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$$z_0 = r e^{j\omega_0} \Rightarrow |z_0| = |r| \cdot |e^{j\omega_0}| = |r| = r, \text{ logo}$$

$$|z| > |z_0| \Leftrightarrow |z| > r$$

$$X(z) = \frac{1}{z(1 - (\frac{z}{z_0})^{-1})} + \frac{1}{z(1 - (\frac{z}{z_0^*})^{-1})} =$$

$$= \frac{1 - z_0^* z^{-1} + 1 - z_0 z^{-1}}{z(1 - z_0 z^{-1})(1 - z_0^* z^{-1})} = \frac{2 - z^{-1} \cdot (z_0 + z_0^*)}{z(1 - z_0 z^{-1})(1 - z_0^* z^{-1})} =$$

$$= \frac{2 - 2r \left( \frac{e^{j\omega_0} + e^{-j\omega_0}}{2} \right) z^{-1}}{z(1 - z_0 z^{-1})(1 - z_0^* z^{-1})} = \frac{1 - r \cos(\omega_0) z^{-1}}{(1 - z_0 z^{-1})(1 - z_0^* z^{-1})}$$

$$\therefore \left| X(z) = \frac{1 - r \cos(\omega_0) z^{-1}}{(1 - r e^{j\omega_0} z^{-1})(1 - r e^{-j\omega_0} z^{-1})}, |z| > r \right|$$