Cálculo III - Atividade 9

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1- $f(x,y,z) = \dfrac{xz}{x^2+y^2+1}$

a) $p(1,0,-1)$

$$\frac{\partial f}{\partial x} = \frac{z(x^2+y^2+1) - 2x^2z}{(x^2+y^2+1)^2} = \frac{z(-x^2+y^2+1)}{(x^2+y^2+1)^2}$$

$$\frac{\partial f}{\partial y} = \frac{-2xyz}{(x^2+y^2+1)^2}$$

$$\frac{\partial f}{\partial z} = \frac{x\cancel{(x^2+y^2+1)}}{(x^2+y^2+1)^{\cancel{2}}} = \frac{x}{x^2+y^2+1}$$

$$\nabla f(x,y,z) = \left(\frac{z(-x^2+y^2+1)}{(x^2+y^2+1)^2}, -\frac{2xyz}{(x^2+y^2+1)^2}, \frac{x}{x^2+y^2+1} \right)$$

$\nabla f(1,0,-1) = \left(0, 0, \dfrac{1}{2}\right)$ // $\rightarrow$ direção de maior variação de f no ponto $(1,0,-1)$

$\|\nabla f(1,0,-1)\| = \sqrt{0^2+0^2+\left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$ // $\rightarrow$ taxa máxima de variação de f no ponto $(1,0,-1)$

b) $r(t) = (1+2t, t, -1+t)$

$r'(t) = (2, 1, 1)$ $\qquad \|r'(t)\| = \sqrt{2^2+1^2+1^2} = \sqrt{6}$

$$u = \frac{1}{\sqrt{6}}(2,1,1) = \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

$$D_{\vec{u}} f(x_0, y_0, z_0) = \nabla f(x, y, z) \cdot \vec{u}$$

$$D_{\vec{u}} f(1, 0, -1) = \left(0, 0, \frac{1}{2}\right) \cdot \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$D_{\vec{u}} f(1, 0, -1) = 0 + 0 + \frac{1}{2\sqrt{6}} = \frac{1}{2\sqrt{6}} //$$ ← taxa de variação de f no ponto $(1, 0, -1)$ na direção do vetor $\vec{u}$.

2- $f(x, y) = x^2 + \operatorname{sen} xy$, $p(1, 0)$

$$f_x = 2x + y \cos xy$$
$$f_y = x \cos xy$$

$$f_x(1, 0) = 2$$
$$f_y(1, 0) = 1$$

$$D_{\vec{u}} f(1, 0) = 2\cos\theta + \operatorname{sen}\theta$$
$$1 = 2\cos\theta + \operatorname{sen}\theta$$
$$(2\cos\theta)^2 = (1 - \operatorname{sen}\theta)^2$$
$$4\cos^2\theta = 1 - 2\operatorname{sen}\theta + \operatorname{sen}^2\theta$$
$$4(1 - \operatorname{sen}^2\theta) = 1 - \operatorname{sen}\theta + \operatorname{sen}^2\theta$$
$$4 - 4\operatorname{sen}^2\theta = 1 - \operatorname{sen}\theta + \operatorname{sen}^2\theta$$
$$5\operatorname{sen}^2\theta - \operatorname{sen}\theta - 3 = 0$$

$u = \operatorname{sen}\theta$

$$u_1 = \frac{1 + \sqrt{61}}{10}, \quad u_2 = \frac{1 - \sqrt{61}}{10}$$

$$\operatorname{sen}\theta = \frac{1 + \sqrt{61}}{10} \text{ e } \operatorname{sen}\theta = \frac{1 - \sqrt{61}}{10}$$

Portanto temos: $\operatorname{sen}\theta = 0{,}8810249$ e $\operatorname{sen}\theta = -0{,}6810249$.

Logo, $\theta = 61{,}77° //$

3- a) $f(x, y) = y + x \operatorname{sen} y$

$$\frac{\partial f}{\partial x} = \operatorname{sen} y = 0$$

$$\frac{\partial f}{\partial y} = 1 + x \cos y = 0$$

$$\begin{cases} \operatorname{sen} y = 0 \rightarrow y = 0 \\ 1 + x \cos y = 0 \end{cases}$$

$\hookrightarrow 1 + x = 0 \rightarrow x = -1$

- ponto crítico: $(-1, 0) //$

- teste da segunda derivada:

$$H(x,y) = \begin{vmatrix} 0 & \cos y \\ \cos y & -x \sin y \end{vmatrix} = -\cos^2 y \qquad H(-1,0) = -1 < 0$$

· Portanto, o ponto $(-1,0)$ é ponto de máximo //

b) $g(x,y) = x+y+4$

$$\left.\begin{aligned} \frac{\partial g}{\partial x} &= 1 = 0 \\ \frac{\partial g}{\partial y} &= 1 = 0 \end{aligned}\right\} \nexists$$

Não existem pontos críticos //

c) $h(x,y) = \dfrac{x}{x^2+y^2+4}$

$$\frac{\partial h}{\partial x} = \frac{x^2+y^2+4-2x^2}{(x^2+y^2+4)^2} = \frac{-x^2+y^2+4}{(x^2+y^2+4)^2} = 0$$

$$\frac{\partial h}{\partial y} = \frac{-2xy}{(x^2+y^2+4)^2} = 0$$

$$\begin{cases} \dfrac{-x^2+y^2+4}{(x^2+y^2+4)^2} = 0 \\ \dfrac{-2xy}{(x^2+y^2+4)^2} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -x^2+y^2+4 = 0 \rightarrow x^2 = y^2+4 \rightarrow x = \pm\sqrt{y^2+4} \\ -2xy = 0 \end{cases}$$

$\hookrightarrow x = 2,\ x = -2$

- pontos críticos: $(0,0)$, $(2,0)$, $(-2,0)$ //

- teste da segunda derivada:

$$H(x,y) = \begin{vmatrix} \dfrac{-2x(-x^2+3y^2+12)}{(x^2+y^2+4)^3} & \dfrac{2y(3x^2-y^2-4)}{(x^2+y^2+4)^3} \\ \dfrac{2y(3x^2-y^2-4)}{(x^2+y^2+4)^3} & \dfrac{-2x(x^2-3y^2+4)}{(x^2+y^2+4)^3} \end{vmatrix}$$

$$H(x,y) = \frac{4x^2(-x^2+3y^2+12)(x^2-3y^2+4)}{(x^2+y^2+4)^6} - \frac{4y^2(3x^2-y^2-4)^2}{(x^2+y^2+4)^6}$$

$\nexists H(0,0)$ $\quad H(2,0) = \dfrac{1}{256} > 0$, portanto $(2,0)$ é ponto de mínimo

$H(-2,0) = \dfrac{1}{256} > 0$, portanto $(-2,0)$ é ponto de mínimo

Concluí-se que os pontos $(-2,0)$ e $(2,0)$ são pontos de mínimos //