Algorithms & Data Structures Notes - SoSe 24

Igor Dimitrov

2024-04-22

Table of contents

Preface

This is a Quarto book.

To learn more about Quarto books visit https://quarto.org/docs/books.

Part I

Introduction

1 Program Run-time Analysis

1.1 Reccurence Relations

Consider a very simple reccurence relation:

$$T(n) := \begin{cases} 1 & n = 1 \\ n + T(n-1), & n > 1 \end{cases}$$

With **mathematical induction** we can formally show that $T(n)$ is quadratic. But there is a simpler & more intuitive way:

$$\begin{aligned}
T(n) &= n + T(n-1) && \text{(Def } T(\cdot)\text{)} \\
&= n + n - 1 + T(n-2) \\
&= \ldots && \text{(Repeat } n-2 \text{ times)} \\
&= n + n - 1 + \cdots + T(1) \\
&= n + n - 1 + \cdots + 1 && \text{(Def } T(1)\text{)} \\
&= \frac{n(n+1)}{2} && \text{(Gauss)} \\
&\in \mathcal{O}(n^2)
\end{aligned}$$

This method can be applied to the more complex divide-and-conquer reccurence relation from the lecture:

$$R(n) := \begin{cases} a, & n = 1 \\ c\dot{n} + d \cdot R(\frac{n}{b}), & n > 1 \end{cases}$$

Applying the above method we expand $R(\cdot)$ repetitively according to its definition until we reach the base case, rearranging terms when necessary:

$$
\begin{aligned}
R(n) &= c \cdot n + d \cdot R(\frac{n}{b}) && \text{(Def } R(\cdot)\text{)} \\
&= c \cdot n + d(c\frac{n}{b} + d \cdot R(\frac{n}{b^2})) \\
&= c \cdot n + d\Big(c\frac{n}{b} + d \cdot \big(c \cdot \frac{n}{b^2} + d \cdot R(\frac{n}{b^2})\big)\Big) \\
&= c \cdot n + d \cdot c\frac{n}{b} + d^2 c\frac{n}{b^2} + d^3 \cdot R(\frac{n}{b^3}) && \text{(Rearrange)} \\
&= c \cdot n \left(1 + \frac{d}{b} + \frac{d^2}{b^2}\right) + d^3 \cdot R(\frac{n}{b^3}) \\
&= \ldots && \text{(Repeat } k\text{-times)} \\
&= c \cdot n \left(1 + \frac{d}{b} + \cdots + \frac{d^{k-1}}{b^{k-1}}\right) + d^k \cdot R(\frac{n}{b^k}) \\
&= c \cdot n \sum_{i=0}^{k-1} \left(\frac{d}{b}\right)^i + d^k \cdot R(\frac{n}{b^k}) \\
&= c \cdot n \sum_{i=0}^{k-1} \left(\frac{d}{b}\right)^i + d^k \cdot R(1) && \text{(Ass } \tfrac{n}{b^k} = 1\text{)} \\
&= c \cdot n \sum_{i=0}^{k-1} \left(\frac{d}{b}\right)^i + a \cdot d^k && \text{(Def } R(1)\text{)}
\end{aligned}
$$

See lecture slides for the complexity analysis of final expression.

Part II

Data Structures

2 Lists

2.1 Sequences as Arrays and Lists

Many terms for same thing: sequence, field, list, stack, string, **file**… Yes, files are simply sequences of bytes!

three views on lists:

- **abstract**: $(2, 3, 5, 7)$
- **functionality**: stack, queue, etc… What operations does it support?
- **representation**: How is the list represented in a given programming model/language/paradigm?

2.2 Applications of Lists

- Storing and processing any kinds of data
- Concrete representation of abstract data types such as: set, graph, etc…

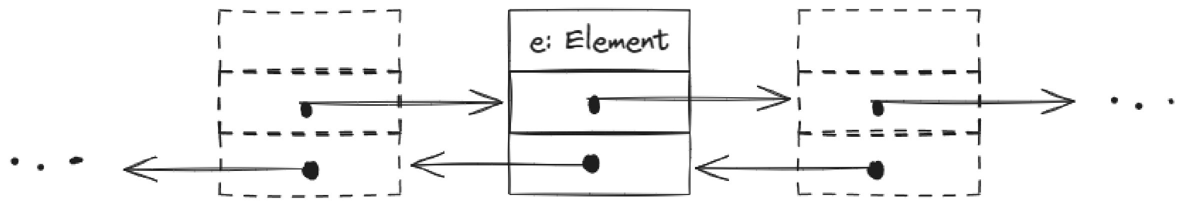
2.3 Linked and Doubly Linked Lists

	simply linked	doubly linked
lecture	`SList`	`List`
c++	`std::forward_list`	`std::list`

Doubly linked lists are usually **simpler** and require "only" double the space at most. Therefore their use is more widespread.

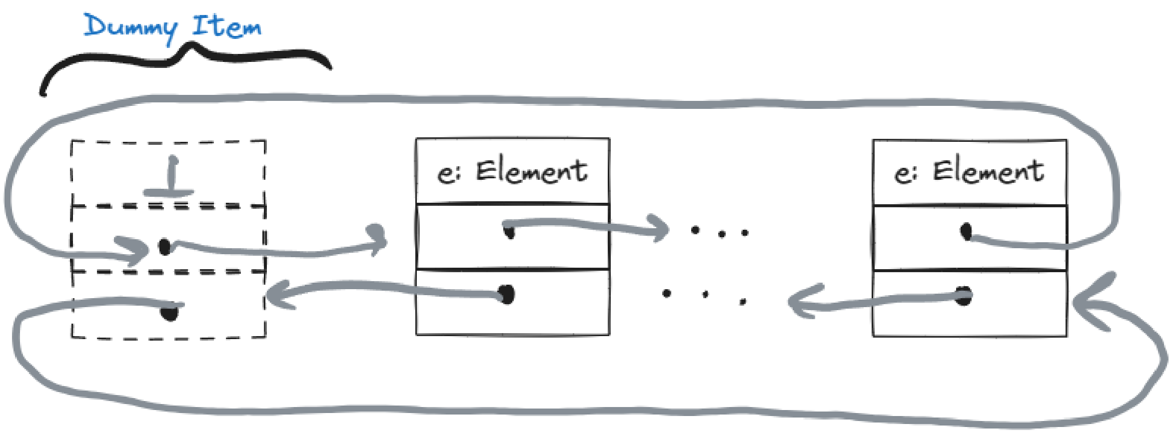
2.3.1 List Items

```
Class Item of T :=
    e: T
    next: *Item //Pointer to Item
    prev: *Item //Pointer to Item
    invariant next->prev = this = prev->next
```



Problem: * predeccessor of first list element? * successor of last list element?

Solution: Dummy Item with an empty data field as follows:



Advatanges of this solution:

- **Invariant** is always satisfied
- Exceptions are avoided, thus making the coding more:
  - simple
  - readable
  - faster
  - elegant

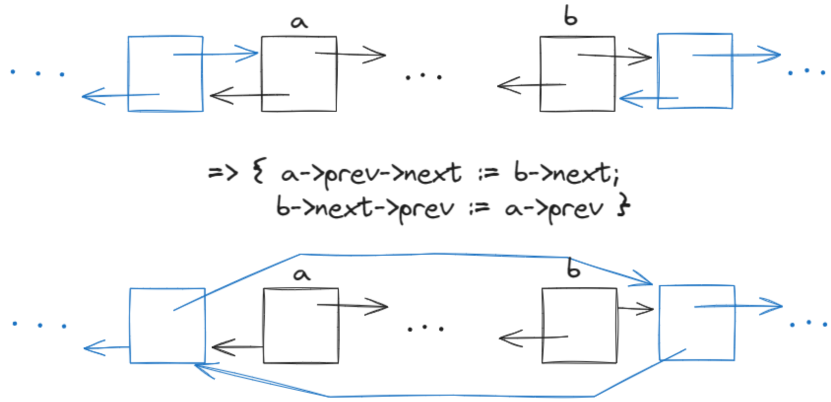
Disadvantages: a little more storage space.

2.3.2 The List Class

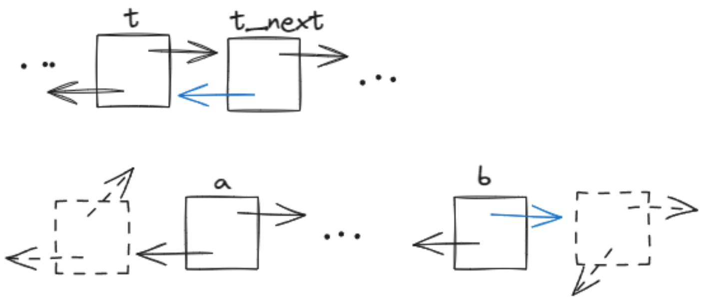
```
Class List of T :=
    dummy := (
        Null : T
        &dummy : *T // initially list is empty, therefore next points to the dummy itself
        &dummy : *T // initially list is empty, therefore prev points to the dummy itself
    ) : Item

    // returns the address of the dummy, which represents the head of the list
    Function head() : *Item :=
        return address of dummy

    // simple access functions
    // returns true iff list empty
    Function is_empty() : Bool :=
        return dummy.next == dummy

    // returns pointer to first Item of the list, given list is not empty
    Function first() : *Item :=
        assert (not is_empty())
        return dummy.next

    // returns pointer to last Item of the list, given list is not empty
    Function last() : *Item :=
        assert (not is_empty())
        return dummy.last

    /* Splice is an all-purpose tool to cut out parts from a list
       Cut out (a, ... b) form this list and insert after t */
    Procedure splice(a, b, t : *Item) :=
        assert (
            b is not before a
            and
            t not between a and b
        )
        // Cut out (a, ... , b)
        a->prev->next := b->next
        b->next->prev := a->prev

        // insert (a, ... b) after t
```

```
t->next->prev := b
b->next := t->next
t->next := a
a->prev := t
```

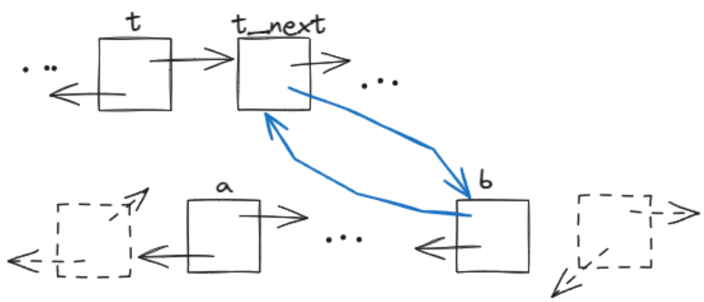
- Dlist cut-out $(a, ..., b)$:



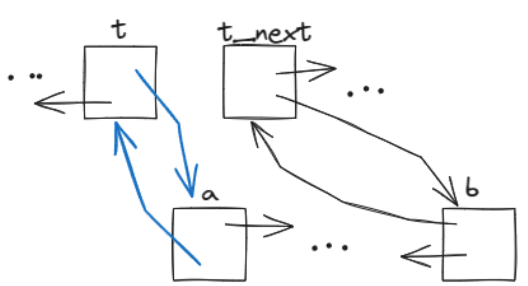
- Dlist insert $(a, ..., b)$ after t:



=>{ t->next->prev := b;
b->next := t->next }



=> {t->next := a
a->prev := t }



==