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Overlapping Communities Detection by Hypergraph Constructing

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What a community is?

- There is no universally accepted definition for community in graphs.
- Graph partitioning problem. Dozens of algorithms and techniques.
- Sets of nodes that the algorithm finds are then called “clusters,” “communities,” “groups,” “classes,” or “modules”.
- This is Ok! The applications are context-depended.
- Do we need one more algorithm? If it has a use case!

Community vs. Cluster

- Two main approaches for network partitioning: by whole network analysis or by local data.
- **Term cluster** is suitable for global approach, when clusters have been recognized by comparison properties of different parts of network.
  - Modularity
  - Betweenness centrality
- **Term community** is suitable for local approach, when a community have been recognized without full network analysis.
  - Clique percolation
  - Label propagation

Natural Community!!

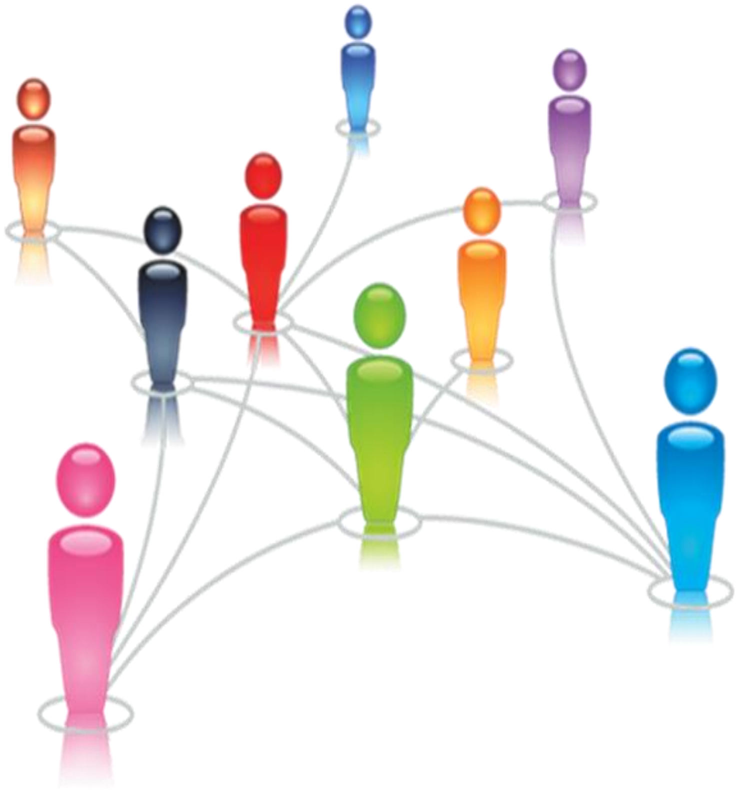
Some intuitive understanding of **overlapping communities** can be derived from social networks…

- Each node “knows” his community’s members: local property.
- Each node may belong to more than one community
- **Topologically**:
  two nodes **“surely”** belong to the same community if they have a significant number of common neighbors.

Nodes Commonality

- *N*(*i*) is the neighborhood of a node *i*
- *commonality*(*N*(*i*), *N*(*j*)) is a function of two nodes to quantify the status of their common neighbors.
- Exists a threshold c_0 , if *commonality*(*i,j*) > c_0 nodes *i,j* “for suer” belongs to the same community.
- For different type of networks, *commonality* may be different functions
- Commonality may be calculated by a pretrained neural network as a probability to be members of the same community.

Commonality - Jaccard coefficient

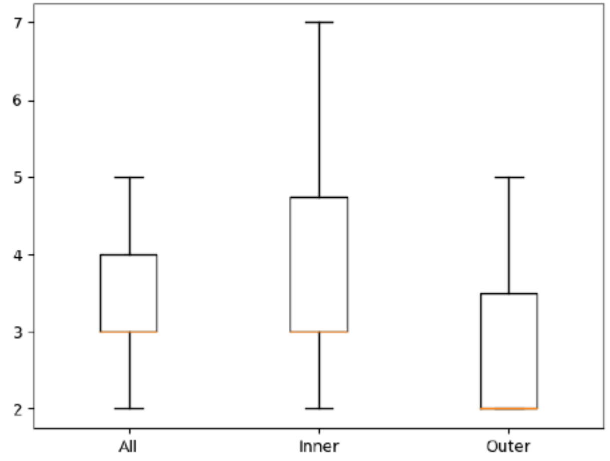


$$c(i,j) = \frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$$

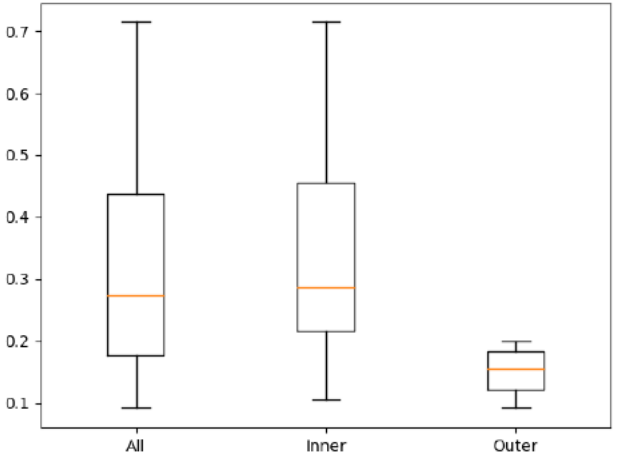
- *commonality c*(*i*,*j*) of two nodes *i* and *j* is a fraction of common neighbors
- The simplest, but may be not the best
- *c*(🔴,🟢)=2/5, *c*(🔴🔵)=3/4

commonality concept test

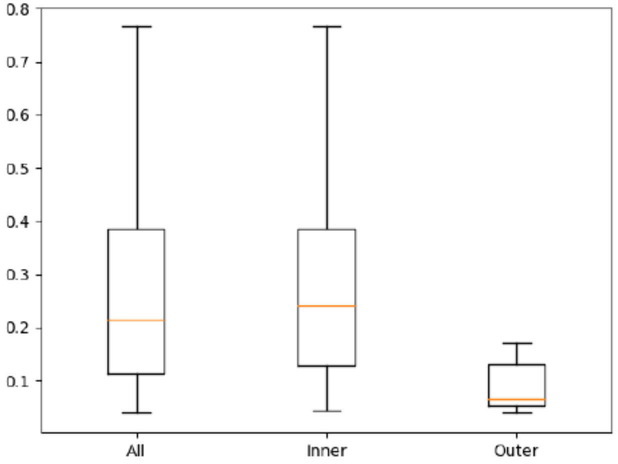
- calculate the commonality for different real-world networks with “ground truth”
- f1= $|N(i) \cap N(j)|$, f2= $\frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$, f3= $\frac{|N(i) \cap N(j)|^2}{|N(i) \cup N(j)|}$
- For Zachary’s Karate Club commonality distribution:



f1



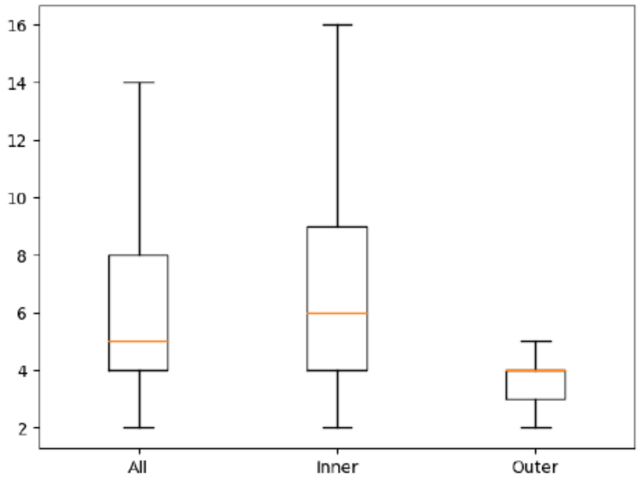
f2



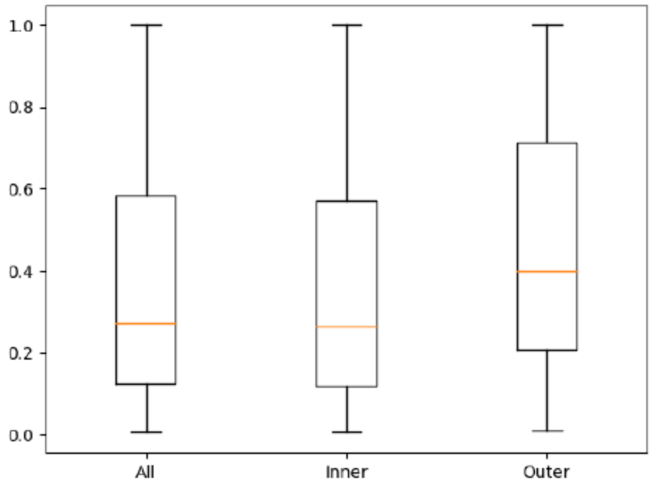
f3

commonality distribution

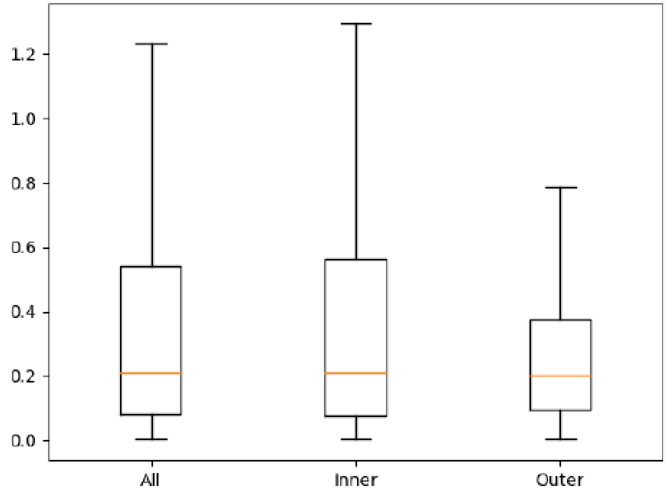
- For DBLP a co-authorship network Jaewon Yang and Jure Leskovec. “Defining and evaluating network communities based on ground-truth”. In: Proceedings of the ACM SIGKDD Workshop on Mining Data Semantics. 2012, pp. 1–8.
- f1= $|N(i) \cap N(j)|$, f2= $\frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$, f3= $\frac{|N(i) \cap N(j)|^2}{|N(i) \cup N(j)|}$



f1

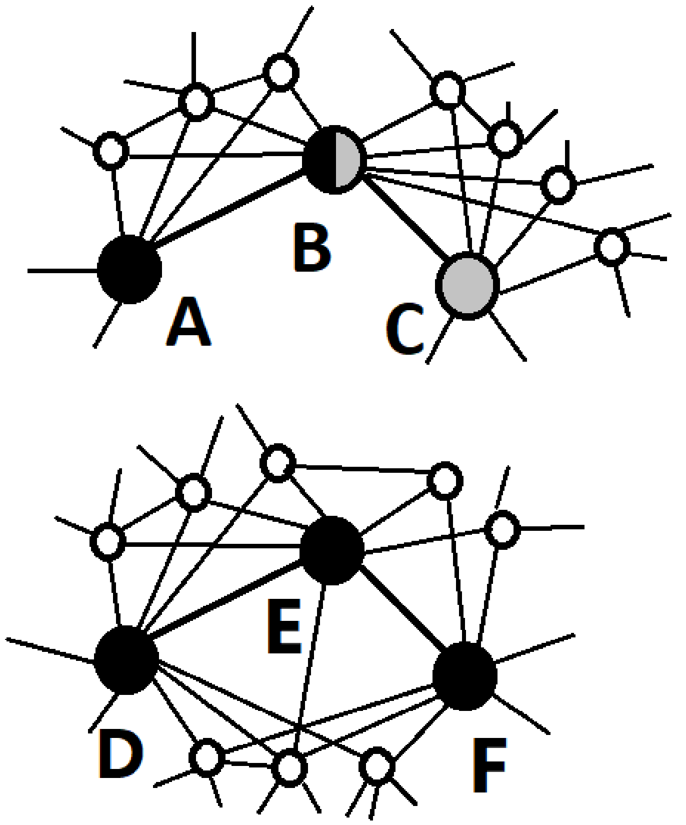


f2



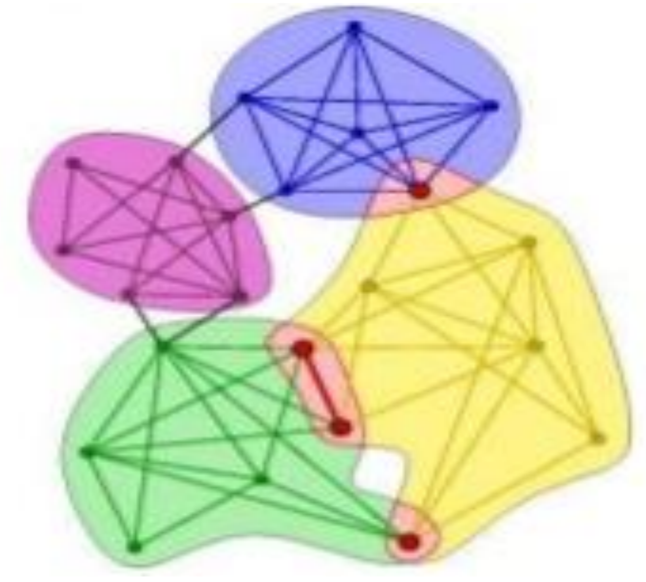
f3

Hypergraph constructing



- A link (i,j) is inside a community "for sure" (inner link) if $c(i,j) > c_0$
  - (A,B), (B,C), (D,E), (E,F) are "for sure" inner links. c(A,B)=3/7, c(B,C)=4/10, c(A,C)=1/13, c(D,E)=4/11, c(E,F)=3/11, c(D,F)=4/13.
- Three nodes having $c(i,j) > c_0$ are a hypernode.
  - (D,E,F) is a hypernode. (A,B,C) is not.
- Two hypernodes having two nodes in common are connected by hyperlink.

Natural Community - definition



- Natural community is a set of nodes belonging to a connected component of the hypergraph.
- Natural communities are overlapping.

Algorithm for a natural community detection

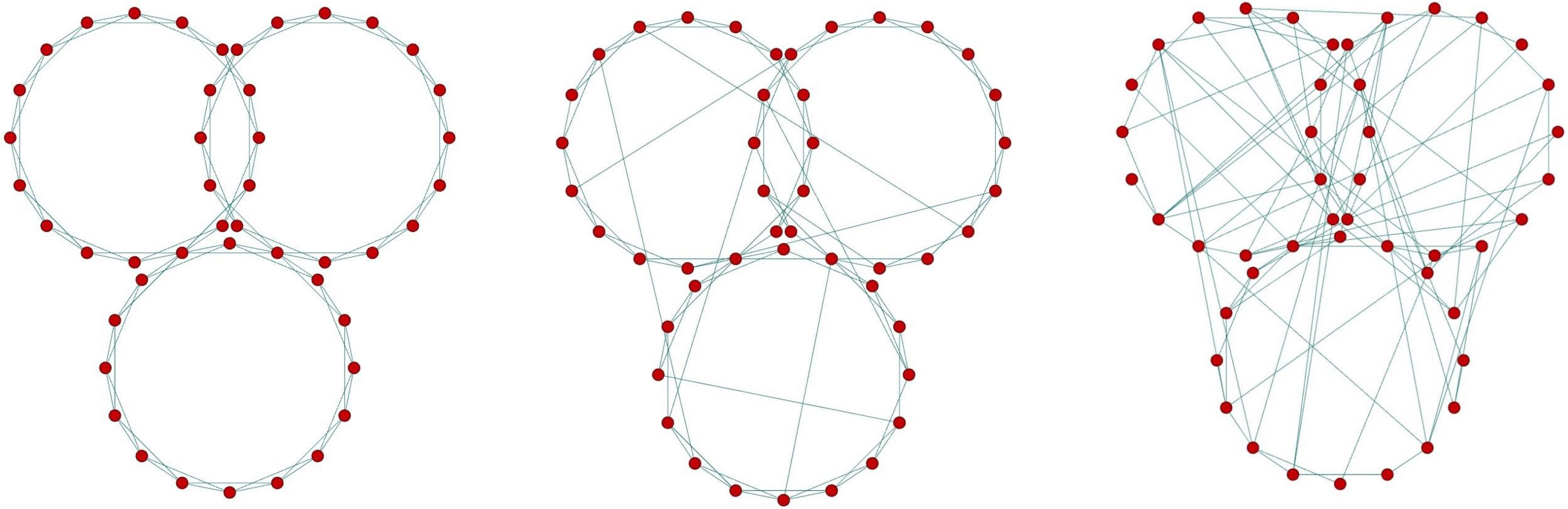
```
*Input:* network $G$=($V$,$E$), threshold value $c_0$;
*Output:* A community $C \subset G$ .
1.   Start with arbitrary node $v$ , C={$v$};
2.   Loop for each $w \in N(v)$
         if $c(v,w) > c_0$ put ($v$,$w$) into queue $Q$; break;
3. loop while $Q$ is not empty
     a. pop ($v$,$w$) from $Q$;
     b. loop for each $u \in N(w)$
          if  ($c(w,u) > c_0$ and $c(w,v) > c_0$ and $u \notin C$ )
               put ($w$, $u$) into $Q$; add $w$ *and* $u$ to $C$;
```

$N(v)$ is set of the node v neighbors

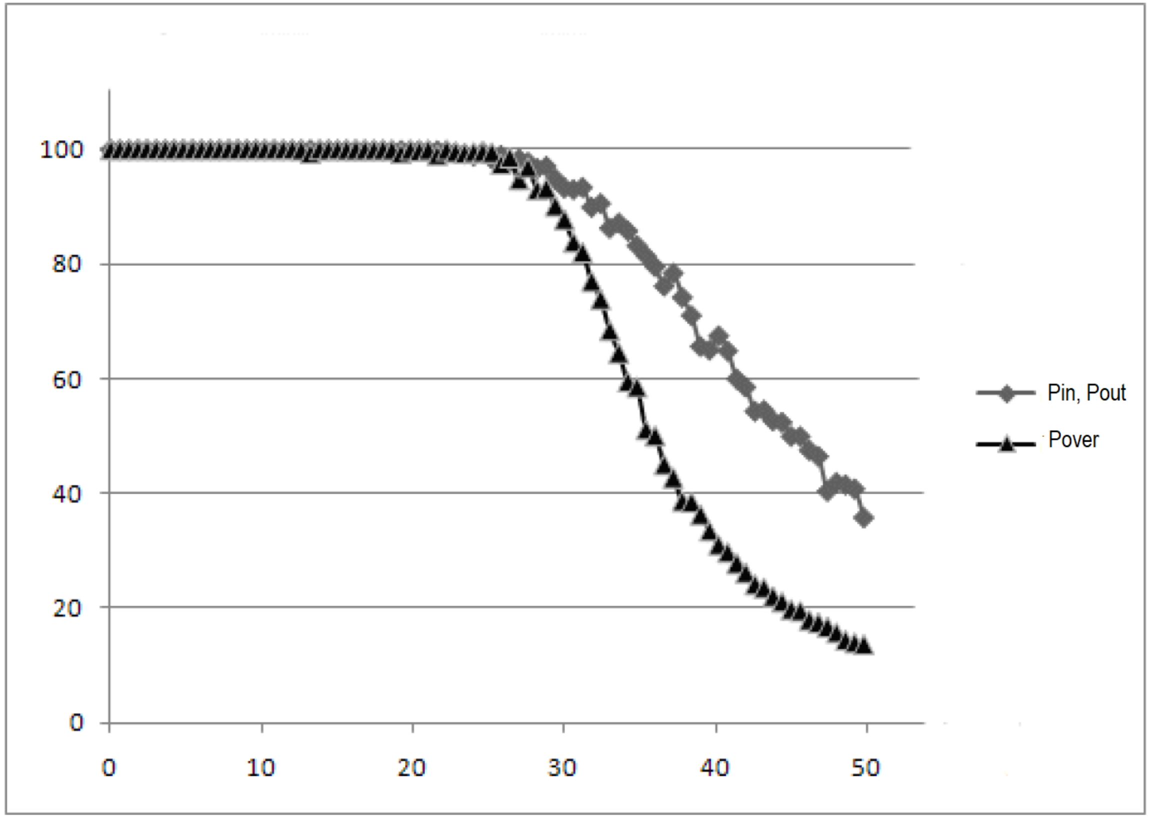
https://github.com/netcomdet/Network-Community-Detection

Synthetic data Test Case: Small World Graph Extension

- Collection of ring lattices with randomly reconnected P_{in} links inside the ring lattice and P_{out} reconnected links between the rings
- For overlapping case P_{over} randomly chosen are common for rings nodes.



Simulation for the Test Case



- % of nodes recognized as correct communities' members as function of % randomized links for 16 rings - communities

Conclusion

- **Commonality** quantifies the potential of two nodes to belong to the same community, based on their shared neighbors.
- A **hypernode** is defined as a set of three nodes having high mutual commonality.
- A **hyperlink** exists between two hypernodes having two nodes in common.
- A **Natural Community** is a connected component of the hypergraph.
- **The algorithm** developed from these definitions is straightforward, utilized local data, efficient and effective. It also exhibits stability in the face of random link perturbations.

Thank you.

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