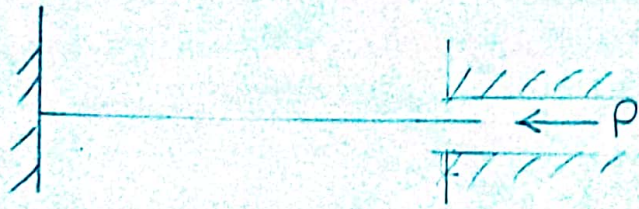# Questão 3:

$$FS = \frac{P_{cr}}{P_r} = 2 \rightarrow P = \frac{P_{cr}}{2}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (200 \times 10^3 \times 306796,16)}{(K \times 5000)^2} = P_{cr} = \frac{24223,65}{K^2}$$

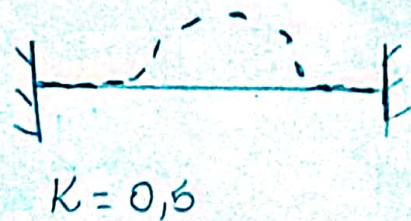- $I = \frac{\pi}{64}(30)^4$

$I \simeq 306796,16 \, mm^4$

$L = 5 \, m = 5000 \, mm$

$E = 200 \times 10^3 \, MPa$

Deformadas

Pano xy

$K = 0,5$

Plano xz

$K = 2$
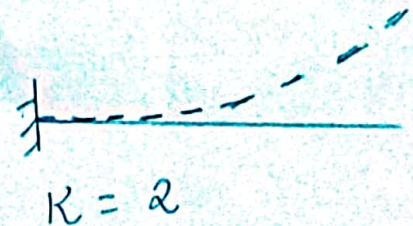
Plano xy

$$P_{cr} = \frac{24223,65}{(0,5)^2} = 96894,6 \, N$$
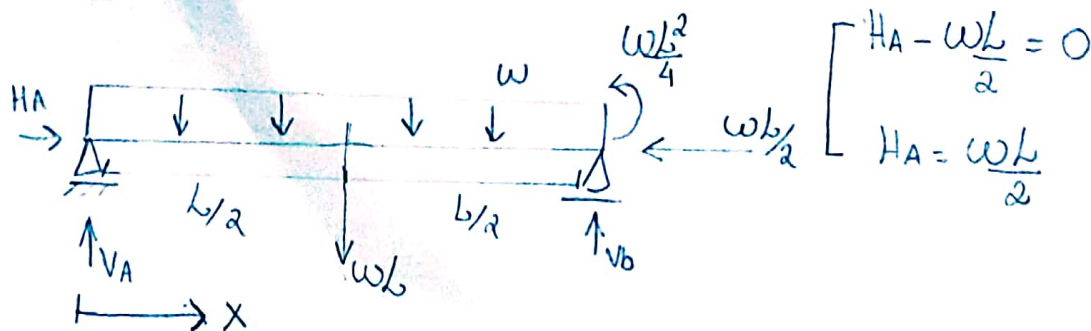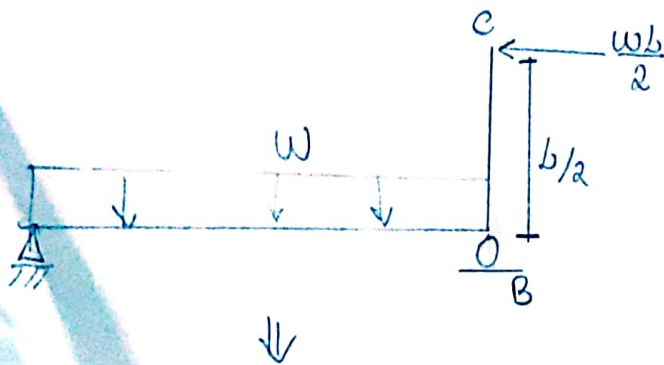
$$P = \frac{P_{cr}}{2} \rightarrow P = 48447,3 \, N$$

Plano xz

$$P_{cr} = \frac{24223,65}{(2)^2} = 6055,91 \, N$$

$$P = \frac{P_{cr}}{2} \rightarrow P = 3027,95 \, N$$

Questão 1:

$$H_A - \frac{\omega L}{2} = 0$$
$$H_A = \frac{\omega L}{2}$$

$$\begin{cases} V_A + V_B - \omega L = 0 \\ V_A + V_B = \omega L \end{cases}$$

$$\sum M_B = 0 = -L V_A + \omega L \cdot \frac{L}{2}$$

$$V_A = -\frac{\omega L^2}{-2L} \rightarrow V_A = \frac{\omega L}{2}$$

Com 'x' indo da esquerda para a direita

$$M(x) = \frac{\omega L}{2} x - \omega x \cdot \frac{x}{2} \Rightarrow M(x) = \frac{\omega L x}{2} - \frac{\omega x^2}{2}$$

$$EI \frac{d^2 v}{dx^2} = M(x)$$

$$EI \theta = \int M(x)\, dx = \frac{\omega L x^2}{4} - \frac{\omega x^3}{6} + C_1$$

$$EI v = \int \theta(x)\, dx = \frac{\omega L x^3}{12} - \frac{\omega x^4}{24} + C_1 x + C_2$$

Condições de contorno: ① $x = 0 \rightarrow v(x) = 0$ / ② $x = L \rightarrow v(x) = 0$

① $v(0) = 0 = \frac{1}{EI}\left(\frac{\omega L 0^3}{12} - \frac{\omega 0^4}{24} + C_1(0) + C_2 \right)$

$$C_2 = 0$$

$$\mathcal{V}(b) = 0 = \frac{\omega L^4}{12} - \frac{\omega L^4}{24} + C_1 b$$

$$C_1 b = -\frac{\omega L^4}{24} \rightarrow C_1 = -\frac{\omega L^3}{24}$$

$$\begin{cases} \mathcal{V}(x) = \frac{1}{EI}\left(\frac{\omega L x^3}{12} - \frac{\omega x^4}{24} - \frac{\omega L^3 x}{24}\right) \\[2mm] \Theta(x) = \frac{1}{EI}\left(\frac{\omega L x^2}{4} - \frac{\omega x^3}{6} - \frac{\omega L^3}{24}\right) \end{cases}$$

$$\mathcal{V}(L/2) = ?$$
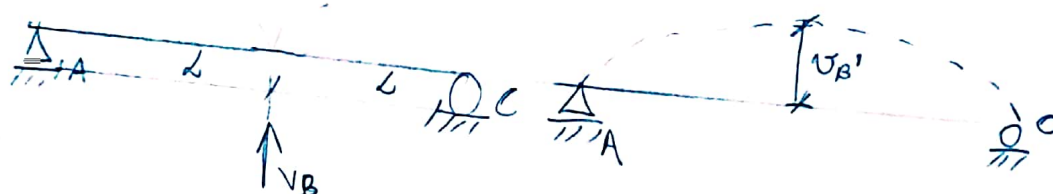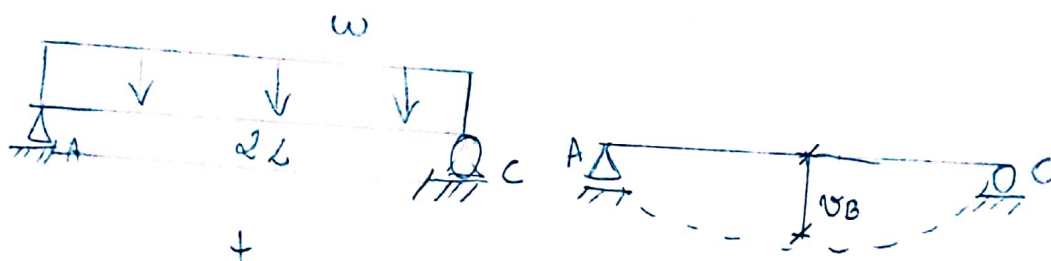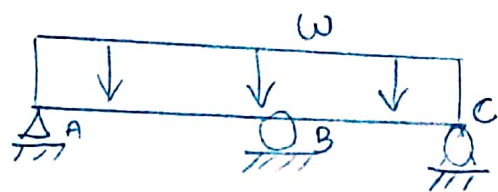
$$\mathcal{V}(L/2) = \frac{1}{EI}\left(\frac{\omega L}{12} \cdot \frac{L^3}{8} - \frac{\omega}{24} \cdot \frac{L^4}{16} - \frac{\omega L^3}{24} \cdot \frac{L}{2}\right)$$

$$\mathcal{V}(L/2) = \frac{1}{EI}\left(\frac{\omega L^4}{96} - \frac{\omega L^4}{384} - \frac{\omega L^4}{48}\right)$$

$$\mathcal{V}(L/2) = \frac{1}{EI}\left(\frac{4\omega L^4 - \omega L^4 - 8\omega L^4}{384}\right)$$

$$\mathcal{V}(L/2) = \frac{1}{EI}\left(\frac{-5\omega L^4}{384}\right)$$

Questão 2

Equação de compatibilidade: $v_B + v_B' = 0$

$v_B = -\dfrac{\omega x}{24EI}(x^3 - 2(2L)x^2 + (2L)^3) \Rightarrow$ para $x = L$

$v_B = -\dfrac{\omega x}{24EI}(x^3 - 4Lx^2 + 8L^3)$

$v_B(L) = -\dfrac{\omega L}{24EI}(L^3 - 4L^3 + 8L^3)$

$v_B(L) = -\dfrac{\omega 5L^4}{24EI}$

$v_B' = \dfrac{V_B x}{48EI}(3(2L)^2 - 4x^2)$

$v_B' = \dfrac{V_B x}{48EI}(12L^2 - 4x^2) \Rightarrow$ para $x = L$
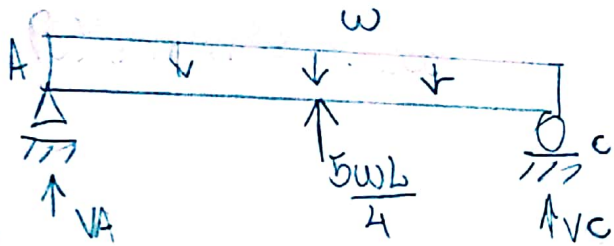
$v_B'(L) = \dfrac{V_B L}{48EI}(12L^2 - 4L^2)$

$v_B'(L) = \dfrac{V_B L^3}{6EI}$

$v_B(L) + v_B'(L) = 0$

$$-\frac{\omega 5 L^4}{24EI} + \frac{V_B L^3}{6EI} = 0 \quad \rightarrow \quad V_B = \frac{5\omega L^4}{24EI} \cdot \frac{6EI}{L^3} \Rightarrow \frac{5\omega L}{4}$$

$$\frac{V_B L^3}{6EI} = \frac{\omega 5 L^4}{24EI}$$

$$\sum F = 0 = V_A + V_C + \frac{5\omega L}{4} - \omega 2 L$$

$$V_A + V_C = \frac{3\omega L}{4}$$

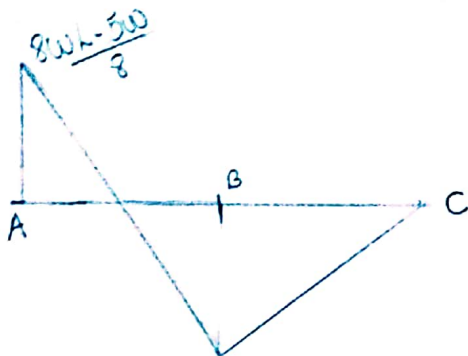$$\sum M_C = 0 = -V_A \cdot 2L + 2\omega L \cdot L - \frac{5\omega L}{4}$$

$$-2 V_A L + 2\omega L^2 - \frac{5\omega L}{4} = 0$$
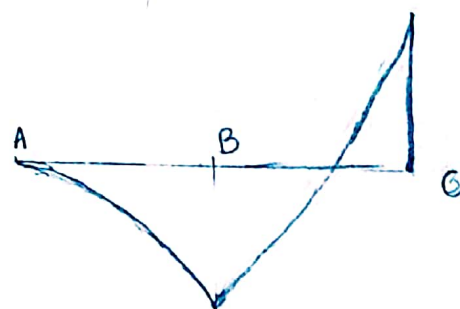
$$V_A = \frac{8\omega L - 5\omega}{8}$$

$$V_A + V_C = \frac{3\omega L}{4}$$

$$V_C = \frac{3\omega L}{4} - \left(\frac{8\omega L - 5\omega}{8}\right)$$
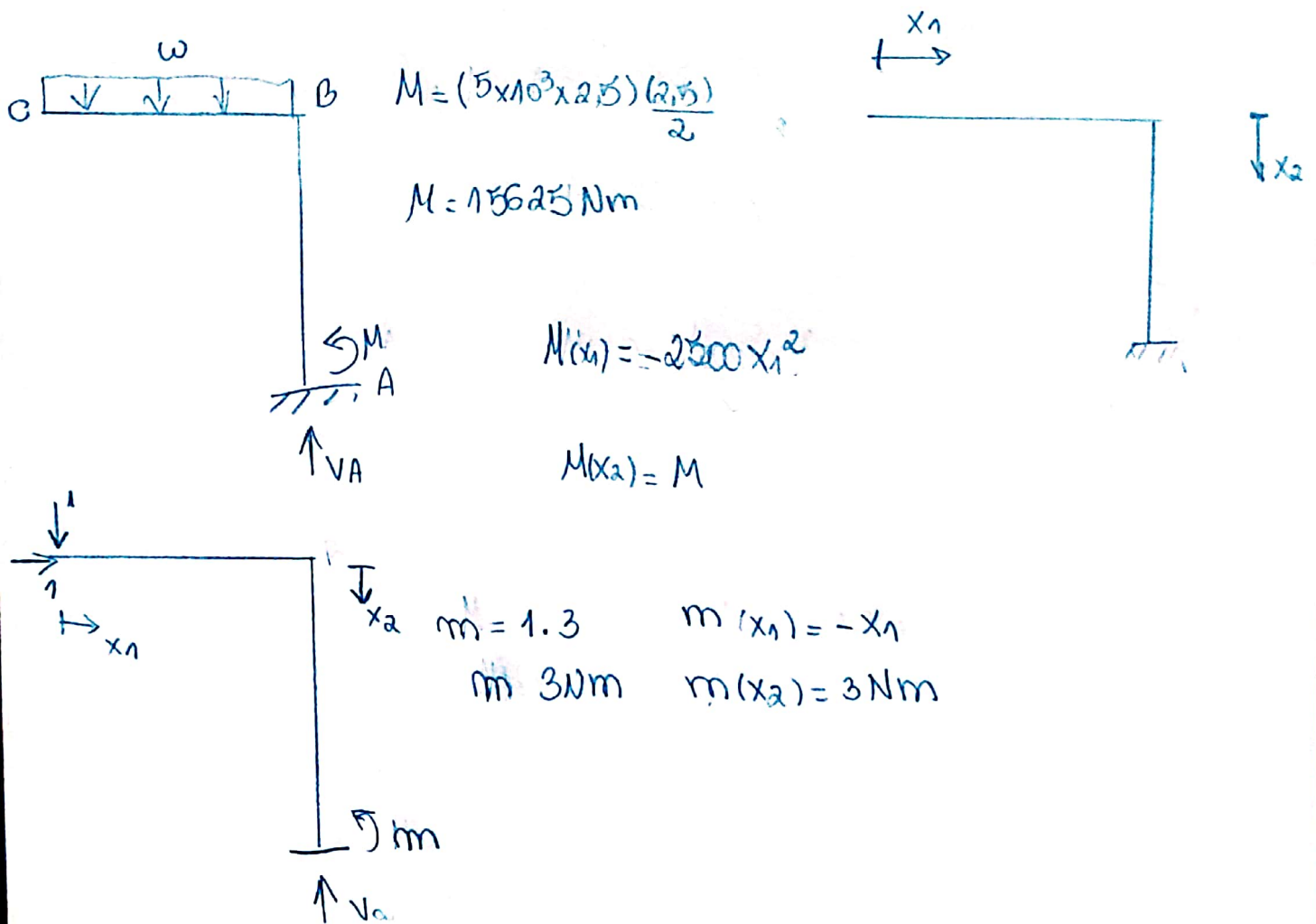
$$V_C = -\frac{2\omega L + 5\omega}{8}$$

$\frac{8\omega L - 5\omega}{8}$

Cortante

$\frac{-5\omega L}{8}$

momento

(5)

Questão 4

C ⊏̲▽̲ ̲▽̲ ̲▽̲⊐ B $\qquad$ $M = \dfrac{(5\times10^3 \times 2,5)(2,5)}{2}$

ω

$M = 15625\,Nm$

$\circlearrowleft M$
A

$M(x_1) = -2500\,x_1^2$

↑ V_A $\qquad$ $M(x_2) = M$

$m = 1.3$ $\qquad$ $m(x_1) = -x_1$

$\dot{m}$ $3Nm$ $\qquad$ $m(x_2) = 3\,Nm$

$\underline{} \circlearrowleft m$

↑ V_a

Horizontal

$$1.\Delta = \int_0^{2,5} \frac{m(x_1)\,M(x_1)}{EI}\,dx_1$$

Vertical.

$$1.\Delta = \int_0^3 \frac{m(x_2)\,M(x_2)}{EI}\,dx_2$$