquids in various industrial sectors. Hence, it is a request for satisfactory performance during loadings. In addition to static loading, tanks can be subjected to dynamic loading (e.g. seismic activity). Therefore, maximum attention should be given to the tank design to cover as many factors whose consequences can cause catastrophe. In the case of dynamic response to these systems, it is necessary to know dynamic properties such as their natural frequencies and mode shapes. The paper deals with the evaluation of natural frequencies and their respective mode shapes of structure and liquid. The aim is to compare the acquired results computed by FE method (in software ANSYS Multiphysics) with the results from the analytical models of the tank with liquid and standard Eurocode 8 respectively.

Keywords: *modal analysis, liquid storage tank, spring-mass model, pendulum model, FEM, Eurocode 8.*

1. Introduction

Large capacity tanks are widely used in each sector of industry (food, petrochemical, chemical etc.). In general, their main purpose is to store a variety of liquids before the subsequent treatment or utilization. Tanks storing water are frequently used as a source of drinking water or water for irrigation in agriculture. These tanks can be used for firefighting protection as well. In addition to water, storage tanks can store other types of liquids, e.g. with toxic explosive nature or dangerous inflammable substances or they are necessary for the function of other devices. These liquids are e.g. chemicals, liquefied natural gas, oil etc.

Liquid storage tanks are important components of liquid transmission and distribution systems and should be properly designed to withstand dynamic loading. Dynamic loading can take several forms (wind loads, seismic waves, base excitation etc.). One of them, which is of interest to many researchers, is the investigation of tanks subjected to earthquakes. Insufficiently designed tanks in the past exposed to strong ground motions were disrupted with negative effects on the environment and human beings.

Satisfactory performance of storage tanks is crucial for avoiding failures in operation, minimizing tank damages and destructions with the purpose of protection from environmental disasters and losses of human lives. Therefore, maximum attention must be paid to the tank design to prevent negative consequences in the future.

In tank design, the emphasis must be laid to capture all possible causes and forms of failures. There is a variety of damages caused by strong ground motion which was described in many publications. Among them are destructions to which tanks were exposed, which were caused by the liquid and its response to the seismic activity.

There are situations when liquid oscillates in unison with the tank and from the point of dynamic analysis it occurs at a fundamental natural frequency of the system. It represents the most unfavorable response to the system which behaves like uniform cantilever flexural beam with a total mass equal to the full mass of the tank and of the liquid.

Other ruptures and accidents arise due to sloshing effects of upper part of the liquid in a tank. The sloshing liquid can damage the roof and the top of the tank wall.

The subject of this paper is to investigate dynamic properties of the system shell – liquid. In other words, the goal is to calculate natural frequencies of the system and of the liquid and determine their mode shapes of them at these frequencies. These properties of a real model of ground supported cylindrical vertical open tank containing liquid will be firstly computed numerically using software based on finite element method and consequently compared with analytical models, which were proposed by the researchers in the past, and by the European standard Eurocode 8.

2. Theory and analytical models

During a seismic action, inertial forces are induced due to the acceleration of tank structure, while hydrodynamic forces are induced due to the acceleration of liquid.

The foregoing analyses were carried out with the aim to find a solution of La Place's equation that satisfies the boundary conditions. With these known solutions, it is possible to derive satisfactory solutions by an approximate method which avoids partial-differential equations and presents solutions in a simple form. In the past, some procedures (analytical models) were proposed for determination of behavior (response) of tanks containing liquid. The widely used of all procedures is the one proposed by George W. Housner, in which the total liquid mass is divided into two zones. The hydrodynamic effects are evaluated approximately as the sum of impulsive and convective part. The impulsive part is a part representing the effects of the portion of liquid which moves in unison with the tank. This part behaves like a mass that is rigidly connected to the tank wall. The convective part represents free surface which moves against the walls and causes the sloshing effects of liquid. This part of liquid moves in long-period motion.

It should be noted that concrete and steel tanks show different behavior under a seismic action. In the case of concrete tanks, the walls are assumed as rigid whereas the steel tank walls are assumed flexible. Tank flexibility affects the hydrodynamics effects and they may be increased significantly. The wall deflection configuration does not remain the same but vary from the base to the top of the tank. In this paper, all analyses are carried out on the steel tank.

A representation of the liquid dynamics inside containers can be approximated by equivalent mechanical systems. The equivalence is taken in the sense of equal resulting forces and moments acting on the tank wall. For linear planar liquid motion, equivalent mechanical models in the form of a series of mass-spring systems or a set of simple pendulums are assumed. For nonlinear sloshing phenomena, other equivalent models such as spherical or compound pendulum were developed to emulate rotational and chaotic sloshing.

In following subchapters, these models do not take into account tank flexibility, so for this paper purposes, these representations are appropriate to determine sloshing frequencies only because the oscillations of the convective effects are dominated by natural frequencies much lower than those characterizing the impulsive effects. Therefore they cannot be affected by tank flexibility and are calculated using procedures for rigid tanks.

At the end of this chapter important terms from a simple seismic procedure for flexible tanks will be introduced in accordance to Eurocode 8.

2.1 Spring–mass modeling

The dynamic analysis can be carried out with models based on G. W. Housner's theories using the concept of generalized single-degree-of-freedom, representing impulsive and convective modes of oscillation (Fig. 1). The response to the SDOF system can be calculated independently and then combined for the purpose of the total response to the system. For practical application, only a few modes of vibration are considered in the analysis.

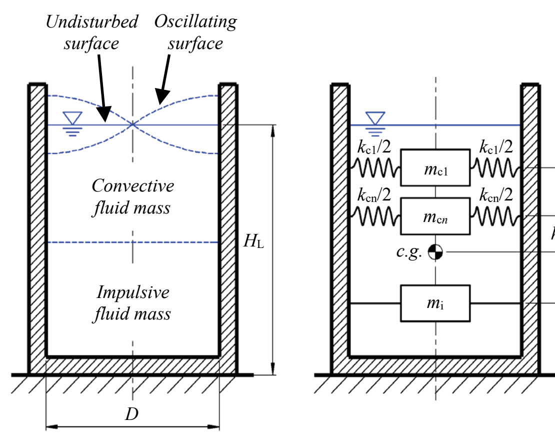

Fig. 1 Spring-mass equivalent model

An equivalent mechanical model consisting of a rigid mass m_i moving in unison with the tank and a system of masses m_{cn} representing the equivalent mass of each sloshing mode is shown in figure 1. Each modal mass m_{cn} is restrained by a spring with the stiffness k_{cn}. If damping is assumed, a dashpot with damping c_{cn} restrains together with springs each modal mass. The spring-mass model should satisfy the following conditions

$$m_f = m_i + \sum_{n=1}^{\infty} m_{cn} \tag{1}$$

The center of mass should be preserved

$$m_i h_i - \sum_{n=1}^{\infty} m_{cn} h_{cn} = 0 \tag{2}$$

Spring constant k_{cn} can be determined from the definition of natural frequencies

$$\omega_n^2 = \frac{k_{cn}}{m_{cn}} = g\frac{\xi_{1n}}{R}\tanh(\xi_{1n}\frac{H_L}{R}) \tag{3}$$

where R is a radius of a tank, H_L is a height of a free surface and ξ_{1n} are values for which the first derivative of Bessel function of the first kind and of the first order are zero.

Other term for calculation of the first natural frequency of the circular vertical ground supported tank containing liquid defined by Housner is

$$\omega_n^2 = \frac{g}{R}\sqrt{\frac{27}{8}}\tanh\left(\sqrt{\frac{27}{8}}\frac{H_{\mathrm{L}}}{R}\right) \tag{4}$$

The position of convective forces can be calculated

$$\frac{h_{cn}}{H_{\mathrm{L}}} = \frac{1}{2}\left(1 - \frac{4R\tanh(\xi_{1n}H_{\mathrm{L}}/2R)}{\xi_{1n}H_{\mathrm{L}}}\right) \tag{5}$$

$$\frac{h_{\mathrm{i}}}{H_{\mathrm{L}}} = \frac{1}{1-\sum_{n=1}^{\infty}(2R/\xi_{1n}H_{\mathrm{L}}(\xi_{1n}^2-1))\tanh(\xi_{1n}H_{\mathrm{L}}/R)}$$

$$\cdot\left[\frac{1}{2(H_{\mathrm{L}}/R)^2} - \sum_{n=1}^{\infty}\frac{\xi_{1n}\tanh(\xi_{1n}H_{\mathrm{L}}/R)+4(R/H_{\mathrm{L}}\cosh(\xi_{1n}H_{\mathrm{L}}/R))}{\xi_{1n}^2(\xi_{1n}^2-1)H_{\mathrm{L}}/R}\right] \tag{6}$$

Modal masses representing convective fluid masses can be obtained

$$\frac{m_{cn}}{m_{\mathrm{f}}} = \frac{2\mathrm{R}}{\xi_{1n}H_{\mathrm{L}}(\lambda_n^2-1)}\tanh(\xi_{1n}H_{\mathrm{L}}/R) \tag{7}$$

Impulsive fluid mass is obtained

$$\frac{m_{\mathrm{i}}}{m_{\mathrm{f}}} = 1 - \sum_{n=1}^{\infty}\frac{m_{cn}}{m_{\mathrm{f}}} \tag{8}$$

2.2 Pendulum modeling

Figure 2 presents a modeling of a tank containing liquid and the model is substituted by its equivalent – system of simple pendulums. Each of pendulums represents another mode shape of convective fluid mass. Individual pendulums consist of convective mass m_{cn} and length l_n plus a rigid mass m_{i}. The support point of the nth pendulum is at distance L_n below the undisturbed free surface and the rigid mass location is at distance L_0.

Mass preservation requires

$$m_{\mathrm{f}} = m_{\mathrm{i}} + \sum_{n=1}^{\infty} m_{cn} \tag{9}$$

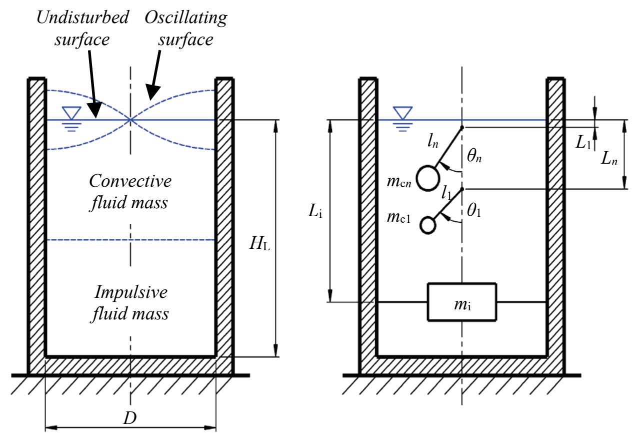

Fig. 2 Pendulum equivalent model

The natural frequency of a simple pendulum

$$\omega_n = \sqrt{\frac{g}{l_n}} \tag{10}$$

where g represents the gravitational acceleration and l_n is the length of the nth pendulum.

Equating the natural frequency of a simple pendulum to the natural frequency of the nth sloshing mode in cylindrical tanks yields the following expressions for the length of the nth pendulum

$$l_n = \frac{R}{\xi_{1n}}\tanh\left(\xi_{1n}\frac{H_{\mathrm{L}}}{R}\right) \tag{11}$$

Expressions for calculation of impulsive fluid mass and series of convective fluid mass are determined in the same manner as in the previous model based on a system of masses and springs. The nth pendulum location is to be assigned from

$$L_n = -\frac{2R}{\xi_{1n}\sinh(\xi_{1n}H_{\mathrm{L}}/R)} \tag{12}$$

Expression for impulsive fluid mass is given

$$L_{\mathrm{i}} = -\left(\frac{H_{\mathrm{L}}}{2} + \frac{1}{m_{\mathrm{i}}}\sum_{n=1}^{\infty} m_n\left(\frac{H_{\mathrm{L}}}{2} - L_n - l_n\right)\right) \tag{13}$$

It is necessary to notice that linear equivalent mechanical models provide a good conformity of the sloshing dynamics when excitation frequency is not close to the sloshing modal frequencies and excitation amplitude is relatively small. In resonance, the liquid free surface experiences complex motions and nonlinear equivalent models are necessary to develop.

2.3 Eurocode 8

Eurocode 8 (Design of structures for earthquake resistance) more precisely its fourth part (Silos, tanks and pipelines) is the European standard specifying principles and application rules for the seismic design of the structural aspects of facilities composed of above-ground and of storage tanks of different types and uses.

The tank-liquid system is modeled by two SDOF systems, where one DOF corresponds to the impulsive component moving together with the flexible wall and the other to the convective component.

Eurocode 8 specifies the natural periods of the impulsive and convective responses to flexible ground-supported circular vertical tanks as follows

$$T_{\mathrm{imp}} = C_{\mathrm{i}} \frac{\sqrt{\rho_{\mathrm{L}}} H_{\mathrm{L}}}{\sqrt{\frac{t}{R}} \sqrt{E}} \tag{14}$$

$$T_{\mathrm{con}} = C_{\mathrm{c}} \sqrt{R} \tag{15}$$

where E is Young's modulus of elasticity of a tank material, t is the equivalent uniform thickness of the tank wall, ρ_{L} is the density of the liquid. Coefficients C_{i} and C_{c} depends on the ratio of fluid height to the radius of a tank and can be found in the standard.

Terms for determination of fluid masses are the same as those for the above-mentioned equivalent models.

In addition to these natural frequencies standard specifies basic seismic characteristics like overturning moment, base shear, vertical displacement of liquid surface etc. which are not covered by this paper.

3. Finite element modeling

Analytical methods are convenient to use only for the simplest systems. Any more complex geometry can be onerous to analyze analytically, therefore, new computation methods have been developed. One of them is a finite element method – numerical method which is used to calculate responses to the structures due to the application of boundary conditions and forcing functions.

Finite element analysis (FEA) is a convenient method to compute responses to the tank containing liquid. Analytical calculation, as it was mentioned, requires finding a solution of partial differential equations. Based on many analyses, the nonviscous compressible homogenous liquid is assumed in FEA. The liquid is in FE software ANSYS modeled using acoustic elements. There are two formulations of finite elements analyzing acoustic problems: pressure and displacement. The most used finite element is the pressure-formulated element and is described in the following section.

In the analysis of tank-liquid systems, it is necessary to couple structural elements with acoustic elements at the interface. The equations related to the structural dynamics are considered along with the mathematical description of acoustics of a system, given by the Navier–Stokes equations of fluid momentum and the flow continuity equation. The fluid momentum and continuity equations are simplified to form the acoustic wave equation, which is used to describe the acoustic response to the fluid.

3.1 Pressure-formulated acoustic elements

The acoustic pressure p within a finite element is defined as

$$p = \sum_{i=1}^{m} N_i p_i \tag{16}$$

where N_{i} is a set of linear shape functions, p_{i} are acoustic nodal pressures at node i and m is the number of nodes forming the element. The finite element equation for the fluid in the matrix form for the pressure-formulated acoustic elements is

$$\mathbf{M_f} \, \ddot{\mathbf{p}} + \mathbf{K_f} \, \mathbf{p} = \mathbf{f_f} \tag{17}$$

where $\mathbf{M_f}$ is the fluid mass matrix, $\mathbf{K_f}$ is the fluid stiffness matrix, $\mathbf{f_f}$ is a vector of applied fluid loads, $\mathbf{u}$ is a vector of unknown nodal displacements and $\ddot{\mathbf{u}}$ is a vector of the second derivative of displacements with respect to time (acceleration of the nodes).

3.2 Fluid-structure interaction

This section describes the matrix equations for the coupled fluid–structure interaction problem. The equations of motion for the structure are

$$\mathbf{M_s} \, \ddot{\mathbf{u}} + \mathbf{K_s} \, \mathbf{u} = \mathbf{f_s} \tag{18}$$

where $\mathbf{M_s}$ is the structural mass matrix, $\mathbf{K_s}$ is the structural stiffness matrix, $\mathbf{f_s}$ is a vector of applied structural loads, $\mathbf{p}$ is a vector of unknown nodal acoustic pressures and $\ddot{\mathbf{p}}$ is a vector of the second derivative of acoustic pressure with respect to time.

The fluid-structure interaction occurs at the interface between the structure and the fluid (acoustic elements). Acoustic pressure exerts a force on the structure and motion of the structure produces a pressure.

To assume the interaction between the structure and the fluid, additional terms are added to the equations of motion and their matrix form can be written as follows

$$\begin{bmatrix} \mathbf{M_s} & 0 \\ \rho_L \mathbf{R}^T & \mathbf{M_f} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C_s} & 0 \\ 0 & \mathbf{C_f} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K_s} & -\mathbf{R} \\ 0 & \mathbf{K_f} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{f_s} \\ \mathbf{f_f} \end{bmatrix} \tag{19}$$

where $\mathbf{R}$ stands for the coupling matrix, $\boldsymbol{\rho}_\mathbf{L}$ is the density of fluid, $\mathbf{C_s}$ represents structural damping matrix and $\mathbf{C_f}$ is acoustic damping matrix.

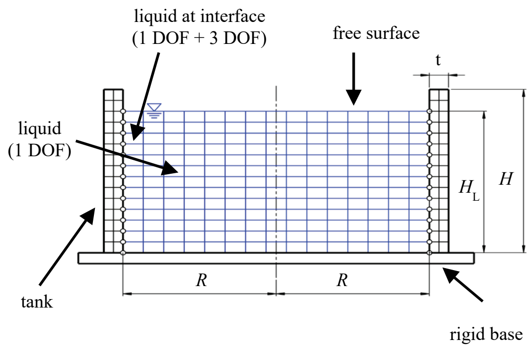

Fig. 3 Scheme of a finite element model with fluid–structure interaction

Figure 3 shows a FE model of the tank containing liquid. Elements representing the liquid (acoustic elements) which are not in contact with tank walls have only 1 degree of freedom (pressure). Structural elements representing tank have displacement degrees of freedom. At the interface between the acoustic fluid (representing liquid) and the structure is a single layer of acoustic elements that have pressure and displacement DOFs.

When using ANSYS it is necessary to define which surface of the structure and the fluid are in contact by using the fluid-structure interface (FSI) flag.

4. Dynamic analysis of liquid storage tank

This section deals with the performance of dynamic analysis to determine dynamic properties of a model of a liquid storage tank. Dynamic properties (natural frequencies and mode shapes) are important to know because of possible resonance formation during dynamic loading, e.g. earthquake and of dynamic stiffness of a system.

The analysis will be carried out on the model of the open ground-supported cylindrical vertical flexible tank with liquid using software based on finite element method. Consequently, the aim is to compare the results obtained from the analysis with results of analytical simplified models and the standard Eurocode 8.

4.1 Model of liquid-storage tank

A model of a ground supported circular steel tank is assumed which is fully anchored to the ground. The design parameters of the tank are defined as follows: radius of the tank R = 2 m, height H = 2 m and thickness of the wall and the bottom t = 0,05 m. Thickness is assumed to be constant along the tank height and the bottom. Figure 4 shows a spatial and planar (cross-sectional) model for which dynamic properties will be determined.

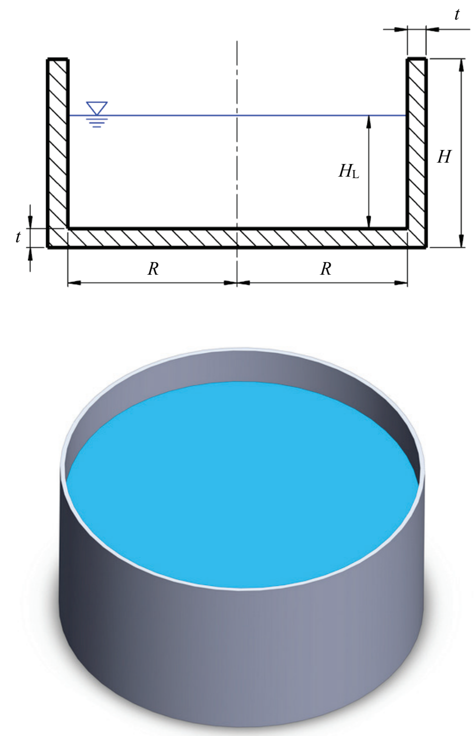

Fig. 4 Planar and spatial representation of tank

The material of the tank is assumed to be made from steel (Young's modulus of elasticity E = 2,1e11 Pa, density ρ_s = 7 850 kgm^{-3}, Poisson's ratio ν = 0,3). Tank contains liquid – water (sonic velocity c = 1482 ms^{-1}, density ρ_L = 1 000 kgm^{-3}). In this case, the free surface height of liquid H_L is equal to the tank height H, so a full tank is assumed.

4.2 FE model of liquid-storage tank and boundary conditions

This section describes modeling of the tank and liquid in FE software ANSYS Multiphysics. The one way is to generate a model in CAD software and import geometry to ANSYS. This option is convenient when the difficult geometry is considered. The second option is to model the tank containing liquid direct in ANSYS using keypoints, lines, areas, nodes etc.

After tank modeling, element types are introduced for model discretization. In analysis, structural finite element SHELL181 for the tank and acoustic pressure-based FLUID30 for the liquid are used.

As it was mentioned in section 3, it is necessary to form a coupling at the interface (fluid-structure interaction). It is a special coupling of some movable or deformable structure with an internal or surrounding fluid. To extract natural frequencies and to expand mode shapes modal analysis must be performed. As we can see in Eq. (19), there are unsymmetrical matrices in fluid-structure interaction formulation. Therefore, unsymmetrical solver is needed to set in analysis settings.

In addition to applying fluid-structure interaction flag (represented by red color in Fig. 5), other boundary conditions are applied to model – fixed displacements at the tank bottom (full anchorage to a rigid foundation) and zero pressure at the free surface. These analysis settings are considered when extracting natural frequencies and expanding mode shapes of the tank with liquid.

In the case of modal analysis of the convective fluid mass, sloshing effect must be considered. It means applying a gravitational effect and free surface flag to FE model.

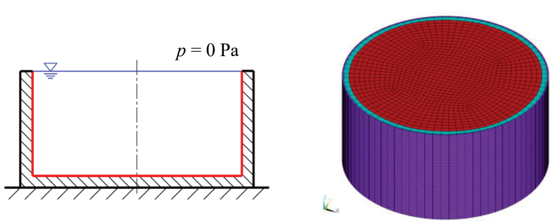

Fig. 5 FE model of liquid storage tank and boundary conditions

4.4 Results of Modal analysis

To perform a modal analysis with the purpose to determine dynamic properties of the tank containing liquid, FE model (Fig. 5) was used.

Figure 6 shows the first 4 mode shapes – responses of the system at their natural frequencies. It is a representation of system behavior during dynamic loading with excitation frequency equal to the natural frequency.

In table 1, there is a comparison of extracted natural frequencies for cases of the full and empty tank. As it can be seen, with total mass increasing, natural frequencies are decreasing.

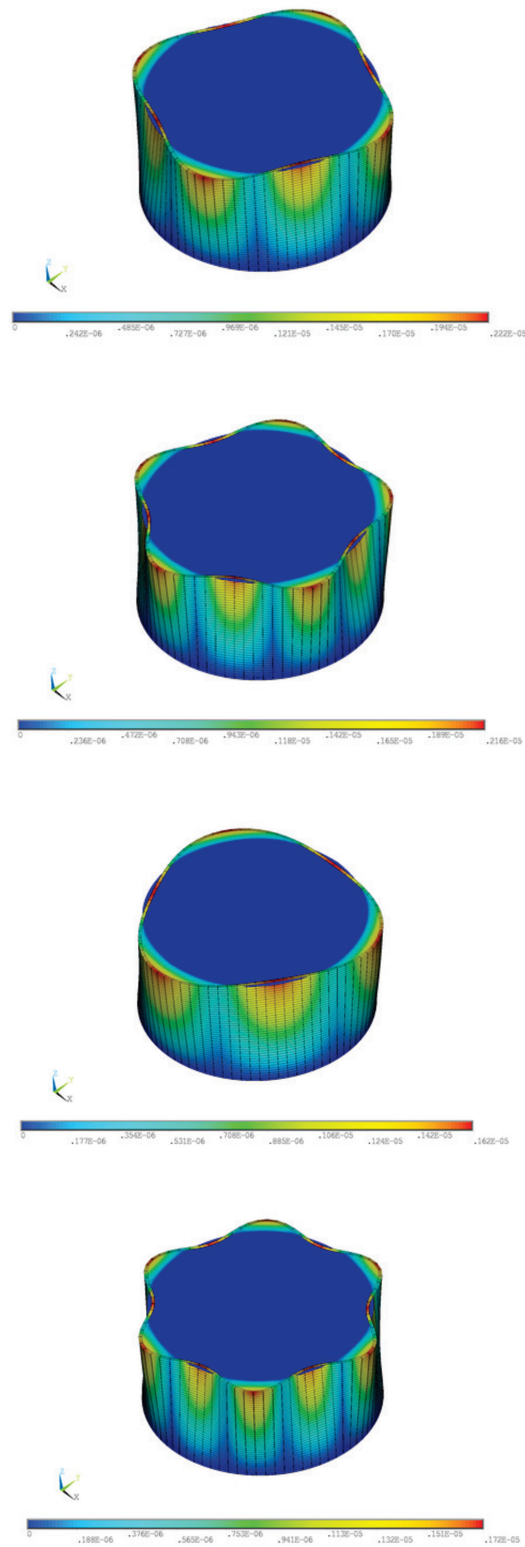

Fig. 6 Mode shapes of liquid storage tank containing water

Tab. 1 Comparison of natural frequencies of full and empty tank

	Full tank	**Empty tank**
1.	63,765 Hz	84,537 Hz
2.	72,901 Hz	93,921 Hz
3.	73,567 Hz	101,51 Hz
4.	96,479 Hz	121,66 Hz
5.	101,79 Hz	149,15 Hz

In the introductory section, the most unfavorable response of a tank-liquid system was described. It is a situation when liquid oscillates in unison with the tank. This mode shape appertains to the first impulsive frequency. Using seismic analysis presented in Eurocode 8 and using Eq. (14), the impulsive frequency of the flexible tank system was computed. This response (Fig. 7) occurs at frequency 180,13 Hz (in ANSYS 161,813 Hz).

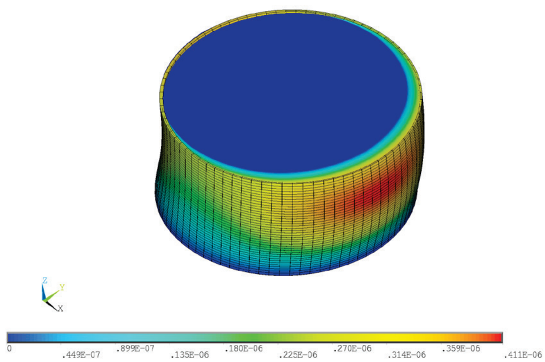

Fig. 7 Mode shape of liquid storage tank filled containing water at the first impulsive frequency

The second task was to extract natural frequencies of the upper part of a fluid mass at which sloshing effects occur. Sloshing of the convective fluid mass must be taken into account in the tank design and using seismic analysis (Eurocode 8) maximum vertical wave displacement be calculated. If this fact is neglected, liquid spilling can be caused by dynamic loading.

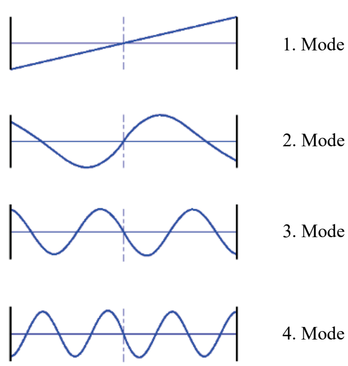

Fig. 8 Theoretical antisymmetric mode shapes of convective fluid mass

Sloshing effects are described by infinite antisymmetric modes shapes. Figure 8 shows first 4 theoretical mode shapes of convective fluid mass.

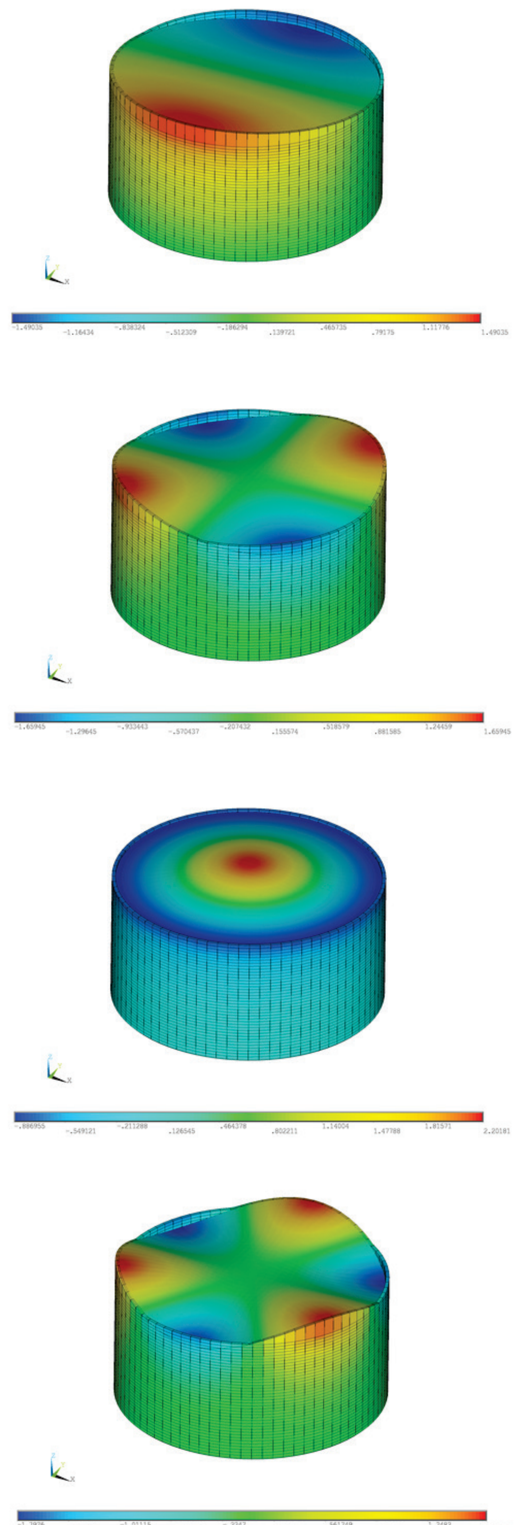

Fig. 9 Mode shapes of convective fluid mass

Responses to the convective fluid mass showed in figure 8 are considered when investigating cross-sectional model. Using the spatial model, ANSYS expands besides planar mode shapes (Fig. 8) spatial mode shapes as well (Fig. 9). Table 2 presents natural frequencies at which mentioned spatial mode shapes occur.

Tab. 2 Natural frequencies of convective fluid mass

	Full Tank
1.	0,46664 Hz
2.	0,61515 Hz
3.	0,69057 Hz
4.	0,72343 Hz
5.	0,81476 Hz

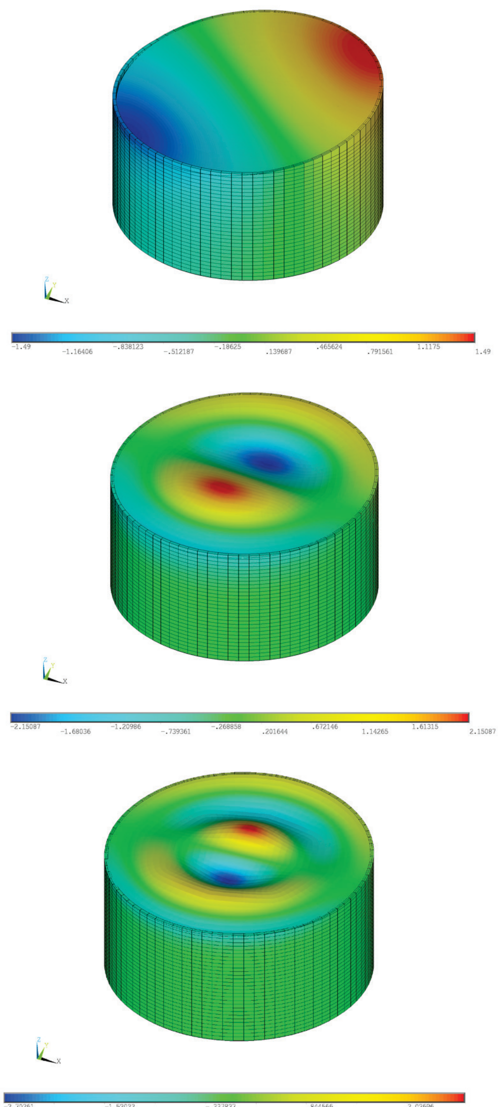

Fig. 10 Mode shapes of convective fluid mass in according to the theory

Figure 10 shows eigenmodes of the convective fluid mass which are in a good conformity with the planar mode shapes showed in figure 8. These mode shapes occur at frequencies listed in table 3 and are obtained by applying proposed theories (spring-mass model, pendulum model and Eurocode 8).

In table 3, there is a comparison of the results obtained by the software ANSYS with the theory from Eurocode 8 and the pendulum model. Eurocode 8 procedure is based on a spring-mass model, therefore, natural frequencies obtained using both approaches are the same.

Tab. 3 Comparison of natural frequencies of convective fluid mass

	ANSYS	**Eurocode 8**	**Pendulum model**
1.	0,46664 Hz	0,4664 Hz	0,4905 Hz
2.	0,81551 Hz	0,8138 Hz	0,8139 Hz
3.	1,03509 Hz	1,0298 Hz	1,0298 Hz

The first natural frequency estimated from Housner's theory represented by Eq. (4) is 0,4658 Hz.

A good conformity of results can be stated when following the results from the FE model compared with analytical models and principles.

5. Conclusions

The aim of this paper was to estimate dynamic properties of the model of the liquid storage tank with liquid (water), namely natural frequencies and mode shapes. These properties are crucial when resonance is investigated during dynamic loading. From the results obtained for liquid storage tank, it is clear that the tank flexibility and presence of liquid influence total response to this system.

For the tank with liquid subjected to the ground motion, it is essential to estimate response to the convective fluid mass due to the sloshing effects. In cases of open tanks and tanks with the roof, it is necessary to determine the sloshing wave height of this fluid mass in the tank design and assume this height above the free surface to prevent fluid spilling.

To sum up following results, a good validity of models proposed in publications and standards can be stated.

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Miloš Musil (*1962) has completed academic study at Slovak University of Technology, Faculty of Mechanical Engineering with honours (Ing.) in study programme Applied Mechanics (1986) and was awarded the Rector's price. Doctorate study was graduated in Applied Mechanics (1994) and he was granted with academic title Ph.D. From 2001 until now he is an Associate Professor at Institute of Applied Mechanics and Mechatronics at Faculty of Mechanical Engineering of the Slovak University of Technology in Bratislava. His work is dedicated to scope of dynamics of machines, their passive and active vibroinsulation, experimental identification of parameters and correction of mathematical models of mechanical and mechatronic system applied to evaluation of seismic resistance of structures, rotor dynamics, instability analysis of break squealing of vehicles or airplane wings, defect detection of mechanical systems or passive or active semiactive vehicle suspension. At the university, he has lectured subjects like Modeling and analyses of mechanical systems, Random vibration, Experimental methods in mechanics and mechatronics, Vibroinsulation and vibrodiagnostics. He is an author of various scientific papers (approx. 70) dedicated to mentioned scopes, which were published in national or international conferences. He is an author and coauthor of 2 monographs (Active and semiactive vehicle suspension, Design of semiactive damper in vehicle suspension considering the tire lift off) and 3 textbooks (Basics of machines dynamics using Matlab, Passive and active vibroinsulation of machines and Random vibration). He was a responsible solutionist of various scientific projects in the previous term. Projects such as Safety increase of equipment in nuclear power plants during seismic activity (funded by the EU structural funds), Influence of thermal loading on dynamics of automotive disc brakes and squeal noise generation (cooperation with the University of Lila in France), Failure detection of machine structures (cooperation with Virginia Polytechnic Institute and State University in USA) belong to his most remarkable projects.

Martin Sivý (*1989) has completed levels of higher education with honours at Slovak University of Technology, Faculty of Mechanical Engineering as follows: bachelor level (Bc.) in study programme Production Systems and Quality Management (2012), master level (Ing.) in study programme Applied Mechanics (2014). On the present, he is a student at doctorate study in study programme Applied Mechanics.