Atividade 4 - Álgebra Linear

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1- a) $B = \{(1,0,1), (0,1,0), (1,2,0)\}$ e $V = \mathbb{R}^3$

i) é LI pois: $a_1(1,0,1) + a_2(0,1,0) + a_3(1,2,0) = (0,0,0)$

$$\Rightarrow \begin{cases} a_1 + a_3 = 0 \rightarrow 0 + a_3 = 0 \rightarrow a_3 = 0 \\ a_2 + 2a_3 = 0 \rightarrow a_2 + 2.0 = 0 \rightarrow a_2 = 0 \\ a_1 = 0 \end{cases}$$

ii) $[(1,0,1), (0,1,0), (1,2,0)] = \{v = a(1,0,1) + b(0,1,0) + c(1,2,0) / a,b,c \in \mathbb{R}\}$
$= \{v = (a+c, b+2c, a) / a,b,c \in \mathbb{R}\}$
$= \{v = (x, y, z) / x,y,z \in \mathbb{R}\} = \mathbb{R}^3$

Logo, $\{(1,0,1), (0,1,0), (1,2,0)\}$ é base de $\mathbb{R}^3$ //

b) $B = \{1, x-1, x^2+2x+1\}$ e $V = P_2(\mathbb{R})$

i) é LI pois: $a(1) + b(x-1) + c(x^2+2x+1) = 0$
$(a-b+c) + (b+2c)x + cx^2 = 0 \Rightarrow \begin{cases} a-b+c = 0 \ (II) \\ b+2c = 0 \ (I) \\ c = 0 \end{cases}$

em I: $b + 2(0) = 0$
$b = 0$ //

em II: $a - 0 + 0 = 0$
$a = 0$ //

ii) $[1, x-1, x^2+2x+1] = P_2(\mathbb{R})$

$\forall p(x) \in P_2(\mathbb{R})$ temos:

$p(x) = a_1(1) + a_2(x-1) + a_3(x^2+2x+1)$; $a_1, a_2, a_3 \in \mathbb{R}$

$p(x) = \underbrace{(a_1 - a_2 + a_3) + (a_2 + 2a_3)x + a_3(x^2)}_{\text{grau} \leq 2}$; $a_1, a_2, a_3 \in \mathbb{R}$

Logo, $\{(1), (x-1), (x^2+2x+1)\}$ é base de $P_2(\mathbb{R})$

2- $V = \{(x,y,z) \in \mathbb{R}^3 / x+y-z=0\}$ e $W = \{(x,y,z) \in \mathbb{R}^3 / x=y\}$

a) i) base e dimensão de V:

$V = \{(x,y,z) \in \mathbb{R}^3 / x+y-z=0\} \rightarrow z = x+y$

$V = \{(x,y,x+y) / x, y \in \mathbb{R}\}$

$V = \{x(1,0,1) + y(0,1,1) / x, y \in \mathbb{R}\}$

$V = [(1,0,1), (0,1,1)]$

$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ Os dois vetores são LI pois estão escalonados

Logo, $\beta_1 = \{(1,0,1), (0,1,1)\}$ é base de V e dim V = 2

ii) base e dimensão de W:

$W = \{(x,y,z) \in \mathbb{R}^3 / x=y\}$

$W = \{(x,x,z) / x, z \in \mathbb{R}\}$

$W = \{x(1,1,0) + z(0,0,1) / x, z \in \mathbb{R}\}$

$W = [(1,1,0), (0,0,1)]$

$\begin{pmatrix} 1, & 1, & 0 \\ 0, & 0, & 1 \end{pmatrix}$ Os 2 vetores são LI pois estão na forma escalonada.

Logo, $\beta_2 = \{(1,1,0), (0,0,1)\}$ é base de W e dim W = 2

b) i) $V+W = [(1,0,1), (0,1,1), (1,1,0), (0,0,1)]$

$$\begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{l} \times(-1) \\ \\ \swarrow + \\ \\ \end{array} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & \boxed{1} & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{l} \\ \times(-1) \\ \swarrow + \\ \\ \end{array} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{pmatrix} \begin{array}{l} \\ \\ \times 2 \\ \swarrow + \end{array} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Assim, $\{(1,0,1), (0,1,1), (0,0,1)\}$ é base de $V+W$ e dim $V+W = 3$

ii) base de $V \cap W$:

$\forall (x, y, z) \in V$ temos:

$$(x, y, z) = a(1,0,1) + b(0,1,1) \Rightarrow \begin{cases} a = x \\ b = y \\ a+b = z \rightarrow x+y = z \end{cases}$$

Então: $(x, y, z) = (x, y, x+y)$

Logo, $V = \{(x, y, z) / z = x+y\}$

Portanto, $(x, y, z) \in V \cap W$ se: $\begin{cases} z = x+y \\ x = y \end{cases} \rightarrow = (x, x, 2x)$

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$\Rightarrow V \cap W = \{(x, x, 2x) / x \in \mathbb{R}\}$

$V \cap W = \{x(1, 1, 2) / x \in \mathbb{R}\} = [(1, 1, 2)]$

$\hookrightarrow$ LI

Portanto $\{(1, 1, 2)\}$ é base de $V \cap W$ e $\dim V \cap W = 1$