Mecânica Aplicada I

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Turma:1

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1 Exercício 1 (Aceleração angular da fig 1.15)

1)

$$^{R}\alpha^{R''} = {}^{R}\alpha^{R'} + {}^{R'}\alpha^{R''} + {}^{R}w^{R'} \times {}^{R'}w^{R''}$$

$$^{R}w^{R'} = w_2 e_2$$

$$^{R'}w^{R''} = w_3 e_3$$

$$^{R}R^{R'} \times {}^{R'}w^{R''} = w_2 . w_3 . e_1$$

$$^{R}\alpha^{R''} = 0 + 0 + w_2 e_2 \times w_3 e_3 = w_2 . w_3 e_1$$

2 Exercício 2 (Exercício 3 da pag 28

2) $\frac{x^2}{9} + \frac{y^2}{4} = 1$, $\dot{x} = 2\,cm/s$ $\ddot{x} = 1{,}5\,cm/s^2$

$\vec{OP} = x\,i + y\,j$, sendo $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$y = \left(1 - \frac{x^2}{9}\right)^{1/2} \cdot 2$

$\vec{OP} = x\,i + 2\left(1 - \frac{x^2}{9}\right)^{1/2} j$, se $x = 1$

$\vec{OP} = 1\,\hat{i} + \frac{4\sqrt{2}}{3}\,\hat{j}$

2) exercício 2 Velocidade

$$^{R}V^{R} = \frac{^{R}d\,\vec{OP}}{dt} = \dot{x}\,\hat{i} + 2\,\frac{1}{2}\cdot\left(1-\frac{x^2}{9}\right)^{-1/2}\cdot\frac{2x\cdot\dot{x}}{9}\,\hat{j}$$

$x = 1$ e $\dot{x} = 2$

$$^{R}V^{P} = 2\hat{i} - \frac{2\cdot 2}{9\cdot\frac{2\sqrt{2}}{3}}\,\hat{j} = 2i - \frac{2\sqrt{2}}{3\,2}\,\hat{j}$$

$$^{R}V^{P} = \left(2i - \frac{\sqrt{2}}{3}\,j\right) cm/s$$

2) Exercício 2 aceleração

$$ {}^{R}a^{P} = \frac{{}^{R}dV^{P}}{dt} = $$

$$ \ddot{x}\,i + \frac{1}{3}\left(-\frac{1}{6}(9-x^2)^{-3/2} \cdot (-2x) \cdot \dot{x} \cdot (-2x)\dot{x} + (9-x^2)^{-1/2} \right.$$

$$ \left. \cdot (-2)\,\dot{x}^2 + (9-x^2)^{-1/2} \cdot (-2x) \cdot \ddot{x}\right) j $$

$$ x = 1 ; \dot{x} = 2 , \ddot{x} = 1{,}5 $$

$$ {}^{R}a^{P} = 1{,}5\,i + \frac{1}{3}\left(-\frac{1}{2} \cdot 8^{-3/2} \cdot (-2) \cdot 2 \cdot (-2) \cdot 2 + 8^{-\frac{1}{2}} \cdot (-2) \cdot 4 \right.$$

$$ \left. + 8^{-\frac{1}{2}} \cdot (-2) \cdot 1{,}5\right) j $$

$$ {}^{R}a^{P} = 1{,}5\,i + \frac{1}{3}\,12\frac{\sqrt{2}}{4} = \left(1{,}5\,i + \sqrt{2}\,j\right)\ cm/s^2 $$

3 Exercício 3 (Exercício 7 da pagina 33)

3) em $t=0$, $v_0=0$, $x_0=0$, $a=-kv^2$, $a=-kv^2$

a) $dv = a \cdot dt$

$dv = kv^2$

$$\frac{dv}{v} = -k\,dt$$

$$\int_{v_0}^{v} \frac{dv}{v^2} = k\int_0^t dt$$

$$-1/v\Big|_{v_0}^{v} = -k\,t\Big|_0^t$$

$$-\frac{1}{v} + \frac{1}{v_0} = kt$$

$$v = 1/[kt + (1/v_0)]$$

b) $dx\ v\,dt$

$dx = dt/[kt + (1/v_0)]$

$$\int_0^x dx = \int_0^t = dt/[kt + (1/v_0)]$$

$$x = \ln\left(kt + \frac{1}{v_0}\right)^{1/k}$$

3) c) $v\,dv = a\,dx$

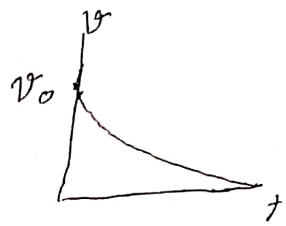
$v\,dv = -k v^2 dx$

$\frac{dv}{v} = -k\,dx$

$\ln v = kx$

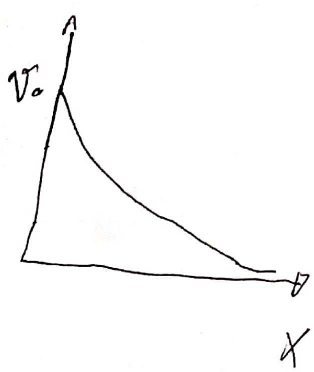
$v = e^{-kx}$

d)



e)

f)



4 Exercício 4 (Exercício 8 da pagina 34)

4) $\vec{a} = -kv\hat{i}$, k constante positiva, $x = 0$

a) $dv = a\,dt$

$dv = -kv\,dt$

$\frac{dv}{v} = -k\,dt$

$v = v_0 e^{-kt}$

b) $dx = v\,dt$

$dx = v_0 e^{-kt}\,dt$

$\int_0^x dx = v_0 \int_0^t e^{-kt}\,dt$

$x = \frac{v_0}{k}\cdot(1 - e^{-kt})$

c) $v\,dv = a\,dx$

$v\,dv = -kv\,dx$

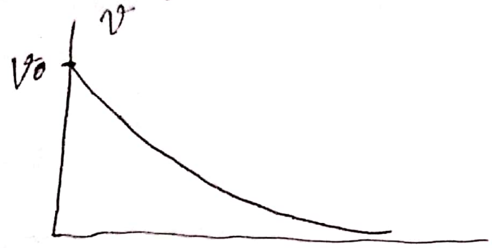
$dv = -k\,dx$

$\int_{v_0}^{v} dv = -k\int_0^x dX$

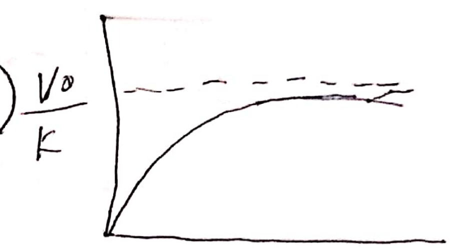
$v = v_0 - kX$

Questão 4)

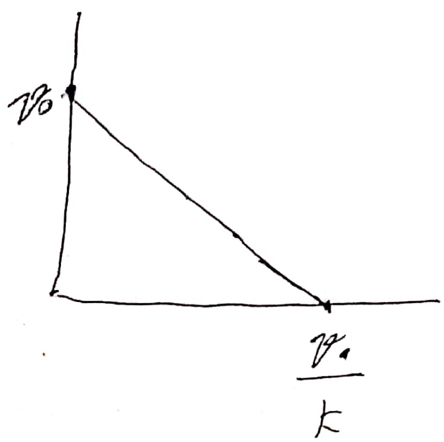
d)



e)



F)



5 Exercício 5 (Exercício 5 da pagina 47)

5) $\dot{V} = V_0 \text{ sen } \theta \, e_r + V_0 \cos\theta \, e_\theta = \dot{r} \, e_r + \dot{r} \, \theta \cdot e_\theta$

$$\frac{b \, \dot{\theta}}{\cos\theta} = V_0 \cos\theta \qquad \boxed{\dot{\theta} = \frac{V_0 \cos^2\theta}{b}}$$

$$\ddot{\theta} = \frac{V_0 \, 2\cos\theta}{b} (-\text{sen}\,\theta)\,\dot{\theta} = \frac{-2 V_0}{b} \cos\theta \text{ sen}\,\theta$$

$$\left(\frac{V_0}{b} \cos^2\theta\right) = \boxed{\frac{-2 V_0^2}{b^2} \cos^3\theta \text{ sen}\,\theta = \ddot{\theta}}$$

$$\frac{-2 V_0^2}{b^2} \cos^3\theta \text{ sen}\,\theta = \ddot{\theta}$$

6 Exercício 6 (Exercício 8 da pagina 49)

6) $\rho = 6(1+\cos\phi)$ $\quad \vec{OP} = \rho\,\hat{j}$

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$\rho + \theta = 90^\circ$; $\quad \cos\phi = \operatorname{sen}\theta$

$$\boxed{\vec{OP} = 6(1+\operatorname{sen}\theta)\,\hat{j}}$$

$$\frac{d\vec{OP}}{d\theta} = 6\dot{\theta}\cos\theta ; \quad \boxed{V^P = 6\,\dot{\theta}\cos\theta\,\hat{j}}$$

$$\frac{dV^P}{d\theta} = (6\,\ddot{\theta}\cos\theta - 6\,\dot{\theta}^2\operatorname{sen}\theta)\,\hat{j}$$

$$\boxed{a^P = 6(\ddot{\theta}\cos\theta - \dot{\theta}^2\operatorname{sen}\theta)\,\hat{j}}$$

7 Exercício 7 (Exercício 9 da pagina 50)

7) $\vec{O}P = P\hat{i}$ $\vec{O}P = 6\phi\hat{i}$

$\theta + \phi = 360°$ $\theta + \phi = 2\pi$

$\phi = 2\pi - \theta$

$$\boxed{\vec{OP} = (2\pi - \theta)\, 6\, \hat{i}}$$

$$v^P = \frac{d\vec{OP}}{d\theta} \qquad a^P = \frac{dv^P}{d\theta}$$

$$\boxed{v^P = -6\dot{\theta}\,\hat{i}} \qquad \boxed{a^P = -6\ddot{\theta}\,\hat{i}}$$