Atividade 10 – Cálculo III

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1- $P(x,y,z) = ?$ $x + 3y + 2z = 6$

$P(0,0,0)$

$$d = \sqrt{x^2+y^2+z^2} \Rightarrow d^2 = x^2+y^2+z^2$$

$f(x,y,z) = x^2+y^2+z$

s.a.: $x+3y+2z-6=0$

$$L(x,y,z,\lambda) = x^2+y^2+z^2+\lambda(x+3y+2z-6)$$

$\frac{\partial L}{\partial x} = 2x + \lambda = 0$

$\frac{\partial L}{\partial y} = 2y + 3\lambda = 0$

$\frac{\partial L}{\partial z} = 2z + 2\lambda = 0$

$\frac{\partial L}{\partial \lambda} = x+3y+2z-6=0$

$$\begin{cases} 2x+\lambda=0 \rightarrow \lambda = -2x \quad (I) \\ 2y+3\lambda=0 \rightarrow \lambda = -\frac{2}{3}y \quad (II) \\ 2z+2\lambda=0 \rightarrow \lambda = -z \quad (III) \\ x+3y+2z-6=0 \quad (IV) \end{cases}$$

• igualando I e III / I e II

$-2x = -z \rightarrow z = 2x$

$-2x = -\frac{2}{3}y \rightarrow y = 3x$

• substituindo em IV:

$x+3y+2z-6=0$

$x+3(3x)+2(2x)-6=0$

$x+9x+4x-6=0$

$14x = 6$

$x = \frac{3}{7}$

• voltando para y e z:

$y = 3x \rightarrow y = 3 \cdot \frac{3}{7} \rightarrow y = \frac{9}{7}$

$z = 2x \rightarrow z = 2 \cdot \frac{3}{7} \rightarrow z = \frac{6}{7}$

$P\left(\frac{3}{7}, \frac{9}{7}, \frac{6}{7}\right)$ //

2- $P(x, y, z) = ?$ $3x + 2y + 4z = 12$

$f(x,y,z) = x^2 + 4y^2 + 5z^2$

$L(x,y,z,\lambda) = x^2 + 4y^2 + 5z^2 + \lambda(3x + 2y + 4z - 12)$

$$\frac{\partial L}{\partial x} = 2x + 3\lambda = 0$$

$$\frac{\partial L}{\partial y} = 8y + 2\lambda = 0$$

$$\frac{\partial L}{\partial z} = 10z + 4\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 3x + 2y + 4z - 12 = 0$$

$$\begin{cases} 2x + 3\lambda = 0 \rightarrow x = -3/2\,\lambda & \text{I} \\ 8y + 2\lambda = 0 \rightarrow y = -1/4\,\lambda & \text{II} \\ 10z + 4\lambda = 0 \rightarrow z = -2/5\,\lambda & \text{III} \\ 3x + 2y + 4z - 12 = 0 & \text{IV} \end{cases}$$

• substituindo I, II e III em IV:

$$3\left(-\frac{3}{2}\lambda\right) + 2\left(-\frac{1}{4}\lambda\right) + 4\left(-\frac{2}{5}\lambda\right) - 12 = 0$$

$$-\frac{9}{2}\lambda - \frac{1}{2}\lambda - \frac{8}{5}\lambda = 12$$

$$-\frac{77}{10}\lambda = 12$$

$$-77\lambda = 120 \rightarrow \lambda = -\frac{120}{77}$$

• voltando para x, y e z:

$$x = -\frac{3}{2}\lambda \rightarrow x = \frac{180}{77}$$

$$y = -\frac{1}{4}\lambda \rightarrow y = \frac{30}{77}$$

$$z = -\frac{2}{5}\lambda \rightarrow z = \frac{48}{77}$$

Portanto, o ponto mínimo é $P\left(\frac{180}{77}, \frac{30}{77}, \frac{48}{77}\right)$ //