Universidade Federal do Rio Grande do Norte
Departamento de Engenharia de Computacão e Automação

Programação Concorrente e Distribuida

Segunda Lista de Exercícios – 3.1-3.11,3.13-3.25

Natal-RN, Brasil
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Professor

Prof. Samuel Xavier - DCA/UFRN

Aluno

Igor Macedo Silva - Bacharelando em Engenharia de Computação

Sumário

Lista de Figuras

Lista de Tabelas

1 Descrição

Lista da 2a unidade Descrição: Apresentar as respostas a todas as questões de exercício do livro texto, com exceção às questões: 3.3, 3.15 e 3.18.

Instruções para resolução (LEIAM!):

- Procure responder corretamente todas as questões da lista;
- Suas respostas serão validadas de forma oral por amostragem - geralmente de 2 à 3 defesas orais;
- Se não conseguir responder alguma questão, procure esclarecer as dúvidas em tempo em sala de aula com o professor, pelo SIGAA, com um colega, ou por e-mail. Se necessário, é possível marcar um horário para tirar dúvidas na sala do professor;
- Não serão aceitas respostas "mágicas", ou seja, quando a resposta está na lista entregue mas você não sabe explicar como chegou a ela. Sua nota nesse caso será 0 (zero). Mesmo que não saiba explicar apenas parte da sua resposta;
- Procure entregar a resolução da lista de forma organizada. Isso pode favorecer a sua nota;
- Os códigos dos programas requisitados (ou as partes relevantes) deverão aparecer no corpo da resolução da questão;
- A resolução da lista deverá ser entregue em formato PDF em apenas 1 (um) arquivo;
- O envio da resolução pode ser feito inúmeras vezes. Utilize-se disso para manter sempre uma versão atualizada das suas respostas e evite problemas com o envio próximo ao prazo de submissão devido a instabilidades no SIGAA;
- A lista com o número das questões respondidas deve aparecer na primeira folha da lista. Não será aceita alteração nessa lista.
- Procure preparar sua defesa oral para cada questão. Explicações diretas e sem arrodeios favorecerão a sua nota;
- A defesa deverá ser agendada com antecedência. Para isso, indique por email (samuel@dca.ufrn.br) no mínimo 3 horários dentro dos intervalos disponíveis em pelo menos 3 turnos diferentes. Caso não tenha disponibilidade em 3 turnos diferentes, deverá apresentar uma justificativa.
- Os horários disponíveis serão disponibilizados em uma notícia na turma virtual e serão atualizados a medida que os agendamentos forem sendo fixados.
- A defesa oral leva apenas de 10 a 15 minutos em horários fixados com antecedência. Não será tolerado que o aluno chegue atrasado para a sua prova.

Período: Inicia em 20/09/2017 às 00h00 e finaliza em 11/10/2017 às 23h59

2 Questões

2.1 Questão 3.1

What happens in the greetings program if, instead of `strlen(greeting) + 1`, we use `strlen(greeting)` for the length of the message being sent by processes `1, 2, ..., comm_sz- 1` ? What happens if we use `MAX_STRING` instead of `strlen(greeting) + 1` ? Can you explain these results?

Neste caso, o `+ 1` indica que o caractere de terminação da string também deve ser incluido no envio da mensagem. Se substituirmos por apenas `strlen(greeting)` a mensagem pode ser impressa corretamente ou não, dependendo do conteúdo presente no buffer de recebimento. Caso o buffer de recebimento esteja preenchido com zeros ("\0"), o comando `printf()` vai conseguir imprimir a mensagem corretamente mesmo que não exista um terminador nulo na mensagem enviada.

Em testes feitos localmente, as mensagens sempre foram exibidas corretamente, pois os buffers estavam sempre sendo iniciados com zero em suas posições de memória.

2.2 Questão 3.2

Modify the trapezoidal rule so that it will correctly estimate the integral even if `comm_sz` doesn't evenly divide n. (You can still assume that n $\geq$ `comm_sz`.)

Se `comm_sz` não divide perfeitamente n, devemos alocar os trapézios restantes nos processos de maneira mais deliberada. o pseudocódigo poderia ser:

```
1  get a, b, n;
2  h = (b - a)/n;
3  local_n = n/comm_sz; //Devemos garantir que a divisao sera inteira
4
5  n_mod_comm = n % comm_sz;
6  local_a = a + (my_rank*local_n* +
7                 my_rank*((int)(my_rank < n_mod_comm))+
8                 n_mod_comm*((int)(my_rank >= n_mod_comm && n_mod_comm
                      > 0)))*h;
9
10
11 local_b = local_a + (local_n + (int)(my_rank < n_mod_comm))*h;
12 local_integral = Trap(local_a, local_b, local_n, h);
```

O trecho da linha 7 se refere ao acrescimo incremental que deve acontecer ao h para cada rank. Isto é, no caso de `n_mod_comm = 3`, o primeiro `local_a` deve receber um acrescimo de 0, o segundo, de 1, o terceiro, de 3 e assim sucessivamente. Isso acontece pois é uma compensação ao `local_b` que está sendo acrescido de 1 até o momento em que todos os trapézios extras (no caso de n não exatamente divisivel por `comm_sz`) forem alocados em algum processo. E isso vai acontecer somente quando `my_rank` $\geq$ `n_mod_comm`

2.3 Questão 3.4

Modify the program that just prints a line of output from each process (`mpi_output.c`) so that the output is printed in process rank order: process 0s output first, then process 1s, and so on.

```
1  #include <stdio.h>
2  #include <mpi.h>
3  #include <string.h> /* For strlen */
4
5  const int MAX_STRING = 100;
6  int main(void) {
7      char phrase[MAX_STRING];
8      int my_rank, comm_sz;
9
10     MPI_Init(NULL, NULL);
11     MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
12     MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
13
14     if (my_rank != 0) {
15         sprintf(phrase, "Proc %d of %d > Does anyone have a toothpick
               ?", my_rank, comm_sz);
16         MPI_Send(phrase, strlen(phrase)+1, MPI_CHAR, 0, 0,
              MPI_COMM_WORLD);
17     } else {
18         printf("Proc %d of %d > Does anyone have a toothpick?\n",
              my_rank, comm_sz);
19         for (int q = 1; q < comm_sz; q++) {
20             MPI_Recv(phrase, MAX_STRING, MPI_CHAR, q, 0,
                  MPI_COMM_WORLD, MPI_STATUS_IGNORE);
21             printf("%s\n",phrase);
22         }
23     }
24
25     MPI_Finalize();
26     return 0;
27 }  /* main */
```

O princípio da resulução desta questão é perceber que para lidar com o não-determinismo do output de um programa MPI, nós devemos mandar todas as mensagens de saída para um único processo, neste caso o processo 0. Dessa forma, apenas um processo é responsável por gerenciar as mensagens de saída e, assim, podemos controlar a ordem de saída das mensagens.

2.4 Questão 3.5

In a binary tree, there is a unique shortest path from each node to the root. The length of this path is often called the depth of the node. A binary tree in which every nonleaf has two children is called a full binary

tree, and a full binary tree in which every leaf has the same depth is sometimes called a complete binary tree. See Figure 3.14. Use the principle of mathematical induction to prove that if T is a complete binary tree with n leaves, then the depth of the leaves is log 2 (n).

Para fazer a indução vamos considerar $\log_2(n) = d$, onde d é a profundidade.
Então,

$$\log_2(n) = d \implies 2^d = n \tag{1}$$

Considerando o caso base em que d = 0,

$$\begin{aligned} 2^d &= n \\ 2^0 &= n \\ 2^0 &= 1 \end{aligned} \tag{2}$$

Tomamos que $d = k$ e assumimos que $2^k = n$ é verdadeiro. Então, aplicamos o passo de indução, onde $d = k + 1$, e para a próxima profundidade, $n_i = n_{i-1} \cdot 2$. Logo,

$$\begin{aligned} 2^{k+1} &= n \cdot 2 \\ &= 2^k 2^1 \\ &= 2^{k+1} \end{aligned} \tag{3}$$

Portanto, $2^d = n$ e, logo $log_2(n) = d$, onde n é o número de folhas e d é a profundidade.

2.5 Questão 3.6

Suppose comm_sz = 4 and suppose that x is a vector with n = 14 components.

a. How would the components of x be distributed among the processes in a program that used a block distribution?

b. How would the components of x be distributed among the processes in a program that used a cyclic distribution?

c. How would the components of x be distributed among the processes in a program that used a block-cyclic distribution with blocksize b = 2?

You should try to make your distributions general so that they could be used regardless of what comm sz and n are. You should also try to make your distributions "fair" so that if q and r are any two processes, the difference between the number of components assigned to q and the number of components assigned to r is as small as possible.

Processos	Bloco	Cíclico	Bloco-Cíclico (tamanho do bloco = 2)
0	0, 1, 2, 3	0, 4, 8, 12	0 1, 8 9
1	4, 5, 6, 7	1, 5, 9, 13	2 3, 10 11
2	8, 9, 10,	2, 6, 10,	4 5, 12 13
3	11, 12, 13,	3, 7, 11,	6 7

Tabela 1: Distribuição dos elementos do vetor

2.6 Questão 3.7

What do the various MPI collective functions do if the communicator contains a single process?

O processo MPI não terá qualquer outro processo para enviar os dados, logo o processamento deve ser feito nesse único processo. É um principio que permite que os processo MPI sejam executados corretamente independente de número de nós/processos disponíveis.

A forma que isso é implementado é enviar os dados para si mesmo, algo que várias funções coletivas do MPI executam. Em alguns casos, para reduzir o movimento desnecessário de dados na memória, o MPI fornece uma flag chamada MPI_IN_PLACE, que permite que o buffer de recebimento seja o mesmo do de envio, melhorando o desempenho. Essa flag não funciona em todas as funções coletivas.

`https://www.mcs.anl.gov/research/projects/mpi/mpi-standard/mpi-report-2.0/node145.htm`

2.7 Questão 3.8

Suppose `comm_sz = 8` and `n = 16`.

a. Draw a diagram that shows how MPI Scatter can be implemented using tree-structured communication with `comm_sz` processes when process 0 needs to distribute an array containing n elements.

b. Draw a diagram that shows how MPI Gather can be implemented using tree-structured communication when an n-element array that has been distributed among `comm_sz` processes needs to be gathered onto process 0.

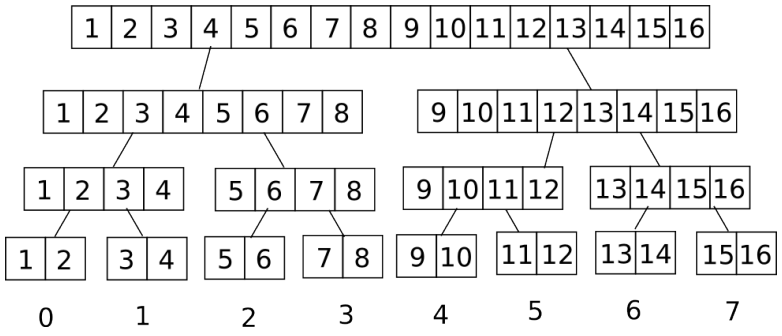


Figura 1: Scatter em comunição baseada em árvore

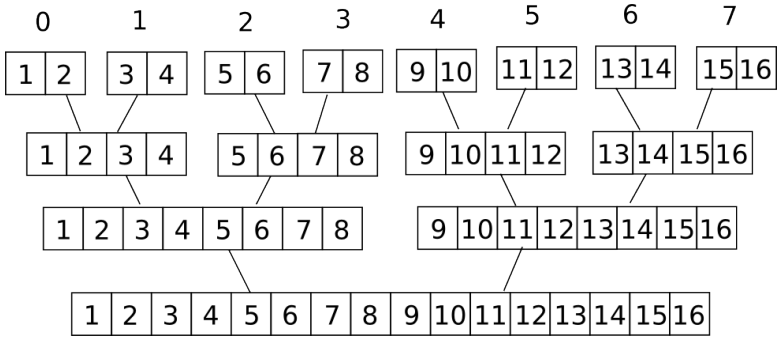


Figura 2: Gather em comunição baseada em árvore

2.8 Questão 3.9

Write an MPI program that implements multiplication of a vector by a scalar and dot product. The user should enter two vectors and a scalar, all of which are read in by process 0 and distributed among the processes. The results are calculated and collected onto process 0, which prints them. You can assume that n, the order of the vectors, is evenly divisible by `comm_sz` .

```
/*Esse codigo foi modificado de mpi_vector_add.c */

/* File:     mpi_vector_add.c
 *
 * Purpose:  Implement parallel vector addition using a block
 *           distribution of the vectors.  This version also
 *           illustrates the use of MPI_Scatter and MPI_Gather.
 *
 * Compile:  mpicc -g -Wall -o mpi_vector_add mpi_vector_add.c
 * Run:      mpiexec -n <comm_sz> ./vector_add
 *
 * Input:    The order of the vectors, n, and the vectors x and y
 * Output:   The sum vector z = x+y
 *
 * Notes:
 * 1.  The order of the vectors, n, should be evenly divisible
 *     by comm_sz
 * 2.  DEBUG compile flag.
 * 3.  This program does fairly extensive error checking.  When
 *     an error is detected, a message is printed and the processes
 *     quit.  Errors detected are incorrect values of the vector
 *     order (negative or not evenly divisible by comm_sz), and
 *     malloc failures.
 *
 * IPP:  Section 3.4.6 (pp. 109 and ff.)
 */

#include <stdio.h>
#include <stdlib.h>
#include <mpi.h>

void Check_for_error(int local_ok, char fname[], char message[],
      MPI_Comm comm);
void Read_n(int* n_p, int* local_n_p, int my_rank, int comm_sz,
      MPI_Comm comm);
void Allocate_vectors(double** local_x_pp, double** local_y_pp,
      double** local_z_pp, int local_n, MPI_Comm comm);
```

```
38 void Read_vector(double local_a[], int local_n, int n, char vec_name
       [],
39         int my_rank, MPI_Comm comm);
40 void Read_scalar(double* scalar_p, int my_rank, int comm_sz, MPI_Comm
       comm);
41 void Print_vector(double local_b[], int local_n, int n, char title[],
42         int my_rank, MPI_Comm comm);
43 void Parallel_vector_dotproduct(double local_x[], double local_y[],
44         double local_z[], double* result, int local_n, int n, MPI_Comm
              comm);
45 void Parallel_scalar_mutiplication(double scalar, double local_x[],
46       double local_z[], int local_n);
47
48
49 /*-------------------------------------------------------------------
       */
50 int main(void) {
51    int n, local_n;
52    int comm_sz, my_rank;
53    double *local_x, *local_y, *local_z;
54    double scalar, result;
55    MPI_Comm comm;
56
57    MPI_Init(NULL, NULL);
58    comm = MPI_COMM_WORLD;
59    MPI_Comm_size(comm, &comm_sz);
60    MPI_Comm_rank(comm, &my_rank);
61
62    Read_n(&n, &local_n, my_rank, comm_sz, comm);
63
64    Allocate_vectors(&local_x, &local_y, &local_z, local_n, comm);
65
66    Read_vector(local_x, local_n, n, "x", my_rank, comm);
67    Print_vector(local_x, local_n, n, "x is", my_rank, comm);
68    Read_vector(local_y, local_n, n, "y", my_rank, comm);
69    Print_vector(local_y, local_n, n, "y is", my_rank, comm);
70
71    Read_scalar(&scalar, my_rank, comm_sz, comm);
72
73    Parallel_scalar_mutiplication(scalar, local_x, local_x, local_n);
74    Print_vector(local_x, local_n, n, "now x is", my_rank, comm);
75    Parallel_vector_dotproduct(local_x, local_y, local_z, &result,
          local_n,
76                               n, comm);
77
```

```
78      if (my_rank == 0) {
79          printf("The result of (scalar*x[]).y[] is %lf \n", result);
80      }
81
82      free(local_x);
83      free(local_y);
84      free(local_z);
85
86      MPI_Finalize();
87
88      return 0;
89  }   /* main */
90
91  /*--------------------------------------------------------------------
92   * Function:  Check_for_error
93   * Purpose:   Check whether any process has found an error.  If so,
94   *            print message and terminate all processes.  Otherwise,
95   *            continue execution.
96   * In args:   local_ok:  0 if calling process has found an error, 1
97   *               otherwise
98   *            fname:     name of function calling Check_for_error
99   *            message:   message to print if there's an error
100  *            comm:      communicator containing processes calling
101  *                       Check_for_error:  should be MPI_COMM_WORLD.
102  *
103  * Note:
104  *    The communicator containing the processes calling
     Check_for_error
105  *    should be MPI_COMM_WORLD.
106  */
107 void Check_for_error(
108       int         local_ok   /* in */,
109       char        fname[]    /* in */,
110       char        message[]  /* in */,
111       MPI_Comm    comm       /* in */) {
112     int ok;
113
114     /* Pega o minimo do vetor
115          Se o minimo for zero, aconteceu algum erro,
116          caso contrario, todos os processos retornaram 1 e estao ok
117     */
118     MPI_Allreduce(&local_ok, &ok, 1, MPI_INT, MPI_MIN, comm);
119     if (ok == 0) {
120        int my_rank;
121        MPI_Comm_rank(comm, &my_rank);
```

```
122         if (my_rank == 0) {
123             fprintf(stderr, "Proc %d > In %s, %s\n", my_rank, fname,
124                   message);
125             fflush(stderr);
126         }
127         MPI_Finalize();
128         exit(-1);
129     }
130 }  /* Check_for_error */
131
132
133 /*---------------------------------------------------------------------
134  * Function:  Read_n
135  * Purpose:   Get the order of the vectors from stdin on proc 0 and
136  *            broadcast to other processes.
137  * In args:   my_rank:    process rank in communicator
138  *            comm_sz:    number of processes in communicator
139  *            comm:       communicator containing all the processes
140  *                        calling Read_n
141  * Out args:  n_p:        global value of n
142  *            local_n_p:  local value of n = n/comm_sz
143  *
144  * Errors:    n should be positive and evenly divisible by comm_sz
145  */
146 void Read_n(
147         int*       n_p         /* out */,
148         int*       local_n_p   /* out */,
149         int        my_rank     /* in  */,
150         int        comm_sz     /* in  */,
151         MPI_Comm   comm        /* in  */) {
152     int local_ok = 1;
153     char *fname = "Read_n";
154
155     if (my_rank == 0) {
156         printf("What's the order of the vectors?\n");
157         scanf("%d", n_p);
158     }
159     MPI_Bcast(n_p, 1, MPI_INT, 0, comm);
160     if (*n_p <= 0 || *n_p % comm_sz != 0) local_ok = 0;
161     Check_for_error(local_ok, fname,
162           "n should be > 0 and evenly divisible by comm_sz", comm);
163     *local_n_p = *n_p/comm_sz;
164 }  /* Read_n */
165
166
```

```
167 /*---------------------------------------------------------------------
168  * Function:  Allocate_vectors
169  * Purpose:   Allocate storage for x, y, and z
170  * In args:   local_n:  the size of the local vectors
171  *            comm:     the communicator containing the calling
     processes
172  * Out args:  local_x_pp, local_y_pp, local_z_pp:  pointers to memory
173  *               blocks to be allocated for local vectors
174  *
175  * Errors:    One or more of the calls to malloc fails
176  */
177 void Allocate_vectors(
178       double**   local_x_pp  /* out */,
179       double**   local_y_pp  /* out */,
180       double**   local_z_pp  /* out */,
181       int        local_n     /* in  */,
182       MPI_Comm   comm        /* in  */) {
183    int local_ok = 1;
184    char* fname = "Allocate_vectors";
185
186    *local_x_pp = malloc(local_n*sizeof(double));
187    *local_y_pp = malloc(local_n*sizeof(double));
188    *local_z_pp = malloc(local_n*sizeof(double));
189
190    if (*local_x_pp == NULL || *local_y_pp == NULL ||
191        *local_z_pp == NULL) local_ok = 0;
192    Check_for_error(local_ok, fname, "Can't allocate local vector(s)",
193          comm);
194 }  /* Allocate_vectors */
195
196
197 /*---------------------------------------------------------------------
198  * Function:   Read_vector
199  * Purpose:    Read a vector from stdin on process 0 and distribute
200  *             among the processes using a block distribution.
201  * In args:    local_n:  size of local vectors
202  *             n:        size of global vector
203  *             vec_name: name of vector being read (e.g., "x")
204  *             my_rank:  calling process' rank in comm
205  *             comm:     communicator containing calling processes
206  * Out arg:    local_a:  local vector read
207  *
208  * Errors:     if the malloc on process 0 for temporary storage
209  *             fails the program terminates
210  *
```

```
211  * Note:
212  *    This function assumes a block distribution and the order
213  *    of the vector evenly divisible by comm_sz.
214  */
215 void Read_vector(
216       double     local_a[]   /* out */,
217       int        local_n     /* in  */,
218       int        n           /* in  */,
219       char       vec_name[]  /* in  */,
220       int        my_rank     /* in  */,
221       MPI_Comm   comm        /* in  */) {
222
223    double* a = NULL;
224    int i;
225    int local_ok = 1;
226    char* fname = "Read_vector";
227
228    if (my_rank == 0) {
229       a = malloc(n*sizeof(double));
230       if (a == NULL) local_ok = 0;
231       Check_for_error(local_ok, fname, "Can't allocate temporary
              vector",
232             comm);
233       printf("Enter the vector %s\n", vec_name);
234       for (i = 0; i < n; i++)
235          scanf("%lf", &a[i]); // reads a double (long float)
236       MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n,
             MPI_DOUBLE, 0,
237             comm);
238       free(a);
239    } else {
240       Check_for_error(local_ok, fname, "Can't allocate temporary
              vector",
241             comm);
242       MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n,
             MPI_DOUBLE, 0,
243             comm);
244    }
245 }  /* Read_vector */
246
247 /*--------------------------------------------------------------------
248  * Function:  Read_scalar
249  * Purpose:   Get the the scalar number from stdin on proc 0 and
250  *            broadcast to other processes.
251  * In args:   my_rank:    process rank in communicator
```

```
252  *            comm_sz:    number of processes in communicator
253  *            comm:       communicator containing all the processes
254  *                        calling Read_n
255  * Out args:  scalar_p:        global value of n
256  *
257  */
258 void Read_scalar(
259       double*    scalar_p        /* out */,
260       int        my_rank    /* in  */,
261       int        comm_sz    /* in  */,
262       MPI_Comm   comm       /* in  */) {
263
264    if (my_rank == 0) {
265       printf("What's the scalar value?\n");
266       scanf("%lf", scalar_p); // reads double
267    }
268    MPI_Bcast(scalar_p, 1, MPI_DOUBLE, 0, comm);
269
270 }  /* Read_scalar */
271
272 /*--------------------------------------------------------------------
273  * Function:  Print_vector
274  * Purpose:   Print a vector that has a block distribution to stdout
275  * In args:   local_b:  local storage for vector to be printed
276  *            local_n:  order of local vectors
277  *            n:        order of global vector (local_n*comm_sz)
278  *            title:    title to precede print out
279  *            comm:     communicator containing processes calling
280  *                      Print_vector
281  *
282  * Error:     if process 0 can't allocate temporary storage for
283  *            the full vector, the program terminates.
284  *
285  * Note:
286  *    Assumes order of vector is evenly divisible by the number of
287  *    processes
288  */
289 void Print_vector(
290       double     local_b[]  /* in */,
291       int        local_n    /* in */,
292       int        n          /* in */,
293       char       title[]    /* in */,
294       int        my_rank    /* in */,
295       MPI_Comm   comm       /* in */) {
296
```

```
297     double* b = NULL;
298     int i;
299     int local_ok = 1;
300     char* fname = "Print_vector";
301
302     if (my_rank == 0) {
303        b = malloc(n*sizeof(double));
304        if (b == NULL) local_ok = 0;
305        Check_for_error(local_ok, fname, "Can't allocate temporary
               vector",
306              comm);
307        MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE
               ,
308              0, comm);
309        printf("%s\n", title);
310        for (i = 0; i < n; i++)
311           printf("%f ", b[i]);
312        printf("\n");
313        free(b);
314     } else {
315        Check_for_error(local_ok, fname, "Can't allocate temporary
               vector",
316              comm);
317        MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE
               , 0,
318              comm);
319     }
320  }  /* Print_vector */
321
322
323  /*--------------------------------------------------------------------
324   * Function:  Parallel_vector_dotproduct
325   * Purpose:   Add a vector that's been distributed among the
        processes
326   * In args:   local_x:  local storage of one of the vectors being
        added
327   *            local_y:  local storage for the second vector being
        added
328   *            local_n:  the number of components in local_x, local_y,
329   *                      and local_z
330   * Out arg:   local_z:  local storage for the sum of the two vectors
331   */
332  void Parallel_vector_dotproduct(
333        double  local_x[]  /* in  */,
334        double  local_y[]  /* in  */,
```

```
335         double  local_z[]  /* out */,
336         double* result     /* out */,
337         int     local_n    /* in  */,
338         int     n          /* in  */,
339         MPI_Comm  comm     /* in */) {
340     int local_i;
341
342     for (local_i = 0; local_i < local_n; local_i++){
343         local_z[local_i] = local_x[local_i] * local_y[local_i];
344     }
345
346     double local_total_z = 0.0;
347     for (local_i = 0; local_i < local_n; local_i++){
348         local_total_z += local_z[local_i];
349     }
350
351     MPI_Reduce(&local_total_z, result, 1, MPI_DOUBLE, MPI_SUM, 0, comm)
        ;
352
353 }  /* Parallel_vector_dotproduct */
354
355 /*--------------------------------------------------------------------
356  * Function:  Parallel_scalar_mutiplication
357  * Purpose:   Add a vector that's been distributed among the
       processes
358  * In args:   scalar:  storage for the scalar value
359  *            local_x:  local storage for the second vector being
       added
360  *            local_n:  the number of components in local_x, local_y,
361  *                      and local_z
362  * Out arg:   local_z:  local storage for the scalar mutiplication of
363  *                      the vector
364  */
365 void Parallel_scalar_mutiplication(
366         double  scalar     /* in  */,
367         double  local_x[]  /* in  */,
368         double  local_z[]  /* out */,
369         int     local_n    /* in  */) {
370     int local_i;
371
372     for (local_i = 0; local_i < local_n; local_i++)
373         local_z[local_i] = local_x[local_i] * scalar;
374 }  /* Parallel_scalar_mutiplication */
```

2.9 Questão 3.10

In the `Read_vector` function shown in Program 3.9, we use `local_n` as the actual argument for two of the formal arguments to `MPI_Scatter` : `send_count` and `recv_count`. Why is it OK to alias these arguments?

É possível alinhar os dois argumentos porque `send_count` deve ser o número de elementos, de acordo com `send_type`, que serão enviados para cada processo. E, de forma semelhante, o `recv_count` deve ser o número de elementos recebidos do procesos raíz, de acordo com `recv_type`. Portanto, neste caso, ambos os argumentos devem receber o valor de `local_n`.

2.10 Questão 3.11

Finding **prefix sums** is a generalization of global sum. Rather than simply finding the sum of n values,

$$x_0 + x_1 + ... + x_{n-1},$$

the prefix sums are the n partial sums

$$x_0, x_0 + x_1, x_0 + x_1 + x_2, ..., x_0 + x_1 + ... + x_{n-1}.$$

a. Devise a serial algorithm for computing the n prefix sums of an array with n elements.

b. Parallelize your serial algorithm for a system with n processes, each of which is storing one of the x_is.

c. Suppose $n = 2^k$ for some positive integer k. Can you devise a serial algorithm and a parallelization of the serial algorithm so that the parallel algorithm requires only k communication phases?

d. MPI provides a collective communication function, `MPI_Scan` , that can be used to compute prefix sums:

```
1 int MPI_Scan(
2       void*          sendbuf_p   /* in  */,
3       void*          recvbuf_p   /* out */,
4       int            count       /* in  */,
5       MPI_Datatype   datatype    /* in  */,
6       MPI_Op         op          /* in  */,
7       MPI_Comm       comm        /* in  */);
```

It operates on arrays with `count` elements; both `sendbuf_p` and `recvbuf_p` should refer to blocks of `count` elements of type `datatype`. The `op` argument is the same as `op` for `MPI_Reduce`. Write an MPI program that generates a random array of count elements on each MPI process, finds the prefix sums, and prints the results.

a.

```
1 #include <stdio.h>
2 #include <time.h>
3 #include <stdlib.h>
4
5 int main(){
6     int n = 15;
```

```
7       srand(time(NULL));
8
9       int vec[n];
10      int prefix_sum[n];
11
12      for(int i = 0; i < n; i++){
13          vec[i] = rand() % 10; // number between 0 and 9
14          printf("%d ", vec[i]);
15      }
16      printf("\n");
17
18      for(int i = 0; i < n; i++){
19          prefix_sum[i] = 0;
20          for(int j = 0; j <= i; j++){
21              prefix_sum[i] += vec[j];
22          }
23          printf("%d ", prefix_sum[i]);
24      }
25  }
```

b.

```
1  #include <stdio.h>
2  #include <time.h>
3  #include <stdlib.h>
4  #include <mpi.h>
5
6  int main(){
7      int comm_sz; /* Number of processes */
8      int my_rank; /* My process rank */
9      MPI_Init(NULL, NULL);
10     MPI_Comm comm = MPI_COMM_WORLD;
11     MPI_Comm_size(comm, &comm_sz);
12     MPI_Comm_rank(comm, &my_rank);
13
14     int vec_i;
15     int prefix_sum = 0;
16     int n = comm_sz;
17     int vec[n];
18
19     if (my_rank == 0){
20         srand(time(NULL));
21         for(int i = 0; i < n; i++){
22             vec[i] = rand() % 10; // number between 0 and 9
23             printf("%d ", vec[i]);
24         }
```

```
25          printf("\n");
26
27          MPI_Scatter(vec, 1, MPI_INT, &vec_i, 1, MPI_INT, 0, comm);
28      }
29      else {
30          MPI_Scatter(vec, 1, MPI_INT, &vec_i, 1, MPI_INT, 0, comm);
31      }
32
33      // envia para todos os processo maiores do que ele mesmo
34      for(int i = my_rank; i < comm_sz; i++){
35          MPI_Send(&vec_i, 1, MPI_INT, i, 0, comm);
36      }
37
38      // recebe de todos os processos menores que ele mesmo
39      prefix_sum += vec_i;
40      for(int i = 0; i < my_rank; i++){
41          int aux_vec_i = 0;
42          MPI_Recv(&aux_vec_i, 1, MPI_INT, i, 0, comm,
                MPI_STATUS_IGNORE);
43          prefix_sum += aux_vec_i;
44      }
45
46
47      // Imprime o resultado na tela
48      if (my_rank != 0) {
49          MPI_Send(&prefix_sum, 1, MPI_INT, 0, 1, comm);
50      } else {
51          printf("%d ", vec_i);
52          for (int q = 1; q < comm_sz; q++) {
53              int aux;
54              MPI_Recv(&aux, 1, MPI_INT, q, 1, comm, MPI_STATUS_IGNORE)
                    ;
55              printf("%d ", aux);
56          }
57          printf("\n");
58      }
59
60
61      MPI_Finalize();
62      return 0;
63  }
```

c. **Serial**

```
1  #include <stdio.h>
2  #include <time.h>
```

```c
#include <stdlib.h>
#include <math.h>

int main(){
    int n = 8;
    srand(time(NULL));

    int vec[n];

    for(int i = 0; i < n; i++){
        vec[i] = 1; //rand() % 10; // number between 0 and 9
        printf("%d ", vec[i]);
    }
    printf("\n");

    for(int k = 0; k < log2(n); k++){
        printf("k is %d\n",k);
        for(int i = pow(2,k) - 1; i < pow(2,log2(n)); i = i + pow(2,k
            +1)){
            printf("i is %d\n",i);
            for(int j = 1; j <= pow(2,k); j++){
                printf("vec[%d] = vec[%d] + vec[%d]\n",i+j,i,i+j);
                vec[i+j] = vec[i] + vec[i+j];
            }
        }
        for(int i = 0; i < n; i++){
            printf("%d ", vec[i]);
        }
        printf("\n");
    }

    for(int i = 0; i < n; i++){
        printf("%d ", vec[i]);
    }
    printf("\n");
}
```

Paralelo

```c
#include <stdio.h>
#include <time.h>
#include <stdlib.h>
#include <mpi.h>
#include <math.h>

void print_vec(int vec[], int n);
```

```
8  int main(){
9      int comm_sz; /* Number of processes */
10     int my_rank; /* My process rank */
11     MPI_Init(NULL, NULL);
12     MPI_Comm comm = MPI_COMM_WORLD;
13     MPI_Comm_size(comm, &comm_sz);
14     MPI_Comm_rank(comm, &my_rank);
15
16     int vec_i;
17     int n = comm_sz;
18     int vec[n];
19
20     if (my_rank == 0){
21         srand(time(NULL));
22         for(int i = 0; i < n; i++){
23             vec[i] = 1; //rand() % 10; // number between 0 and 9
24         }
25         print_vec(vec,n);
26         MPI_Scatter(vec, 1, MPI_INT, &vec_i, 1, MPI_INT, 0, comm);
27     }
28     else {
29         MPI_Scatter(vec, 1, MPI_INT, &vec_i, 1, MPI_INT, 0, comm);
30     }
31
32     for(int k = 0; k < log2(n); k++){
33         for(int i = pow(2,k) - 1; i < pow(2,log2(n)); i = i + pow(2,k
               +1)){
34             for(int j = 1; j <= pow(2,k); j++){
35                 if (my_rank == i){
36                     MPI_Send(&vec_i, 1, MPI_INT, i+j, 0, comm);
37                     printf("This is %d, Sending to %d\n", my_rank, i+
                           j);
38                 } else if (my_rank == i + j){
39                     int aux_vec_i = 0;
40                     printf("This is %d, Receiving from %d\n",my_rank,
                           i);
41                     MPI_Recv(&aux_vec_i, 1, MPI_INT, i, 0, comm,
                           MPI_STATUS_IGNORE);
42                     vec_i += aux_vec_i;
43                 }
44             }
45         }
46     }
47
48     int vec_sum[n];
```

```
49      if (my_rank == 0) {
50          MPI_Gather(&vec_i, 1, MPI_INT, vec_sum, 1, MPI_INT, 0, comm);
51          print_vec(vec_sum,n);
52      } else {
53           MPI_Gather(&vec_i, 1, MPI_INT, vec_sum, 1, MPI_INT, 0, comm);
54      }
55
56      MPI_Finalize();
57      return 0;
58  }
59
60  void print_vec(int vec[], int n){
61      for(int i = 0; i < n; i++){
62          printf("%d ",vec[i]);
63      }
64      printf("\n");
65  }
```

d.

```
1   #include <stdio.h>
2   #include <stdlib.h>
3   #include <time.h>
4   #include <mpi.h>
5
6   void print_vec(int vec[], int n);
7
8   int main(void) {
9       int comm_sz; /* Number of processes */
10      int my_rank; /* My process rank */
11      MPI_Init(NULL, NULL);
12      MPI_Comm comm = MPI_COMM_WORLD;
13      MPI_Comm_size(comm, &comm_sz);
14      MPI_Comm_rank(comm, &my_rank);
15
16      int count = comm_sz;
17      int vec[count];
18      int sum[count];
19
20      srand(my_rank+1);
21      for(int i = 0; i < count; i++){
22          vec[i] = rand() % 10; // number between 0 and 9
23          sum[i] = 0;
24      }
25
26      MPI_Scan(vec, sum, count, MPI_INT, MPI_SUM, comm);
```

```
27
28      if (my_rank != 0) {
29          MPI_Send(vec, count, MPI_INT, 0, 0, comm);
30          MPI_Send(sum, count, MPI_INT, 0, 0, comm);
31      } else {
32          printf("rank %d - [", my_rank );
33          print_vec(vec,count);
34          printf("] => [");
35          print_vec(sum,count);
36          printf("]\n");
37          for (int q = 1; q < comm_sz; q++) {
38              MPI_Recv(vec, count, MPI_INT, q, 0, comm,
                    MPI_STATUS_IGNORE);
39              MPI_Recv(sum, count, MPI_INT, q, 0, comm,
                    MPI_STATUS_IGNORE);
40              printf("rank %d - [", q);
41              print_vec(vec,count);
42              printf("] => [");
43              print_vec(sum,count);
44              printf("]\n");
45          }
46      }
47
48      MPI_Finalize();
49      return 0;
50  }
51
52  void print_vec(int vec[], int n){
53      for(int i = 0; i < n; i++){
54          printf("%d ",vec[i]);
55      }
56  }
```

Output: [vec] => [sum]
rank 0 - [3 6 7 5 3] => [3 6 7 5 3]
rank 1 - [0 9 8 5 1] => [3 15 15 10 4]
rank 2 - [6 5 8 0 5] => [9 20 23 10 9]
rank 3 - [1 3 4 6 3] => [10 23 27 16 12]
rank 4 - [5 5 0 2 6] => [15 28 27 18 18]

2.11 Questão 3.12 *

An alternative to a butterfly-structured allreduce is a ring-pass structure. In a ring-pass, if there are p processes, each process q sends data to process q + 1, except that process p - 1 sends data to process 0. This is repeated until each process has the desired result. Thus, we can implement allreduce with the

following code:

```
sum = temp_val = my_val;
  for (i = 1; i < p; i++) {
    MPI_Sendrecv_replace(&temp_val, 1, MPI_INT, dest,
    sendtag, source, recvtag, comm, &status);
    sum += temp_val;
}
```

a. Write an MPI program that implements this algorithm for allreduce. How does its performance compare to the butterfly-structured allreduce?

b. Modify the MPI program you wrote in the first part so that it implements prefix sums.

2.12 Questão 3.13

`MPI_Scatter` and `MPI_Gather` have the limitation that each process must send or receive the same number of data items. When this is not the case, we must use the MPI functions `MPI_Gatherv` and `MPI_Scatterv`. Look at the man pages for these functions, and modify your vector sum, dot product program so that it can correctly handle the case when n isn't evenly divisible by `comm_sz`.

```
/*Esse codigo foi modificado de mpi_vector_add.c */

/* File:     mpi_vector_add.c
 *
 * Purpose:  Implement parallel vector addition using a block
 *           distribution of the vectors.  This version also
 *           illustrates the use of MPI_Scatter and MPI_Gather.
 *
 * Compile:  mpicc -g -Wall -o mpi_vector_add mpi_vector_add.c
 * Run:      mpiexec -n <comm_sz> ./vector_add
 *
 * Input:    The order of the vectors, n, and the vectors x and y
 * Output:   The sum vector z = x+y
 *
 * Notes:
 * 1.  The order of the vectors, n, should be evenly divisible
 *     by comm_sz
 * 2.  DEBUG compile flag.
 * 3.  This program does fairly extensive error checking.  When
 *     an error is detected, a message is printed and the processes
 *     quit.  Errors detected are incorrect values of the vector
 *     order (negative or not evenly divisible by comm_sz), and
 *     malloc failures.
 *
 * IPP:  Section 3.4.6 (pp. 109 and ff.)
 */

```

```c
#include <stdio.h>
#include <stdlib.h>
#include <mpi.h>

void Check_for_error(int local_ok, char fname[], char message[],
      MPI_Comm comm);
void Read_n(int* n_p, int* local_n_p, int* rest, int my_rank, int
   comm_sz,
      MPI_Comm comm);
void Allocate_vectors(double** local_x_pp, double** local_y_pp,
      double** local_z_pp, int local_n, MPI_Comm comm);
void Read_vector(int rest, double local_a[], int local_n, int n, char
    vec_name[],
      int my_rank, MPI_Comm comm);
void Read_scalar(double* scalar_p, int my_rank, int comm_sz, MPI_Comm
    comm);
void Print_vector(double local_b[], int local_n, int n, char title[],
      int my_rank, MPI_Comm comm);
void Parallel_vector_dotproduct(double local_x[], double local_y[],
      double local_z[], double* result, int local_n, int n, MPI_Comm
         comm);
void Parallel_scalar_mutiplication(double scalar, double local_x[],
    double local_z[], int local_n);


/*-------------------------------------------------------------------
   */
int main(void) {
   int n, local_n, rest;
   int comm_sz, my_rank;
   double *local_x, *local_y, *local_z;
   double scalar, result;
   MPI_Comm comm;

   MPI_Init(NULL, NULL);
   comm = MPI_COMM_WORLD;
   MPI_Comm_size(comm, &comm_sz);
   MPI_Comm_rank(comm, &my_rank);

   Read_n(&n, &local_n, &rest, my_rank, comm_sz, comm);

   Allocate_vectors(&local_x, &local_y, &local_z, local_n, comm);

   Read_vector(rest, local_x, local_n, n, "x", my_rank, comm);
   Print_vector(local_x, local_n, n, "x is", my_rank, comm);
```

```
68      printf("Done ok" );
69      Read_vector(rest, local_y, local_n, n, "y", my_rank, comm);
70      Print_vector(local_y, local_n, n, "y is", my_rank, comm);
71
72      Read_scalar(&scalar, my_rank, comm_sz, comm);
73
74      Parallel_scalar_mutiplication(scalar, local_x, local_x, local_n);
75      Print_vector(local_x, local_n, n, "now x is", my_rank, comm);
76      Parallel_vector_dotproduct(local_x, local_y, local_z, &result,
            local_n,
77                                      n, comm);
78
79      if (my_rank == 0) {
80          printf("The result of (scalar*x[]).y[] is %lf \n", result);
81      }
82
83      free(local_x);
84      free(local_y);
85      free(local_z);
86
87      MPI_Finalize();
88
89      return 0;
90  }   /* main */
91
92  /*---------------------------------------------------------------------
93   * Function:  Check_for_error
94   * Purpose:   Check whether any process has found an error.  If so,
95   *            print message and terminate all processes.  Otherwise,
96   *            continue execution.
97   * In args:   local_ok:  0 if calling process has found an error, 1
98   *               otherwise
99   *            fname:     name of function calling Check_for_error
100  *            message:   message to print if there's an error
101  *            comm:      communicator containing processes calling
102  *                       Check_for_error:  should be MPI_COMM_WORLD.
103  *
104  * Note:
105  *    The communicator containing the processes calling
       Check_for_error
106  *    should be MPI_COMM_WORLD.
107  */
108 void Check_for_error(
109       int         local_ok   /* in */,
110       char        fname[]    /* in */,
```

```
111         char        message[]   /* in */,
112         MPI_Comm    comm        /* in */) {
113     int ok;
114
115     /* Pega o minimo do vetor
116           Se o minimo for zero, aconteceu algum erro,
117           caso contrario, todos os processos retornaram 1 e estao ok
118     */
119     MPI_Allreduce(&local_ok, &ok, 1, MPI_INT, MPI_MIN, comm);
120     if (ok == 0) {
121         int my_rank;
122         MPI_Comm_rank(comm, &my_rank);
123         if (my_rank == 0) {
124             fprintf(stderr, "Proc %d > In %s, %s\n", my_rank, fname,
125                   message);
126             fflush(stderr);
127         }
128         MPI_Finalize();
129         exit(-1);
130     }
131 }  /* Check_for_error */
132
133
134 /*---------------------------------------------------------------------
135  * Function:  Read_n
136  * Purpose:   Get the order of the vectors from stdin on proc 0 and
137  *            broadcast to other processes.
138  * In args:   my_rank:    process rank in communicator
139  *            comm_sz:    number of processes in communicator
140  *            comm:       communicator containing all the processes
141  *                        calling Read_n
142  * Out args:  n_p:        global value of n
143  *            local_n_p:  local value of n = n/comm_sz
144  *
145  * Errors:    n should be positive and evenly divisible by comm_sz
146  */
147 void Read_n(
148         int*        n_p         /* out */,
149         int*        local_n_p   /* out */,
150         int*        rest        /* out */,
151         int         my_rank     /* in  */,
152         int         comm_sz     /* in  */,
153         MPI_Comm    comm        /* in  */) {
154     int local_ok = 1;
155     char *fname = "Read_n";
```

```
156
157     if (my_rank == 0) {
158         printf("What's the order of the vectors?\n");
159         scanf("%d", n_p);
160     }
161     MPI_Bcast(n_p, 1, MPI_INT, 0, comm);
162     if (*n_p <= 0 ) local_ok = 0;
163     Check_for_error(local_ok, fname,
164           "n should be > 0. ", comm);
165     *local_n_p = *n_p/comm_sz + 1;
166     *rest = *n_p % comm_sz;
167 }   /* Read_n */
168
169
170 /*---------------------------------------------------------------------
171  * Function:   Allocate_vectors
172  * Purpose:    Allocate storage for x, y, and z
173  * In args:    local_n:  the size of the local vectors
174  *             comm:     the communicator containing the calling
     processes
175  * Out args:   local_x_pp, local_y_pp, local_z_pp:  pointers to memory
176  *                blocks to be allocated for local vectors
177  *
178  * Errors:     One or more of the calls to malloc fails
179  */
180 void Allocate_vectors(
181       double**    local_x_pp  /* out */,
182       double**    local_y_pp  /* out */,
183       double**    local_z_pp  /* out */,
184       int         local_n     /* in  */,
185       MPI_Comm    comm        /* in  */) {
186     int local_ok = 1;
187     char* fname = "Allocate_vectors";
188
189     *local_x_pp = malloc(local_n*sizeof(double));
190     *local_y_pp = malloc(local_n*sizeof(double));
191     *local_z_pp = malloc(local_n*sizeof(double));
192
193     if (*local_x_pp == NULL || *local_y_pp == NULL ||
194         *local_z_pp == NULL) local_ok = 0;
195     Check_for_error(local_ok, fname, "Can't allocate local vector(s)",
196           comm);
197 }   /* Allocate_vectors */
198
199
```

```
200 /*---------------------------------------------------------------------
201  * Function:    Read_vector
202  * Purpose:     Read a vector from stdin on process 0 and distribute
203  *              among the processes using a block distribution.
204  * In args:     local_n:  size of local vectors
205  *              n:        size of global vector
206  *              vec_name: name of vector being read (e.g., "x")
207  *              my_rank:  calling process' rank in comm
208  *              comm:     communicator containing calling processes
209  * Out arg:     local_a:  local vector read
210  *
211  * Errors:      if the malloc on process 0 for temporary storage
212  *              fails the program terminates
213  *
214  * Note:
215  *    This function assumes a block distribution and the order
216  *    of the vector evenly divisible by comm_sz.
217  */
218 void Read_vector(
219       int         rest         /* in  */,
220       double      local_a[]    /* out */,
221       int         local_n      /* in  */,
222       int         n            /* in  */,
223       char        vec_name[]   /* in  */,
224       int         my_rank      /* in  */,
225       MPI_Comm    comm         /* in  */) {
226
227    double* a = NULL;
228    int i;
229    int local_ok = 1;
230    char* fname = "Read_vector";
231
232    int comm_sz;
233    MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
234    int* sendcounts = malloc(sizeof(int)*comm_sz);
235    int* displs = malloc(sizeof(int)*comm_sz);
236
237    // Calculating send counts and displacement
238    int sum = 0;
239    int rem = rest;
240     for (int i = 0; i < comm_sz; i++) {
241         sendcounts[i] = n/comm_sz;
242         if (rem > 0) {
243             sendcounts[i]++;
244             rem--;
```

```
245             }
246
247             displs[i] = sum;
248             sum += sendcounts[i];
249         }
250
251         // print calculated send counts and displacements for each
                process
252         if (0 == my_rank) {
253             for (int i = 0; i < comm_sz; i++) {
254                 printf("sendcounts[%d] = %d\tdispls[%d] = %d\n", i,
                        sendcounts[i], i, displs[i]);
255             }
256         }
257
258         a = malloc(n*sizeof(double));
259         if (a == NULL) local_ok = 0;
260         Check_for_error(local_ok, fname, "Can't allocate temporary vector
                ",
261                 comm);
262
263         if (my_rank == 0) {
264           printf("Enter the vector %s\n", vec_name);
265           for (i = 0; i < n; i++)
266             scanf("%lf", &a[i]); // reads a double (long float)
267         }
268
269         // divide the data among processes as described by sendcounts and
                displs
270         // MPI_Scatterv(&data, sendcounts, displs, MPI_CHAR, &rec_buf,
                100, MPI_CHAR, 0, MPI_COMM_WORLD);
271         MPI_Scatterv(a, sendcounts, displs, MPI_DOUBLE, local_a, local_n,
                MPI_DOUBLE, 0, comm);
272
273         // print what each process received
274          printf("%d: ", my_rank);
275          for (int i = 0; i < sendcounts[my_rank]; i++) {
276              printf("%lf\t", local_a[i]);
277          }
278          printf("\n");
279
280          free(a);
281          free(sendcounts);
282          free(displs);
283     }  /* Read_vector */
```

```
284
285 /*---------------------------------------------------------------------
286  * Function:  Read_scalar
287  * Purpose:   Get the the scalar number from stdin on proc 0 and
288  *            broadcast to other processes.
289  * In args:   my_rank:    process rank in communicator
290  *            comm_sz:    number of processes in communicator
291  *            comm:       communicator containing all the processes
292  *                        calling Read_n
293  * Out args:  scalar_p:        global value of n
294  *
295  */
296 void Read_scalar(
297       double*    scalar_p        /* out */,
298       int        my_rank     /* in  */,
299       int        comm_sz     /* in  */,
300       MPI_Comm   comm        /* in  */) {
301
302    if (my_rank == 0) {
303       printf("What's the scalar value?\n");
304       scanf("%lf", scalar_p); // reads double
305    }
306    MPI_Bcast(scalar_p, 1, MPI_DOUBLE, 0, comm);
307
308 }  /* Read_scalar */
309
310 /*---------------------------------------------------------------------
311  * Function:  Print_vector
312  * Purpose:   Print a vector that has a block distribution to stdout
313  * In args:   local_b:  local storage for vector to be printed
314  *            local_n:  order of local vectors
315  *            n:        order of global vector (local_n*comm_sz)
316  *            title:    title to precede print out
317  *            comm:     communicator containing processes calling
318  *                      Print_vector
319  *
320  * Error:     if process 0 can't allocate temporary storage for
321  *            the full vector, the program terminates.
322  *
323  * Note:
324  *    Assumes order of vector is evenly divisible by the number of
325  *    processes
326  */
327 void Print_vector(
328       double     local_b[]  /* in */,
```

```
329         int         local_n     /* in */,
330         int         n           /* in */,
331         char        title[]     /* in */,
332         int         my_rank     /* in */,
333         MPI_Comm    comm        /* in */) {
334
335     double* b = NULL;
336     int i;
337     int local_ok = 1;
338     char* fname = "Print_vector";
339
340     int comm_sz;
341     MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
342     int* recvcounts = malloc(sizeof(int)*comm_sz);
343     int* displs = malloc(sizeof(int)*comm_sz);
344
345     // Calculating send counts and displacement
346     int sum = 0;
347     int rem = n%comm_sz;
348     for (int i = 0; i < comm_sz; i++) {
349         recvcounts[i] = n/comm_sz;
350         if (rem > 0) {
351             recvcounts[i]++;
352             rem--;
353         }
354
355         displs[i] = sum;
356         sum += recvcounts[i];
357     }
358
359     if (0 == my_rank) {
360         for (int i = 0; i < comm_sz; i++) {
361             printf("recvcounts[%d] = %d\tdispls[%d] = %d\n", i,
                    recvcounts[i], i, displs[i]);
362         }
363     }
364
365
366     b = malloc(n*sizeof(double));
367     if (b == NULL) local_ok = 0;
368     Check_for_error(local_ok, fname, "Can't allocate temporary vector
        ",
369             comm);
370
371     if (my_rank == 0) {
```

```
372         MPI_Gatherv(local_b, recvcounts[my_rank], MPI_DOUBLE, b,
                recvcounts, displs,
373               MPI_DOUBLE, 0, comm);
374         printf("%s\n", title);
375         for (i = 0; i < n; i++)
376            printf("%f ", b[i]);
377         printf("\n");
378
379      } else {
380         MPI_Gatherv(local_b, recvcounts[my_rank], MPI_DOUBLE, b,
                recvcounts, displs,
381                    MPI_DOUBLE, 0, comm);
382      }
383
384      free(b);
385   }  /* Print_vector */
386
387
388   /*--------------------------------------------------------------------
389    * Function:   Parallel_vector_dotproduct
390    * Purpose:    Add a vector that's been distributed among the
        processes
391    * In args:    local_x:  local storage of one of the vectors being
        added
392    *             local_y:  local storage for the second vector being
        added
393    *             local_n:  the number of components in local_x, local_y,
394    *                       and local_z
395    * Out arg:    local_z:  local storage for the sum of the two vectors
396    */
397   void Parallel_vector_dotproduct(
398         double  local_x[]  /* in  */,
399         double  local_y[]  /* in  */,
400         double  local_z[]  /* out */,
401         double* result     /* out */,
402         int     local_n    /* in  */,
403         int     n          /* in  */,
404         MPI_Comm  comm     /* in */) {
405      int local_i;
406
407     for (local_i = 0; local_i < local_n; local_i++){
408         local_z[local_i] = local_x[local_i] * local_y[local_i];
409     }
410
411     double local_total_z = 0.0;
```

```
412    for (local_i = 0; local_i < local_n; local_i++){
413        local_total_z += local_z[local_i];
414    }
415
416    int my_rank;
417    MPI_Comm_rank(comm, &my_rank);
418    printf("rank: %d > %lf\n", my_rank, local_total_z );
419
420    MPI_Reduce(&local_total_z, result, 1, MPI_DOUBLE, MPI_SUM, 0, comm)
       ;
421
422 }  /* Parallel_vector_dotproduct */
423
424 /*--------------------------------------------------------------------
425  * Function:  Parallel_scalar_mutiplication
426  * Purpose:   Add a vector that's been distributed among the
       processes
427  * In args:   scalar:  storage for the scalar value
428  *            local_x:  local storage for the second vector being
       added
429  *            local_n:  the number of components in local_x, local_y,
430  *                      and local_z
431  * Out arg:   local_z:  local storage for the scalar mutiplication of
432  *                      the vector
433  */
434 void Parallel_scalar_mutiplication(
435       double  scalar     /* in  */,
436       double  local_x[]  /* in  */,
437       double  local_z[]  /* out */,
438       int     local_n    /* in  */) {
439    int local_i;
440
441    for (local_i = 0; local_i < local_n; local_i++)
442       local_z[local_i] = local_x[local_i] * scalar;
443 }  /* Parallel_scalar_mutiplication */
```

2.13 Questão 3.14

a. Write a serial C program that defines a two-dimensional array in the main function. Just use numeric constants for the dimensions: `int two d[3][4];`
Initialize the array in the main function. After the array is initialized, call a function that attempts to print the array. The prototype for the function should look something like this.
`void Print two d(int two d[][], int rows, int cols);`
After writing the function try to compile the program. Can you explain why it won't compile?

b. After consulting a C reference (e.g., Kernighan and Ritchie [29]), modify the program so that it will

compile and run, but so that it still uses a two-dimensional C array.

O código abaixo não compila pois o compilador precisa saber as dimensões para fazer a aritmética de ponteiros corretamente. Isto é, se o compilador não tiver as dimensões do array quando ele é passado para uma função (e passado como ponteiro), a expressão array[x][y] não pode ser calculada, pois não há como sabe onde uma linha ou coluna começa ou termina.

```c
#include <stdio.h>
#include <stdlib.h>

void Print_two_d(int two_d[][], int rows, int cols);

int main(){
    int two_d[3][4] = {{1,1,1,1},
                       {2,2,2,2},
                       {3,3,3,3}};

   Print_two_d(two_d,3,4);
   return 0;
}

void Print_two_d(int two_d[][], int rows, int cols){
    int i, j;
    for(i = 0; i < rows; i++){
        for(j = 0; j < cols; j++){
            printf("%d\n", two_d[i][j]);
        }
    }
}
```

```c
#include <stdio.h>
#include <stdlib.h>

void Print_two_d(int two_d[][4], int rows, int cols);

int main(){
    int two_d[3][4] = {{1,1,1,1},
                       {2,2,2,2},
                       {3,3,3,3}};

   Print_two_d(two_d,3,4);
   return 0;
}

void Print_two_d(int two_d[][4], int rows, int cols){
    int i, j;
```

```
17      for(i = 0; i < rows; i++){
18          for(j = 0; j < cols; j++){
19              printf("%d\n", two_d[i][j]);
20          }
21      }
22  }
```

2.14 Questão 3.16

Suppose `comm_sz` = 8 and the vector x = (0, 1, 2, ... , 15) has been distributed among the processes using a block distribution. Draw a diagram illustrating the steps in a butterfly implementation of allgather of x.

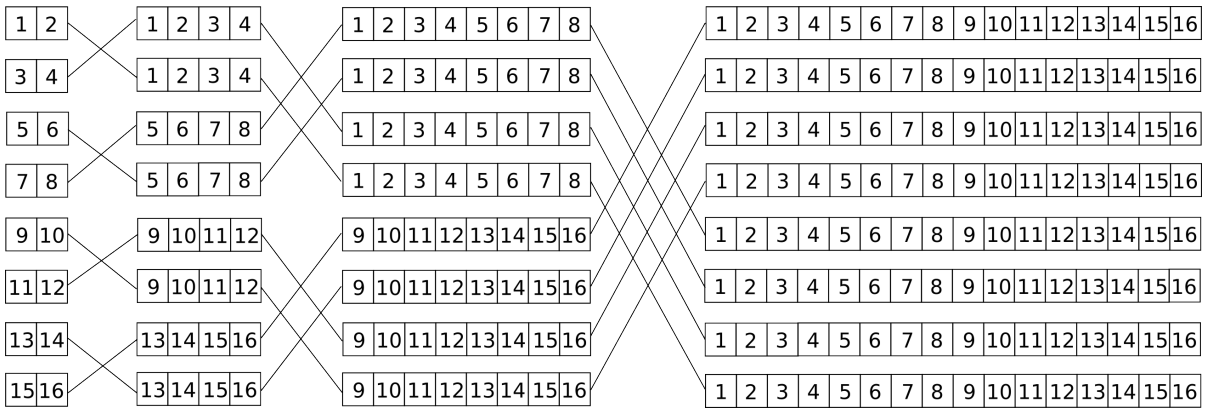


Figura 3: Allgather em comunicação butterfly

2.15 Questão 3.17

`MPI_Type_contiguous` can be used to build a derived datatype from a collection of contiguous elements in an array. Its syntax is

```
int MPI_Type_contiguous(
    int           count        /* in */,
    MPI_Datatype  old_mpi_t    /* in */,
    MPI_Datatype* new_mpi_t_p  /* out */);
```

Modify the `Read_vector` and `Print_vector` functions so that they use an MPI datatype created by a call to `MPI_Type_contiguous` and a count argument of 1 in the calls to `MPI_Scatter` and `MPI_Gather`.

```
void Read_vector(
      double     local_a[]   /* out */,
      int        local_n     /* in  */,
      int        n           /* in  */,
      char       vec_name[]  /* in  */,
      int        my_rank     /* in  */,
```

```
7         MPI_Comm   comm          /* in   */) {
8
9     double* a = NULL;
10    int i;
11    int local_ok = 1;
12    char* fname = "Read_vector";
13
14    MPI_Datatype CONTIGUOUS;
15    MPI_Type_contiguous(local_n, MPI_DOUBLE, &CONTIGUOUS);
16    MPI_Type_commit(&CONTIGUOUS);
17
18    if (my_rank == 0) {
19       a = malloc(n*sizeof(double));
20       if (a == NULL) local_ok = 0;
21       Check_for_error(local_ok, fname, "Can't allocate temporary
             vector",
22             comm);
23       printf("Enter the vector %s\n", vec_name);
24       for (i = 0; i < n; i++)
25          scanf("%lf", &a[i]); // reads a double (long float)
26       MPI_Scatter(a, 1, CONTIGUOUS, local_a, 1, CONTIGUOUS, 0, comm);
27       free(a);
28    } else {
29       Check_for_error(local_ok, fname, "Can't allocate temporary
             vector",
30             comm);
31       MPI_Scatter(a, 1, CONTIGUOUS, local_a, 1, CONTIGUOUS, 0, comm);
32    }
33 }  /* Read_vector */
```

```
1  void Print_vector(
2        double     local_b[]  /* in */,
3        int        local_n    /* in */,
4        int        n          /* in */,
5        char       title[]    /* in */,
6        int        my_rank    /* in */,
7        MPI_Comm   comm       /* in */) {
8
9     double* b = NULL;
10    int i;
11    int local_ok = 1;
12    char* fname = "Print_vector";
13
14    MPI_Datatype CONTIGUOUS;
15    MPI_Type_contiguous(local_n, MPI_DOUBLE, &CONTIGUOUS);
```

```
16      MPI_Type_commit(&CONTIGUOUS);
17
18      if (my_rank == 0) {
19          b = malloc(n*sizeof(double));
20          if (b == NULL) local_ok = 0;
21          Check_for_error(local_ok, fname, "Can't allocate temporary
                vector",
22                  comm);
23          MPI_Gather(local_b, 1, CONTIGUOUS, b, 1, CONTIGUOUS, 0, comm);
24          printf("%s\n", title);
25          for (i = 0; i < n; i++)
26              printf("%f ", b[i]);
27          printf("\n");
28          free(b);
29      } else {
30          Check_for_error(local_ok, fname, "Can't allocate temporary
                vector",
31                  comm);
32          MPI_Gather(local_b, 1, CONTIGUOUS, b, 1, CONTIGUOUS, 0, comm);
33      }
34  }  /* Print_vector */
```

2.16 Questão 3.19

`MPI_Type_indexed` can be used to build a derived datatype from arbitrary array elements. Its syntax is

```
1  int MPI_Type_indexed(
2      int           count                        /* in */,
3      int           array_of_blocklengths[]      /* in */,
4      int           array_of_displacements[]     /* in */,
5      MPI_Datatype  old_mpi_t                    /* in */,
6      MPI_Datatype* new_mpi_t_p                  /* out */);
```

Unlike `MPI_Type_create_struct`, the displacements are measured in units of `old_mpi_t` - not bytes. Use `MPI_Type_indexed` to create a derived datatype that corresponds to the upper triangular part of a square matrix. For example, in the 4 x 4 matrix

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \\ 12 & 13 & 14 & 15 \end{pmatrix}$$

the upper triangular part is the elements 0, 1, 2, 3, 5, 6, 7, 10, 11, 15. Process 0 should read in an n x n matrix as a one-dimensional array, create the derived datatype, and send the upper triangular part with a single call to `MPI_Send`. Process 1 should receive the upper triangular part with a single call to `MPI_Recv` and then print the data it received.

```c
#include <stdio.h>
#include <stdlib.h>
#include <mpi.h>

int main(int argc, char *argv[])
{
    int rank, size, i;
    MPI_Datatype type;
    int n = 4;

    int blocklen[n], displacement[n];
    for (int i = n, j = 0;
         i > 0 && j < n; i--, j++)
         { blocklen[j] = i;}

    for (int i = 0, j = 0;
         i < n, j < n; i+=5, j++)
         { displacement[j] = i;}

    int buffer[n*n];
    MPI_Init(&argc, &argv);
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
    MPI_Comm_size(MPI_COMM_WORLD, &size);

    if (size < 2)
    {
        printf("Please run with 2 processes.\n");
        MPI_Finalize();
        return 1;
    }

    MPI_Type_indexed(n, blocklen, displacement, MPI_INT, &type);
    MPI_Type_commit(&type);

    if (rank == 0)
    {
        printf("Enter the matrix: ");
        fflush(stdout);
        for (int i = 0; i < n*n; i++){
            scanf("%d", &buffer[i]);
        }

        MPI_Send(buffer, 1, type, 1, 123, MPI_COMM_WORLD);
    }

```

```
46      if (rank == 1)
47      {
48          for (i=0; i<n*n; i++)
49              buffer[i] = -1;
50          MPI_Recv(buffer, 1, type, 0, 123, MPI_COMM_WORLD,
                MPI_STATUS_IGNORE);
51          for (i=0; i<n*n; i++)
52              printf("buffer[%d] = %d\n", i, buffer[i]);
53          fflush(stdout);
54      }
55
56      MPI_Finalize();
57      return 0;
58  }
```

2.17 Questão 3.20

The functions `MPI_Pack` and `MPI_Unpack` provide an alternative to derived datatypes for grouping data. `MPI_Pack` copies the data to be sent, one block at a time, into a user-provided buffer. The buffer can then be sent and received. After the data is received, `MPI_Unpack` can be used to unpack it from the receive buffer. The syntax of `MPI_Pack` is

```
1       int MPI_Pack(
2             void*          in_buf          /* in     */,
3             int            in_buf_count    /* in     */,
4             MPI_Datatype   datatype        /* in     */,
5             void*          pack_buf        /* out    */,
6             int            pack_buf_sz     /* in     */,
7             int*           position_p      /* in/out */,
8             MPI_Comm       comm            /* in     */);
```

We could therefore pack the input data to the trapezoidal rule program with the following code:

```
1       char pack_buf[100];
2       int position = 0;
3
4       MPI_Pack(&a, 1, MPI_DOUBLE, pack_buf, 100, &position, comm);
5       MPI_Pack(&b, 1, MPI_DOUBLE, pack_buf, 100, &position, comm);
6       MPI_Pack(&n, 1, MPI_INT, pack_buf, 100, &position, comm);
```

The key is the position argument. When `MPI_Pack` is called, position should refer to the first available slot in pack buf . When `MPI_Pack` returns, it refers to the first available slot after the data that was just packed, so after process 0 executes this code, all the processes can call`MPI_Bcast` :

```
1       MPI_Bcast(pack_buf, 100, MPI_PACKED, 0, comm);
```

Note that the MPI datatype for a packed buffer is `MPI_PACKED`. Now the other processes can unpack the data using: `MPI_Unpack` :

```
int MPI_Unpack(
      void*          pack_buf       /* in     */,
      int            pack_buf_sz    /* in     */,
      int*           position_p     /* in/out */,
      void*          out_buf        /* out    */,
      int            out_buf_count  /* in     */,
      MPI_Datatype   datatype       /* in     */,
      MPI_Comm       comm           /* in     */);
```

This can be used by "reversing" the steps in `MPI_Pack` , that is, the data is unpacked one block at a time starting with position = 0. Write another Get input function for the trapezoidal rule program. This one should use `MPI_Pack` on process 0 and `MPI_Unpack` on the other processes.

```
void Get_input(int my_rank, int comm_sz, double* a_p, double* b_p,
      int* n_p) {
   //int dest;

   MPI_Comm comm = MPI_COMM_WORLD;
   char pack_buf[100];
   int position = 0;

   if (my_rank == 0) {
      printf("Enter a, b, and n\n");
      scanf("%lf %lf %d", a_p, b_p, n_p);

      printf("rank %d: %d\n", my_rank, *n_p);
      printf("rank %d: %lf\n", my_rank, *b_p);
      printf("rank %d: %lf\n", my_rank, *a_p);

      MPI_Pack(a_p, 1, MPI_DOUBLE, pack_buf, 100, &position, comm);
      MPI_Pack(b_p, 1, MPI_DOUBLE, pack_buf, 100, &position, comm);
      MPI_Pack(n_p, 1, MPI_INT, pack_buf, 100, &position, comm);

      MPI_Bcast(pack_buf, 100, MPI_PACKED, 0, comm);
   }
   else { /* my_rank != 0 */
      MPI_Bcast(pack_buf, 100, MPI_PACKED, 0, comm);

      MPI_Unpack(pack_buf, 100, &position, a_p, 1, MPI_DOUBLE, comm);
      MPI_Unpack(pack_buf, 100, &position, b_p, 1, MPI_DOUBLE, comm);
      MPI_Unpack(pack_buf, 100, &position, n_p, 1, MPI_INT, comm);
   }
}  /* Get_input */
```

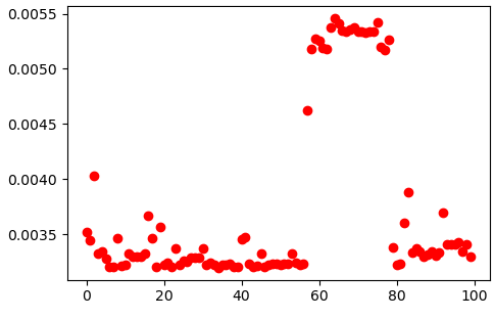
2.18 Questão 3.21

How does your system compare to ours? What run-times does your system get for matrix-vector multiplication? What kind of variability do you see in the times for a given value of `comm_sz` and n? Do the results tend to cluster around the minimum, the mean, or the median?

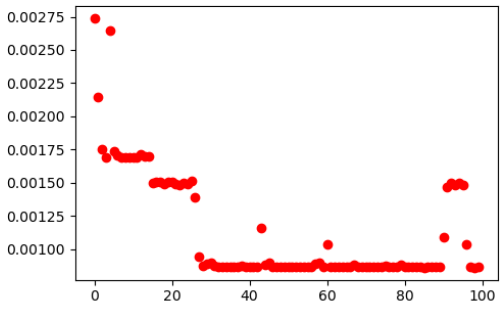
	Ordem da Matriz				
comm_sz	1024	2048	4096	8192	16384
1	3.7561	12.8853	52.0139	210.4686	846.9655
2	2.0700	10.2071	41.1858	107.3081	438.0811
4	1.7275	6.8219	25.0998	64.2229	245.5204
8	1.7007	4.1896	14.3932	56.5015	219.8544
16	2.1960	6.0016	20.8711	76.8699	236.1451

Tabela 2: Tempo de execução

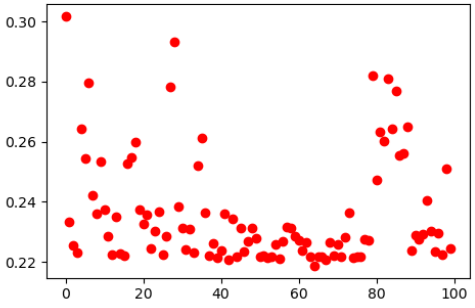
A medida que o comm_sz aumenta, é possível ver que o tempo de execução diminui. Contudo, a execução nem sempre tem um comportamento constante. Isso pode ser observado na Figura 4.



(a) `comm_sz` = 1, Ordem = 1024, (s)



(b) `comm_sz` = 4, Ordem = 1024, (s)



(c) `comm_sz` = 16, Ordem = 16384, (s)

Figura 4: Comportamento dos tempos de execução

2.19 Questão 3.22

Time our implementation of the trapezoidal rule that uses `MPI_Reduce`. How will you choose n, the number of trapezoids? How do the minimum times compare to the mean and median times? What are the speedups? What are the efficiencies? On the basis of the data you collected, would you say that the

trapezoidal rule is scalable?

Recall that programs that can maintain a constant efficiency without increasing the problem size are sometimes said to be strongly scalable. Programs that can maintain a constant efficiency if the problem size increases at the same rate as the number of processes are sometimes said to be weakly scalable.

A escolha do n é feita de forma que o algoritmo tome tempo de execução suficiente para uma medição segura e que ele possa ser dobrado a cada rodada. Em 100 rodadas de cada configuração, montamos as tabelas abaixo.

Em análise da Tabela 7 vemos que o algoritmo tem um momento de escalabilidade fraca entre p = 2 e p = 4, mas volta a cair em p = 8. Provavelmente, o overhead de comunicação ainda é bem alto para esses p's, mas deve ser superado em p's maiores.

	Ordem da Matriz			
comm_sz	1024	2048	4096	8192
1	1.58214537e-05	3.064394e-05	6.87956822e-05	0.000125017164
2	1.09386417e-05	1.88255289e-05	5.77306725e-05	6.33239711e-05
4	2.1030905e-05	2.27093698e-05	2.16841662e-05	3.60274319e-05
8	1.62243825e-05	3.75008588e-05	2.96688098e-05	2.9134753e-05

Tabela 3: Tempo de execução (s) (média)

	Ordem da Matriz			
comm_sz	1024	2048	4096	8192
1	1.597404e-05	3.004074e-05	6.890297e-05	0.0001249313
2	1.096725e-05	2.002716e-05	5.793571e-05	6.29425e-05
4	1.907349e-05	2.217293e-05	2.098083e-05	3.504753e-05
8	1.597404e-05	2.908707e-05	2.598763e-05	2.598763e-05

Tabela 4: Tempo de execução (mediana)

	Ordem da Matriz			
comm_sz	1024	2048	4096	8192
1	1.478195e-05	2.884865e-05	5.984306e-05	0.0001239777
2	1.001358e-05	1.478195e-05	5.698204e-05	6.198883e-05
4	1.788139e-05	2.193451e-05	2.098083e-05	3.385544e-05
8	1.28746e-05	2.69413e-05	2.31266e-05	2.479553e-05

Tabela 5: Tempo de execução (s) (mínimo)

	Ordem da Matriz			
comm_sz	1024	2048	4096	8192
1	1.0	1.0	1.0	1.0
2	1.44638193058	1.62778640445	1.191666045465	1.97424706361
4	0.752295429036	1.34939631834	3.17262289753	3.47005482786
8	0.975165230479	0.81715301944	2.31878806948	4.29099790206

Tabela 6: Tempo de Speedup da média

	Ordem da Matriz			
comm_sz	1024	2048	4096	8192
1	1.0	1.0	1.0	1.0
2	0.723190965291	0.813893202225	0.595833022732	0.987123531803
4	0.188073857259	0.337349079586	0.793155724383	0.867513706965
8	0.12189565381	0.10214412743	0.289848508685	0.536374737757

Tabela 7: Tempo de eficiência da média

2.20 Questão 3.23

Although we don't know the internals of the implementation of `MPI_Reduce`, we might guess that it uses a structure similar to the binary tree we discussed. If this is the case, we would expect that its run-time would grow roughly at the rate of $log_2(p)$, since there are roughly $log_2(p)$ levels in the tree. (Here, p = `comm_sz`.) Since the run-time of the serial trapezoidal rule is roughly proportional to n, the number of trapezoids, and the parallel trapezoidal rule simply applies the serial rule to n/p trapezoids on each process, with our assumption about `MPI_Reduce`, we get a formula for the overall run-time of the parallel trapezoidal rule that looks like

$$T_{parallel}(n,p) \approx a \times \frac{n}{p} + b \cdot \log_2(p)$$

for some constants a and b.

a. Use the formula, the times you've taken in Exercise 3.22, and your favorite R program for doing mathematical calculations (e.g., MATLAB) to get a least-squares estimate of the values of a and b.

b. Comment on the quality of the predicted run-times using the formula and the values for a and b computed in part (a).

a. Script utilizado para fazer o fit

```
import pandas as pd
import numpy as np
import lmfit

df = pd.DataFrame({
  'n'       : pd.Series([1024, 2048, 4096 , 8192]*4),
  'p'       : pd.Series([1, 1, 1, 1, 2, 2, 2, 2, 4, 4, 4, 4, 8, 8, 8,
     8]),
```

```
    't_par' : pd.Series([1.58214537e-05, 3.064394e-05, 6.87956822e-05
        , 0.000125017164,1.09386417e-05 , 1.88255289e-05 , 5.77306725e
        -05 , 6.33239711e-05, 2.1030905e-05 , 2.27093698e-05 ,
        2.16841662e-05 , 3.60274319e-05,1.62243825e-05 , 3.75008588e-05
        , 2.96688098e-05 , 2.9134753e-05])
})

print "p: " + str(df['p'])
print "n: " + str(df['n'])
print "t_par: " +str(df['t_par'])

def fun(A, B, p1 = 1, p2 = 1):
    return p1 * np.exp(A) + p2 * np.exp(B)

def t_parallel(n, p, a = 1, b = 1):
    return a * (n/p) + b*np.log2(p)

model = lmfit.Model(t_parallel, independent_vars=['n', 'p'])
fit = model.fit(df['t_par'], n = df['n'], p = df['p'])

print fit.fit_report()
print fit.values
```

Output

```
[[Model]]
    Model(t_parallel)
[[Fit Statistics]]
    # function evals   = 11
    # data points      = 16
    # variables        = 2
    chi-square         = 0.000
    reduced chi-square = 0.000
    Akaike info crit   = -373.692
    Bayesian info crit = -372.147
[[Variables]]
    a:    1.5364e-08 +/- 7.61e-10 (4.96%) (init= 1)
    b:    6.5503e-06 +/- 1.11e-06 (16.90%) (init= 1)
[[Correlations]] (unreported correlations are <  0.100)
    C(a, b)                      = -0.259

>>> print fit.values
{'a': 1.5364234860843803e-08, 'b': 6.5502917499154895e-06}
```

b. Com a predição

	Ordem da Matriz			
comm_sz	1024	2048	4096	8192
1	1.58214537e-05	3.064394e-05	6.87956822e-05	0.000125017164
2	1.09386417e-05	1.88255289e-05	5.77306725e-05	6.33239711e-05
4	2.1030905e-05	2.27093698e-05	2.16841662e-05	3.60274319e-05
8	1.62243825e-05	3.75008588e-05	2.96688098e-05	2.9134753e-05

Tabela 8: Tempo de execução Real (s) (média)

	Ordem da Matriz			
comm_sz	1024	2048	4096	8192
1	1.5732e-05	3.1465e-05	6.2931e-05	12.5863e-05
2	1.4416e-05	2.2283e-05	3.8016e-05	6.9482e-05
4	1.7033e-05	2.0967e-05	2.8833e-05	4.4566e-05
8	2.1617e-05	2.3584e-05	2.7517e-05	3.5383e-05

Tabela 9: Tempo de execução Predito (s) (média)

Com os dados das Tabelas 8 e 9 vemos que a predição consegue razoavelmente seguir os valores da curva real. Isto acontece especialmente quando p = 1 e p = 2. A medida que p aumenta a predição fica mais precária. Porém a predição também melhora a medida que n aumenta.

2.21 Questão 3.24

Take a look at Programming Assignment 3.7. The code that we outlined for timing the cost of sending messages should work even if the count argument is zero. What happens on your system when the count argument is 0? Can you explain why you get a nonzero elapsed time when you send a zero-byte message?

Quando a mensagem é enviada com count = 0, o procedimento do sistema é o mesmo, porém 0 bytes são enviados. O tempo que o sistema leva para enviar 0 bytes é geralmente tido como a definição da latência do sistema, que envolve toda a preparação que o software precisa fazer para enviar a mensagem, assim como o hardware também. Nesse caso, a mensagem, apesar de ter 0 bytes, possui informações outras como a tag e o comunicator, que levam algum tempo para serem enviadas.

2.22 Questão 3.25

If `comm_sz = p`, we mentioned that the "ideal" speedup is p. Is it possible to do better?

a. Consider a parallel program that computes a vector sum. If we only time the vector sum - that is, we ignore input and output of the vectors - how might this program achieve speedup greater than p?

b. A program that achieves speedup greater than p is said to have super- linear speedup. Our vector sum example only achieved superlinear speedup by overcoming certain "resource limitations." What were these resource limitations? Is it possible for a program to obtain superlinear speedup without overcoming resource limitations?

Uma forma de ter um speedup maior do que p é quando acontece superação de limitação de recursos. Ou seja se durante o aumento do problema ou do número de nós, acontece que uma limitação de hardware é superada.

a. Neste caso, se os vetores não conseguem ser alocados (em linha ou coluna) na cache de um único processo, mas conseguem ser alocados (em linha ou coluna) quando existem vários processos, é possível que aconteça um speedup melhor que o linear. Nesse caso, o acesso aos dados na cache seria mais rápido que o acesso dos dados na memória principal, por isso haveria um ganho de velocidade.

b. Nesse caso, acontece de que a limitação de cache foi superada, e o speedup super-linear foi possível. Um possível caso de speedup superlinear, sem superação de limitação de recursos, pode ser considerado quando a modelagem do problema e os dados necessariamente permitisse que ao paralelizar o algoritmo, o software consiga o super speedup. Por exemplo, em um pesquisa em estrutura de árvore, onde a paralelização permita que os dados sejam encontrados mais rápido.