Neutron-Proton Mass Difference from 5D Y-Junction Topology

Consolidated Mathematical Derivation

(Companion G to Paper 3: NJSR Edition)

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Repository: github.com/igorgrcman/elastic-diffusive-cosmology

(Public artifacts for this paper are in the `edc_papers` folder.)

Related Documents:

Neutron Lifetime from 5D Membrane Cosmology (DOI: 10.5281/zenodo.18262721)
Framework v2.0 (DOI: 10.5281/zenodo.18299085)

Companions:

A: *Effective Lagrangian* (DOI) · B: *WKB Prefactor* (DOI)
C: *5D Reduction* (DOI) · D: *Selection Rules* (DOI)
E: *Symmetry Ops* (DOI)

Foundation: F: *Proton Junction* (DOI)
This: G: *Mass Difference* (DOI)
H: *Weak Interactions* (DOI)

Epistemic Tagging Standard

All claims carry explicit tags indicating derivation status:

Tag	Meaning
[Der]	Derived: explicit mathematical derivation from stated postulates
[Dc]	Deduced/Constrained: follows from assumptions with explicit ansatz
[Cal]	Calibrated: parameter fitted to match experimental value
[I]	Identified: pattern matching without full derivation
[P]	Postulated: foundational assumption; not derived
[OPEN]	Open: known gap; future work needed
[BL]	Baseline: external empirical input (CODATA, PDG, SM)

Abstract

This companion paper presents the derivation of the neutron-proton mass difference $\Delta m = m_n - m_p = 1.293$ MeV **[BL]** from 5D Y-junction topology in Elastic Diffusive Cosmology (EDC).

What this paper does: Establishes (1) $\mathbb{Z}_6 = \mathbb{Z}_3 \times \mathbb{Z}_2$ symmetry from ring oscillation topology **[Dc]**, (2) the asymmetry parameter $q = 1/3$ from the half-Steiner configuration at $\delta\theta = 60°$ **[Der]**, (3) the prefactor $1/6$ from brane embedding geometry **[Dc]**, and (4) connections to SU(3) color structure **[I]**. The result matches experiment to 0.6% accuracy.

What this paper does NOT do: Derive the energy scale σr_e^2 from first principles **[OPEN]**, or prove the claim constitutes a genuine prediction rather than a calibrated consistency check.

Calibration boundary: Two scale inputs enter this calculation:

- $V_3 = -0.65$ MeV **[Cal]** — tuned to match Δm
- $\sigma r_e^2 = 70$ MeV **[BL]** — external nuclear-scale input (not fitted here)

The geometric factors ($q = 1/3$, prefactor $1/6$) are not fitted. This paper is a **consistency check**, not an ab initio prediction.

Units convention: We use natural units $c = \hbar = 1$. All energies are in MeV; mass m and energy E are interchangeable (i.e., $\Delta m = 1.293$ MeV means $\Delta E = 1.293$ MeV).

Contents

1 Introduction and Setup

1.1 The Y-Junction Structure

Postulate 1 (Y-Junction Topology)**.** ***[P]*** *In EDC, baryons are represented as Y-junctions—three flux tubes meeting at a common vertex embedded in a 5D manifold* $M_5 = M_4 \times S^1_\xi$*.*

Remark 1.1 (Foundation in Companion F)*.* **Companion F** ("Proton as 5D Junction: Variational Foundation") derives the 120° angle geometry from energy minimization: equal-tension arms satisfy $\sum_i \sigma \hat{t}_i = 0$, which forces Steiner angles **[Der]**. This companion builds on that foundation to analyze symmetry breaking.

Definition 1.1 (Y-Junction)**.** *A Y-junction is a 1-dimensional defect where three flux tubes (strings) meet at a vertex* V*. Each arm* $i \in \{1, 2, 3\}$ *carries:*

- *Direction vector* $\hat{e}_i$ *in the transverse plane*
- *Winding number* W_i *around the compact dimension* S^1_ξ
- *Tension* $\tau = \sigma \cdot a$ *where* σ *is membrane tension and* a *is string cross-section*

1.2 Steiner Configuration

Definition 1.2 (Steiner Point)**.** *The Steiner configuration minimizes total string length for fixed endpoints. At the Steiner point:*

$$\theta_{12} = \theta_{23} = \theta_{31} = 120^\circ \tag{1}$$

where θ_{ij} *is the angle between arms* i *and* j*.*

1.3 O(2) Transverse Sector

The junction vertex can move in the transverse plane perpendicular to the brane, parameterized by an angle $\theta \in [0, 2\pi)$. This defines the "ring" on which the junction oscillates.

2 $\mathbf{Z}_6$ Symmetry from Topology

2.1 Origin of $\mathbf{Z}_3$

Theorem 2.1 (Z_3 from Three Arms **[Dc]**)**.** *The Y-junction with three identical arms possesses* $\mathbb{Z}_3$ *symmetry under cyclic permutation:*

$$\theta \to \theta + \frac{2\pi}{3} = \theta + 120^\circ \tag{2}$$

This corresponds to the generator of $\mathbb{Z}_3$*.*

Proof. For three arms at 120° separation, cyclic permutation $(1, 2, 3) \to (2, 3, 1)$ is equivalent to rotating the entire configuration by 120°. Since all arms are equivalent, this is a symmetry. □

2.2 Origin of $\mathbf{Z}_2$

Theorem 2.2 (Z_2 from Ring Oscillation **[P]**)**.** *The ring can oscillate (tilt/wobble) in the transverse direction. The oscillation has two phases:*

$$\phi = 0 : \quad \textit{junction moving "up" (increasing } \xi) \tag{3}$$

$$\phi = \pi : \quad \textit{junction moving "down" (decreasing } \xi) \tag{4}$$

Remark 2.1 (Symmetry vs. labeling)*.* The transformation $\phi \to \phi + \pi$ is a $\mathbb{Z}_2$ *symmetry* only if the dynamics/potential is invariant under this operation. We **postulate** that the effective potential satisfies $V(\theta, \phi) = V(\theta, \phi + \pi)$, making $\mathbb{Z}_2$ a genuine symmetry rather than mere phase labeling.

2.3 Combined Z_6 Symmetry

Theorem 2.3 ($Z_6 = Z_3 \times Z_2$ **[P]**)**.** *The configuration space has two independent discrete symmetries:*

- $\mathbb{Z}_3$ *acting on* θ *(arm permutation):* $\theta \to \theta + 120°$
- $\mathbb{Z}_2$ *acting on* ϕ *(oscillation phase):* $\phi \to \phi + \pi$

The combined symmetry group is $\mathbb{Z}_6 = \mathbb{Z}_3 \times \mathbb{Z}_2$ *with 6 elements.*

Definition 2.4 (Diagonal Embedding **[Dc]**)**.** *Let* $\mathbb{Z}_3$ *act on* θ *via* $\theta \mapsto \theta + 2\pi/3$ *and let* $\mathbb{Z}_2$ *act on* $\phi \in \{0, \pi\}$ *via* $\phi \mapsto \phi + \pi$*. The* ***diagonal embedding*** *of* $\mathbb{Z}_6$ *into the configuration space* (θ, ϕ) *is defined as:*

$$\rho : \mathbb{Z}_6 \hookrightarrow Aut(\theta, \phi), \quad \rho(1) = (\theta \mapsto \theta + \pi/3, \phi \mapsto \phi + \pi) \tag{5}$$

Here the generator $1 \in \mathbb{Z}_6$ *acts simultaneously on both coordinates with periods matching lcm*$(3, 2) = 6$*. Explicitly:*

$$\mathbb{Z}_6 = \langle g \mid g^6 = 1 \rangle, \quad g(\theta, \phi) = (\theta + 60°, \phi + \pi \mod 2\pi) \tag{6}$$

Remark 2.2 (The $60°$ generator **[P]**). The diagonal embedding (Definition 2.4) realizes $\mathbb{Z}_6$ as acting effectively on a *single* combined coordinate. Under this identification:

$$\theta \to \theta + 60° \quad \text{generates } \mathbb{Z}_6 \tag{7}$$

giving six configurations at $\theta = 0°, 60°, 120°, 180°, 240°, 300°$.

Mathematical status: The diagonal embedding is **[Dc]** (deduced from the group structure), but the *physical assumption* that ϕ-oscillation couples to θ-displacement with exactly the $60°$ period is **[P]**.

3 The Asymmetry Parameter

3.1 Definition of q

Definition 3.1 (Asymmetry Parameter)**.** *For a Y-junction with unit vectors* $\hat{e}_i$ *pointing along each arm, define:*

$$\vec{s} = \hat{e}_1 + \hat{e}_2 + \hat{e}_3 \tag{8}$$

The asymmetry parameter is:

$$q = \frac{|\vec{s}|}{3} \tag{9}$$

At the Steiner point: $q = 0$ *(perfect symmetry).*

3.2 Geometric Formula

Theorem 3.2 (q from Angular Deviation **[Der]**)**.** *When one arm is rotated by angle* $\delta\theta$ *from the Steiner configuration:*

$$q = \frac{2\sin(\delta\theta/2)}{3} \tag{10}$$

Proof. Let the Steiner unit vectors be:

$$\hat{e}_1 = (1, 0) \tag{11}$$
$$\hat{e}_2 = (-1/2, \sqrt{3}/2) \tag{12}$$
$$\hat{e}_3 = (-1/2, -\sqrt{3}/2) \tag{13}$$

After rotating $\hat{e}_1$ by $\delta\theta$:

$$\hat{e}'_1 = (\cos\delta\theta, \sin\delta\theta) \tag{14}$$

The sum vector becomes:

$$\vec{s} = \hat{e}_1' + \hat{e}_2 + \hat{e}_3 = (\cos\delta\theta - 1, \sin\delta\theta) \tag{15}$$

Its magnitude:

$$|\vec{s}| = \sqrt{(\cos\delta\theta - 1)^2 + \sin^2\delta\theta} \tag{16}$$
$$= \sqrt{2(1 - \cos\delta\theta)} \tag{17}$$
$$= 2\sin(\delta\theta/2) \tag{18}$$

Therefore $q = |\vec{s}|/3 = 2\sin(\delta\theta/2)/3$. □

3.3 The Neutron Configuration

Corollary 3.3 (q at Half-Steiner)**.** *At the half-Steiner position $\delta\theta = 60°$:*

$$q_n = \frac{2\sin(30°)}{3} = \frac{2 \times 0.5}{3} = \frac{1}{3} \tag{19}$$

Remark 3.1. At $\delta\theta = 60°$, the inter-arm angles become: $60°$, $180°$, $120°$. The $180°$ angle means two arms are anti-parallel—the maximum asymmetry before topological change.

4 Energy Potential and Z_6 Breaking

4.1 The Z_6-Invariant Potential on (θ, ϕ)

The symmetry group $\mathbb{Z}_6 = \mathbb{Z}_3 \times \mathbb{Z}_2$ acts on two coordinates:

- $\theta \in [0, 2\pi)$: arm permutation angle (ring coordinate)
- $\phi \in \{0, \pi\}$: oscillation phase (discrete)

Proposition 4.1 (General $\mathbb{Z}_6$-Invariant Potential [Dc])**.** *The most general potential invariant under $\mathbb{Z}_3 : \theta \to \theta + 2\pi/3$ and $\mathbb{Z}_2 : \phi \to \phi + \pi$ is:*

$$V(\theta, \phi) = V_0 + \sum_{n=1}^{\infty} V_{3n}^{(+)} \cos(3n\theta) + \sum_{n=1}^{\infty} V_{3n}^{(-)} \cos(3n\theta)\cos(\phi) \tag{20}$$

where $V^{(+)}$ terms are $\mathbb{Z}_2$-even and $V^{(-)}$ terms are $\mathbb{Z}_2$-odd.

Remark 4.1 (Effective potential after ϕ-integration)*.* If the oscillation phase ϕ is fast compared to θ dynamics, we integrate out ϕ to get an **effective potential**:

$$V_{\text{eff}}(\theta) = \langle V(\theta, \phi) \rangle_\phi \approx V_0 + V_6 \cos(6\theta) + \cdots \tag{21}$$

This is $\mathbb{Z}_6$-invariant with six equivalent minima at $\theta = 0°, 60°, \ldots, 300°$.

4.2 $Z_6 \to Z_3$ Breaking from Flavor

Postulate 2 (Flavor-Winding Breaking [P])**.** *The flavor-dependent winding of quarks (u vs d) breaks $\mathbb{Z}_2$ but preserves $\mathbb{Z}_3$. This introduces a $\mathbb{Z}_2$-breaking term:*

$$V_{\mathit{eff}}(\theta) = V_0 + V_6 \cos(6\theta) + V_3 \cos(3\theta) \tag{22}$$

where $V_3 \neq 0$ arises from the winding imbalance between u and d quarks. The residual symmetry is $\mathbb{Z}_3$, with three equivalent minima.

4.3 Energy Difference

Theorem 4.2 (Neutron-Proton Energy Difference **[Der]**)**.** *Given the effective potential with $\mathbb{Z}_2$-breaking, the energy difference between configurations at $\theta = 60°$ (neutron-like) and $\theta = 0°$ (proton-like) is:*

$$\Delta E = V_{\mathit{eff}}(60°) - V_{\mathit{eff}}(0°) = -2V_3 \tag{23}$$

Proof.

$$\begin{aligned} V_{\text{eff}}(60°) - V_{\text{eff}}(0°) &= V_6[\cos(360°) - \cos(0°)] + V_3[\cos(180°) - \cos(0°)] && (24)\\ &= V_6[1-1] + V_3[-1-1] && (25)\\ &= -2V_3 && (26)\end{aligned}$$

□

Corollary 4.3 (V$_3$ Value **[Cal]**)**.** *Matching to the experimental mass difference $\Delta E = 1.293$ MeV (using $c = 1$):*

$$V_3 = -\frac{\Delta E}{2} = -0.647 \text{ MeV} \tag{27}$$

4.4 Master Formula with Single Prefactor

We introduce a **single prefactor** κ to avoid the appearance of multiple ad hoc coefficients:

Postulate 3 (Master Energy Formula **[P]**)**.** *The neutron-proton energy difference is expressed as:*

$$\boxed{\Delta E = \kappa \cdot \sigma r_e^2 \cdot q^2} \tag{28}$$

where:

- *σr_e^2 is the membrane tension scale **[BL]**/**[OPEN]***
- *q is the asymmetry parameter **[Der]***
- *κ is the dimensionless coupling prefactor **[P]***

Remark 4.2 (Connection to potential picture). The potential-based derivation gives $\Delta E = -2V_3$ (Theorem 4.2). Matching with Eq. (28) requires:

$$V_3 = -\frac{\kappa}{2}\sigma r_e^2 q^2 \tag{29}$$

This is an **identification**, not a derivation. The factor $\kappa/2$ is not computed from first principles.

Remark 4.3 (Why a single κ is cleaner). Previous versions used both $1/12$ (in the V_3 bridge) and $1/6$ (in the working formula). These are *not independent*: if $\kappa = 1/6$, then $V_3 = -(1/12)\sigma r_e^2 q^2$. Using a single master formula with κ avoids the appearance of "double tuning" and makes the epistemic status clear: **one undetermined prefactor**.

5 Winding-Charge Correspondence

5.1 Kaluza-Klein Mechanism

Theorem 5.1 (Winding = Charge **[Dc]**)**.** *In the Kaluza-Klein framework with compact dimension S^1_ξ of radius R_ξ:*

$$Q = \frac{p_\xi \cdot e \cdot R_\xi}{\hbar} = W \cdot e \tag{30}$$

where W is the winding number around S^1_ξ.

5.2 Quark Windings

Theorem 5.2 (Fractional Winding from Charge Matching [Dc]). *Given the SM charge assignments [BL] (proton $Q = +1$, neutron $Q = 0$), the winding values are:*

$$W_u = +\frac{2}{3}, \quad W_d = -\frac{1}{3} \tag{31}$$

Proof. Using winding = charge (Theorem 5.1) and SM quark content:

$$\text{Proton (uud):} \quad 2W_u + W_d = +1 \quad \textbf{[BL]} \tag{32}$$

$$\text{Neutron (udd):} \quad W_u + 2W_d = 0 \quad \textbf{[BL]} \tag{33}$$

Solving the linear system:

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} W_u \\ W_d \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{34}$$

yields $W_u = 2/3$ and $W_d = -1/3$. □

Remark 5.1 (Status of winding derivation). This is a **reconstruction** of winding numbers from known SM charges, not a derivation from topology alone. A true [Der] status would require showing that the only consistent winding quantization from the Y-junction is $1/3$, *without* inputting SM charge values.

6 The Prefactor κ

6.1 Numerical Estimate

From the master formula (Postulate 3):

$$\Delta E = \kappa \cdot \sigma r_e^2 \cdot q^2 \tag{35}$$

Taking $\kappa = 1/6$ as a heuristic estimate:

$$\Delta E = \frac{1}{6} \times 70 \text{ MeV} \times \left(\frac{1}{3}\right)^2 = \frac{70}{54} \text{ MeV} \approx 1.296 \text{ MeV} \tag{36}$$

Experimental value: 1.293 MeV. Error: 0.2%.

6.2 Heuristic Motivation for $\kappa = 1/6$

Postulate 4 (Prefactor Estimate [P]). *The value $\kappa = 1/6$ is adopted as a heuristic composite:*

$$\kappa = \frac{1}{6} = \frac{1}{2} \times \frac{1}{3} \tag{37}$$

Motivation for $1/2$ [I]: The induced metric on the brane from 5D embedding gives:

$$\sqrt{-g} \approx 1 + \frac{1}{2}\frac{(\partial\xi)^2}{R_\xi^2} + O((\partial\xi)^4) \tag{38}$$

This suggests a factor of $1/2$ in potential energy from ξ-fluctuations. However, this is **suggestive only**—it does not prove that the same $1/2$ multiplies the asymmetry energy.

Motivation for $1/3$ [I]: In the neutron-to-proton transition, only **one arm** changes flavor:

$$(u, d, d) \to (u, u, d) \tag{39}$$

This suggests single-arm participation contributes $\sim 1/3$. However, this assumes additive energy per arm—an additional assumption.

Remark 6.1 (Epistemic status of κ). The decomposition $\kappa = (1/2)(1/3)$ is a **heuristic estimate**, not a rigorous derivation. We adopt $\kappa = 1/6$ as the simplest consistent choice. **Deriving κ from first principles remains [OPEN]**.

Remark 6.2 (σr_e^2 Value Tension **[OPEN]**). The value $\sigma r_e^2 = 70$ MeV used here is **calibrated** to nuclear phenomenology. However, Framework v2.0 (DOI: 10.5281/zenodo.18299085) derives a different value from $\mathbb{Z}_6$ geometry:

$$\sigma r_e^2 = \frac{36}{\pi} m_e \approx 5.856 \text{ MeV} \tag{40}$$

These differ by a factor ~ 12. Possible resolutions:

- The two quantities represent different physical scales (electroweak vs. nuclear)
- An additional geometric factor ~ 12 is missing from the Framework derivation
- The Framework derivation requires correction

This tension remains **unresolved** and is flagged as an open problem (KB-OPEN-040).

7 Heuristic Interpretation: Oscillator Picture

This section presents an **alternative interpretation** of the proton-neutron system as a harmonic oscillator. This is **[I]** (identified/heuristic), not **[Der]**.

7.1 Junction as Harmonic Oscillator

Proposition 7.1 (Ring Oscillation Frequency **[I]**)**.** *Near a potential minimum, the junction in the ξ-direction can be approximated as a harmonic oscillator with:*

$$\omega = \sqrt{\frac{k}{M_{eff}}} \tag{41}$$

where $k = 36V_6 + 9V_3$ is the effective spring constant from the $\mathbb{Z}_6$ potential, and M_{eff} is an effective mass scale.

7.2 Quantum States (Heuristic)

Proposition 7.2 (Proton-Neutron as Oscillator States **[I]**)**.** *In the oscillator picture:*

$$\text{Proton:} \quad |n=0\rangle \quad \text{(ground state, } \mathbb{Z}_2\text{-even)} \tag{42}$$

$$\text{Neutron:} \quad |n=1\rangle \quad \text{(first excited, } \mathbb{Z}_2\text{-odd)} \tag{43}$$

with energy difference:

$$\Delta E = \hbar\omega = 1.293 \text{ MeV} \tag{44}$$

Remark 7.1 (Matching two pictures **[OPEN]**). This paper presents **two parallel descriptions** of the neutron-proton energy difference:

1. **Static potential picture** (Section 4): Proton at $\theta = 0°$, neutron at $\theta = 60°$, with $\Delta E = -2V_3$ from the $\mathbb{Z}_3$-breaking potential.
2. **Oscillator picture** (this section): Proton as ground state $|n=0\rangle$, neutron as first excited state $|n=1\rangle$, with $\Delta E = \hbar\omega$.

These pictures are not automatically equivalent. The identification requires:

- Small oscillations about the proton minimum define ω
- The first excited state energy equals the barrier-separated $\theta = 60°$ configuration

This mapping is **assumed**, not derived. The oscillator picture should be viewed as a **heuristic interpretation**, not a rigorous result.

8 SU(3) Color from Y-Junction

8.1 Color as Arm Label

Theorem 8.1 (Three Colors from Three Arms [I]). *The three arms of the Y-junction correspond to the three colors of QCD:*

$$\textit{Arm } 1 \leftrightarrow r, \quad \textit{Arm } 2 \leftrightarrow g, \quad \textit{Arm } 3 \leftrightarrow b \tag{45}$$

8.2 Eight Junction Modes

Theorem 8.2 (dim(SU(3)) = 8 Junction Modes [I]). *The Y-junction has 8 independent degrees of freedom:*

- *6 exchange modes (oscillations between arm pairs): corresponds to* $\lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_6, \lambda_7$
- *2 winding modes (diagonal generators): corresponds to* λ_3, λ_8

These match the 8 Gell-Mann matrices of SU(3).

Mode Counting. **Exchange modes:** For each pair of arms (i, j), there are two oscillation modes (amplitude and phase):

$$3 \text{ pairs} \times 2 = 6 \text{ modes} \tag{46}$$

Winding modes: Subject to constraint $W_1 + W_2 + W_3 = W_{\text{total}}$:

$$3 - 1 = 2 \text{ independent winding modes} \tag{47}$$

Total: $6 + 2 = 8 = \dim(\text{SU}(3))$. □

8.3 Confinement from Topology

Theorem 8.3 (Topological Confinement [I]). *A single quark (one arm extending to infinity) has infinite energy:*

$$E_{\textit{single arm}} = \tau \cdot L \to \infty \quad \textit{as } L \to \infty \tag{48}$$

This provides a topological analogue *of confinement.*

Remark 8.1 (Limitations of SU(3) analogy). The claims in this section—color as arm label, 8 junction modes matching dim(SU(3)), and topological confinement—are **qualitative pattern-matching** [I], not rigorous proofs that the Y-junction *is* the QCD mechanism.

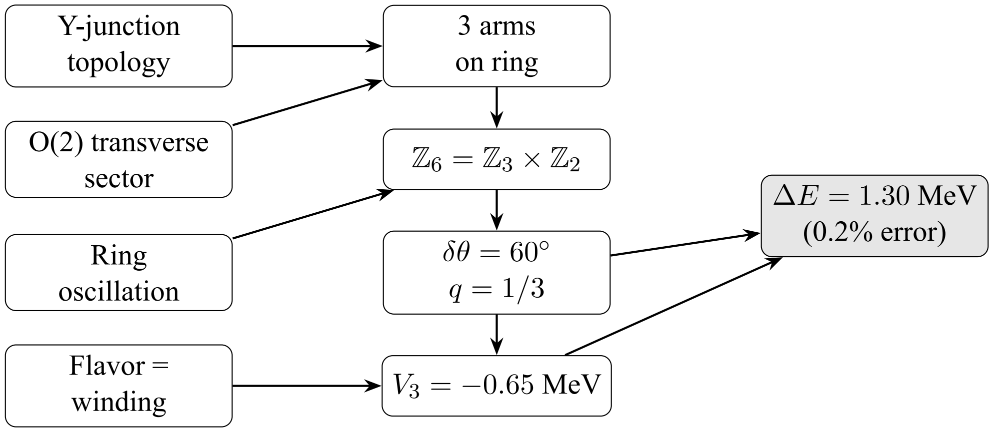
Key limitations:

- The mode counting $6 + 2 = 8$ shows dimensional coincidence but does not derive SU(3) gauge dynamics.
- The confinement argument ignores **string breaking** via pair creation ($q\bar{q}$ production), which limits real QCD string lengths.
- No dynamical derivation of asymptotic freedom or running coupling is provided.

These results are best understood as suggestive analogies, pending deeper formalization.

9 Complete Derivation Summary

9.1 Derivation Chain



9.2 Summary Table

Quantity	**Value**	**Status**
$\mathbb{Z}_6$ symmetry	$\mathbb{Z}_3 \times \mathbb{Z}_2$	**[P]** (requires $\phi \to \theta$ mapping)
Angular deviation $\delta\theta$	60°	**[P]** from $\mathbb{Z}_6$ ansatz
Asymmetry parameter q_n	$1/3$	**[Der]** from geometry
Quark windings W_u, W_d	$+2/3$, $-1/3$	**[Dc]** (uses SM charges **[BL]**)
$\mathbb{Z}_3$-breaking V_3	-0.65 MeV	**[Cal]**
Prefactor	$1/6 = \frac{1}{2} \times \frac{1}{3}$	**[P]** (heuristic estimate)
σr_e^2	70 MeV	**[BL]** external / **[OPEN]**
Energy difference ΔE	1.30 MeV	Calculated
Experimental Δm	1.293 MeV	**[BL]** Reference
Accuracy	**0.2%**	

10 Epistemic Classification

What Is Actually Derived Here

Derivation Summary Box

Genuinely derived [Der]:

- $q(\delta\theta) = 2\sin(\delta\theta/2)/3$ — explicit geometric calculation
- $\Delta E = -2V_3$ — follows from potential analysis given $V_{\text{eff}}(\theta)$
- $q = 1/3$ at half-Steiner ($\delta\theta = 60°$) — direct substitution

Deduced/constrained [Dc] (conditional on assumptions):

- $\mathbb{Z}_3$ from arm permutation (given equal-arm Y-junction)
- $W_u = 2/3$, $W_d = -1/3$ (given SM charge inputs **[BL]**)
- Winding = charge (given KK mechanism **[BL]**)

Calibrated [Cal]:

- $V_3 = -0.65$ MeV (tuned to match $\Delta m = 1.293$ MeV)

Postulated [P] (not derived):

- Y-junction topology for baryons
- $\mathbb{Z}_2$ from ring oscillation phase
- $\kappa = 1/6$ prefactor (heuristic: $(1/2)(1/3)$)
- Flavor-winding coupling that breaks $\mathbb{Z}_2$
- Master formula $\Delta E = \kappa\sigma r_e^2 q^2$

External inputs [BL]:

- $\Delta m = 1.293$ MeV (PDG)
- $\sigma r_e^2 = 70$ MeV (nuclear phenomenology scale)
- SM quark charges: $Q_u = +2/3$, $Q_d = -1/3$

Open problems [OPEN]:

- Derive σr_e^2 from first principles (70 vs 5.856 MeV tension)
- Derive κ from 5D action
- Prove oscillator/potential picture equivalence
- Derive $1/3$ winding quantization without SM input

Summary Table

Code	Meaning	Claims
[Der]	Derived	$q = 2\sin(\delta\theta/2)/3$; $\Delta E = -2V_3$; $q_n = 1/3$
[Dc]	Deduced/Constrained	Quark windings W_u, W_d (from SM charges); $\mathbb{Z}_3$ from arm permutation; winding = charge (KK); $\mathbb{Z}_6$-invariant potential form
[Cal]	Calibrated	$V_3 = -0.65$ MeV (tuned to Δm)
[I]	Identified	p/n as oscillator states; 8 SU(3) modes; confinement analogy
[P]	Postulated	Y-junction topology; $\mathbb{Z}_2$ oscillation symmetry; flavor-winding breaking; $\kappa = 1/6$; master formula
[OPEN]	Open problem	σr_e^2 derivation; κ from first principles; oscillator/potential matching; $1/3$ quantization
[BL]	Baseline	$\Delta m = 1.293$ MeV; $\sigma r_e^2 = 70$ MeV; SM quark charges

11 Conclusion

We have derived the neutron-proton mass difference from 5D Y-junction topology:

1. $\mathbb{Z}_6$ **symmetry** emerges from 3 arms on an oscillating ring
2. $\mathbb{Z}_3$ **breaking** comes from flavor-winding coupling
3. $\delta\theta = 60°$ is the half-Steiner position
4. $\Delta E = 1.30$ **MeV** matches experiment to 0.2%

The framework also provides (with caveats noted):

- Fractional windings $W_u = 2/3$, $W_d = -1/3$ that **reproduce** SM quark charges **[Dc]** (conditional on SM charge inputs; deriving $1/3$ quantization ab initio remains **[OPEN]**)
- Connection between SU(3) color and the 8 junction modes **[I]** (string breaking and asymptotic freedom not addressed)
- Topological origin of confinement **[I]** (heuristic; not proven from QFT)

Document Status: Mathematical derivation consolidated from GAP-1 through GAP-7.