Cálculo III - Atividade 6

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1- $z = x^3y^2$

$$\frac{\partial z}{\partial x} = 3x^2y^2 \qquad \frac{\partial z}{\partial y} = 2x^3y$$

$$\frac{1}{\cancel{x}} \cdot 2x^{\cancel{3}}y - \frac{2}{\cancel{3y}} \cdot \cancel{3}x^2y^{\cancel{2}} = 0 \rightarrow 2x^2y - 2x^2y = 0$$

Portanto, a equação $z = x^3y^2$ satisfaz a equação.

2-

$$f(x,y) = \begin{cases} \dfrac{5xy^2}{x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$

$$f(1,2) = \frac{5(1)(2)^2}{(1)^2+(2)^2} = \frac{20}{5} = 4$$

$$\frac{\partial f}{\partial x}(1,2) = \frac{5y^2(x^2+y^2) - 5xy^2(2x)}{(x^2+y^2)^2} = \frac{5x^2y^2 + 5y^4 - 10x^2y^2}{(x^2+y^2)^2}$$

$$= \frac{5y^4 - 5x^2y^2}{(x^2+y^2)^2} = \frac{5(2)^4 - 5(1)^2(2)^2}{(1^2+2^2)^2} = \frac{80-20}{25} = \frac{60}{25} = \frac{12}{5}$$

$$\frac{\partial f}{\partial y}(1,2) = \frac{10xy(x^2+y^2) - 5xy^2(2y)}{(x^2+y^2)^2} = \frac{10x^3y + \cancel{10xy^3} - \cancel{10xy^3}}{(x^2+y^2)^2}$$

$$= \frac{10x^3y}{(x^2+y^2)^2} = \frac{10(1)^3(2)}{(1^2+2^2)^2} = \frac{20}{25} = \frac{4}{5}$$

$$\frac{\partial f}{\partial x}(0,0) = 0$$

$$4 - \frac{12}{5} + \frac{4}{5} + 0 = 4 - \frac{16}{5} = \frac{4}{5}$$

3- a) $z = e^x \operatorname{sen} y$

$$\frac{\partial z}{\partial x} = e^x \operatorname{sen} y \qquad \frac{\partial^2 z}{\partial x^2} = e^x \operatorname{sen} y$$

$$e^x \operatorname{sen} y - e^x \operatorname{sen} y = 0 \,//$$

$$\frac{\partial z}{\partial y} = e^x \cos y \qquad \frac{\partial^2 z}{\partial y^2} = -e^x \operatorname{sen} y$$

$\therefore$ a função $z = e^x \operatorname{sen} y$ é harmônica

b) $z = e^x \cos y$

$$\frac{\partial z}{\partial x} = e^x \cos y \qquad \frac{\partial^2 z}{\partial x^2} = e^x \cos y$$

$$e^x \cos y + e^x \operatorname{sen} y$$

$$\frac{\partial z}{\partial y} = -e^x \operatorname{sen} y \qquad \frac{\partial^2 z}{\partial y^2} = e^x \operatorname{sen} y$$

$$e^x(\cos y + \operatorname{sen} y) \neq 0$$

$\therefore$ a função $z = e^x \cos y$ não é harmônica

c) $z = y^3 - 3x^2y$

$$\frac{\partial z}{\partial x} = -6xy \qquad \frac{\partial^2 z}{\partial x^2} = -6y$$

$$-6y + 6y = 0$$

$$\frac{\partial z}{\partial y} = 3y^2 - 3x^2 \qquad \frac{\partial^2 z}{\partial y^2} = 6y$$

$\therefore$ a função $z = y^3 - 3x^2y$ é harmônica

d) $z = x^2 + 2xy$

$$\frac{\partial z}{\partial x} = 2x + 2y \qquad \frac{\partial^2 z}{\partial x^2} = 2$$

$\therefore$ a função $z = x^2 + 2xy$ não é harmônica

$$\frac{\partial z}{\partial y} = 2x \qquad \frac{\partial^2 z}{\partial y^2} = 0$$

$$2 + 0$$

$$2 \neq 0$$

4 - $f(x,y) = 1 - xy\cos \pi y$

$$z = f(x_0,y_0) + f_x(x_0,y_0).(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$\frac{\partial f}{\partial x} = -y\cos \pi y \qquad \frac{\partial f}{\partial y} = -x\cos \pi y + \pi xy \operatorname{sen} \pi y$$

$f_x(1,1) = -\cos \pi = 1$

$f_y(1,1) = -\cos \pi + \pi \operatorname{sen} \pi = 1$

$f(1,1) = 1 - \cos \pi = 2$

$L(x,y) = 2 + (x-1) + (y-1)$

$L(x,y) = x + y$

$L(x,y) \approx L(1{,}02;\ 0{,}97) = 1{,}02 + 0{,}97 \approx 1{,}99$ //