Atividade 5 - Álgebra Linear

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1- $E = \{v_1, v_2, v_3\} = \{(1,1,1), (2,3,2), (1,5,4)\}$

$F = \{u_1, u_2, u_3\} = \{(1,1,0), (1,2,0), (1,2,1)\}$

a) $P_{E \to F}$

$(1,1,0) = \alpha_{11}(1,1,1) + \alpha_{21}(2,3,2) + \alpha_{31}(1,5,4)$ (I)

$(1,2,0) = \alpha_{12}(1,1,1) + \alpha_{22}(2,3,2) + \alpha_{32}(1,5,4)$ (II)

$(1,2,1) = \alpha_{13}(1,1,1) + \alpha_{23}(2,3,2) + \alpha_{33}(1,5,4)$ (III)

$$(I) \rightarrow \begin{cases} \alpha_{11} + 2\alpha_{21} + \alpha_{31} = 1 \quad \times(-1) \\ \alpha_{11} + 3\alpha_{21} + 5\alpha_{31} = 1 \\ \alpha_{11} + 2\alpha_{21} + 4\alpha_{31} = 0 \end{cases} \sim \begin{cases} \alpha_{11} + 2\alpha_{21} + \alpha_{31} = 1 \rightarrow \alpha_{11} = -4/3 \\ \alpha_{21} + 4\alpha_{31} = 0 \rightarrow \alpha_{21} = 4/3 \\ 3\alpha_{31} = -1 \rightarrow \alpha_{31} = -1/3 \end{cases}$$

$$(II) \Rightarrow \begin{cases} \alpha_{12} + 2\alpha_{22} + \alpha_{32} = 1 \quad \times(-1) \\ \alpha_{12} + 3\alpha_{22} + 5\alpha_{32} = 2 \\ \alpha_{12} + 2\alpha_{22} + 4\alpha_{32} = -1 \end{cases} \sim \begin{cases} \alpha_{12} + 2\alpha_{22} + \alpha_{32} = 1 \rightarrow \alpha_{12} = -10/3 \\ \alpha_{22} + 4\alpha_{32} = 1 \rightarrow \alpha_{22} = 7/3 \\ 3\alpha_{32} = -1 \rightarrow \alpha_{32} = -1/3 \end{cases}$$

$$(III) \Rightarrow \begin{cases} \alpha_{13} + 2\alpha_{23} + \alpha_{33} = 1 \quad \times(-1) \\ \alpha_{13} + 3\alpha_{23} + 5\alpha_{33} = 2 \\ \alpha_{13} + 2\alpha_{23} + 4\alpha_{33} = 1 \end{cases} \sim \begin{cases} \alpha_{13} + 2\alpha_{23} + \alpha_{33} = 1 \rightarrow \alpha_{13} = -1 \\ \alpha_{23} + 4\alpha_{33} = 1 \rightarrow \alpha_{23} = 1 \\ 3\alpha_{33} = 0 \rightarrow \alpha_{33} = 0 \end{cases}$$

$$P_{E \to F} = \begin{pmatrix} -4/3 & -10/3 & -1 \\ 4/3 & 7/3 & 1 \\ -1/3 & -1/3 & 0 \end{pmatrix}$$

b) i) coordenada da u:

$u = 3v_1 + 2v_2 - v_3$ $\qquad (u)_E = P_{E\to F} \cdot (u)_F$

$(u)_E = (3, 2, -1)$

$$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -4/3 & -10/3 & -1 \\ 4/3 & 7/3 & 1 \\ -1/3 & -1/3 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{cases} -\frac{4}{3}x - \frac{10}{3}y - z = 3 \quad \times(-\frac{1}{4}) \times 1 \\ \frac{4}{3}x + \frac{7}{3}y + z = 2 \quad + \\ -\frac{1}{3}x - \frac{1}{3}y = -1 \quad + \end{cases} \sim \begin{cases} -\frac{4}{3}x - \frac{10}{3}y - z = 3 \\ -y = 5 \quad \times(\frac{1}{2}) \\ \frac{1}{2}y + \frac{1}{4}z = -\frac{7}{4} \quad + \end{cases}$$

$$\sim \begin{cases} -\frac{4}{3}x - \frac{10}{3}y - z = 3 \to x = 8 \\ -y = 5 \to y = -5 \\ \frac{1}{4}z = \frac{3}{4} \to z = 3 \end{cases}$$

$(u)_F = (8, -5, 3) \to u = 8u_1 - 5u_2 + 3u_3$ //

ii) coordenadas de v:

$(-3, 2, 3) = x(1,1,0) + y(1,2,0) + z(1,2,1)$

$(-3, 2, 3) = (x, x, 0) + (y, 2y, 0) + (z, 2z, z)$

$$\begin{cases} x + y + z = -3 \quad \times(-1) \\ x + 2y + 2z = 2 \quad + \\ z = 3 \end{cases} \sim \begin{cases} x + y + z = -3 \to x = -8 \\ y + z = 5 \to y = 2 \\ z = 3 \end{cases}$$

$(v)_F = (-8, 2, 3) \to v = -8u_1 + 2u_2 + 3u_3$ //