

Atividade 6 - Álgebra Linear

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1- i) sejam $A = (x_1, y_1, z_1)$ e $B = (x_2, y_2, z_2) \in \mathbb{R}^3$

$$\begin{aligned} T(A+B) &= T(x_1+x_2, y_1+y_2, z_1+z_2) \stackrel{\text{HIP}}{=} (x_1+x_2+z_1+z_2, 2(x_1+x_2)-(y_1+y_2)+z_1+z_2) \\ T(A)+T(B) &= T(x_1, y_1, z_1) + T(x_2, y_2, z_2) \stackrel{\text{HIP}}{=} (x_1+z_1, 2x_1-y_1+z_1) + (x_2+z_2, 2x_2-y_2+z_2) \\ &= (x_1+x_2+z_1+z_2, 2x_1+2x_2-y_1-y_2+z_1+z_2) = (x_1+x_2+z_1+z_2, 2(x_1+x_2)-(y_1+y_2)+z_1+z_2) \\ &= T(A+B) \end{aligned}$$

ii) seja $A = (x, y, z) \in \mathbb{R}^3$ e $b \in \mathbb{R}$

$$T(b \cdot A) = T(bx, by, bz) \stackrel{\text{HIP}}{=} (bx+bz, 2bx-by+bz) = b(x+z, 2x-y+z) = bT(A)$$

Portanto, T é transformação linear de $\mathbb{R}^3 \rightarrow \mathbb{R}^2$.

2- i) sejam $A = (x_1, y_1)$ e $B = (x_2, y_2) \in \mathbb{R}^2$

$$\begin{aligned} T(A+B) &= T(x_1+x_2, y_1+y_2) \stackrel{\text{HIP}}{=} (x_1+x_2+a, y_1+y_2+b) \\ T(A)+T(B) &= T(x_1, y_1) + T(x_2, y_2) \stackrel{\text{HIP}}{=} (x_1+a, y_1+b) + (x_2+a, y_2+b) \\ &= (x_1+x_2+2a, y_1+y_2+2b) \end{aligned}$$

$T(A+B) \neq T(A)+T(B)$, portanto T não é transformação linear.

3-

$$R^3 \rightarrow R^2$$

$$S(3,2,1) = (1,1), S(0,1,0) = (0,-2), S(0,0,1) = (0,0)$$

$$\begin{pmatrix} 1 & 1 \\ 0 & -2 \\ 0 & 0 \end{pmatrix} \rightarrow \text{systema est excludens, potentes i base}$$

$$\forall (x,y,z) \in R^2 : (x,y,z) = a(3,2,1) + b(0,1,0) + c(0,0,1)$$

$$\begin{cases} 3a = x \rightarrow a = x/3 \\ 2a + b = y \text{ (I)} \\ a + c = z \text{ (II)} \end{cases}$$

$$(I) - 2\left(\frac{x}{3}\right) + b = y \rightarrow \frac{2x}{3} + b = y$$

$$b = \frac{3y - 2x}{3} //$$

$$(II) - \frac{x}{3} + c = z \rightarrow c = \frac{3z - x}{3}$$

$$\text{Cossim, } S(x,y,z) = \frac{x}{3} S(3,2,1) + \frac{3y-2x}{3} S(0,1,0) + \frac{3z-x}{3} S(0,0,1)$$

$$\therefore S(x,y,z) = \frac{x}{3} (1,1) + \frac{3y-2x}{3} (0,-2) + \frac{3z-x}{3} (0,0)$$

$$\text{Cossim, } S(x,y,z) = \left(\frac{x}{3}, \frac{x}{3} - \frac{2(3y-2x)}{3} \right)$$

$$S(x,y,z) = \left(\frac{x}{3}, \frac{x}{3} - \frac{6y+4x}{3} \right) = \left(\frac{x}{3}, \frac{-6y+5x}{3} \right) //$$