Cálculo III - Atividade 7

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1- $z = \sqrt{20 - x^2 - 7y^2}$, ponto $(2,1,3)$

$$z = f(x_0, y_0) + f_x(x_0, y_0).(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$f_x = \frac{-x}{\sqrt{20 - x^2 - 7y^2}} \qquad f_y = \frac{-7y}{\sqrt{20 - x^2 - 7y^2}}$$

$$f(2,1) = \sqrt{9} = 3$$

$$f_x(2,1) = -\frac{2}{3}$$

$$f_y(2,1) = -\frac{7}{3}$$

$$L(x,y) = 3 + \left(\frac{-2}{3}\right)(x-2) + \left(\frac{-7}{3}\right)(y-1)$$

$$L(x,y) = 3 - \frac{2}{3}x + \frac{4}{3} - \frac{7}{3}y + \frac{7}{3}$$

$$L(x,y) = -\frac{2}{3}x - \frac{7}{3}y + \frac{20}{3}$$

$$L(x,y) = -2x - 7y + 20$$

2- $P = \frac{V^2}{R}$ $\quad P_V = 2\frac{V}{R}$ $\quad P_R = \frac{-V^2}{R^2}$ $\quad dV = -0{,}001$

$dR = 0{,}02$

$$dP = \left(2\frac{V}{R}\right).dV + \left(-\frac{V^2}{R^2}\right)dR$$

$$dP = \left(2\frac{V}{R}\right).(-0{,}001) + \left(-\frac{V^2}{R^2}\right)(0{,}02)$$

$$dP = \left(\frac{2 \cdot 120}{12}\right)(-0{,}001) + \left(-\frac{120^2}{12^2}\right)(0{,}02)$$

$$dP = -0{,}02 + (-2) = -2{,}02$$

3- $f(x,y) = \dfrac{x}{y} + e^{xy}$, $x(t) = \dfrac{1}{t}$ e $y(t) = \sqrt{t}$. $h(t) = f(x(t), y(t))$

$\dfrac{dh}{dt} = ?$

$\dfrac{\partial f}{\partial x} = \dfrac{1}{y} + ye^{xy}$ $\dfrac{\partial x(t)}{\partial t} = -\dfrac{1}{t^2}$

$\dfrac{\partial f}{\partial y} = -\dfrac{x}{y^2} + xe^{xy}$ $\dfrac{\partial y(t)}{\partial t} = \dfrac{1}{2\sqrt{t}}$

$$\frac{dh}{dt} = \left(\frac{1}{y} + ye^{xy}\right)\left(-\frac{1}{t^2}\right) + \left(-\frac{x}{y^2} + xe^{xy}\right)\left(\frac{1}{2\sqrt{t}}\right)$$

$$\frac{dh}{dt} = \left(\frac{1}{\sqrt{t}} + \sqrt{t}\, e^{\frac{1}{t}\sqrt{t}}\right)\left(-\frac{1}{t^2}\right) + \left(\frac{-1/t}{(\sqrt{t})^2} + \sqrt{t}\, e^{\frac{1}{t}\sqrt{t}}\right)\left(\frac{1}{2\sqrt{t}}\right)$$

$$\frac{dh}{dt} = \left(\frac{1 + te^{\sqrt{t}/t}}{\sqrt{t}}\right)\left(-\frac{1}{t^2}\right) + \left(\frac{-1 + t^{5/2}e^{\sqrt{t}/t}}{t^2}\right)\left(\frac{1}{2\sqrt{t}}\right)$$

$$\frac{dh}{dt} = \frac{-1 - te^{\sqrt{t}/t}}{t^{5/2}} + \frac{-1 + t^{5/2}e^{\sqrt{t}/t}}{2t^{5/2}} \quad //$$

4- $h(t) = f(e^{2t}, \cos t)$

a) $h'(t) = ?$

$\dfrac{\partial x(t)}{\partial t} = 2e^{2t}$ $\dfrac{\partial y(t)}{\partial t} = -\operatorname{sen} t$

$$h'(t) = \frac{\partial f}{\partial x} \cdot \frac{\partial x(t)}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y(t)}{\partial t}$$

$$h'(t) = \frac{\partial f}{\partial x} \cdot 2e^{2t} + \frac{\partial f}{\partial y} \cdot (-\operatorname{sen} t) \quad //$$

b) $f_x(e^{2\pi}, -1) = \dfrac{1}{e^{2\pi}}$, $h'(\pi) = ?$

$$h'(\pi) = \frac{\partial f}{\partial x} \cdot 2e^{2\pi} + \cancel{\frac{\partial f}{\partial y} \cdot (-\operatorname{sen} \pi)} \to 0$$

$$h'(\pi) = \frac{1}{\cancel{e^{2\pi}}} \cdot 2\cancel{e^{2\pi}} = 2 \,//$$