Selection Rules for Neutron β^- Decay in Elastic Diffusive Cosmology

A 5D Topological Necessity of a Neutral Mode with $p^\xi \neq 0$

(Companion D to Paper 3: NJSR Edition)

Igor Grčman

January 2026

DOI: 10.5281/zenodo.18299855

Repository: github.com/igorgrcman/elastic-diffusive-cosmology

(Public artifacts for this paper are in the `edc_papers` folder.)

Related Documents:

Neutron Lifetime from 5D Membrane Cosmology (DOI: 10.5281/zenodo.18262721)
Framework v2.0 (DOI: 10.5281/zenodo.18299085)

Companions:

A: *Effective Lagrangian* (DOI) · B: *WKB Prefactor* (DOI)
C: *5D Reduction* (DOI) · E: *Symmetry Ops* (DOI)
F: *Proton Junction* (DOI) · G: *Mass Difference* (DOI)
H: *Weak Interactions* (DOI)

Abstract

This companion note establishes the *selection rules* for neutron β^- decay within the 5D Elastic Diffusive Cosmology (EDC) framework. We derive, from topological charge conservation and kinematic constraints, that the decay must produce exactly two outputs: (i) a brane-bound excitation carrying $Q = -1$ (topologically confined to $\xi = 0$), and (ii) a neutral mode with nonzero fifth momentum $p^\xi \neq 0$. **Both outputs are created on the brane**; the distinction is kinematic—the neutral mode can carry momentum in the fifth dimension while the charged mode cannot. The note addresses channel legality and budget bookkeeping only; it does *not* compute the decay lifetime τ_n (treated in Paper 3 via WKB methods). No Standard Model dynamical parameters are used as inputs; particle labels appear only as observational identifiers. All conclusions are conditional on the stated assumptions.

1 Introduction

Neutron β^- decay is one of the most precisely measured processes in nuclear physics. In the Standard Model, it is mediated by W-boson exchange within the electroweak framework. This note takes a different approach: we ask what constraints on the decay channel arise from the 5D topological structure of Elastic Diffusive Cosmology (EDC), *independent* of any Standard Model mechanism.

1.1 Scope and Two-Sided Presentation

Throughout this note, we adopt a **two-sided rule**:

Left side: 5D EDC cause	Right side: 3D observational evidence
Topological constraint	Measured quantity **[BL]**
Conservation law **[Dc]**/**[Der]**	Experimental signature

This separation ensures that Standard Model facts appear only as *observational targets* to be matched, never as theoretical inputs to the derivation.

1.2 What This Note Does and Does Not Do

Scope Declaration

This note addresses:

- Channel selection rules (which output configurations are permitted)
- Charge, winding, and momentum budget bookkeeping
- The necessity of a neutral mode with $p^\xi \neq 0$ ("antineutrino")

This note does NOT:

- Compute the neutron lifetime τ_n (see the main paper [1])
- Derive SM coupling constants (G_F, V_{ud}, etc.)
- Model collider cross-sections or high-energy processes
- Derive the effective Lagrangian $\mathcal{L}_{\text{eff}}$ from first principles (see companion note [2])

2 Assumptions, Definitions, and Epistemic Legend

2.1 Epistemic Tag Legend

All statements in this note carry explicit epistemic tags:

Tag	Meaning	Example
[I]	Empirical input (observation)	Continuous e^- spectrum
[Def]	Definition introduced here	Collective coordinate q
[P]	Working hypothesis	Junction unwinding chirality
[Dc]	Decisively constrained (conditional)	Mode classification
[Der]	Derived within stated equations	Charge budget
[OPEN]	Deferred / not yet derived	Junction chirality origin
[BL]	Baseline observational fact	$Q_{\text{decay}} = 1.293$ MeV

2.2 Assumption Box

Assumptions for This Note

Geometric/Topological Assumptions:

A1. [P] The observable universe is a 3-brane Σ embedded in 5D spacetime $\mathcal{M}_5$.

A2. [P] Baryons are Y-junctions of three flux tubes meeting on Σ.

A3. [Def] Electric charge Q is a topological winding number: $Q = \frac{1}{2\pi}\oint A_\phi \, d\phi$.

A4. [P] Topological winding is conserved across decay processes.

A5. [P] Fifth momentum p^ξ is conserved (with Plenum recoil absorption).

Empirical Inputs:

I1. [I] The electron spectrum in β^- decay is continuous (not monoenergetic).

I2. [I] Neutrinos/antineutrinos are electrically neutral ($Q_\nu = 0$).

I3. [I] The decay releases $Q_{\text{decay}} = 1.293$ MeV [BL].

I4. [I] The proton carries charge $Q_p = +1$; neutron carries $Q_n = 0$ [BL].

Classification Convention:

C1. [Def]/[P] Particle vs. antiparticle is distinguished by the combination $(h_5, \text{sign}(p^\xi))$ where h_5 is the 5D helicity.

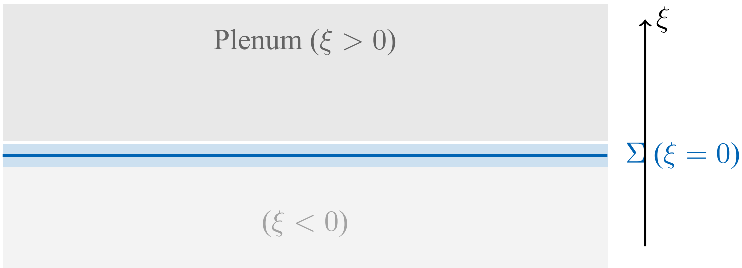
3 The 5D Stage and Topological Defects

3.1 Spacetime Structure

In EDC, spacetime is 5-dimensional:

$$\mathcal{M}_5 = \{(x^\mu, \xi) : x^\mu \in \mathbb{R}^{1,3},\ \xi \in \mathbb{R}\} \qquad ([\mathrm{P}])$$

The brane Σ at $\xi = 0$ hosts observable matter. The bulk ($\xi > 0$) is the Plenum, an energy reservoir with positive pressure $P_{\text{bulk}} > 0$.



3.2 Baryons as Y-Junctions

Definition 1 (Y-Junction)**.** **[Def]** A baryon is a Y-junction where three flux tubes meet on the brane Σ. The junction is characterized by three unit vectors $\hat{e}_1, \hat{e}_2, \hat{e}_3$ pointing along the flux tubes.

Definition 2 (Collective Coordinate q)**.** **[Def]** The junction asymmetry is measured by:

$$q \equiv \frac{|\hat{e}_1 + \hat{e}_2 + \hat{e}_3|}{3} \in [0, 1] \tag{1}$$

- $q = 0$: Symmetric Steiner configuration (120° angles) → proton (stable)
- $q = q_n \approx 0.31$: Asymmetric configuration → neutron (unstable)

Definition 3 (Topological Charge)**.** **[Def]** Electric charge is defined as a topological winding number:

$$Q = \frac{1}{2\pi} \oint A_\phi \, d\phi \in \mathbb{Z} \tag{2}$$

Defect	Q	Baryonic W	Status
Proton	$+1$	$+1$	Brane-bound [BL]
Neutron	0	$+1$	Brane-bound [BL]
Electron	-1	0	Brane-bound [BL]
Neutral mode	0	0	$p^\xi \neq 0$ allowed [Dc]

Important Clarification: Where Particles Are Created

All decay products are created on the brane ($\xi = 0$).
The distinction between "brane-bound" and "$p^\xi \neq 0$ allowed" is *kinematic*, not about origin:

- **Brane-bound** ($Q \neq 0$): Topological charge requires a localized core; the mode *cannot* carry fifth momentum. It is confined to $\xi = 0$.
- $p^\xi \neq 0$ **allowed** ($Q = 0$): No topological obstruction; the mode *can* carry momentum in the ξ-direction, even though it is created on the brane.

The antineutrino does not "live in the bulk"—it is created on the brane like all other decay products. However, it can carry energy-momentum into the fifth dimension, which is why fifth-momentum conservation requires its existence.

4 Conserved Budgets Across the Membrane

The decay process $n \to p + X$ must satisfy several conservation constraints. We analyze each budget systematically.

4.1 Charge Budget

Proposition 1 (Charge Closure)**.** ***[Der]*** *The decay outputs must carry total charge $Q = -1$.*

Proof. By charge conservation (Assumption A4):

$$Q_n = Q_p + \sum_i Q_i \tag{3}$$

Substituting known values **[BL]**:

$$0 = +1 + \sum_i Q_i \quad \Longrightarrow \quad \boxed{\sum_i Q_i = -1} \tag{4}$$

□

Two-sided check:

5D cause	3D evidence
Winding conservation **[Der]**	Electron carries $Q = -1$ **[BL]**

4.2 Baryonic Winding Budget

Proposition 2 (Winding Closure)**.** ***[Der]*** *The decay outputs carry no net baryonic winding.*

Proof.

$$W_n = W_p + \sum_i W_i \quad \Longrightarrow \quad +1 = +1 + \sum_i W_i \quad \Longrightarrow \quad \boxed{\sum_i W_i = 0} \tag{5}$$

□

Implication: Outputs are not baryons (leptons only).

4.3 Kinematic/Spectral Constraint

Proposition 3 (Three-Body Necessity)**.** ***[I]/[Dc]*** *The decay cannot be a pure two-body process; a third channel is required.*

Proof. Empirical input **I1**: The electron spectrum is continuous, not monoenergetic.

A two-body decay $n \to p + e^-$ would produce a monoenergetic electron with:

$$E_e = \frac{(m_n - m_p)^2 + m_e^2}{2m_n} c^2 \approx 1.29\ \text{MeV} \tag{6}$$

The observed continuous spectrum **[I]** rules out pure two-body kinematics. Therefore, a third participant is required to absorb the missing energy-momentum. □

Two-sided check:

5D cause	3D evidence
Energy-momentum balance **[Dc]**	Continuous e^- spectrum **[I]**

4.4 Fifth Momentum Budget

Proposition 4 (p^ξ Balance)**.** ***[Dc]/[P]*** *Under Assumption A5, at least one output must have $p^\xi \neq 0$.*

Proof. Fifth momentum conservation with Plenum recoil:

$$p_n^\xi = p_p^\xi + \sum_i p_i^\xi + \Delta p_{\text{Plenum}}^\xi \tag{7}$$

Since neutron and proton are both brane-bound: $p_n^\xi = p_p^\xi = 0$.
If the Plenum absorbs nonzero recoil ($\Delta p_{\text{Plenum}}^\xi \neq 0$), then:

$$\sum_i p_i^\xi = -\Delta p_{\text{Plenum}}^\xi \neq 0 \quad \Longrightarrow \quad \boxed{\exists\, p_i^\xi \neq 0} \tag{8}$$

□

Implication: At least one output mode carries nonzero fifth momentum $p^\xi \neq 0$.

5 Selection Rules: Formal Propositions

5.1 Mode Classification

Theorem 1 (Brane Localization)**.** ***[Dc]*** *If $Q \neq 0$, then $p^\xi = 0$ (mode is brane-bound).*

Proof. Within the stated assumptions:

1. Non-zero winding ($Q \neq 0$) requires a topological core.
2. Cores are localized at brane junctions (by construction of defects).
3. Bulk waves ($\xi > 0$) have no localized core structure.
4. Therefore: $Q \neq 0 \Rightarrow$ brane-localized $\Rightarrow p^\xi = 0$.

□

Theorem 2 (Fifth Momentum Permitted for $Q = 0$)**.** ***[Dc]*** *If $Q = 0$, then $p^\xi \neq 0$ is kinematically allowed.*

Proof. The 5D dispersion relation is:

$$E^2 = |\vec{p}|^2 c^2 + (p^\xi c)^2 + m_0^2 c^4 \tag{9}$$

For $Q = 0$ (no topological mass contribution from winding), if $E > |\vec{p}|c$, then:

$$p^\xi = \frac{1}{c}\sqrt{E^2 - |\vec{p}|^2 c^2 - m_0^2 c^4} \neq 0 \tag{10}$$

is kinematically permitted. □

5.2 Unique Minimal Configuration

Theorem 3 (Two-Output Selection Rule)**.** ***[Dc]*** *Under the stated assumptions, the unique minimal output configuration is:*

1. *One brane-bound mode with $Q = -1$, $W = 0$, $p^\xi = 0$.*
2. *One neutral mode with $Q = 0$, $W = 0$, $p^\xi \neq 0$ (carries fifth momentum).*

Proof. Combining all constraints:

- Proposition 4.1: $\sum_i Q_i = -1$ (charge closure)
- Proposition 4.2: $\sum_i W_i = 0$ (winding closure)
- Proposition 4.3: Three-body required (spectral constraint)

- Proposition 4.4: $\exists p_i^\xi \neq 0$ (fifth momentum required)
- Theorem 5.1: $Q \neq 0 \Rightarrow p^\xi = 0$ (brane localization)

The minimal solution satisfying all constraints:

$$\text{Output 1:} \quad Q_1 = -1,\ W_1 = 0,\ p_1^\xi = 0 \quad \text{(brane soliton)} \tag{11}$$

$$\text{Output 2:} \quad Q_2 = 0,\ W_2 = 0,\ p_2^\xi \neq 0 \quad \text{(neutral, carries } p^\xi\text{)} \tag{12}$$

Verification: $Q_1 + Q_2 = -1 + 0 = -1$ ✓; $W_1 + W_2 = 0$ ✓; $p^\xi \neq 0$ mode exists ✓. □

5.3 Excluded Alternatives

Alternative	Violated Constraint	Status
$n \to p$ only	Energy conservation	Forbidden
$n \to p + e^-$ (two-body)	Spectral shape [I]	Forbidden
$n \to p + \nu$ only	Charge conservation	Forbidden
$n \to p + e^- + e^+ + e^-$	Kinematic threshold	Forbidden
$n \to p+$ 3+ outputs	Action minimality [P]	Suppressed

6 Antineutrino vs. Neutrino: Derived vs. Defined

The preceding selection rules establish that a neutral mode with $p^\xi \neq 0$ is required. This section addresses whether this mode should be labeled "neutrino" (ν) or "antineutrino" ($\bar{\nu}$).

6.1 Angular Momentum Constraint

Proposition 5 (L^{35} Balance). ***[Dc]/[P]*** *Under the assumption that 5D angular momentum L^{AB} is approximately conserved, and that the junction unwinding releases angular momentum with definite chirality, the two outputs must have opposite 5D helicities:*

$$h_5^{(1)} + h_5^{(2)} \approx 0 \quad \Longrightarrow \quad h_5^{(1)} = -h_5^{(2)} \tag{13}$$

Status: This proposition is **[Dc]** (decisively constrained) conditional on the assumptions. The effective Lagrangian derivation is provided in the companion note [2].

6.2 Helicity Assignment

Proposition 6 (Junction Chirality). ***[P]*** *When the neutron junction unwinds ($q : q_n \to 0$), the released angular momentum has $h_5 < 0$, which is absorbed by the brane-bound mode (electron).*

Status: This is a working hypothesis **[P]**. The specific sign of h_5 for the electron is not derived from first principles in this note.

Consequence:

$$h_5^{(\text{electron})} = -1 \quad \textbf{[P]} \tag{14}$$

$$h_5^{(\text{neutral mode})} = +1 \quad \text{(by Proposition 6.1)} \tag{15}$$

6.3 Particle/Antiparticle Classification

Definition 4 (Classification Rule). **[Def]/[P]** For neutral modes with $p^\xi \neq 0$, the particle/antiparticle label is determined by the combination $(h_5, \text{sign}(p^\xi))$:

h_5	sign(p^ξ)	Classification
$+1$	$+1$ (outward)	Antiparticle ($\bar{\nu}$)
-1	$+1$ (outward)	Particle (ν)

Status: This classification convention **[Def]** is consistent with CPT structure but is introduced here as a definition, not derived from the 5D action.

6.4 Result: Antineutrino Identification

Applying the classification:

$$h_5^{(\text{neutral})} = +1, \quad p^\xi > 0 \quad \Longrightarrow \quad \text{Antiparticle label: } \bar{\nu} \tag{16}$$

Key Result (Conditional)

Within the stated assumptions, the neutral mode with $p^\xi \neq 0$ is labeled "antineutrino" ($\bar{\nu}$), not "neutrino" (ν).

A neutrino would require $h_5 = -1$ with $p^\xi > 0$, but this would violate Proposition 6.1 since the brane-bound mode (electron) already carries $h_5 = -1$.

Caveat: This conclusion is conditional on assumptions A1–A5, C1, and Proposition 6.2 [P].

6.5 Two-Sided Summary

Output	5D Properties	3D Label [BL]
Brane soliton	$Q = -1$, $p^\xi = 0$, $h_5 = -1$	Electron (LH)
Neutral mode	$Q = 0$, $p^\xi \neq 0$, $h_5 = +1$	Antineutrino (RH)

7 Relation to the Main Paper

This companion note and the main paper [1] address complementary aspects of neutron β^- decay in EDC:

Aspect	This Note	Main Paper
Channel selection	✓	—
Budget bookkeeping	✓	—
WKB tunneling rate	—	✓
Lifetime computation	—	✓
Barrier potential V_B	—	✓
Prefactor A_0	—	✓

Interface: This note establishes *what* channels are permitted; the main paper [1] computes *how fast* the transition occurs via $\Gamma = A_0 \exp(-B/\hbar)$.

No feedback loop: The selection rules here do not depend on the WKB computation, and vice versa. The two analyses are logically independent.

No SM parameter import: Neither this note nor the main paper uses Standard Model parameters (G_F, V_{ud}, etc.) as theoretical inputs. SM facts appear only as observational baselines [BL] to be matched.

8 Limitations and Falsifiable Next Steps

8.1 What Remains Open

O1. [Der] **Derive $\mathcal{L}_{\text{eff}}$ from 5D action:** Completed in companion note [2]. The effective Lagrangian $L_{\text{eff}}(q, \dot{q}) = \frac{1}{2}M(q)\dot{q}^2 - V(q)$ is derived from the 5D Einstein-Hilbert action via Israel junction conditions.

O2. [OPEN] **Derive junction chirality from first principles:** Proposition 6.2 ($h_5^{(\text{electron})} = -1$) is postulated, not derived.

O3. [OPEN] **Derive particle/antiparticle classification:** Definition 6.1 is a convention; a derivation from CPT in 5D is needed.

O4. [OPEN] Flavor structure: Why "electron neutrino" specifically? The note does not address lepton flavor.

O5. [OPEN] Neutrino mass: The note does not derive or constrain m_ν.

8.2 Falsifiable Predictions

Within the assumptions stated, the following predictions can be tested:

F1. Decay channel: Free neutron decay produces exactly $p + e^- + \bar{\nu}_e$ (no additional particles below threshold).

F2. Helicity correlation: The electron is predominantly left-handed; the antineutrino is right-handed.

F3. No baryon-number violation: Outputs carry zero net baryonic winding ($\Delta W = 0$).

All three are consistent with observation **[BL]**, providing necessary (not sufficient) validation.

9 Summary

1. **Charge closure [Der]**: Outputs must carry $Q = -1$.
2. **Winding closure [Der]**: Outputs carry $W = 0$ (non-baryonic).
3. **Spectral constraint [I]/[Dc]**: Three-body decay required.
4. **Fifth momentum mode [Dc]/[P]**: At least one neutral mode with $p^\xi \neq 0$.
5. **Mode classification [Dc]**: $Q \neq 0 \Rightarrow$ brane-bound ($p^\xi = 0$); $Q = 0 \Rightarrow p^\xi \neq 0$ allowed.
6. **Antineutrino label [Dc]/[P]**: Conditional on helicity assignment and classification convention.

No Standard Model dynamical parameters were used as inputs.

Labels "electron" and "antineutrino" are observational identifiers [BL], not theoretical constructs.

References

[1] Igor Grčman. "Neutron Stability in Elastic Diffusive Cosmology: A 5D Geometric Approach". In: *[Preprint]* (2026). Paper 3 main document; this companion note provides detailed topology derivation.

[2] Igor Grčman. *Derivation of the Effective Lagrangian $L_{\text{eff}}(q, \dot{q})$ from the 5D Einstein-Hilbert Action*. Companion Note to Paper 3. Complete derivation of L_{eff} from first principles. 2026.