Atividade 2 - Álgebra Linear

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1- $\begin{cases} x - y + z = 2 \\ x + 2z = 1 \\ x + 2y + mz = 0 \end{cases}$

a) $\det A = 0 + 2 + (-2) - 0 - 4 + m = 0$

$\det A = 0 = -4 + m \rightarrow m = 4$

Para que o sistema seja de Cramer, é necessário que $\det A \neq 0$, para isso $m \neq 0$. Portanto $\{m \in \mathbb{R} / m \neq 4\}$.

b) $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & m \end{pmatrix}$, $X = A^{-1}b$, $m = 1$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \times(-1)\times(-1) \\ + \\ + \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 3 & 0 & -1 & 0 & 1 \end{array}\right) \begin{matrix} \\ \times(-3) \\ + \end{matrix}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & -3 & 2 & -3 & 1 \end{array}\right) \begin{matrix} + \\ \times 1 \\ \times(-\frac{1}{3}) \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 1 & -\frac{1}{3} \end{array}\right) \begin{matrix} + \\ + \\ \times(-2)\times(-1) \end{matrix}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{4}{3} & -1 & \frac{2}{3} \\ 0 & 1 & 0 & -\frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & -\frac{2}{3} & 1 & -\frac{1}{3} \end{array}\right)$$

A^{-1}

$$X = \begin{pmatrix} \frac{4}{3} & -1 & \frac{2}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{2}{3} & 1 & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{8}{3} + (-1) + 0 \\ -\frac{2}{3} + 0 + 0 \\ -\frac{4}{3} + 1 + 0 \end{pmatrix} = \begin{pmatrix} \frac{5}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix} //$$

2-a)

$$Ax = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 3 & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} x + y + 3z = 2 & \times(-1)\ \times(-1) \\ x + 2y + 4z = 3 & + \\ x + 3y + az = b & + \end{cases}$$

$$\sim \begin{cases} x + y + 3z = 2 \\ 0 \quad y + z = 1 & \times(-2) \\ 0 \quad 2y + (a-3)z = b-2 & + \end{cases} \sim \begin{cases} x + y + 3z = 2 \\ y + z = 1 \\ (a-5)z = b-4 \end{cases}$$

↳ sistema escalonado

· Se $a=5$ e $b \neq 4$, então o sistema é impossível, pois $0 = b-4$.

· Se $a = 5$ e $b = 4$, então o sistema será possível e indeterminado, pois $0=0$ e assim o sistema terá 2 equações e 3 incógnitas.

· Se $a \neq 5$ e $b = 4$, então o sistema será possível e determinado, pois o sistema terá 3 equações e 3 incógnitas.

Substituindo $a=5$ e $b=4$ no sistema:

$$\begin{cases} x + y + 3z = 2 \rightarrow x + 1 - z + 3z = 2 \rightarrow x = 1 - 2z \\ y + z = 1 \rightarrow y = 1 - z \\ 0 = 0 \end{cases}$$

$$C(S) = \{(1-2z, 1-z, z) \,/\, z \in \mathbb{R}\}_{//}$$

$$I = AA^t$$

3-

$$A^t = \begin{pmatrix} 1 & 0 & x \\ 0 & 1/\sqrt{2} & y \\ 0 & 1/\sqrt{2} & z \end{pmatrix}$$

$$AA^t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ x & y & z \end{pmatrix} \begin{pmatrix} 1 & 0 & x \\ 0 & 1/\sqrt{2} & y \\ 0 & 1/\sqrt{2} & z \end{pmatrix} = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \\ x & \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} & x^2+y^2+z^2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & x \\ 0 & 1 & \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \\ x & \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} & x^2+y^2+z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{cases} x = 0 \\ \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0 \rightarrow y = -z \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

$$\hookrightarrow 0^2 + (-z)^2 + z^2 = 1$$

$$2z^2 = 1$$

$$z = \pm \frac{1}{\sqrt{2}}$$

$$C(S) = \left\{ (0, 1/\sqrt{2}, -1/\sqrt{2}) \text{ ou } (0, -1/\sqrt{2}, 1/\sqrt{2}) \right\}_{//}$$