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Atividade 9

Exercício 1 - Seja $T: \mathbb{R}^4 \to \mathbb{R}^4$, dada por $T(x,y,z,t) = (3x-4z, 3y+5z, -z, -t)$

a) Determine a matriz $[T]_\alpha$, sendo α a base canônica do $\mathbb{R}^4$, o polinômio característico e os autovalores de T

Temos que:

$$\alpha = \{(1,0,0,0); (0,1,0,0); (0,0,1,0), (0,0,0,1)\}$$

$$\begin{cases} T(1,0,0,0) = (3,0,0,0) & (I) \\ T(0,1,0,0) = (0,3,0,0) & (II) \\ T(0,0,1,0) = (-4,5,-1,0) & (III) \\ T(0,0,0,1) = (0,0,0,-1) & (IV) \end{cases}$$

i) $(3,0,0,0) = a(1,0,0,0) + b(0,1,0,0) + c(0,0,1,0) + d(0,0,0,1)$

$(3,0,0,0) = (a,b,c,d)$

$a = 3,\ b = 0,\ c = 0,\ d = 0$

ii) $(0,3,0,0) = e(1,0,0,0) + f(0,1,0,0) + g(0,0,1,0) + h(0,0,0,1)$

$(0,3,0,0) = (e,f,g,h)$

$e = 0,\ f = 3,\ g = 0,\ h = 0$

iii) $(-4,5,-1,0) = i(1,0,0,0) + j(0,1,0,0) + k(0,0,1,0) + l(0,0,0,1)$

$i = -4,\ j = 5,\ k = -1,\ l = 0$

iv) $(0,0,0,-1) = m(1,0,0,0) + n(0,1,0,0) + o(0,0,1,0) + p(0,0,0,1)$

$(0,0,0,-1) = (m, n, o, p)$

$m = 0, \; n = 0, \; o = 0, \; p = -1$

$$T_\alpha = \begin{pmatrix} 3 & 0 & -4 & 0 \\ 0 & 3 & 5 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} //$$

$$P_T(\lambda) = \det \begin{pmatrix} 3-\lambda & 0 & -4-\lambda & 0 \\ 0 & 3-\lambda & 5-\lambda & 0 \\ 0 & 0 & -1-\lambda & 0 \\ 0 & 0 & 0 & -1-\lambda \end{pmatrix} = (3-\lambda)^2(-1-\lambda)^2 //$$

autovalores são iguais as raízes $P_T(\lambda)$

$P_T(\lambda) = 0$

$(3-\lambda)^2(-1-\lambda)^2 = 0$

$\hookrightarrow -1-\lambda = 0 \rightarrow \lambda = -1 //$

$3-\lambda = 0 \rightarrow \lambda = 3 //$

b) Determine o conjunto de autovetores, o subespaço próprio e as multiplicidades algébrica e geométrica.

$\lambda = 3$ e $\lambda = -1$ são autovalores

$ma(3) = 2 //$ e $ma(-1) = 2 //$

· $\lambda = 3$

$$\begin{pmatrix} 0 & 0 & -7 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 0 = 0 \ \forall x \in \mathbb{R} \\ 0 = 0 \ \forall y \in \mathbb{R} \\ z = 0 \\ t = 0 \end{cases}$$

autovetores de $\lambda = 3$: $v = (x, y, 0, 0)$//

subespaço próprio: $V(3) = \{(x, y, 0, 0) / x, y \in \mathbb{R}\}$

$V(3) = [(1, 0, 0, 0), (0, 1, 0, 0)]$//

$mg(3) = 2$//

· $\lambda = -1$

$$\begin{pmatrix} 3 & 0 & -4 & 0 \\ 0 & 3 & 5 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = -1 \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \Rightarrow \begin{cases} 3x - 4z = -x \to 4x = 4z \to x = z \\ 3y + 5z = -y \to y = -\frac{5}{4}z \\ -z = -z \\ -t = -t \end{cases}$$

autovetores de $\lambda = -1$: $v = (z, -\frac{5}{4}z, z, t)$//

subespaço próprio: $V(-1) = \{(z, -\frac{5}{4}z, z, t) / z, t \in \mathbb{R}\}$

$V(-1) = [(1, -5/4, 1, 0), (0, 0, 0, 1)]$//

$mg(-1) = 2$//

c) Verifique se a transformação linear T é diagonalizável, justificando sua resposta. Se for, apresente a matriz diagonal D associada e a matriz M que diagonaliza B.

T é diagonalizável, pois ma(3) = mg(3) e ma(-1) = mg(-1).

$$D = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad e \quad M = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -5/4 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercício 2- Verifique se $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ dada por:

$$T(p(x)) = p''(x) - 2p'(x) + p(x) \text{ é diagonalizável.}$$

base de $P_2(\mathbb{R}) = \{1, x, x^2\}$ e $\dim P_2(\mathbb{R}) = 3$

$T(1) = (1)'' - 2(1)' + 1 = 0 - 0 + 1 = 0$
$T(x) = (x)'' - 2(x)' + x = 0 - 2 + x = -2 + x$
$T(x^2) = (x^2)'' - 2(x^2)' + x^2 = 2 - 4x + x^2$

$$T_\alpha = \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_T(\lambda) = \det \begin{pmatrix} 1-\lambda & -2-\lambda & 2-\lambda \\ 0 & -1-\lambda & -4-\lambda \\ 0 & 0 & 1-\lambda \end{pmatrix} = (1-\lambda)^3$$

autovalores são iguais as raízes $P_T(\lambda)$

$P_T(\lambda) = 0$

$(1-\lambda)^3 = 0$

$\hookrightarrow \lambda = 1$ $\qquad ma(1) = 3$

$V(1) = \{ v \in P_2(\mathbb{R}) / T(v) = 1.v \}$

seja $v = a + bx + cx^2 = p(x)$ então:

$T(v) = T(p(x)) = (p(x))'' - 2(p(x))' + p(x)$

se $p(x) = a + bx + cx^2$ então $p'(x) = b + 2cx$ e $p''(x) = 2c$

$T(v) = 2c - 2(b + 2cx) + a + bx + cx^2$

$T(v) = (2c - 2b + a) + (b - 4c)x + cx^2$

assim, $T(x) = 1v$

$(2c - 2b + a) + (b - 4c)x + \cancel{cx^2} = 1(a + bx + \cancel{cx^2})$

$(2c - 2b + a) + (b - 4c)x = a + bx$

$$\begin{cases} 2c - 2b + a = a \rightarrow 2b = 2c \rightarrow b = 0 \\ b + 4c = b \rightarrow c = 0 \end{cases}$$

autovetores de $\lambda = 1$: $V(1) = \{ a + bx + cx^2 / a \in \mathbb{R}, b = c = 0 \}$

subespaço próprio: $V(1) = [1]$ $\qquad mg = 1$

Como $ma(1) \neq mg(1)$, T não é diagonalizável.