Atividade 8 - Álgebra Linear

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1- T é automorfismo?

$Ker(T) = \{(x, y, z) \in \mathbb{R}^3 / T(x, y, z) = 0\}$

$Ker(T) = \{(x, y, z) \in \mathbb{R}^3 / (x, x-y, 2x-y-z) \overset{*}{=} (0,0,0)\}$

$$* \begin{cases} x = 0 \\ x - y = 0 \\ 2x - y - z = 0 \end{cases} \Rightarrow \begin{cases} x - y = 0 \Rightarrow y = 0 \\ 2x - y - z = 0 \Rightarrow z = 0 \end{cases}$$

$$(x, y, z) = (0, 0, 0)$$

Portanto, $Ker(T) = \{(0,0,0)\}$, assim T é injetora.

$$\dim Ker(T) + \dim Im(T) = \dim \mathbb{R}^3$$
$$0 + \dim Im(T) = 3$$

$\dim Im(T) = 3 = \dim \mathbb{R}^3$, então T é sobrejetora.

Assim, T é automorfismo do $\mathbb{R}^3$ //

determinar T^{-1}

$\forall (a, b, c) \in \mathbb{R}^3$, existe (x, y, z) tal que:

$T(x, y, z) = (a, b, c)$

$(x, x-y, 2x-y-z) = (a, b, c)$

assim, $\begin{cases} x = a \\ x - y = b \ (\text{I}) \\ 2x - y - z = c \ (\text{II}) \end{cases}$

$x = a$ em I:

$a - y = b$

$y = a - b$

$x = a$ e $y = a - b$ em II:

$2a - (a-b) - z = c$

$z = a + b - c$

voltando para todo $(a, b, c) \in \mathbb{R}^3$, existe $(a, a-b, a+b-c) \in \mathbb{R}^3$ tal que:

$T(a, a-b, a+b-c) = (a, b, c)$

logo, $T^{-1}(a, b, c) = (a, a-b, a+b-c)$ ou seja,

$T^{-1}(x, y, z) = (x, x-y, x+y-z)$ //

2- a) seja $\vec{v} = (x, y) \in \mathbb{R}^2$

$(x, y) = a(1, -1) + b(0, 2)$

$\begin{cases} a = x \\ -a + 2b = y \end{cases} \rightarrow b = \dfrac{y + a}{2} \rightarrow b = \dfrac{y + x}{2}$

Portanto, $(v)_B = \begin{pmatrix} x \\ \frac{y+x}{2} \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ \frac{y+x}{2} \end{pmatrix} = \begin{pmatrix} x \\ x + \frac{y+x}{2} \\ -\frac{y+x}{2} \end{pmatrix} = \begin{pmatrix} x \\ \frac{3x+y}{2} \\ -\frac{(y+x)}{2} \end{pmatrix}$$

Logo, $T(v) = x(1,0,-1) + \frac{3x+y}{2}(0,1,2) - \frac{(y+x)}{2}(1,2,0)$

$T(v) = (x,0,-x) + \left(0, \frac{3x+y}{2}, 3x+y\right) + \left(\frac{-y-x}{2}, -y-x, 0\right)$

$T(v) = \left(x - \frac{y-x}{2}, \frac{3x+y}{2} - x - y, -x+3x+y\right)$

$T(v) = \left(\frac{x-y}{2}, \frac{x-y}{2}, 2x+y\right)$ //

b) $S(1,-1) = (-2,2,1) = a(1,0,-1) + b(0,1,2) + c(1,2,0)$ (I)

$S(0,2) = (4,-2,0) = d(1,0,-1) + e(0,1,2) + f(1,2,0)$ (II)

I - $(a,0,-a) + (0,b,2b) + (c,2c,0) = (-2,2,1)$

$(a+c, b+2c, -a+2b) = (-2,2,1)$

$\begin{cases} a + c = -2 \\ b + 2c = 2 \\ -a + 2b = 1 \end{cases} \sim \begin{cases} a + c = -2 \\ b + 2c = 2 \times (-2) \\ 2b + c = -1 \end{cases} \sim \begin{cases} a + c = -2 \to a = -2 - \frac{5}{3} = \frac{-11}{3} \\ b + 2c = 2 \to b = \frac{-4}{3} \\ -3c = -5 \to c = \frac{5}{3} \end{cases}$

$c = \frac{5}{3}, \quad b = \frac{-4}{3}, \quad a = \frac{-11}{3}$

II - $\begin{cases} d + f = 4 \\ e + 2f = -2 \\ -d + 2e = 0 \end{cases} \sim \begin{cases} d + f = 4 \\ e + 2f = -2 \times (-2) \\ 2e + f = 4 \end{cases} \sim \begin{cases} d + f = 4 \\ e + 2f = -2 \\ -3f = 8 \to f = \frac{-8}{3} \end{cases}$

$f = \frac{-8}{3}, \quad e = -2 - 2 \cdot \frac{-8}{3} = -2 + \frac{16}{3} = \frac{10}{3}, \quad d = 4 + \frac{8}{3} = \frac{20}{3}$

Assim, $(S)_{B,C} = \frac{1}{3}\begin{pmatrix} -11 & 20 \\ -4 & 10 \\ 5 & -8 \end{pmatrix}$ //

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c) $(V)_B = (x, y)$

$(x, y) = a(1, -1) + b(0, 2)$

$$\begin{cases} a = x \\ -a + 2b = y \rightarrow b = \frac{y + x}{2} \end{cases} \qquad (V)_B = \begin{pmatrix} x \\ \frac{y+x}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \frac{y+x}{2} \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ \frac{y+x}{2} \end{pmatrix}$$

Logo, $T(v) = x(1, 0, 0) + 0(0, 1, 0) + \left(\frac{y + x}{2}\right)(0, 0, 1)$

$$T(v) = \left(x, \ 0, \ \frac{x + y}{2}\right)$$

$T(1, -1) = (1, 0, 0)$

$T(0, 2) = (0, 0, 1)$

$(1, 0, 0) = a(1, 0, 0) + b(0, 1, 0) + c(0, 0, 1)$

$(0, 0, 1) = d(1, 0, 0) + e(0, 1, 0) + f(0, 0, 1)$

$a = 1, \ b = 0, \ c = 0, \ d = 0, \ e = 0, \ f = 1$

Assim, $D = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ //