Atividade 7 - Álgebra Linear

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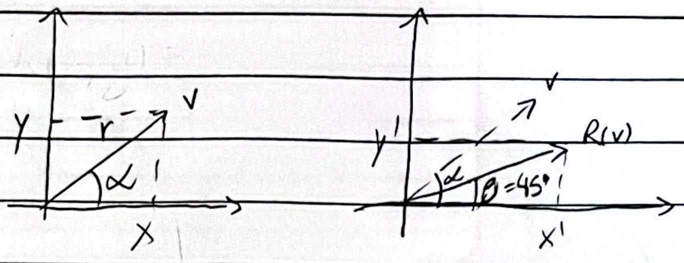
1- plano $\mathbb{R}^2$



$R_{45°}(x,y) = (x', y')$

$$\begin{cases} x = r\cos\alpha \\ y = r\,\text{sen}\,\alpha \end{cases} \rightarrow \begin{cases} x' = r\cos(\alpha - 45°) = r\cos\alpha\cos 45° + r\,\text{sen}\,\alpha\,\text{sen}\,45° \\ y' = r\,\text{sen}(\alpha - 45°) = r\,\text{sen}\,\alpha\cos 45° - r\,\text{sen}\,45°\cos\alpha \end{cases}$$

$$\rightarrow \begin{cases} x' = x\cos 45° + y\,\text{sen}\,45° \\ y' = y\cos 45° - x\,\text{sen}\,45° \end{cases} \rightarrow \begin{cases} x' = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \\ y' = \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2}x \end{cases}$$

$$R_{45°}(x,y) = \left(\frac{\sqrt{2}}{2}(x+y), \frac{\sqrt{2}}{2}(y-x)\right) //$$

2- $T(x,y,z) = (x-3y-2z, y-4z, z)$

a) $Ker(T) = \{u \in \mathbb{R}^3 / T(u) = 0\}$

$(x-3y-2z, y-4z, z) = (0,0,0)$

$$\begin{cases} x-3y-2z = 0 \rightarrow x = 0 \\ y-4z = 0 \rightarrow y = 0 \\ z = 0 \end{cases}$$

$Ker(T) = [(0,0,0)]$, $\dim Ker(T) = 0 //$

$(0,0,a) = (0,0,0)$

b)

$Im(T) = \{(x-3y-2z,\ y-4z,\ z) / x, y, z \in \mathbb{R}\}$

$Im(T) = \{x(1,0,0) + y(-3,1,0) + z(-2,-4,1)\}$

$Im(T) = [(1,0,0), (-3,1,0), (-2,-4,1)]$ //

$\dim Im(T) = 3$ //

3 - $T: \mathbb{R}^3 \to \mathbb{R}^3$

$T(x,y,z) = (x,\ x-y,\ 2x-y-z)$

$Im(T) = \{(x,\ x-y,\ 2x-y-z) / x, y, z \in \mathbb{R}\}$

$Im(T) = \{x(1,1,2), y(0,-1,-1) + z(0,0,-1)\}$

$Im(T) = [(1,1,2), (0,-1,-1), (0,0,-1)]$

Os vetores são LI, pois já está escalonado.

Logo, $Im(T)$ é base do $\mathbb{R}^3$. Assim, T é sobrejetora, pois $Im(T) = [(1,1,2), (0,-1,-1), (0,0,-1)] = \mathbb{R}^3$ //

É injetora?

$Ker(T) = \{u \in \mathbb{R}^3 / T(u) = 0\}$

$T(x,y,z) = (0,0,0)$

$(x,\ x-y,\ 2x-y-z) = (0,0,0) \to \begin{cases} x = 0 \\ x-y = 0 \to y = 0 \\ 2x-y-z = 0 \to z = 0 \end{cases}$

$Ker(T) = [(0,0,0)]$ e $\dim Ker(T) = 0$ é injetora

Portanto, T é injetora e sobrejetora.

tilibra