Cálculo III - Atividade 8

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1- $y = \frac{1}{x}$, $p_0(1,1)$ e $p_1\left(2, \frac{1}{2}\right)$

$f(x,y) = y - \frac{1}{x}$

$\nabla f(x,y) = \left(\frac{1}{x^2}, 1\right)$

$\frac{\partial f}{\partial x} = \frac{1}{x^2}$

$\frac{\partial f}{\partial y} = 1$

$\nabla f(1,1) = (1,1)$ $\quad m = -1/-1 = 1$

$\nabla f(2,1/2) = \left(\frac{1}{4}, 1\right)$ $\quad m = -1/-1/4 = 4$

· reta p/ ponto $p_0(1,1)$:

$y - 1 = 1(x-1)$

$y - 1 = x - 1$

$y = x$ //

· reta p/ ponto $p_1\left(2, \frac{1}{2}\right)$

$y - 1/2 = 4(x-2)$

$y - 1/2 = 4x - 8$

$y = 4x - \frac{15}{2}$ //

2- $F(x,y,z) = x^2 + 7y^2 + z^2 - 20 = 0$, $p(2,1,3)$

$\frac{\partial F}{\partial x} = 2x$

$\frac{\partial F}{\partial y} = 14y$

$\frac{\partial F}{\partial z} = 2z$

$\nabla f(x,y,z) = (2x, 14y, 2z)$

$\nabla f(2,1,3) = (4, 14, 6)$

· Plano tangente:

$4(x-2) + 14(y-1) + 6(z-3) = 0$

$4x + 14y + 6z - 40 = 0$ //

· reta normal: $Q = (x, y, z)$, $\overrightarrow{PQ} = (x-2, y-1, z-3)$

$(x-2, y-1, z-3) = \lambda(4, 14, 6)$; $\lambda \in \mathbb{R}$

$$\begin{cases} x-2 = 4\lambda \\ y-1 = 14\lambda \\ z-3 = 6\lambda \end{cases} \Rightarrow \begin{cases} x = 2 + 4\lambda \\ y = 1 + 14\lambda \\ z = 3 + 6\lambda \end{cases} ; \ \lambda \in \mathbb{R} \quad //$$

3 - $z = f(x, y)$ e $F(x, y, z) = xyz - x - y + x^2 - 3 = 0$

$$\frac{\partial F}{\partial x} = yz - 1 + 2x$$

$$\frac{\partial F}{\partial y} = xz - 1$$

$$\frac{\partial F}{\partial z} = xy$$

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}(x, y)}{\frac{\partial F}{\partial z}(x, y)} = -\frac{yz - 1 + 2x}{xy} \quad //$$

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}(x, y)}{\frac{\partial F}{\partial z}(x, y)} = \frac{xy - 1}{xy} \quad //$$