



# Overlapping Communities Detection by Commonality

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# What a community is?

- There is no universally accepted definition for community in graphs.
- Graph partitioning problem. Dozens of algorithms and techniques.
- Sets of nodes that the algorithm finds are then called “clusters,” “communities,” “groups,” “classes,” or “modules”.
- This is Ok! The applications are context-depended.
- Do we need one more algorithm? If it has a use case!
- Some intuitive understanding of **overlapping communities** can be derived from social networks...

# Natural Community!!



- Each node “knows” his community’s members: local property.
- Each node may belong to more than one community
- **Topologically:**  
two nodes “**surely**” belong to the same community if they have a significant number of common neighbors.

# Nodes Commonality

- $N(i)$  is the neighborhood of a node  $i$
- $commonality(N(i), N(j))$  is a function of two nodes to quantify the status of their common neighbors.
- Exists a threshold  $c_0$ , if  $commonality(i,j) > c_0$  nodes  $i,j$  “for suer” belongs to the same community.
- For different type of networks,  $commonality$  may be different functions
- Commonality may be a pretrained neural network.

# Commonality - Jaccard coefficient

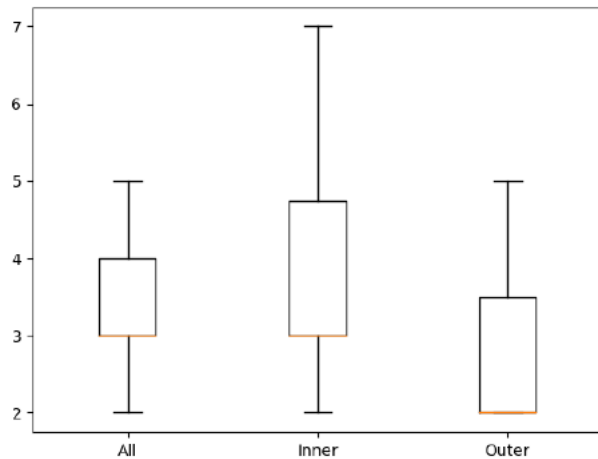
$$c(i, j) = \frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$$



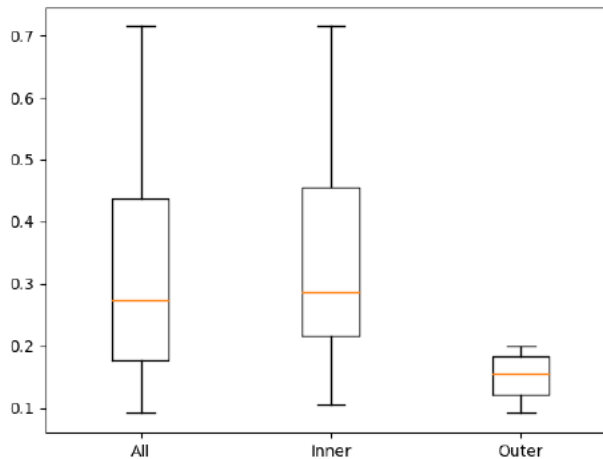
- *commonality*  $c(i, j)$  of two nodes  $i$  and  $j$  is a fraction of common neighbors
- The simplest, but may be not the best
- $c(\text{pink}, \text{green}) = 2/5$ ,  $c(\text{pink}, \text{dark blue}) = 3/4$

# commonality concept test

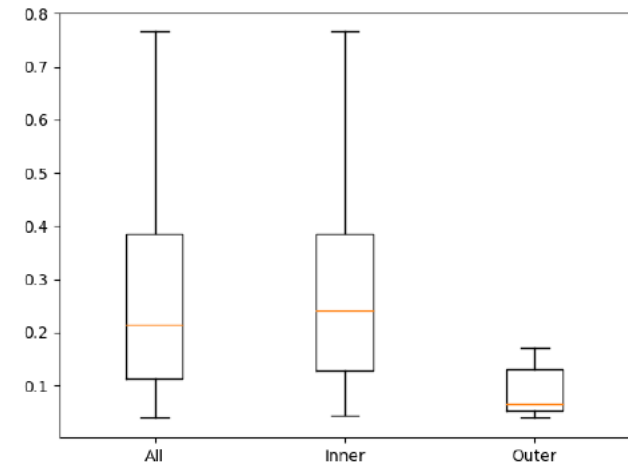
- calculate the commonality for different real-world networks with “ground truth”
- $f1 = |N(i) \cap N(j)|$  ,  $f2 = \frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$  ,  $f3 = \frac{|N(i) \cap N(j)|^2}{|N(i) \cup N(j)|}$
- For Zachary’s Karate Club commonality distribution:



f1



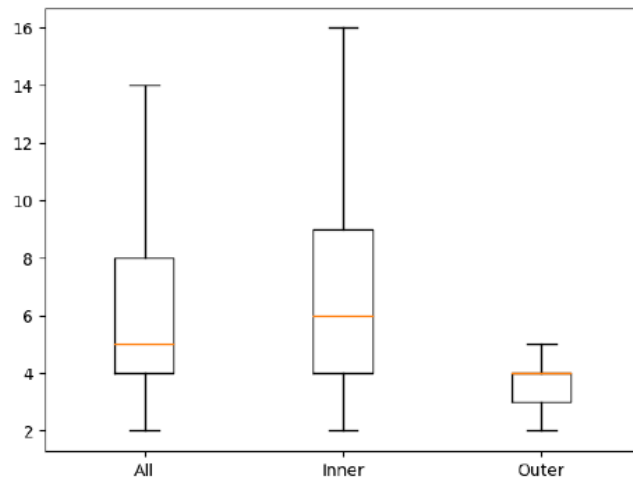
f2



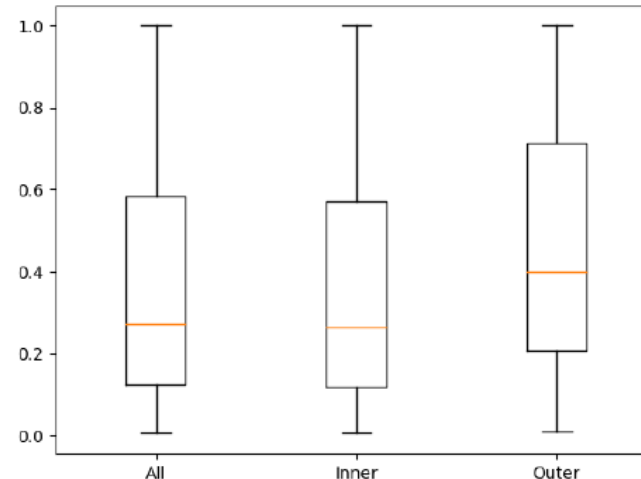
f3

# commonality distribution

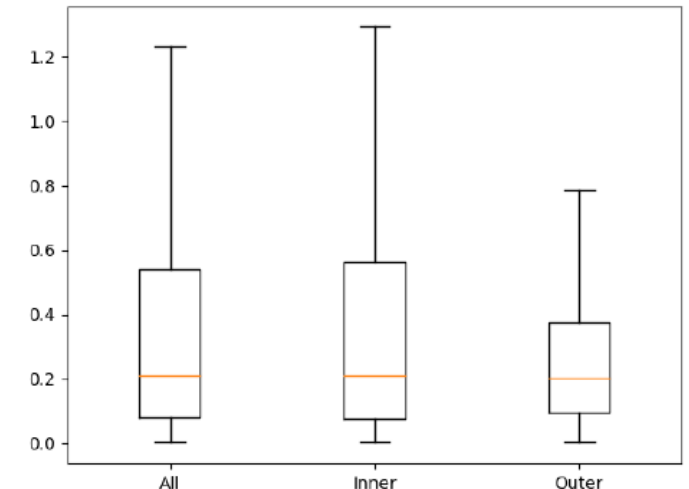
- For DBLP a co-authorship network Jaewon Yang and Jure Leskovec. “Defining and evaluating network communities based on ground-truth”. In: Proceedings of the ACM SIGKDD Workshop on Mining Data Semantics. 2012, pp. 1–8.
- $f1 = \frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$ ,  $f2 = \frac{|N(i) \cap N(j)|^2}{|N(i) \cup N(j)|}$



f1

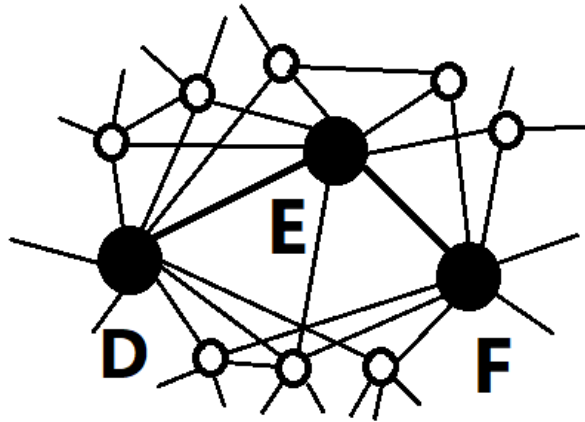
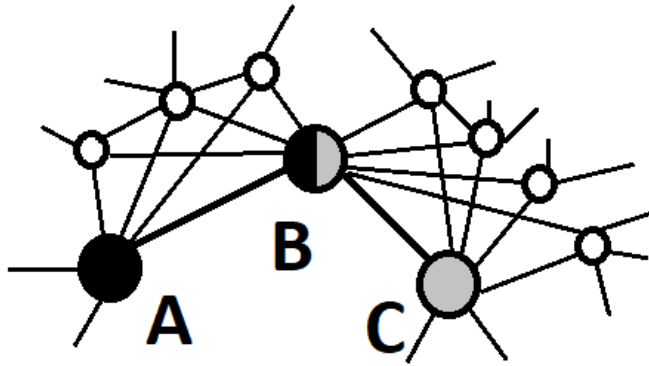


f2



f3

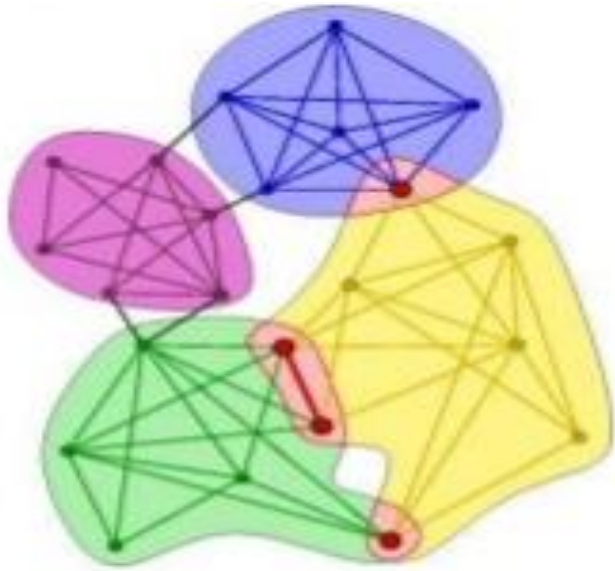
# inner community path



- A link  $(i,j)$  is inside a community “for sure” (inner link) if  $c(i,j) > c_0$
- $(A,B)$ ,  $(B,C)$ ,  $(D,E)$ ,  $(E,F)$  are "for sure" inner links.  $c(A,B)=3/7$ ,  $c(B,C)=4/10$ ,  $c(A,C)=1/13$ ,  $c(D,E)=4/11$ ,  $c(E,F)=3/11$ ,  $c(D,F)=4/13$ .
- A path is inside a community “for sure” (inner path) if it consists of inner links and for each couple of nodes  $(p,l)$  on distance two  $c(p,l) > c_0$
- $(D,E,F)$  is inner path.



# Natural Community - definition



- Natural community is a set of nodes having an inner path between each couple of them.
- Natural communities are overlapping.
- Clusters and communities may be different
- Some of links inside a community may have a small  $c$  value. They are not part of inner path.

# Algorithm for a native community detection

*Input:* network  $G=(V,E)$ , threshold value  $c_0$ ;

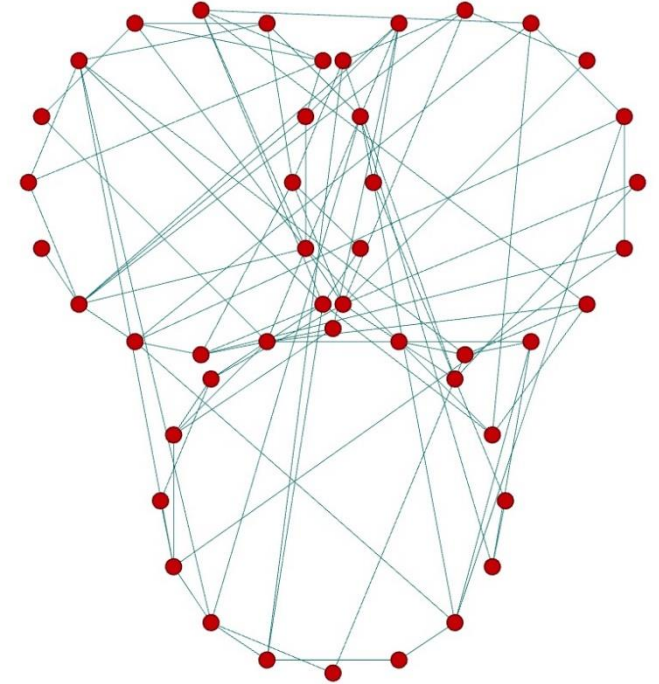
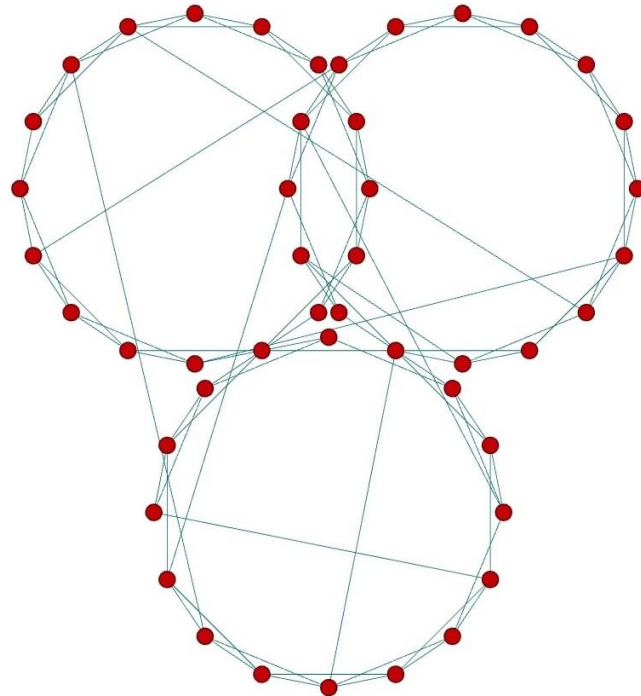
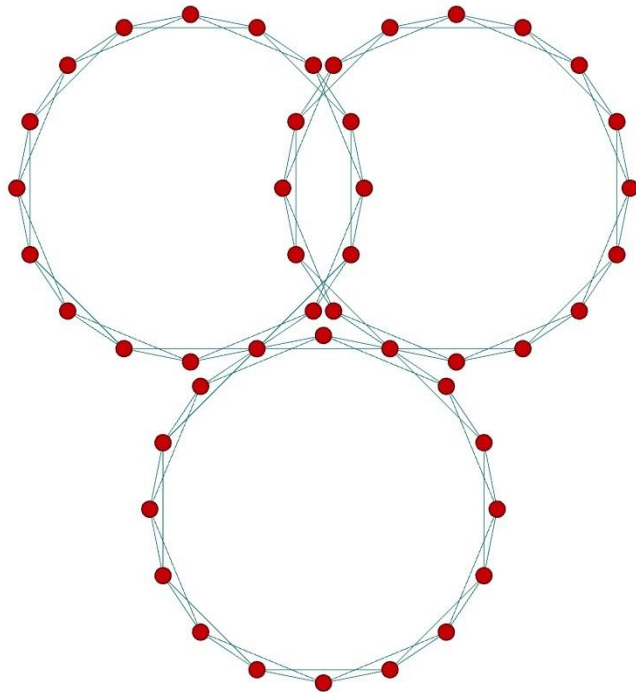
*Output:* A community  $C \subset G$ .

1. Start with arbitrary node  $v$ ,  $C=\{v\}$ ;
2. Loop for each  $w \in N(v)$   
    if  $c(v,w) > c_0$  put  $(v,w)$  into queue  $Q$ ; break;
3. loop while  $Q$  is not empty
  - a. pop  $(v,w)$  from  $Q$ ; add  $w$  to  $C$ ;
  - b. loop for each  $u \in N(w)$   
        if  $(c(w,u) > c_0$  and  $c(w,v) > c_0$  and  $u \notin C)$   
            put  $(w, u)$  into  $Q$ ;

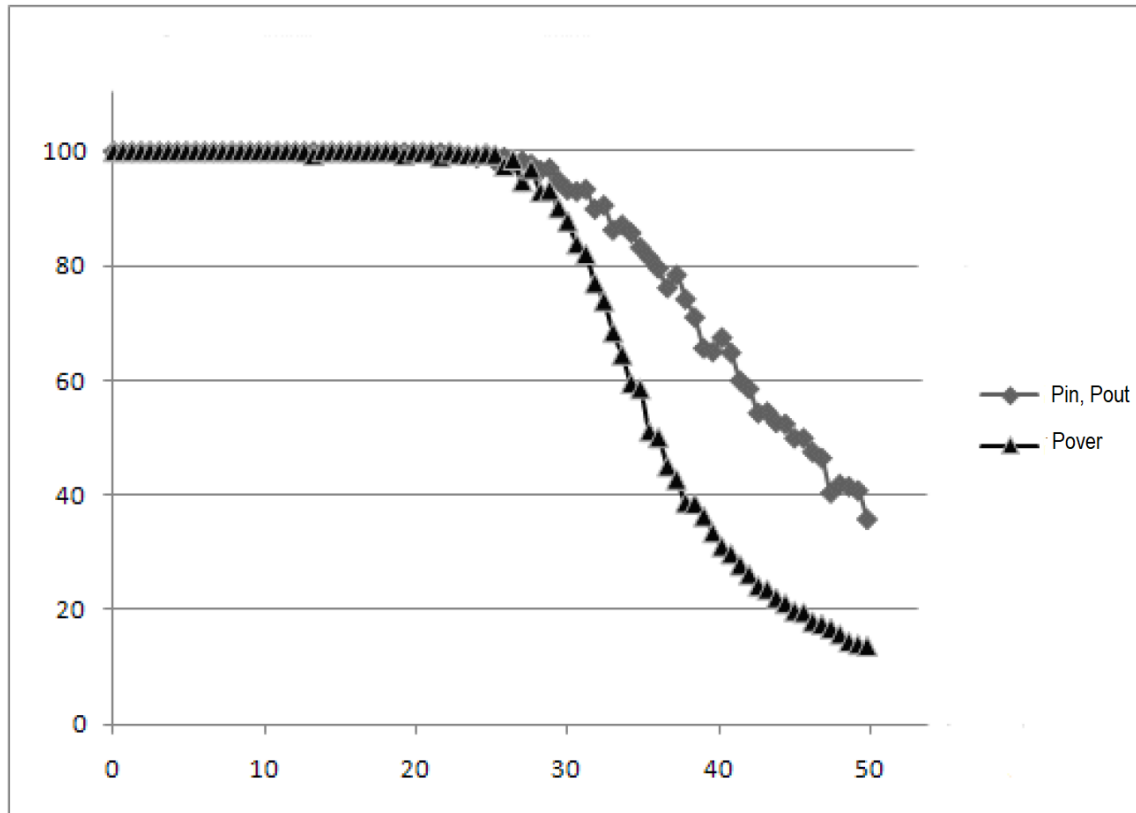
$N(v)$  is set of the  
node  $v$  neighbors

# Synthetic data Test Case: Small World Graph Extension

- Collection of ring lattices with randomly reconnected  $P_{in}$  links inside the ring lattice and  $P_{out}$  reconnected links between the rings
- For overlapping case  $P_{over}$  randomly chosen common for rings nodes.



# Simulation for the Test Case



- % of nodes recognized as correct communities' members as function of % randomized links for 16 rings - communities

# Conclusion

- **Commonality** quantifies the potential of two nodes to belong to the same community, based on their shared neighbors.
- An **Inner Path** is defined as a path within a community where each set of three adjacent nodes exhibits high commonality.
- A **Natural Community** is a collection of nodes interconnected by inner paths.
- **The algorithm** developed from these definitions is straightforward, utilized local data, efficient and effective. It also exhibits stability in the face of random link perturbations.

**Thank you.**

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