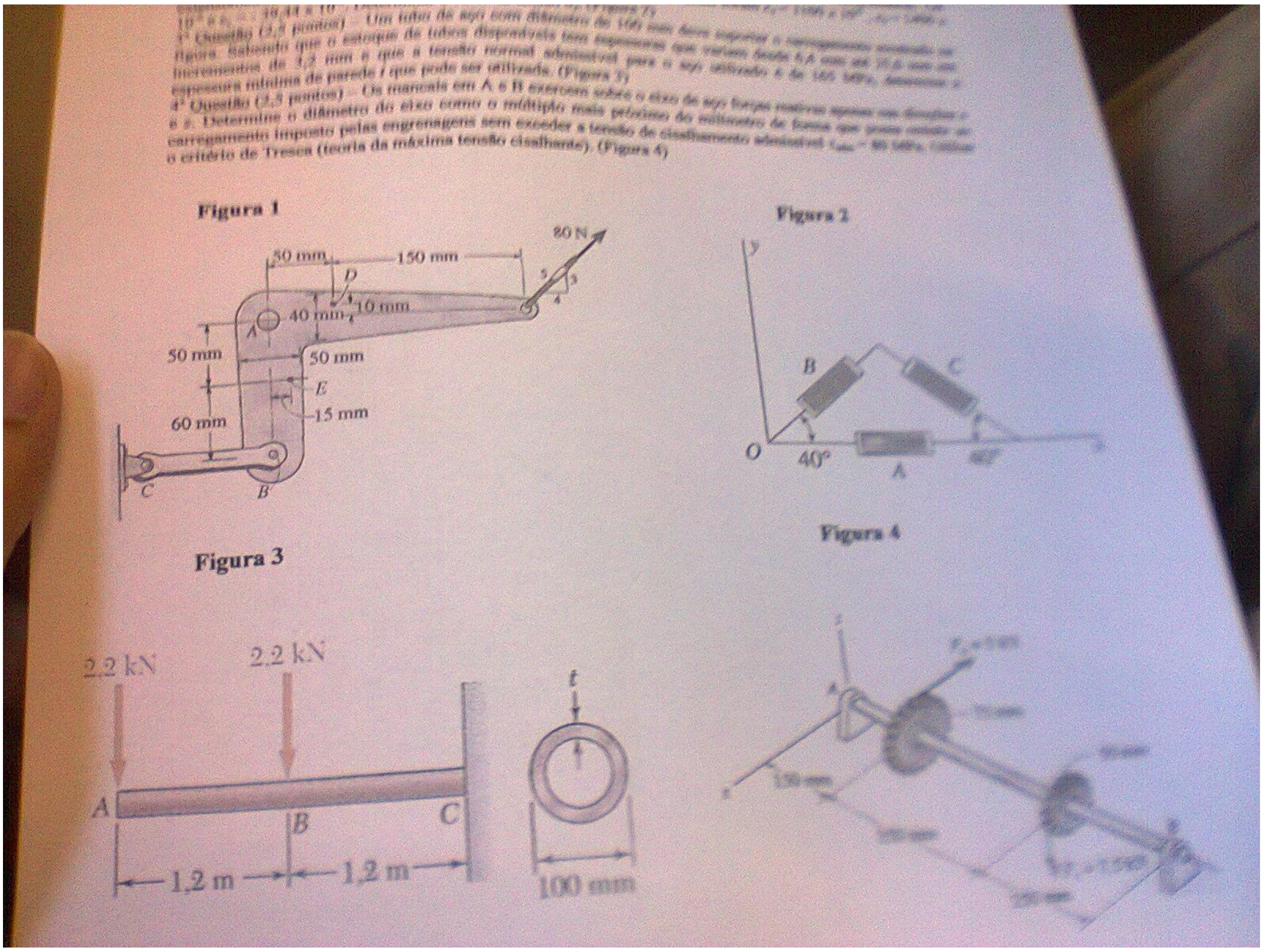
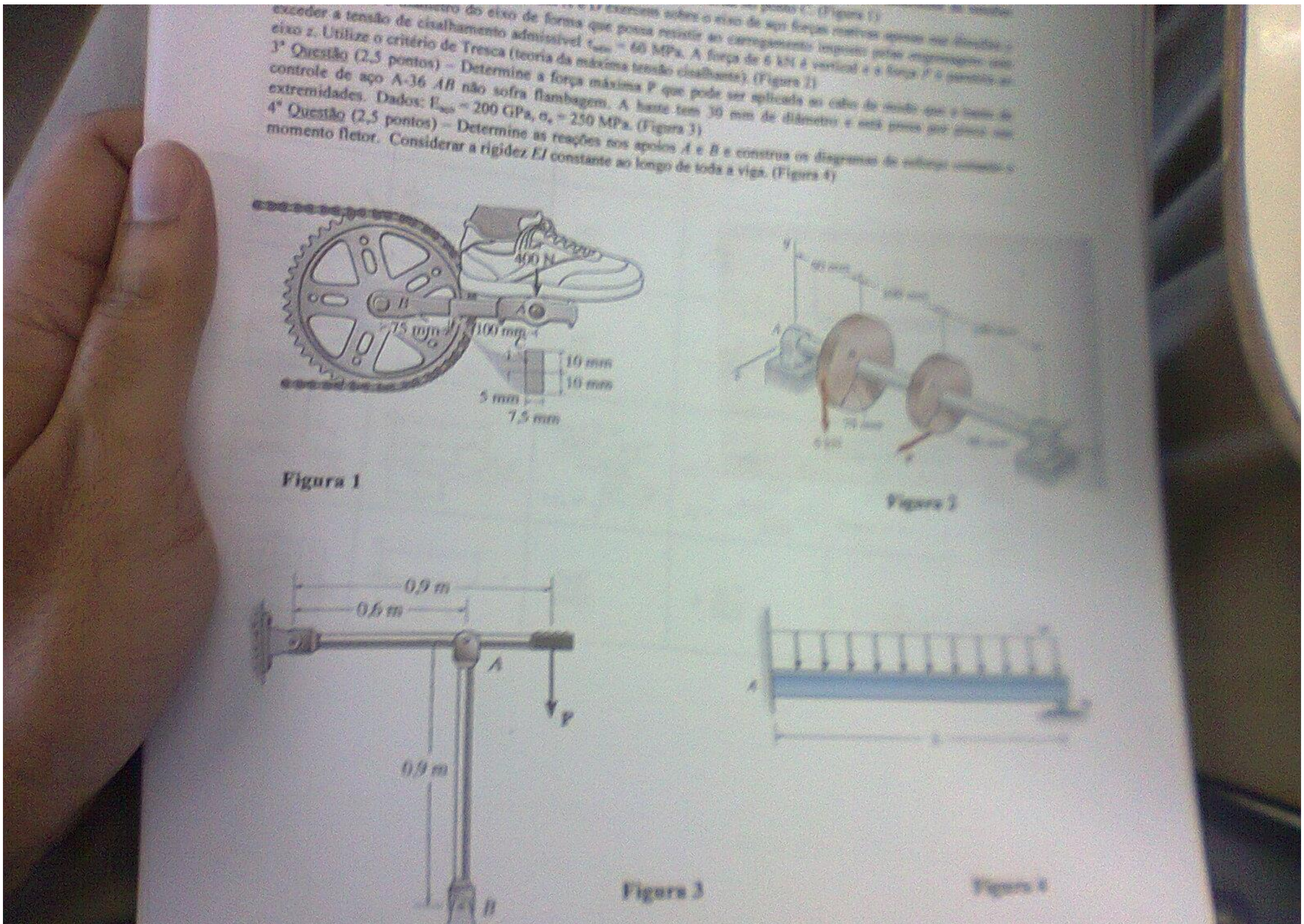
Prova – Reposição P1



Prova – PF 2012_1



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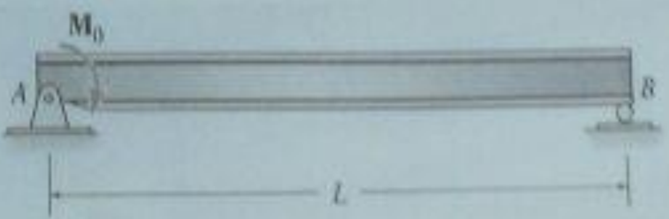
DEPARTAMENTO DE ENGENHARIA MECÂNICA

<u>2ª PROVA DE RESISTÊNCIA DOS MATERIAIS VII – 2013-1</u>

1ª <u>Questão</u> (2,5 pontos) – Determine a equação da linha elástica para a viga bi apoiada sujeita ao momento fletor M_o indicado. Calcule a inclinação em A e o deslocamento máximo. Considerar EI constante. (Figura 1)
2ª <u>Questão</u> (2,5 pontos) – Para a viga uniforme (EI = constante) e o carregamento mostrado na figura determine as reações de apoio. Utilizar a superposição de efeitos (tabela anexa). (Figura 2)
3ª <u>Questão</u> (2,5 pontos) – Uma carga de 60 kN é suportada por um tirante AB e uma escora tubular BC. O tirante tem um diâmetro de 30 mm e é feito de aço com E = 210 GPa e σ_e = 360 MPa. A escora tubular tem diâmetro interno de 50 mm e espessura de 15 mm, e é feita de uma liga de alumínio com E = 73 GPa e σ_e = 280 MPa. Determine o fator de segurança do projeto indicando se é limitado por escoamento ou flambagem da estrutura. (Figura 3)
4ª <u>Questão</u> (2,5 pontos) – Determine, utilizando um método de energia, o deslocamento do ponto B da viga mostrada na figura. Considere EI constante. (Figura 4)



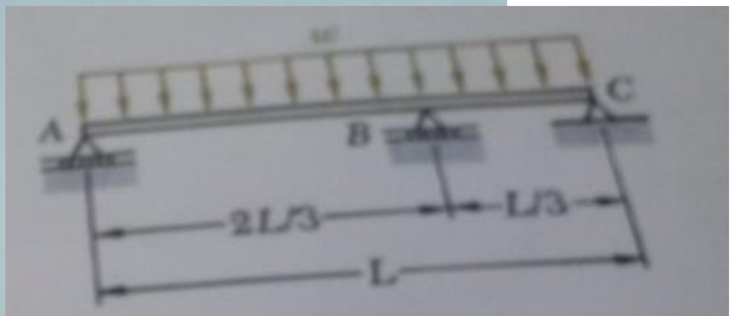
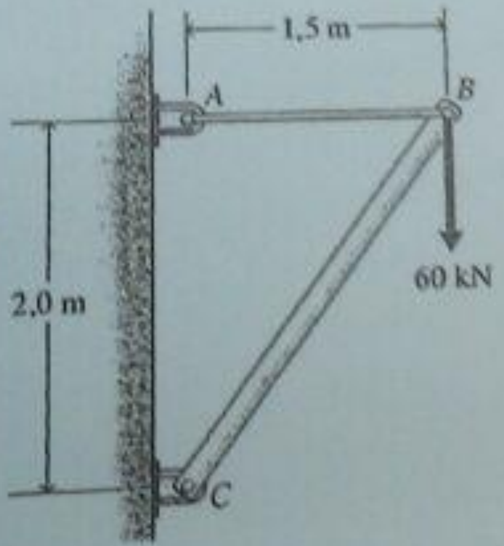


Figura 2

Figura 1



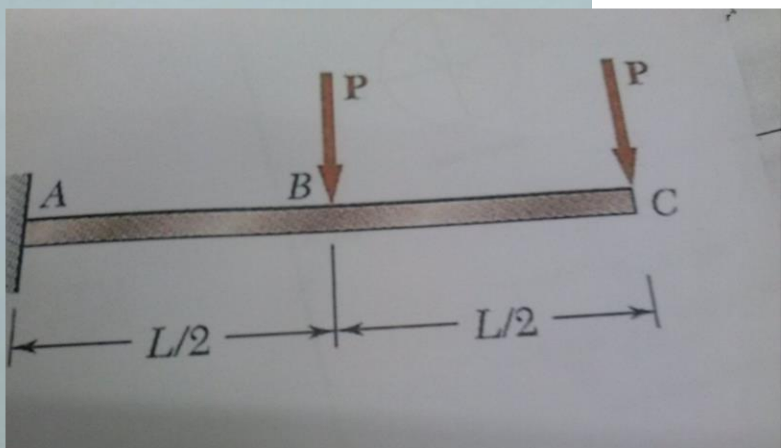


Figura 4

Figura 3

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2ª PROVA DE RESISTÊNCIA DOS MATERIAIS VII – 2013-1

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4ª Questão (2,5 pontos) – Determine, utilizando um método de energia, o deslocamento do ponto B da viga mostrada na figura. Considere EI constante. (Figura 4)

w

1ª Questão

B

M(x) =

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1ª PROVA DE RESISTÊNCIA DOS MATERIAIS VII – 2013-2

Questão (2,5 pontos) –Duas forças P_1 e P_2 de intensidade 15 kN e 18 kN, respectivamente, são aplicadas à barra 3 que está soldada a um eixo cilíndrico *BD* de raio $r = 20$ mm. Determine: (a) o estado de tensões no ponto *K*;) as tensões principais e a tensão de cisalhamento máximo no ponto *K*. (Figura 1)

Questão (2,5 pontos) – As deformações na superfície de um dispositivo experimental feito de alumínio puro (E 70 GPa, $\nu = 0{,}33$) e testado em um ônibus espacial foram medidas por meio de extensômetros. Os tensômetros foram orientados conforme a figura e as deformações medidas foram $\varepsilon_a = 1100 \times 10^{-6}$, $\varepsilon_b = 1496 \times$ 0^{-6} e $\varepsilon_c = -39{,}44 \times 10^{-6}$. Determinar o valor da tensão σ_y. (Figura 2)

Questão (2,5 pontos) – Um eixo tem 150 mm de diâmetro *d* e é feito de aço com limite de escoamento $\sigma_e = 360$ MPa para tração e compressão. As cargas aplicadas são $P = 2200$ kN e $T = 38$ kN.m. Determine o coeficiente de segurança à falha por escoamento de acordo com a teoria da máxima energia de distorção. (Figura 3)

Questão (2,5 pontos) – Um eixo *AB* gira a 360 rpm e transmite 20 kW do motor às máquinas-ferramentas acopladas às engrenagens *E* e *F*. Sabendo que $\tau_{adm} = 45$ MPa e considerando que são transmitidos 10 kW em cada engrenagem, determinar o menor diâmetro admissível para o eixo *AB*. Utilize o critério de Tresca. (Figura 4)

Figura 1

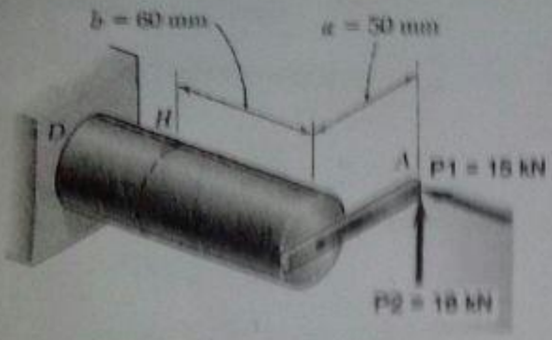


Figura 2

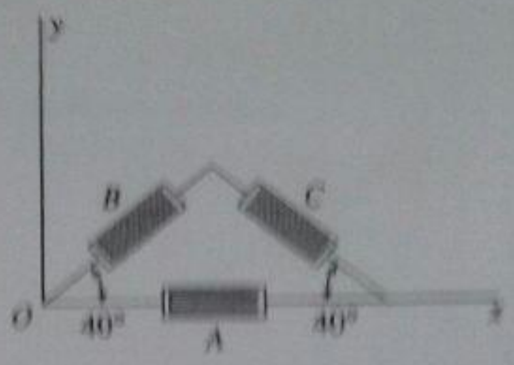
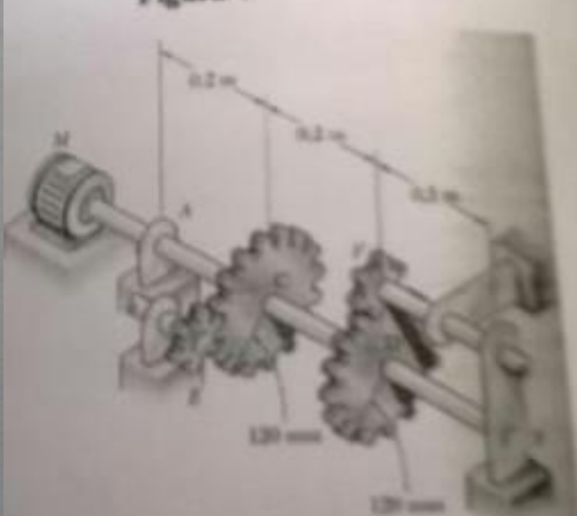


Figura 3

Figura 4



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EXAME FINAL DE RESISTÊNCIA DOS MATERIAIS VII – 2013-1

1ª Questão (2,5 pontos) – Duas forças P_1 e P_2 de intensidade 15 kN e 18 kN, respectivamente, são aplicadas à barra *AB* que está soldada a um eixo cilíndrico *BD* de raio $r = 20$ mm. Determine: (a) o estado de tensões no ponto *K*; (b) as tensões principais e a tensão de cisalhamento máximo no ponto *K*. (Figura 1)

2ª Questão (2,5 pontos) – Os mancais em A e D exercem sobre o eixo de aço forças reativas apenas nas direções *y* e *z*. Determine o diâmetro do eixo de forma que possa resistir ao carregamento imposto pelas engrenagens sem exceder a tensão de cisalhamento admissível $\tau_{adm} = 60$ MPa. A força de 6 kN é vertical e a força *P* é paralela ao eixo *z*. Utilize o critério de Tresca (teoria da máxima tensão cisalhante). (Figura 2)

3ª Questão (2,5 pontos) – Determine a força máxima P que pode ser aplicada ao cabo de modo que a haste de controle de aço A-36 *AB* não sofra flambagem. A haste tem 30 mm de diâmetro e está presa por pinos nas extremidades. Dados: $E_{aço} = 200$ GPa, $\sigma_e = 250$ MPa. (Figura 3)

4ª Questão (2,5 pontos) – Determine as reações nos apoios *A* e *B* e construa os diagramas de esforço cortante e momento fletor. Considerar a rigidez *EI* constante ao longo de toda a viga. (Figura 4)

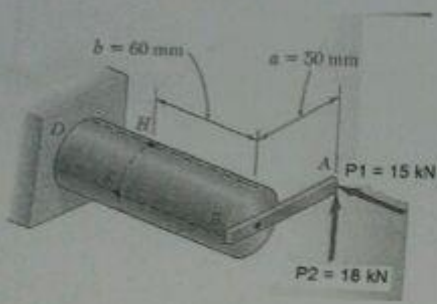


Figura 1

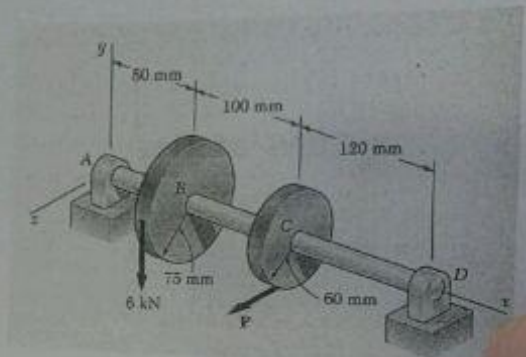


Figura 2

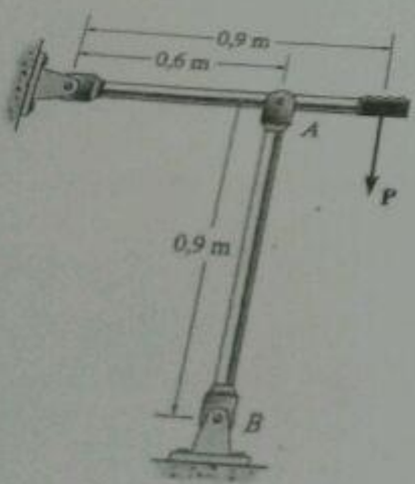


Figura 3

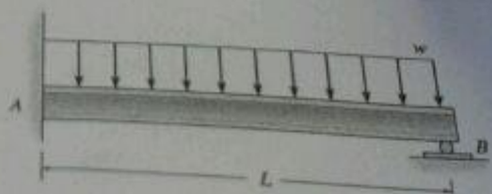


Figura 4

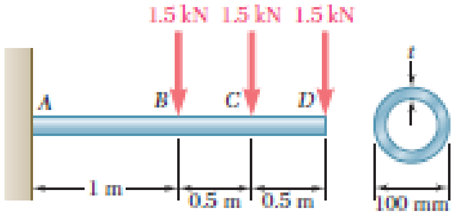


Fig. P5.82

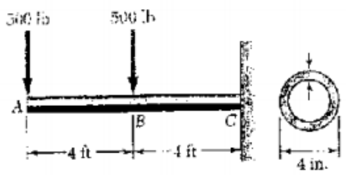
5.82 A steel pipe of 100-mm diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from 6 mm to 24 mm in 3-mm increments, and that the allowable normal stress for the steel used is 150 MPa, determine the minimum wall thickness t that can be used.

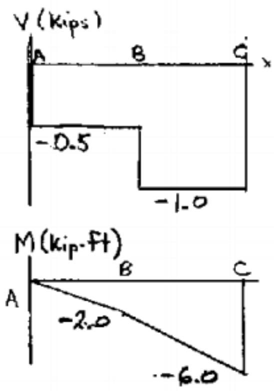
5.83 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

5-79 Um tubo de aço, de diâmetro de 4 Polegadas é para suportar o carregamento imediata. Sabendo que o estoque de tubos disponíveis tem espessuras variando de 1/4 polegada de 1 polegada polegada 1/8 polegada. incrementos, e que a tensão admissível normal para o aço utilizado é de 24 KSI, determinar a espessura mínima de parede T que pode ser usado.

Problem 5.82

5.82 A steel pipe of 4-in. diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from $\frac{1}{4}$ in. to 1 in. in $\frac{1}{8}$ - in. increments, and that the allowable normal stress for the steel used is 24 ksi, determine the minimum wall thickness t that can be used.





Shear: A to B $\quad V = -500$ lb $= -0.5$ kip

B to C $\quad V = -500 - 500 = -1000$ lb $= -1.0$ kip

Areas: A to B $\quad (4)(-0.5) = -2.0$ kip·ft

B to C $\quad (4)(-1.0) = -4.0$ kip·ft

Bending moments: $\quad M_A = 0$

$M_B = 0 - 2.0 = -2.0$ kip·ft

$M_C = -2.0 - 4.0 = -6.0$ kip·ft

Maximum $|M| = 6.0$ kip·ft $= 72$ kip·in

$\sigma_{all} = 24$ ksi

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{72}{24} = 3 \text{ in}^3$$

$I = \frac{\pi}{4}(c_2^4 - c_1^4) \qquad c = c_2 \qquad c_2 = \frac{1}{2}d = 2.0$ in.

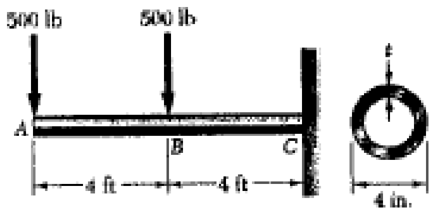
$$S = \frac{I}{c} = \frac{\pi}{4}\frac{c_2^4 - c_1^4}{c_2} = \frac{\pi}{4}\frac{2^4 - c_1^4}{2} = 3 \text{ in}^3$$

$c_1^4 = 2^4 - \frac{(4)(2)(3)}{\pi} = 8.3606$ in^4 $\qquad c_1 = 1.7004$ in.

$t_{min} = c_2 - c_1 = 2.0 - 1.7004 = 0.2996$ in

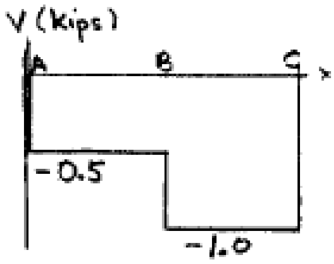
Using $\frac{1}{8}$ in. increments for design $\quad t = \frac{3}{8}$ in.

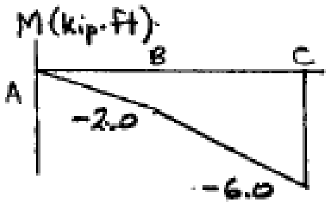
PROBLEM 5.92



5.92 A steel pipe of 4-in. diameter is to support the loading shown. Knowing that the stock of pipes available have thicknesses varying from $\frac{1}{4}$ in. to 1 in. in $\frac{1}{8}$- in. increments, and that the allowable normal stress for the steel used is 24 ksi, determine the minimum wall thickness t that can be used.

SOLUTION





Shear: A to B $\quad V = -500 \text{ lb} = -0.5 \text{ kip}$

B to C $\quad V = -500 - 500 = -1000 \text{ lb} = -1.0 \text{ kip}$

Areas: A to B $\quad (4)(-0.5) = -2.0 \text{ kip·ft}$

B to C $\quad (4)(-1.0) = -4.0 \text{ kip·ft}$

Bending moments: $\quad M_A = 0$

$M_B = 0 - 2.0 = -2.0 \text{ kip·ft}$

$M_C = -2.0 - 4.0 = -6.0 \text{ kip·ft}$

Maximum $|M| = 6.0 \text{ kip·ft} = 72 \text{ kip·in}$

$\sigma_{all} = 24 \text{ ksi}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{72}{24} = 3 \text{ in}^3$$

$I = \frac{\pi}{4}(c_2^4 - c_1^4) \qquad c = c_2 \qquad c_2 = \frac{1}{2}d = 2.0 \text{ in.}$

$$S = \frac{I}{c} = \frac{\pi}{4}\,\frac{c_2^4 - c_1^4}{c_2} = \frac{\pi}{4}\,\frac{2^4 - c_1^4}{2} = 3 \text{ in}^3$$

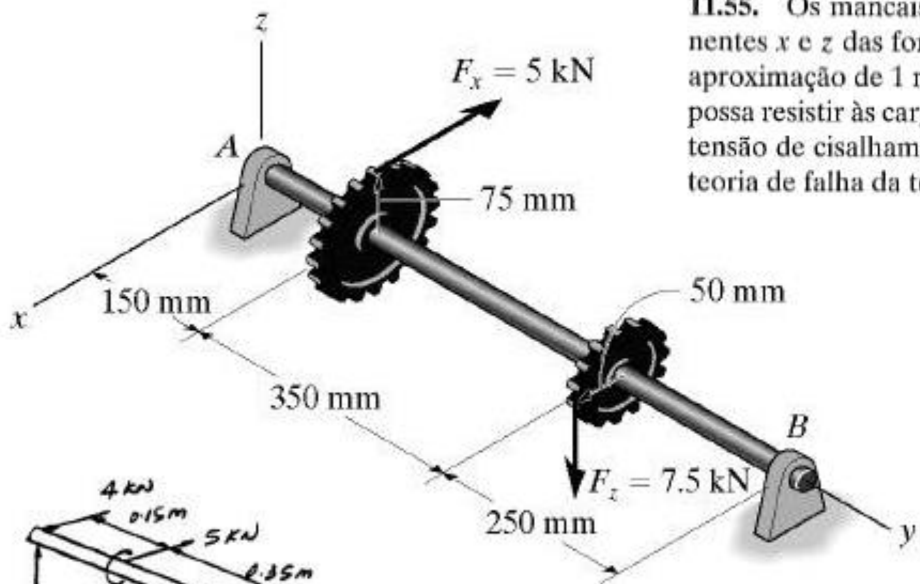
$c_1^4 = 2^4 - \frac{(4)(2)(3)}{\pi} = 8.3606 \text{ in}^4 \qquad c_1 = 1.7004 \text{ in.}$

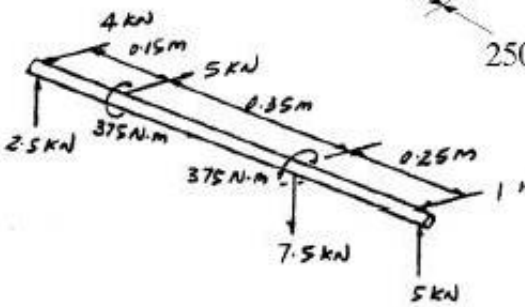
$t_{min} = c_2 - c_1 = 2.0 - 1.7004 = 0.2996 \text{ in}$

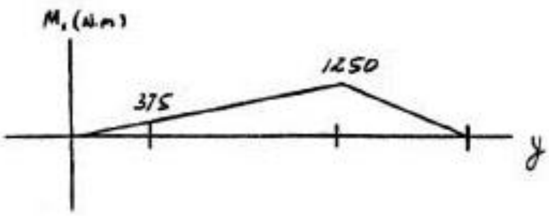
Using $\frac{1}{8}$ in. increments for design $\quad t = \frac{3}{8}$ in. ◀

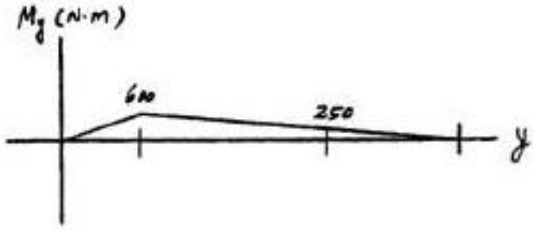
11–55. The bearings at A and B exert only x and z components of force on the steel shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings of the gears without exceeding an allowable shear stress of $\tau_{\text{allow}} = 80$ MPa. Use the maximum-shear-stress theory of failure.

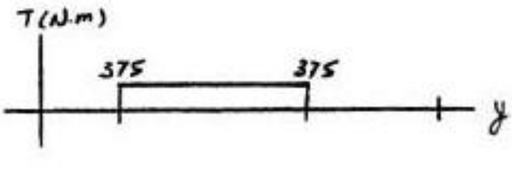
11.55. Os mancais em A e B exercem somente as componentes x e z das forças sobre o eixo de aço. Determine, com aproximação de 1 mm, o diâmetro do eixo, de modo que ele possa resistir às cargas das engrenagens sem ultrapassar uma tensão de cisalhamento admissível de $\tau_{\text{adm}} = 80$ MPa. Use a teoria de falha da tensão de cisalhamento máxima.











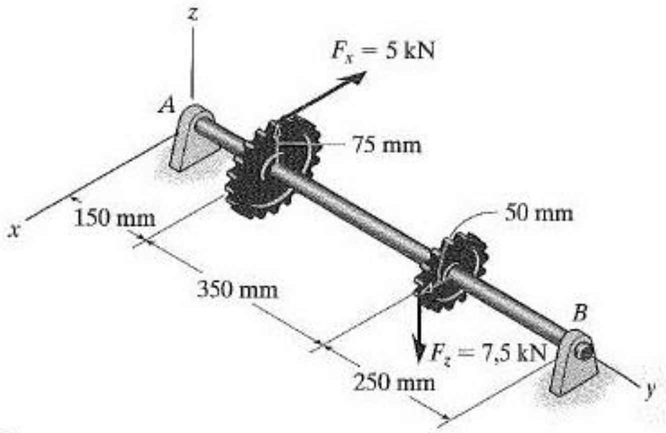
Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N}\cdot\text{m}$

$$c = [\frac{2}{\pi\,\tau_{\text{allow}}}\sqrt{M^2 + T^2}]^{\frac{1}{3}} = [\frac{2}{\pi(80)(10^6)}\sqrt{1274.75^2 + 375^2}]^{\frac{1}{3}} = 0.0219\ \text{m}$$

$d = 2c = 0.0439\ \text{m} = 43.9\ \text{mm}$

Use $d = 44$ **mm** **Ans**

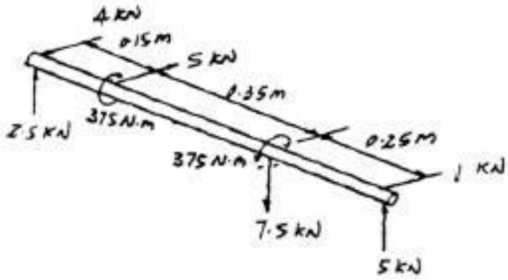
*11.56. Os mancais em A e B exercem somente as componentes x e z das forças sobre o eixo de aço. Determine, com aproximação de 1 mm, o diâmetro do eixo, de modo que ele possa resistir às cargas das engrenagens sem ultrapassar uma tensão de cisalhamento admissível $\tau_{adm} = 80$ MPa. Use a teoria de falha da energia de distorção máxima com $\sigma_{adm} = 200$ MPa.



a 11.56

Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75 \text{ N}\cdot\text{m}$

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$



Let $a = \dfrac{\sigma_x}{2}, b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

$$\sigma_1 = a + b, \qquad \sigma_2 = a - b$$

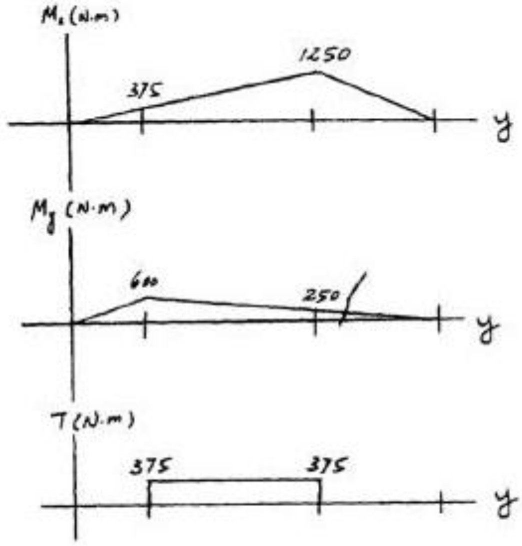
Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{allow}^2$$

$$a^2 + 3b^2 = \sigma_{allow}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{allow}^2$$



$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{allow}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{allow}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{allow}^2$$

$$c^6 = \frac{16}{\sigma_{allow}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{allow}^2\pi^2}$$

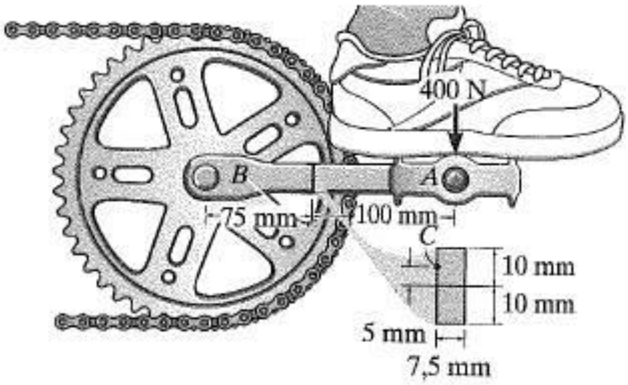
$$c = \left[\frac{4}{\sigma_{allow}^2\pi^2}(4M^2 + 3\,T^2)\right]^{\frac{1}{6}}$$

$$= \left[\frac{4}{(200(10^6))^2(\pi)^2}\,(4(1274.75)^2 + 3(375)^2)\right]^{\frac{1}{6}}$$

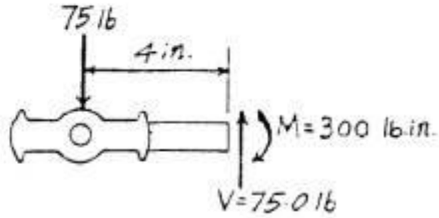
$$= 0.0203 \text{ m} = 20.3 \text{ mm}$$

$d = 40.6$ mm **Ans**

*9.84. A manivela do pedal de uma bicicleta tem a seção transversal mostrada na figura. Se ela estiver presa à engrenagem em B e não girar quando submetida a uma força de 400 N, determine as tensões principais no material na seção transversal no ponto C.



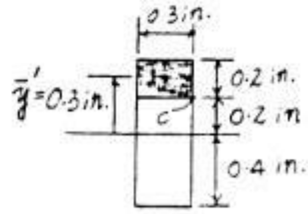
Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I = \frac{1}{12}(0.3)\left(0.8^3\right) = 0.0128 \text{ in}^4$$

$$Q_C = \bar{y}'A' = 0.3(0.2)(0.3) = 0.0180 \text{ in}^3$$

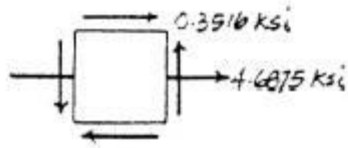


Normal Stress: Applying the flexure formula.

$$\sigma_C = -\frac{My}{I} = -\frac{-300(0.2)}{0.0128} = 4687.5 \text{ psi} = 4.6875 \text{ ksi}$$

Shear Stress: Applying the shear formula.

$$\tau_C = \frac{VQ_C}{It} = \frac{75.0(0.0180)}{0.0128(0.3)} = 351.6 \text{ psi} = 0.3516 \text{ ksi}$$



Construction of the Circle: In accordance with the sign convention. $\sigma_x = 4.6875$ ksi. $\sigma_y = 0$, and $\tau_{xy} = 0.3516$ ksi. Hence.

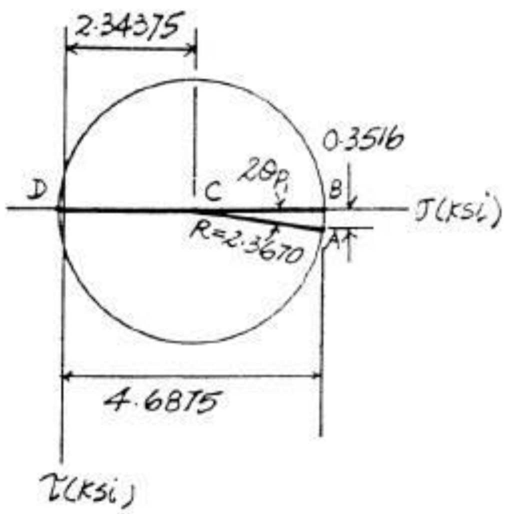
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{4.6875 + 0}{2} = 2.34375 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(4.6875,\ 0.3516) \qquad C(2.34375,\ 0)$$

The radius of the circle is

$$R = \sqrt{(4.6875 - 2.34375)^2 + 0.3516^2} = 2.3670 \text{ ksi}$$



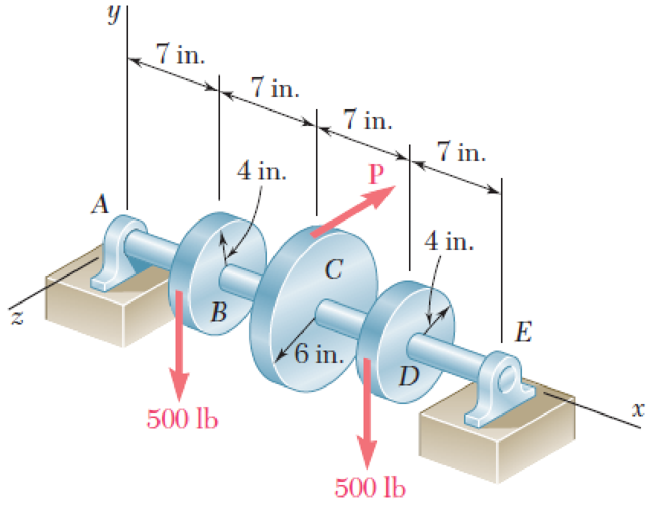
In-Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 2.34375 + 2.3670 = 4.71 \text{ ksi} \qquad \textbf{Ans}$$

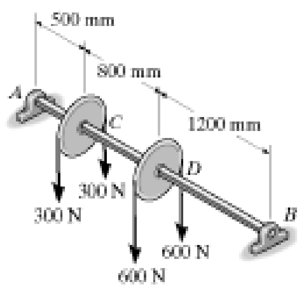
$$\sigma_2 = 2.34375 - 2.3670 = -0.0262 \text{ ksi} \qquad \textbf{Ans}$$

8.16 The two 500-lb forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 8$ ksi, determine the smallest permissible diameter of the solid shaft AE.

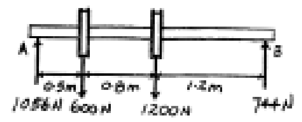
524



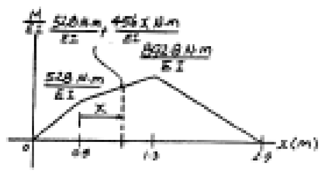
12–75. Determine the maximum deflection of the 50-mm-diameter A-36 steel shaft. It is supported by bearings at its ends A and B which only exert vertical reactions on the shaft.



Moment - Area Theorems:



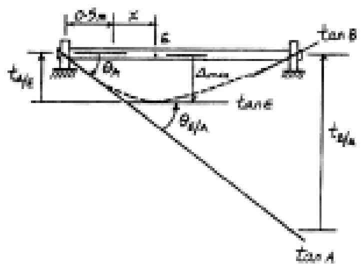
$$t_{B/A} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(0.8) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.4667)$$
$$+\left(\frac{528}{EI}\right)(0.8)(1.6) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(2.1667)$$
$$= \frac{1604.4 \text{ N}\cdot\text{m}^3}{EI}$$



$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{1604.4 \text{ N}\cdot\text{m}^3}{EI}}{2.5 \text{ m}} = \frac{641.76 \text{ N}\cdot\text{m}^2}{EI}$$

The maximum displacement occurs at point E, where $\theta_E = 0$.

$$\theta_{E/A} = \frac{1}{2}\left(\frac{528}{EI}\right)(0.5) + \left(\frac{528}{EI}\right)x + \frac{1}{2}\left(\frac{456}{EI}x\right)x$$
$$= \frac{1}{EI}\left(228x^2 + 528x + 132\right)$$

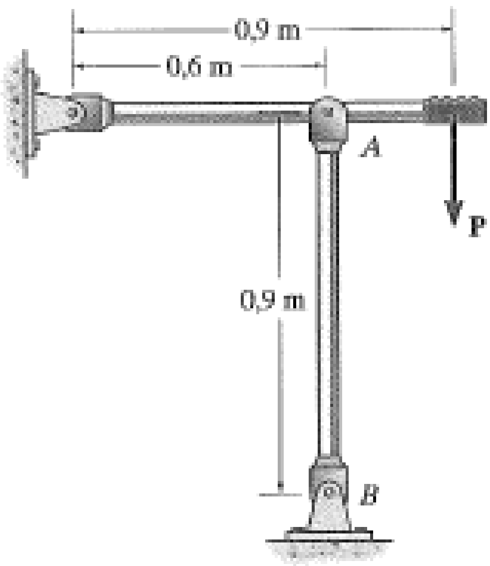


$$\theta_E = \theta_A + \theta_{E/A}$$
$$0 = -\frac{641.76}{EI} + \frac{1}{EI}\left(228x^2 + 528x + 132\right)$$
$$x = 0.7333 \text{ m} < 0.8 \text{ m} \quad (O.K!)$$

The maximum displacement is,

$$\Delta_{max} = |t_{A/E}| = \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(0.3333) + \left(\frac{528}{EI}\right)(0.7333)(0.8666)$$
$$+\frac{1}{2}\left(\frac{456}{EI}\right)(0.7333^2)(0.9888)$$
$$= \frac{500.76 \text{ N}\cdot\text{m}^3}{EI}$$
$$= \frac{500.76}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)}$$
$$= 0.008161 \text{ m} = 8.16 \text{ mm} \downarrow \qquad \textbf{Ans}$$

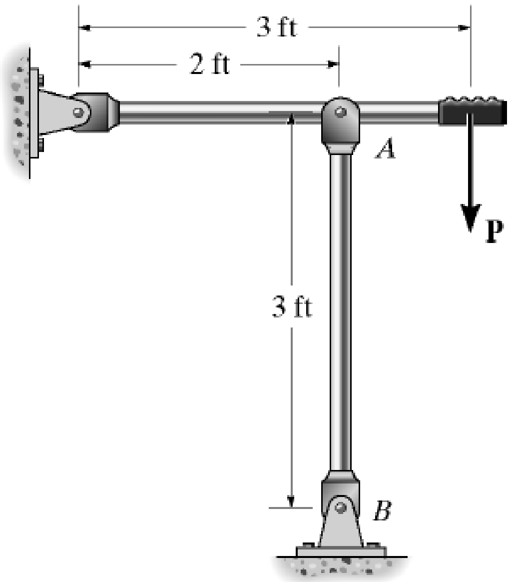
*13.12. Determine a força máxima P que pode ser aplicada ao cabo, de modo que a haste de controle de aço A-36 AB não sofra flambagem. A haste tem diâmetro de 30 mm e está presa por pinos nas extremidades.



Problema 13.12

Res.Mat. Hibbeler 7ªed

***13–12.** Determine the maximum force P that can be applied to the handle so that the A-36 steel control rod AB does not buckle. The rod has a diameter of 1.25 in. It is pin connected at its ends.



$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(2) - P(3) = 0$

$$P = \frac{2}{3}F_{AB} \qquad (1)$$

Bucking load for rod AB:

$$I = \frac{\pi}{4}(0.625^4) = 0.1198 \text{ in}^4$$

$$A = \pi(0.625^2) = 1.2272 \text{ in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = P_{cr} = \frac{\pi^2(29)(10^3)(0.1198)}{[1.0(3)(12)]^2} = 26.46 \text{ kip}$$

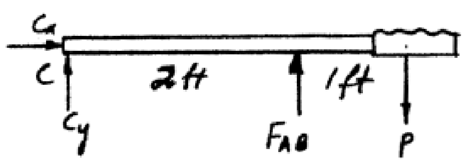
From Eq. (1)

$$P = \frac{2}{3}(26.46) = 17.6 \text{ kip} \qquad \textbf{Ans}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26.46}{1.2272} = 21.6 \text{ ksi} < \sigma_Y \text{ OK}$$

Therefore, Euler's formula is valid.



9.5 STATICALLY INDETERMINATE BEAMS

In the preceding sections, our analysis was limited to statically determinate beams. Consider now the prismatic beam AB (Fig. 9.23*a*), which has a fixed end at A and is supported by a roller at B. Drawing the free-body diagram of the beam (Fig. 9.23*b*), we note that the reactions involve four unknowns, while only three equilibrium equations are available, namely

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0 \tag{9.37}$$

Since only A_x can be determined from these equations, we conclude that the beam is *statically indeterminate*.

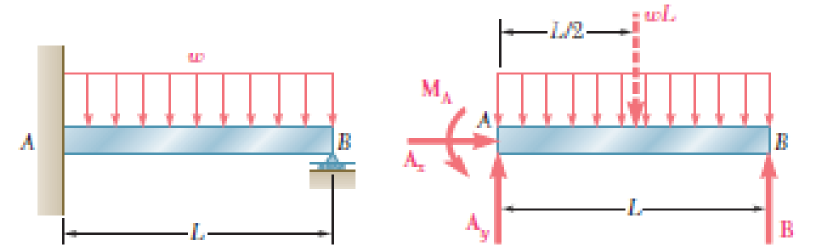


Fig. 9.23 Statically indeterminate beam.

However, we recall from Chaps. 2 and 3 that, in a statically indeterminate problem, the reactions can be obtained by considering the *deformations* of the structure involved. We should, therefore, proceed with the computation of the slope and deformation along the beam. Following the method used in Sec. 9.3, we first express the bending moment $M(x)$ at any given point of AB in terms of the distance x from A, the given load, and the unknown reactions. Integrating in x, we obtain expressions for θ and y which contain two additional unknowns, namely the constants of integration C_1 and C_2. But altogether six equations are available to determine the reactions and the constants C_1 and C_2; they are the three equilibrium equations (9.37) and the three equations expressing that the boundary conditions are satisfied, i.e., that the slope and deflection at A are zero, and that the deflection at B is zero (Fig. 9.24). Thus, the reactions at the supports can be determined, and the equation of the elastic curve can be obtained.

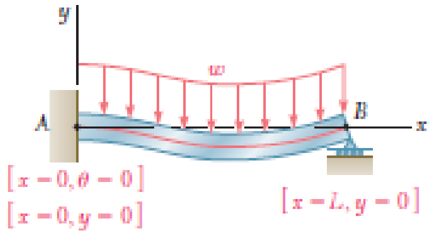


Fig. 9.24 Boundary conditions for beam of Fig. 9.23.

EXAMPLE 9.05

Determine the reactions at the supports for the prismatic beam of Fig. 9.23*a*.

Equilibrium Equations. From the free-body diagram of Fig. 9.23*b* we write

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \qquad A_x = 0$$
$$+\uparrow \Sigma F_y = 0: \qquad A_y + B - wL = 0 \tag{9.38}$$
$$+\circlearrowleft \Sigma M_A = 0: \qquad M_A + BL - \tfrac{1}{2}wL^2 = 0$$

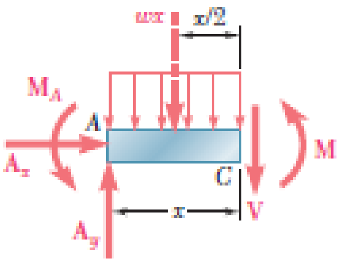


Fig. 9.25

Equation of Elastic Curve. Drawing the free-body diagram of a portion of beam AC (Fig. 9.25), we write

$$+\circlearrowleft \sum M_C = 0: \qquad M + \tfrac{1}{2}wx^2 + M_A - A_y x = 0 \tag{9.39}$$

Solving Eq. (9.39) for M and carrying into Eq. (9.4), we write

$$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + A_y x - M_A$$

Integrating in x, we have

$$EI\,\theta = EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}A_y x^2 - M_A x + C_1 \tag{9.40}$$

$$EI\,y = -\frac{1}{24}wx^4 + \frac{1}{6}A_y x^3 - \frac{1}{2}M_A x^2 + C_1 x + C_2 \tag{9.41}$$

Referring to the boundary conditions indicated in Fig. 9.24, we make $x = 0$, $\theta = 0$ in Eq. (9.40), $x = 0$, $y = 0$ in Eq. (9.41), and conclude that $C_1 = C_2 = 0$. Thus, we rewrite Eq. (9.41) as follows:

$$EI\,y = -\tfrac{1}{24}wx^4 + \tfrac{1}{6}A_y x^3 - \tfrac{1}{2}M_A x^2 \tag{9.42}$$

But the third boundary condition requires that $y = 0$ for $x = L$. Carrying these values into (9.42), we write

$$0 = -\tfrac{1}{24}wL^4 + \tfrac{1}{6}A_y L^3 - \tfrac{1}{2}M_A L^2$$

or

$$3M_A - A_y L + \tfrac{1}{4}wL^2 = 0 \tag{9.43}$$

Solving this equation simultaneously with the three equilibrium equations (9.38), we obtain the reactions at the supports:

$$A_x = 0 \qquad A_y = \tfrac{5}{8}wL \qquad M_A = \tfrac{1}{8}wL^2 \qquad B = \tfrac{3}{8}wL$$

EXAMPLE 9.08

Determine the reactions at the supports for the prismatic beam and loading shown in Fig. 9.33. (This is the same beam and loading as in Example 9.05 of Sec. 9.5.)

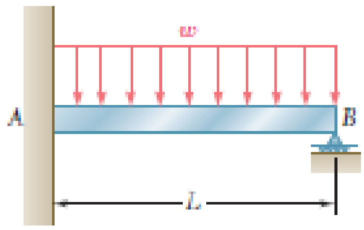
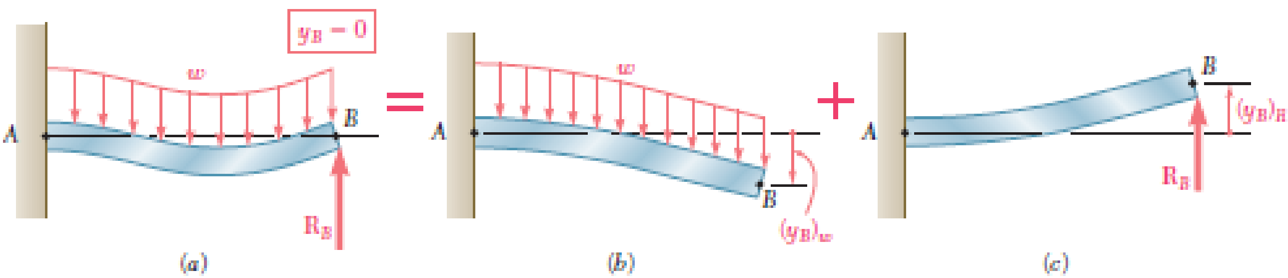


Fig. 9.33

We consider the reaction at B as redundant and release the beam from the support. The reaction $\mathbf{R}_B$ is now considered as an unknown load (Fig. 9.34*a*) and will be determined from the condition that the deflection of the beam at B must be zero. The solution is carried out by considering separately the deflection $(y_B)_w$ caused at B by the uniformly distributed load w (Fig. 9.34*b*) and the deflection $(y_B)_R$ produced at the same point by the redundant reaction $\mathbf{R}_B$ (Fig. 9.34*c*).

From the table of Appendix D (cases 2 and 1), we find that

$$(y_B)_w = -\frac{wL^4}{8EI} \qquad (y_B)_R = +\frac{R_B L^3}{3EI}$$



EXAMPLE 9.14

Determine the reaction at the supports for the prismatic beam and loading shown (Fig. 9.66).

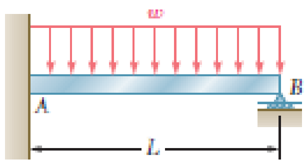


Fig. 9.66

We consider the couple exerted at the fixed end A as redundant and replace the fixed end by a pin-and-bracket support. The couple $\mathbf{M}_A$ is now considered as an unknown load (Fig. 9.67a) and will be determined from the condition that the tangent to the beam at A must be horizontal. It follows that this tangent must pass through the support B and, thus, that the tangential deviation $t_{B/A}$ of B with respect to A must be zero. The solution is carried out by computing separately the tangential deviation $(t_{B/A})_w$ caused by the uniformly distributed load w (Fig. 9.67b) and the tangential deviation $(t_{B/A})_M$ produced by the unknown couple $\mathbf{M}_A$ (Fig. 9.67c).

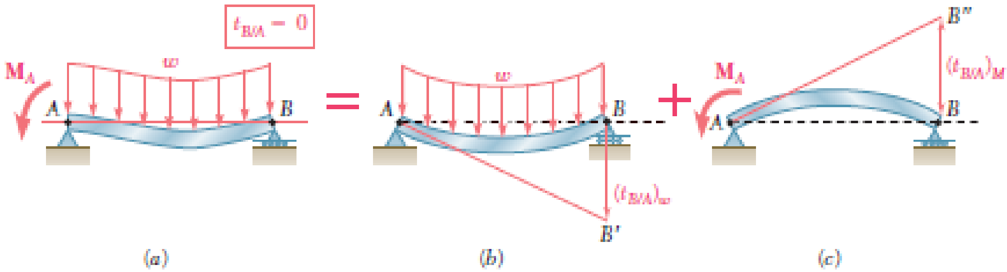


Fig. 9.67

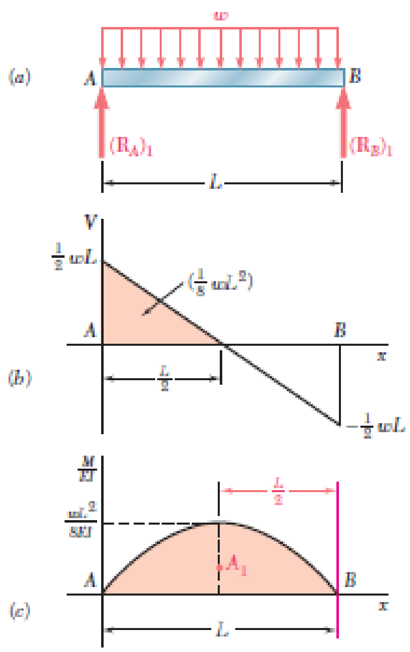


Fig. 9.68

Considering first the free-body diagram of the beam under the known distributed load w (Fig. 9.68a), we determine the corresponding reactions at the supports A and B. We have

$$(\mathbf{R}_A)_1 = (\mathbf{R}_B)_1 = \tfrac{1}{2}wL \uparrow \tag{9.64}$$

We can now draw the corresponding shear and (M/EI) diagrams (Figs. 9.68b and c). Observing that M/EI is represented by an arc of parabola, and recalling the formula, $A = \frac{2}{3}bh$, for the area under a parabola, we compute the first moment of this area about a vertical axis through B and write

$$(t_{B/A})_w = A_1\left(\frac{L}{2}\right) = \left(\frac{2}{3}L\frac{wL^2}{8EI}\right)\left(\frac{L}{2}\right) = \frac{wL^4}{24EI} \tag{9.65}$$

Considering next the free-body diagram of the beam when it is subjected to the unknown couple $\mathbf{M}_A$ (Fig. 9.69a), we determine the corresponding reactions at A and B:

$$(\mathbf{R}_A)_2 = \frac{M_A}{L}\uparrow \qquad (\mathbf{R}_B)_2 = \frac{M_A}{L}\downarrow \tag{9.66}$$

Drawing the corresponding (M/EI) diagram (Fig. 9.69b), we apply again the second moment-area theorem and write

$$(t_{B/A})_M = A_2\left(\frac{2L}{3}\right) = \left(-\frac{1}{2}L\frac{M_A}{EI}\right)\left(\frac{2L}{3}\right) = -\frac{M_AL^2}{3EI} \tag{9.67}$$

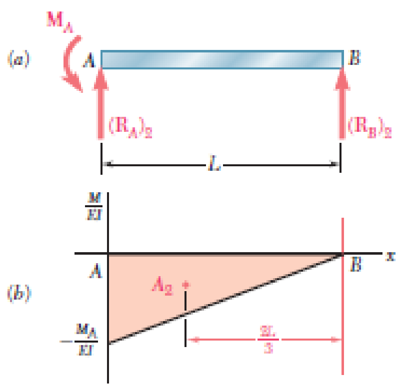


Fig. 9.69

Combining the results obtained in (9.65) and (9.67), and expressing that the resulting tangential deviation $t_{B/A}$ must be zero (Fig. 9.67), we have

$$t_{B/A} = (t_{B/A})_w + (t_{B/A})_M = 0$$

$$\frac{wL^4}{24EI} - \frac{M_AL^2}{3EI} = 0$$

and, solving for M_A,

$$M_A = +\tfrac{1}{8}wL^2 \qquad \mathbf{M}_A = \tfrac{1}{8}wL^2 ⟲$$

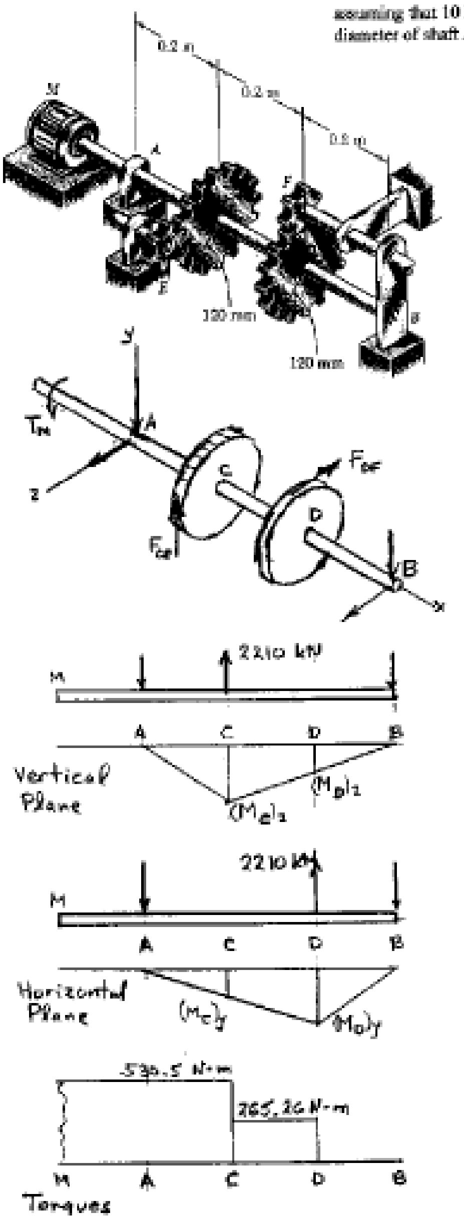
Substituting for M_A into (9.66), and recalling (9.64), we obtain the values of R_A and R_B:

$$R_A = (R_A)_1 + (R_A)_2 = \tfrac{1}{2}wL + \tfrac{1}{8}wL = \tfrac{5}{8}wL$$

$$R_B = (R_B)_1 + (R_B)_2 = \tfrac{1}{2}wL - \tfrac{1}{8}wL = \tfrac{3}{8}wL$$

Problem 8.29

8.29 The solid shaft AB rotates at 360 rpm and transmits 20 kW from the motor M to machine tools connected to gears E and F. Knowing that $\tau_{all} = 45$ MPa and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft AB.



$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$T_M = \frac{P_M}{2\pi f} = \frac{20\times 10^3}{2\pi(6)} = 530.5 \text{ N·m}$$

$$T_C = \frac{P_C}{2\pi f} = \frac{10\times 10^3}{2\pi(6)} = 265.26 \text{ N·m}$$

$$T_D = \frac{P_D}{2\pi f} = \frac{10\times 10^3}{2\pi(6)} = 265.26 \text{ N·m}$$

$$F_{CE} = \frac{T_C}{r_C} = \frac{265.26}{120\times 10^{-3}} = 2.210\times 10^3 \text{ N}$$

$$F_{DF} = \frac{T_D}{r_D} = \frac{265.26}{120\times 10^{-3}} = 2.210\times 10^3 \text{ N}$$

$$(M_C)_z = \frac{(0.2)(0.4)(2.210\times 10^3)}{0.6} = 294.7 \text{ N·m}$$

$$(M_D)_z = \tfrac{1}{2}(M_C)_z = 147.37 \text{ N·m}$$

$$(M_D)_y = \frac{(0.4)(0.2)(2.210\times 10^3)}{0.6} = 294.7 \text{ N·m}$$

$$(M_C)_y = \tfrac{1}{2}(M_D)_y = 147.37 \text{ N·m}$$

Torques in shaft.

$T_{MAC} = 530.5$ N·m

$T_{CD} = 265.26$ N·m

$T_{DB} = 0$

Just to the left of gear C.

$$\max\sqrt{M_y^2 + M_z^2 + T^2} = 624.5 \text{ N·m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\max\sqrt{M_y^2+M_z^2+T^2}}{\tau_{max}}$$

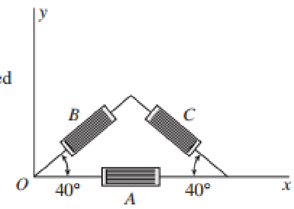
$$= \frac{624.5}{45\times 10^6} = 13.878\times 10^{-6} \text{ m}^3$$

$$c = 20.67\times 10^{-3} \text{ m} = 20.67 \text{ mm}$$

$$d = 2c = 41.3 \text{ mm}$$

Problem 7.7-22 The strains on the surface of an experimental device made of pure aluminum (E = 70 Gpa, ν = 0.33) and tested in a space shuttle were measured by means of strain gages. The gages were oriented as shown in the figure, and the measured strains were $\epsilon_a = 1100 \times 10^{-6}$, $\epsilon_b = 1496 \times 10^{-6}$, and $\epsilon_c = -39.44 \times 10 \times {}^{-6}$.

What is the stress σ_x in the x direction?



Solution 7.7-22 40-40-100° strain rosette

Pure aluminum: E = 70 GPa $\quad \nu = 0.33$

STRAIN GAGES

Gage A at $\theta = 0°$ $\quad \varepsilon_A = 1100 \times 10^{-6}$

Gage B at $\theta = 40°$ $\quad \varepsilon_B = 1496 \times 10^{-6}$

Gage C at $\theta = 140°$ $\quad \varepsilon_C = -39.44 \times 10^{-6}$

FOR $\theta = 0°$: $\quad \varepsilon_x = \varepsilon_A = 1100 \times 10^{-6}$

FOR $\theta = 40°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_{x_1} = \varepsilon_B = 1496 \times 10^{-6}$ and $\varepsilon_x = 1100 \times 10^{-6}$; then simplify and rearrange:

$$0.41318\varepsilon_y + 0.49240\gamma_{xy} = 850.49 \times 10^{-6} \qquad (1)$$

FOR $\theta = 140°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_{x_1} = \varepsilon_c = -39.44 \times 10^{-6}$ and $\varepsilon_x = 1100 \times 10^{-6}$; then simplify and rearrange:

$$0.41318\varepsilon_y - 0.49240\gamma_{xy} = -684.95 \times 10^{-6} \qquad (2)$$

SOLVE EQS. (1) AND (2):

$\varepsilon_y = 200.3 \times 10^{-6} \qquad \gamma_{xy} = 1559.2 \times 10^{-6}$

HOOKE'S LAW

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = 91.6 \text{ MPa} \quad \leftarrow$$

EXAMPLE 8.01

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$, of magnitude $P_1 = 15$ kN and $P_2 = 18$ kN, are applied as shown to the end A of bar AB, which is welded to a cylindrical member BD of radius $c = 20$ mm (Fig. 8.21). Knowing that the distance from A to the axis of member BD is $a = 50$ mm and assuming that all stresses remain below the proportional limit of the material, determine (*a*) the normal and shearing stresses at point K of the transverse section of member BD located at a distance $b = 60$ mm from end B, (*b*) the principal axes and principal stresses at K, (*c*) the maximum shearing stress at K.

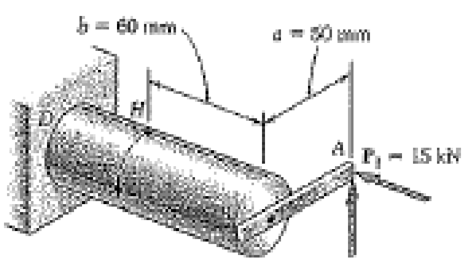


Fig. 8.21

Internal Forces in Given Section. We first replace the forces $\mathbf{P}_1$ and $\mathbf{P}_2$ by an equivalent system of forces and couples applied at the center C of the section containing point K (Fig. 8.22). This system, which represents the internal forces in the section, consists of the following forces and couples:

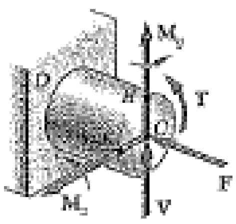


Fig. 8.22

1. A centric axial force $\mathbf{F}$ equal to the force $\mathbf{P}_1$, of magnitude

$$F = P_1 = 15 \text{ kN}$$

2. A shearing force $\mathbf{V}$ equal to the force $\mathbf{P}_2$, of magnitude

$$V = P_2 = 18 \text{ kN}$$

3. A twisting couple $\mathbf{T}$ of torque T equal to the moment of $\mathbf{P}_2$ about the axis of member BD:

$$T = P_2 a = (18 \text{ kN})(50 \text{ mm}) = 900 \text{ N} \cdot \text{m}$$

4. A bending couple $\mathbf{M}_y$, of moment M_y equal to the moment of $\mathbf{P}_1$ about a vertical axis through C:

$$M_y = P_1 a = (15 \text{ kN})(50 \text{ mm}) = 750 \text{ N} \cdot \text{m}$$

5. A bending couple $\mathbf{M}_z$, of moment M_z equal to the moment of $\mathbf{P}_2$ about a transverse, horizontal axis through C:

$$M_z = P_2 b = (18 \text{ kN})(60 \text{ mm}) = 1080 \text{ N} \cdot \text{m}$$

The results obtained are shown in Fig. 8.23.

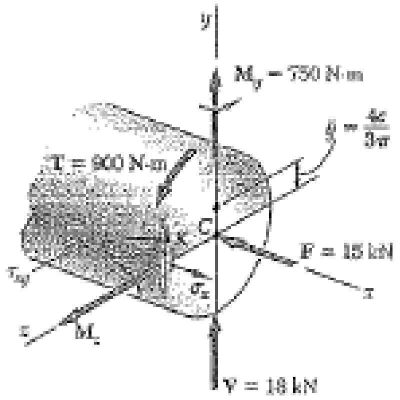


Fig. 8.23

a. Normal and Shearing Stresses at Point K. Each of the forces and couples shown in Fig. 8.23 can produce a normal or shearing stress at point K. Our purpose is to compute separately each of these stresses, and then to add the normal stresses and add the shearing stresses. But we must first determine the geometric properties of the section.

Geometric Properties of the Section. We have

$$A = \pi c^2 = \pi(0.020 \text{ m})^2 = 1.257 \times 10^{-3} \text{ m}^2$$
$$I_y = I_z = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi(0.020 \text{ m})^4 = 125.7 \times 10^{-9} \text{ m}^4$$
$$J_C = \tfrac{1}{2}\pi c^4 = \tfrac{1}{2}\pi(0.020 \text{ m})^4 = 251.3 \times 10^{-9} \text{ m}^4$$

We also determine the first moment Q and the width t of the area of the cross section located above the z axis. Recalling that $\bar{y} = 4c/3\pi$ for a semicircle of radius c, we have

$$Q = A'\bar{y} = \left(\frac{1}{2}\pi c^2\right)\left(\frac{4c}{3\pi}\right) = \frac{2}{3}c^3 = \frac{2}{3}(0.020 \text{ m})^3$$
$$= 5.33 \times 10^{-6} \text{ m}^3$$

and

$$t = 2c = 2(0.020 \text{ m}) = 0.040 \text{ m}$$

Normal Stresses. We observe that normal stresses are produced at K by the centric force $\mathbf{F}$ and the bending couple $\mathbf{M}_y$, but that the couple $\mathbf{M}_z$ does not produce any stress at K, since K is located on the neutral axis corresponding to that couple. Determining each sign from Fig. 8.23, we write

$$\sigma_x = -\frac{F}{A} + \frac{M_y c}{I_y} = -11.9 \text{ MPa} + \frac{(750 \text{ N} \cdot \text{m})(0.020 \text{ m})}{125.7 \times 10^{-9} \text{ m}^4}$$
$$= -11.9 \text{ MPa} + 119.3 \text{ MPa}$$
$$\sigma_x = +107.4 \text{ MPa}$$

Shearing Stresses. These consist of the shearing stress $(\tau_{xy})_V$ due to the vertical shear **V** and of the shearing stress $(\tau_{xy})_{twist}$ caused by the torque **T**. Recalling the values obtained for Q, t, I_z, and J_C, we write

$$(\tau_{xy})_V = +\frac{VQ}{I_z t} = +\frac{(18 \times 10^3\ \text{N})(5.33 \times 10^{-6}\ \text{m}^3)}{(125.7 \times 10^{-9}\ \text{m}^4)(0.040\ \text{m})}$$
$$= +19.1\ \text{MPa}$$

$$(\tau_{xy})_{twist} = -\frac{Tc}{J_C} = -\frac{(900\ \text{N}\cdot\text{m})(0.020\ \text{m})}{251.3 \times 10^{-9}\ \text{m}^4} = -71.6\ \text{MPa}$$

Adding these two expressions, we obtain τ_{xy} at point K.

$$\tau_{xy} = (\tau_{xy})_V + (\tau_{xy})_{twist} = +19.1\ \text{MPa} - 71.6\ \text{MPa}$$
$$\tau_{xy} = -52.5\ \text{MPa}$$

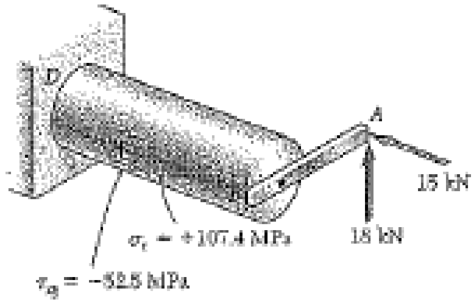


Fig. 8.24

In Fig. 8.24, the normal stress σ_x and the shearing stresses and τ_{xy} have been shown acting on a square element located at K on the surface of the cylindrical member. Note that shearing stresses acting on the longitudinal sides of the element have been included.

b. Principal Planes and Principal Stresses at Point K. We can use either of the two methods of Chap. 7 to determine the principal planes and principal stresses at K. Selecting Mohr's circle, we plot point X of coordinates $\sigma_x = +107.4$ MPa and $-\tau_{xy} = +52.5$ MPa and point Y of coordinates $\sigma_y = 0$ and $+\tau_{xy} = -52.5$ MPa and draw the circle of diameter XY (Fig. 8.25). Observing that

$$OC = CD = \tfrac{1}{2}(107.4) = 53.7\ \text{MPa} \qquad DX = 52.5\ \text{MPa}$$

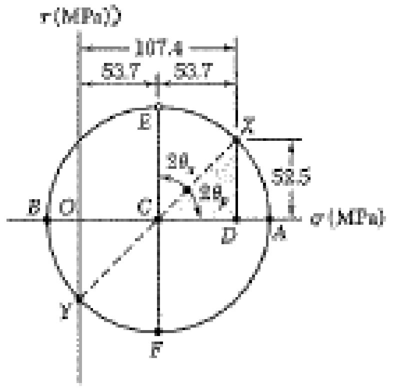


Fig. 8.25

we determine the orientation of the principal planes:

$$\tan 2\theta_p = \frac{DX}{CD} = \frac{52.5}{53.7} = 0.97765 \qquad 2\theta_p = 44.4^\circ \downarrow$$
$$\theta_p = 22.2^\circ \downarrow$$

We now determine the radius of the circle,

$$R = \sqrt{(53.7)^2 + (52.5)^2} = 75.1\ \text{MPa}$$

and the principal stresses,

$$\sigma_{max} = OC + R = 53.7 + 75.1 = 128.8\ \text{MPa}$$
$$\sigma_{min} = OC - R = 53.7 - 75.1 = -21.4\ \text{MPa}$$

The results obtained are shown in Fig. 8.26.

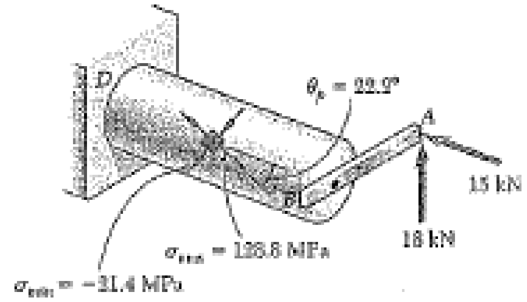


Fig. 8.26

c. Maximum Shearing Stress at Point K. This stress corresponds to points E and F in Fig. 8.25. We have

$$\tau_{max} = CE = R = 75.1\ \text{MPa}$$

Observing that $2\theta_s = 90^\circ - 2\theta_p = 90^\circ - 44.4^\circ = 45.6^\circ$, we conclude that the planes of maximum shearing stress form an angle $\theta_p = 22.8^\circ$ ↺ with the horizontal. The corresponding element is shown in Fig. 8.27. Note that the normal stresses acting on this element are represented by OC in Fig. 8.25 and are thus equal to $+53.7$ MPa.

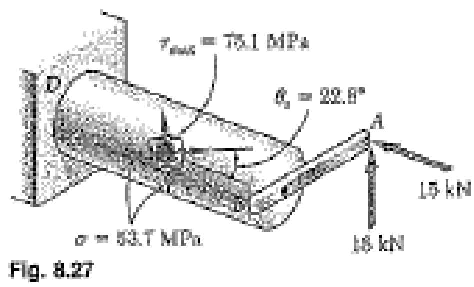


Fig. 8.27

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1ª PROVA DE RESISTÊNCIA DOS MATERIAIS VII – 2017-1

1ª Questão (2,5 pontos) – A alavanca angular da figura é rotulada em A e suportada por um pequeno elemento de ligação BC. Se ela é submetida à força de 80 N, determine as tensões principais no ponto D. A alavanca é construída a partir de uma placa de alumínio com espessura de 20 mm. (Figura 1)

2ª Questão (2,5 pontos) – Na superfície de um componente estrutural em um veículo espacial, as deformações são monitoradas por meio de três extensômetros como ilustrado na figura. Durante uma certa manobra, as seguintes deformações foram obtidas: $\varepsilon_a = 1100 \times 10^{-6}$, $\varepsilon_b = 200 \times 10^{-6}$ e $\varepsilon_c = 200 \times 10^{-6}$. Determine as deformações principais e as tensões principais no material que é uma liga de magnésio ($E = 41$ GPa, $\nu = 0{,}35$). (Figura 2)

3ª Questão (2,5 pontos) – Uma viga de aço tem uma tensão normal admissível $\sigma_{adm} = 140$ MPa e uma tensão cisalhante admissível $\tau_{adm} = 90$ MPa. Determine a carga máxima P que a viga pode suportar com segurança. (Figura 3)

4ª Questão (2,5 pontos) – Os mancais em A e D exercem sobre o eixo de aço forças reativas apenas nas direções y e z. Determine o diâmetro do eixo de forma que possa resistir ao carregamento imposto pelas engrenagens sem exceder a tensão de cisalhamento admissível $\tau_{adm} = 60$ MPa. A força de 6 kN é vertical e a força P é paralela ao eixo z. Utilize o critério de Tresca (teoria da máxima tensão cisalhante). (Figura 4)

Figura 1

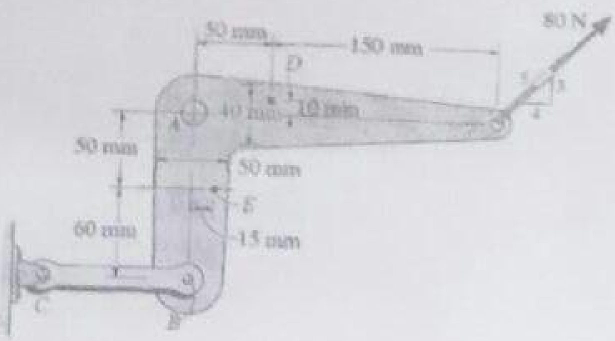


Figura 2

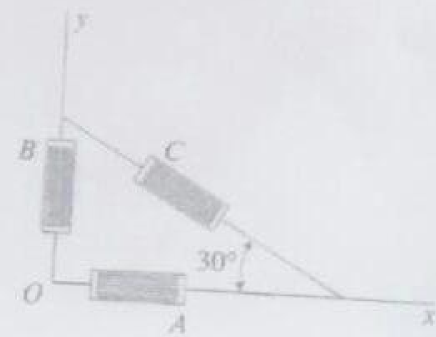


Figura 3

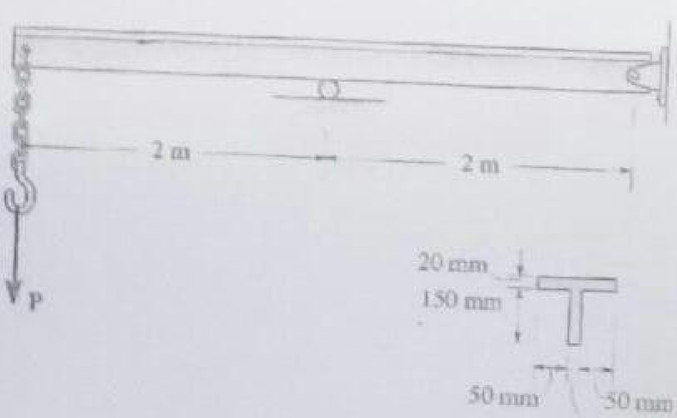
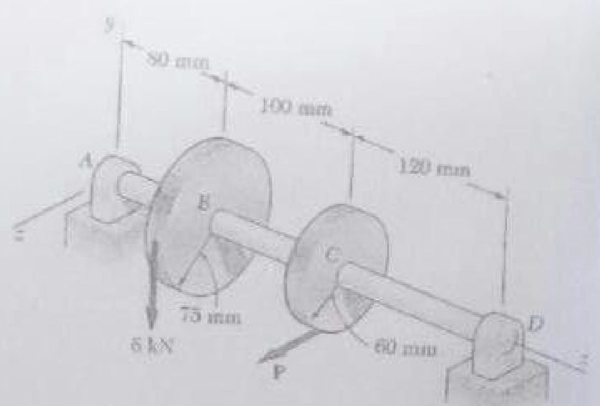


Figura 4



…RIO DE JANEIRO
…DE ENGENHARIA
…ENTO DE ENGENHARIA MECÂNICA
…A DE REPOSIÇÃO DE RESISTÊNCIA DOS MATERIAIS VII (P1) – 2017-1

1ª Questão (2,5 pontos) – O elemento de máquina mostrado está carregado em um plano de simetria. Determine e mostre em um esboço com as respectivas orientações angulares, as tensões principais e a tensão cisalhante máxima no ponto A que está localizado na alma, imediatamente acima da junção entre a mesa e a alma. (Figura 1)

2ª Questão (2,5 pontos) – Uma barra circular sólida de diâmetro $d = 40$ mm está submetida a uma força axial P e a um torque T. Os extensômetros A e B montados na superfície da barra fornecem as leituras $\varepsilon_a = 100 \times 10^{-6}$ e $\varepsilon_b = -55 \times 10^{-6}$. A barra é feita de aço tendo $E = 200$ GPa e $\nu = 0{,}29$. Determine a força axial P e o torque T. (Figura 2)

3ª Questão (2,5 pontos) – Um eixo tem 150 mm de diâmetro d e é feito de aço com limite de escoamento $\sigma_y = 360$ MPa para tração e compressão. As cargas aplicadas são $P = 2200$ kN e $T = 38$ kN.m. Determine o coeficiente de segurança à falha por escoamento de acordo com a teoria da máxima energia de distorção. (Figura 3)

4ª Questão (2,5 pontos) – Os mancais em A e B exercem sobre o eixo de aço forças reativas apenas nas direções x e z. Determine o diâmetro do eixo como o múltiplo mais próximo do milímetro de forma que possa resistir ao carregamento imposto pelas engrenagens sem exceder a tensão de cisalhamento admissível $\tau_{adm} = 80$ MPa. Utilize o critério de Tresca (teoria da máxima tensão cisalhante). (Figura 4)

Figura 1

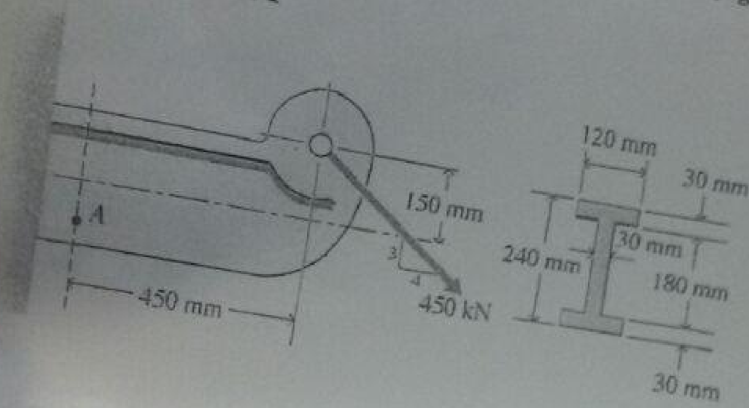


Figura 2

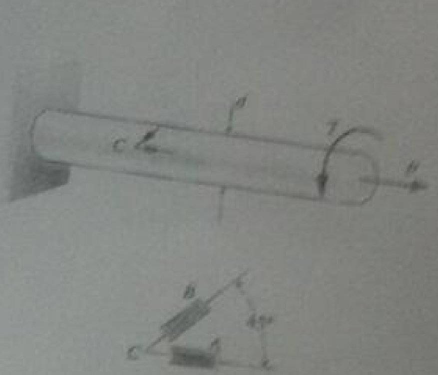


Figura 3

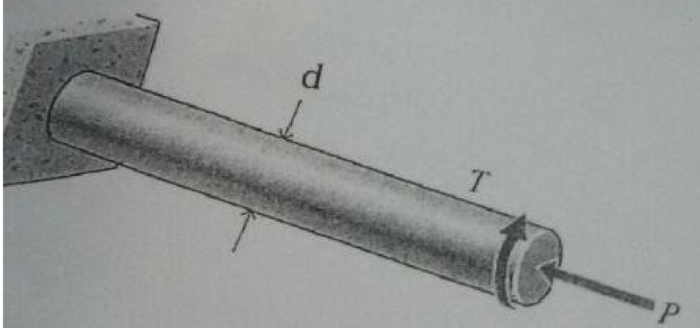
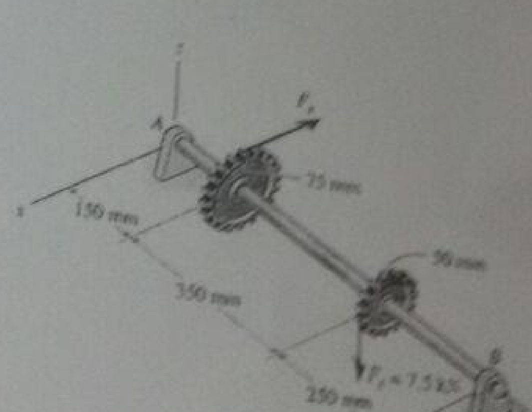


Figura 4



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PROVA DE REPOSIÇÃO DE RESISTÊNCIA DOS MATERIAIS VII (P2) – 2017-1

1ª Questão (2,5 pontos) – Determinar as equações da linha elástica utilizando as coordenadas x_1 e x_2. Considerar EI constante. (Figura 1)

2ª Questão (2,5 pontos) – Determine as reações nos apoios e trace os diagramas de esforço cortante e momento fletor. Utilizar a superposição de efeitos (tabela anexa). EI = constante. (Figura 2)

3ª Questão (2,5 pontos) – Uma carga de 60 kN é suportada por um tirante AB e uma escora tubular BC. O tirante tem um diâmetro de 30 mm e é feito de aço com E = 210 GPa e σ_e = 360 MPa. A escora tubular tem diâmetro interno de 50 mm e espessura de 15 mm, e é feita de uma liga de alumínio com E = 73 GPa e σ_e = 280 MPa. Determine o fator de segurança do projeto indicando se é limitado por escoamento ou flambagem da estrutura. (Figura 3)

4ª Questão (2,5 pontos) – Dois eixos de aço com seções retas circulares são conectados pelas engrenagens mostradas na figura. Utilizando a conservação da energia, determinar o ângulo de rotação da extremidade D do eixo CD quando T = 820 N.m. Dado: G = 77 GPa. (Figura 4)

Figura 1

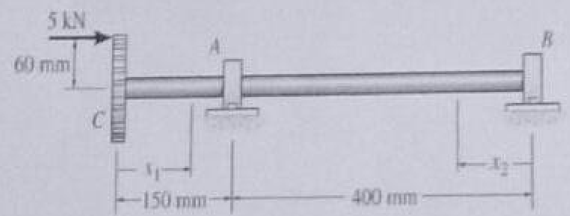


Figura 2

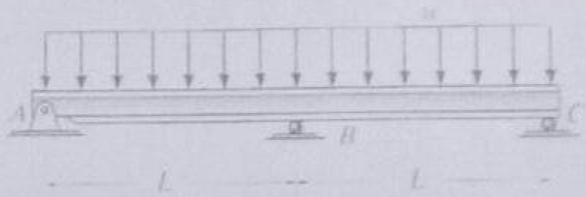


Figura 3

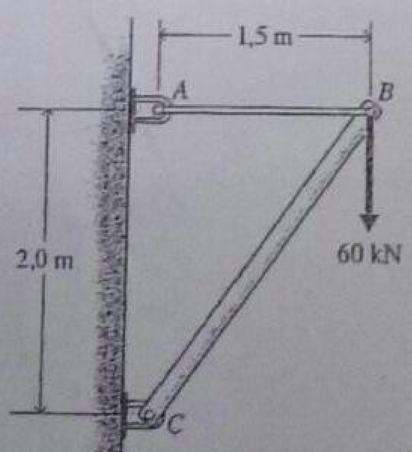
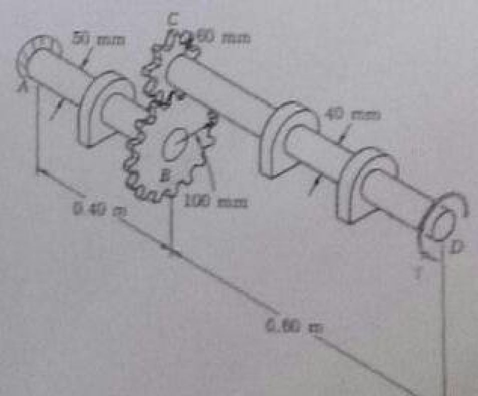


Figura 4



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EXAME FINAL DE RESISTÊNCIA DOS MATERIAIS VII – 2017-1

1ª Questão (2,5 pontos) – Duas forças P_1 e P_2 de intensidade 15 kN e 18 kN, respectivamente, são aplicadas à barra *AB* que está soldada a um eixo cilíndrico *BD* de raio $r = 20$ mm. Determine: (a) o estado de tensões no ponto *K*; (b) as tensões principais e a tensão de cisalhamento máximo no ponto *K*. (Figura 1)

2ª Questão (2,5 pontos) – O eixo cheio *ABC* e as engrenagens mostradas na figura transmitem 10 kW do motor *M* para uma máquina-ferramenta conectada à engrenagem *D*. Sabendo que o motor gira a 240 rpm e que $\tau_{adm} = 60$ MPa, determine o menor diâmetro admissível para o eixo *ABC*. (Figura 2)

3ª Questão (2,5 pontos) – Determine a força máxima P que pode ser aplicada ao cabo de modo que a haste de controle de aço A-36 *AB* não sofra flambagem. A haste tem 30 mm de diâmetro e está presa por pinos nas extremidades. Dados: $E_{aço} = 200$ GPa, $\sigma_e = 250$ MPa. (Figura 3)

4ª Questão (2,5 pontos) – Determine as reações nos apoios *A* e *B* e construa os diagramas de esforço cortante e momento fletor. Considerar a rigidez *EI* constante ao longo de toda a viga. (Figura 4)

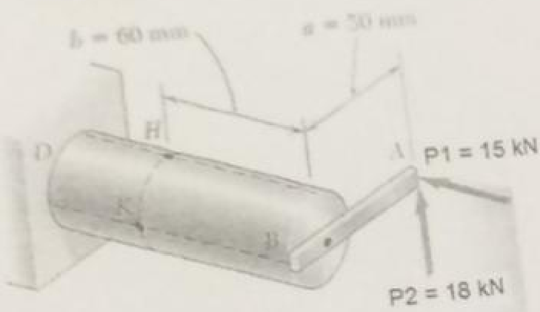


Figura 1

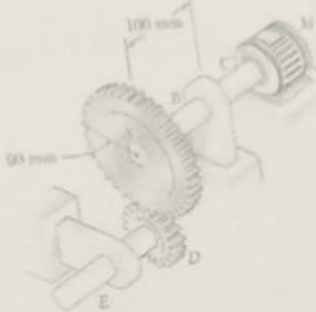


Figura 2

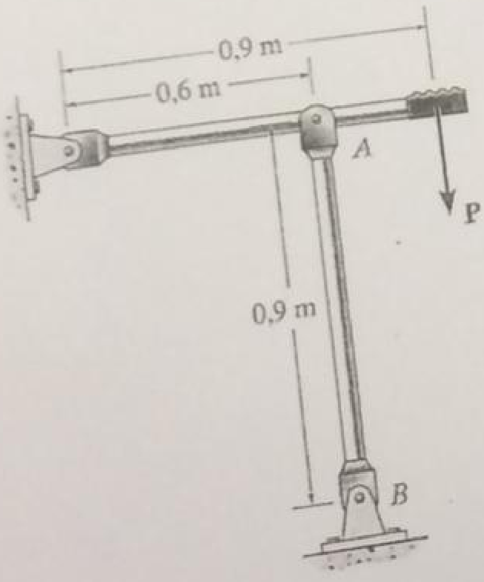


Figura 3

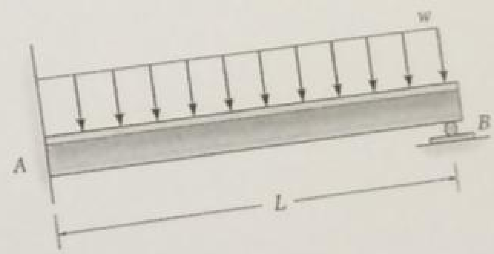


Figura 4

11.21. A viga de aço tem uma tensão de flexão admissível σ_{adm} = 140 MPa e uma tensão de cisalhamento admissível τ_{adm} = 90 MPa. Determine a carga máxima que ela pode suportar com segurança.

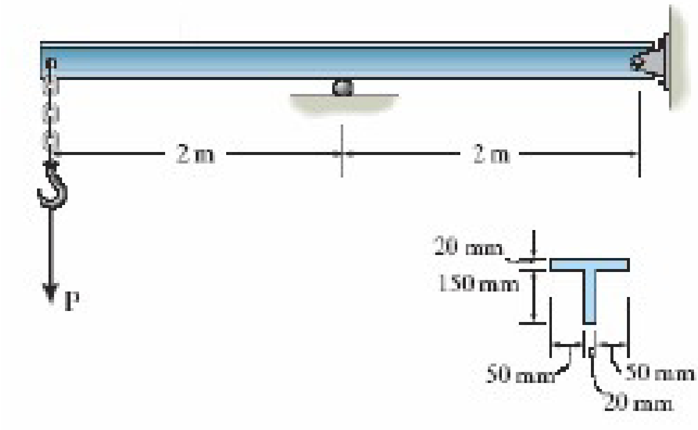
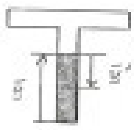


Figura 11.21

Reações:



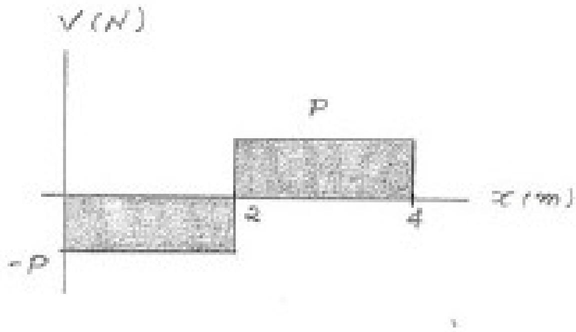
$$\curvearrowleft +\sum M_2 = 0 \quad ; \quad 4P - 2R_1 = 0 \quad \therefore \quad R_1 = 2P$$

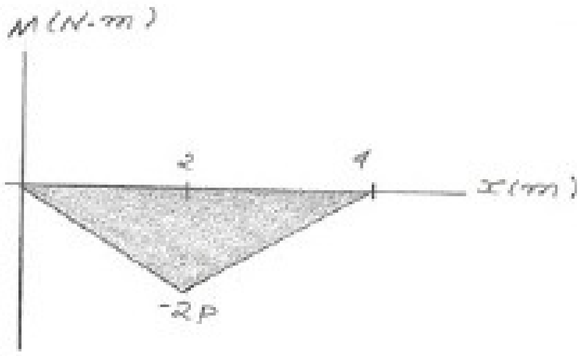
$$\uparrow + \sum F_y = 0 \quad ; \quad -R_2 + 2P - P = 0 \quad \therefore \quad R_2 = P$$

$$y_{CG} = \frac{20 \times 150 \times 75 + 120 \times 20 \times 160}{20 \times 150 + 20 \times 120} = 112{,}777 \text{ mm (centroide da seção transversal)}$$

$$I = \left(\frac{0{,}020 \times 0{,}150^3}{12} + 0{,}020 \times 0{,}150 \times 0{,}03777^2\right) + \left(\frac{0{,}120 \times 0{,}020^3}{12} + 0{,}120 \times 0{,}020 \times 0{,}047223^2\right) = 1{,}533833 \times 10^{-5} \text{ m}^4$$

$$Q_{máx} = 0{,}112777 \times 0{,}02 \times 0{,}0563885 = 1{,}271865 \times 10^{-4} \text{ m}^3$$





$$|M_{máx}| = 2P \quad ; \quad |V_{máx}| = P$$

$$S_{req} = \frac{M_{máx}}{\sigma_{adm}} = \frac{I}{c} \quad \therefore \quad \frac{2P}{140 \times 10^6} = \frac{1{,}533833 \times 10^{-5}}{0{,}112777} \quad \therefore \quad \mathbf{P = 9{,}52\ kN}$$

$$\tau_{máx} = \frac{V_{máx}Q_{máx}}{It} = \frac{(9520 \times 10^3)(1{,}271865 \times 10^{-4})}{(1{,}533833 \times 10^{-5})(0{,}020)} = 3{,}95 \text{ MPa} < \tau_{adm} = 90 \text{ MPa OK!}$$