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Ex 1 -

$$A = \begin{pmatrix} -1 & 1 & 3 \\ 8 & 3 & -5 \\ 1 & -2 & 7 \end{pmatrix} \qquad A^t = \begin{pmatrix} -1 & 8 & 1 \\ 1 & 3 & -2 \\ 3 & -5 & 7 \end{pmatrix}$$

Por hipótese, A não é antissimétrica, pois $-A \neq A^t$ e A^t também não é antissimétrica, pois $-(A^t) \neq (A^t)^t$.

$$A - A^t = \begin{pmatrix} -1-(-1) & 1-8 & 3-1 \\ 8-1 & 3-3 & -5-(-2) \\ 1-3 & -2-(-5) & 7-7 \end{pmatrix} = \begin{pmatrix} 0 & -7 & 2 \\ 7 & 0 & -3 \\ -2 & 3 & 0 \end{pmatrix}$$

$$(A - A^t)^t = \begin{pmatrix} 0 & 7 & -2 \\ -7 & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$$

Verifica-se que $A - A^t$ é uma matriz antissimétrica, pois $-(A - A^t) = (A - A^t)^t$

Ex 2 -

$$A = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} \qquad \begin{matrix} A^2 = A \cdot A \\ A^3 = A^2 \cdot A \end{matrix}$$

$$A^2 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} + \frac{1}{4} & -\frac{1}{4} + (-\frac{1}{4}) \\ -\frac{1}{4} - \frac{1}{4} & \frac{1}{4} + \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$A^3 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Nota-se que ao aumentar o índice da potência, o resultado será o mesmo, portanto $A^n = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Ex 3- a)

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \qquad \det A = 2 + 1 + 0 - 0 - 0 - 0 = 3$$

b)

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \underset{L_1 \leftrightarrow L_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{array}\right) \begin{array}{l} \times(-1) \\ \\ + \end{array}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 & -1 \end{array}\right) \begin{array}{l} \\ \times(-1) \\ + \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 & -1 & -1 \end{array}\right) \begin{array}{l} + \\ + \\ \times\left(-\frac{1}{3}\right) \times\left(\frac{1}{3}\right) \times\left(\frac{2}{3}\right) \end{array}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & 1/3 & 2/3 & -1/3 \\ 0 & 0 & 1 & -1/3 & 1/3 & 1/3 \end{array}\right) \qquad A^{-1} = \frac{1}{3} \cdot \begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$