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## Vehicle-to-vehicle connectivity analysis for vehicular ad-hoc networks

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## ABSTRACT

Vehicle-to-vehicle (V2V) communication in Vehicular Ad hoc Networks (VANETs) is of importance in the Intelligent Transportation System (ITS) in which vehicles enlisted with wireless devices can communicate with each other. Many applications can save people's life or time on traffic such as accident alerts or congestion prediction, etc. However, network communication over VANETs is inherently unstable because of the high mobility of vehicles. In this paper, we analyze vehicle to vehicle wireless connectivity by using mathematic models. We consider the effect of headway distance, acceleration, association time (i.e. connection setup time), relative speed of vehicles, transmission range and message/data size in short range based V2V communications in the models. The numerical results in simulations validate the analysis.

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## 1. Introduction

Traffic accidents and jams are causing deaths and waste of fuel and productive hours [1]. These statistics could be reduced by propagating upcoming traffic information in a timely manner using automated process in vehicular ad hoc networks (VANET) [2,3]. Implementation of an automatic mechanism for disseminating traffic information to drivers in a timely manner can help drivers make informed decisions.

VANET is regarded as a central component for Intelligent Transportation System (ITS) where vehicles exchange upcoming traffic information to provide comfort to passengers, enhance traffic efficiency and safety. It is important in VANET to automate message propagation using vehicle-to-vehicle (V2V) and/or vehicle-to-roadside (V2R) with possible roadside-to-roadside (R2R) communications. In V2R communications, delay is a major concern since message passes from vehicle to roadside and then roadside to destination vehicles. Where as V2V communication, delay may not be an issue but connectivity and security are major concerns as these parameters rely on neighboring participating vehicles. Safety-related applications (such as collision warning, road merging, emergency braking, etc.) short delay which can be achieved by V2V communications. Furthermore, vehicles can avoid collision with other vehicles if they talk to each other using periodic status messages wirelessly [4]. For instance, automatic cruising could be

done based on the local vehicle interactions to avoid any possible collisions.

Note that there is an IEEE 802.11p standard for vehicular communication in 5.850 GHz to 5.925 GHz (75 MHz RF) band as the Dedicated Short Range Communication (DSRC) to enable vehicular communication for safety and infotainment applications [5,6]. This standard allows vehicle to transmit up to 1000 m with 32 dBm power.

In this paper, we investigate connectivity for short range V2V communications. We study the effect of association time, message size, transmission range and relative speed of vehicles for successful message forwarding in vehicular network. Performance is evaluated using analytical and numerical results obtained from simulations for V2V communications. Short range based wireless access technologies are also discussed for V2V communications. Note that short range based V2V communications provide VANET security from outside attackers as a byproduct [7–9] as road side attacker may not be able to insert malicious messages or eavesdrop the conversation.

The remainder of the paper is organized as follows: we provide a succinct survey of papers that have addressed similar issues in 2. We also present the system model and formally state the problem in Section 3 and 4 followed by an estimation of the expected value for successful message exchange in Section 5. We illustrate our analytical results by simulation and numerical results obtained from simulations in Section 6, and present final conclusions in Section 7.

## 2. Related work

Short-range wireless communication for VANET was proposed in [10–14]. The work in [12] proposes NOTICE architecture where

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sensor belts mounted under the road every mile or so collect and transmit traffic-related messages. Vehicles are assumed to be equipped with two wireless radios (one for association and the other for message transmission). Using standard technologies association and message exchange for NOTICE architecture have been analyzed in [15]. In [14], authors have proposed so called “cat’s eye” architecture in which roadside units are involved in message collecting and forwarding in vehicular network.

Panichpapiboon et al. [16] studied links and routing paths on the basis of signal strength. Kiese et al. [17] adapted received power levels and improved antenna gains to find better links. Yang et al. [18] and Kesting et al. [19] proposed using statistical and real-time density data to select wireless links in VANET. Bai et al. [20] proposed a model for path duration distribution in MANET. Based on experiments with Dynamic Source Routing (DSR), they proposed an approximate probability density function (exponential distribution) for the path duration. However, the assumption of path duration was not validated in [20].

Gruber and Hui [21] assumed link durations to be independent, exponentially distributed random variables; with this assumption, they derived the probability distributions of path duration which, not surprisingly, is exponential as well.<sup>1</sup> However, the details of the underlying mobility model supporting such an assumption were not discussed in [21].

Pascoe et al. [22] derived the time duration of an  $n$ -node path. However, their mobility model only includes velocity without considering acceleration. Nekovee [23] proposed a model to determine the probability of a link in VANET under the assumptions that (1) the headway distance is constant, (2) the radio propagation model only accounts for slow fading and ignores path loss due to distance, and (3) vehicle mobility patterns are ignored. Nekovee et al. [24] assumed that car velocities are normally distributed. From this assumption, the throughput is modeled by various formulas. The path-loss is also formalized as an exponential function of velocity. This formula is also the basis of Nekovee’s work [23].

Su et al. [25] proposed an analytical model for the probability density function (pdf) of link lifetime. Their model is based on several assumptions, namely that nodes are equally spaced, and that speed is normally distributed. Building on these assumptions, Sun et al. [25] computed the probability of link lifetime. However, their first assumption is not reasonable since, as widely known, inter-vehicle distance is a random variable and certainly not a constant. It is worth noting that both of these architectures are based on V2R communications and do not provide insight of association (connection setup), physical layer specifications, effect of data size, data rate, etc. Furthermore, work in [10,11] focus on V2V based communication without considering association time, relative speed, data size, etc.

### 3. V2V communication system model

We assume that the vehicles are traveling on the road are equipped with computing and communication devices according to the U.S. National Highway Transportation Safety Administration (NHTSA) ruling [2,26]. Using those computing and communication devices, vehicles can communicate with each other using V2V based communications. We also assume that vehicles periodically broadcast their geolocations, speeds, etc. information in VANET.

Broadly speaking, there are two types of scenarios for V2V communications: (i) vehicles traveling in *opposite* directions as shown in Fig. 1(a) and (ii) vehicles traveling in *same* direction as shown in Fig. 1(b). Note that the time interval during which two vehicles can communicate directly depends in their relative speed. It is

clear that the time for information exchange using V2V communication, for a given transmission range  $r$  and relative speed  $v_{rel}$ , can be computed using  $t = \frac{r}{v_{rel}}$ . This implies that for a given transmission range the time interval during which the vehicles can communicate decreases when their relative speed  $v_{rel}$  increases and vice versa. In other words, when the relative speed is smaller, vehicles remain within communication range for long time and they have longer time for information exchange using direct communication. It is worth noting that the relative speed of vehicles becomes smaller (zero) when vehicles move in same direction (with equal speed). Although this situation is relative rare in urban area, it could be more normal on highways. In the case of zero relative speed V2V communication is similar to communication in *static* peer-to-peer network. It can provide much better reliable communication in this situation and therefore it is important for reliable V2V communication. However, when vehicles move in opposite direction, their relative speed is high since the relative speed is the sum of the speeds of two communicating vehicles and thus V2V network becomes highly *dynamic* ad hoc network. Note for a given transmission range and relative speed, there will be a fixed time during which vehicles remain within communication range of each other. This time must be used for both association and data exchange.

In V2V communication, wireless devices that are mounted in vehicles should be associated before exchanging the actual information or messages. That is, after successful association, vehicles are ready to exchange messages. When the vehicles travel in the same direction, because of small (or zero) relative speed, devices have sufficient time to get associated and to transmit messages. However, when the vehicles travel in opposite directions at highway speeds, they remain within communication range for a short amount of time which might not be enough to get associated and for a meaningful exchange the messages. It is well documented that different wireless technologies (such as Wi-Fi, ZigBee, Bluetooth, UWB, etc.) have different association times and data rates. These parameters play a major role in successful communication in VANET.

In the next section, we investigate the effect of association time, data rate, transmission range and relative speed of vehicles for successful V2V communications.

### 4. The probability analysis of V2V connectivity

#### 4.0.1. Path-loss

The *path-loss model* [28] is a radio propagation model that predicts the signal attenuation (in dB) at a distance  $X$  from the transmitter. Visser et al. [29] used a patch antenna and studied the path-loss of a DSRC link. The path-loss relevant to inter-vehicle communication can be modeled by *two-ray model* which takes the reflection signal from the road itself into consideration. This suggests defining the path-loss in dB as a random variable  $L(X)$  defined by writing [30]:

**Lemma 1.** Assuming that  $X \in \log N(\mu, \sigma)$ , the random variable  $L(X) = 40 \log X - (10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$  is normally distributed.

where<sup>2</sup>  $G_t$  and  $G_r$  are the antenna gains of the transmitter and the receiver, respectively;  $h_t$  and  $h_r$  are, respectively, the heights of the transmitting and receiving antennas [31]. we note that since  $L(X)$  is normally distributed, its probability density function (pdf) reads  $l(z) \in \frac{1}{40} N(\frac{\mu}{40} + b, \sigma)$  where  $b = -(10 \log G_t +$

<sup>1</sup> Recall that the minimum of several independent exponential random variables is also exponential.

<sup>2</sup> Here, and in the remainder of this paper, we use  $\log$  to represent  $\log_{10}$  and  $\ln$  to represent the natural logarithm  $\log_e$ .

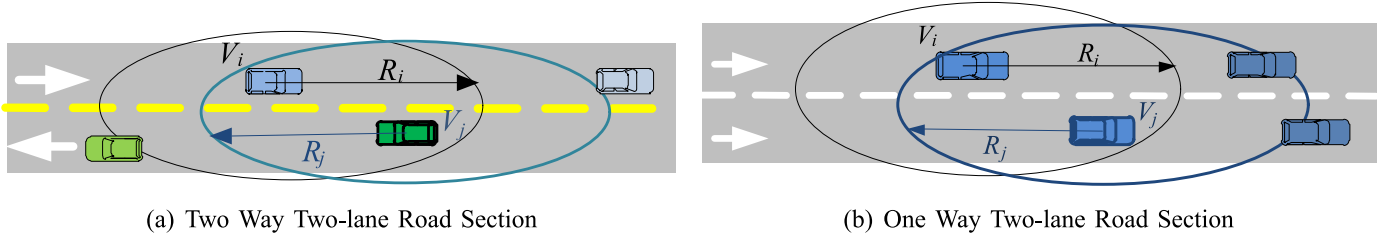


Fig. 1. VANETs with vehicles (e.g.,  $i$  and  $j$ ) moving in their corresponding directions and with their corresponding transmission ranges (e.g.,  $R_i$  and  $R_j$ ) [27].

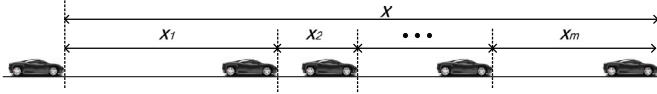


Fig. 2. Illustrating the convolution  $X = X_1 + X_2 + \dots + X_m$ .  $X_1, X_2, \dots, X_m$  are independent to each other and are verified as log-normal distribution [30].

$10 \log G_r + 20 \log h_t + 20 \log h_r$ ). To simplify the notation, we write  $L(X) = Z \in aN(\mu_z, \sigma_z^2)$ , where  $a = \frac{1}{40}$ ,  $\mu_z = a\mu + b$  and  $\sigma_z^2 = \sigma^2$ .

#### 4.0.2. On the link distance

We are interested in the link distance  $X$ , the distance between a source vehicle and a destination vehicle (i.e., between a sender and a receiver). Write  $X = \sum_{i=1}^m X_i$  where the  $X_i$ s are independent log-normal random variables with a common distribution, specifically  $X_i \in \log N(\mu_i, \sigma_i)$ . We notice that  $X_i$  is the headway distance between two vehicles which can be located in different lanes of road. As illustrated in Fig. 2,  $X$  represents the convolution of  $m$  independent headway distances. As it turns out [32,33],  $X$  is approximately log-normal; the commonly-used Fenton-Wilkinson approximation [34] of  $X$  is obtained by setting

$$\sigma_X^2 = \log \left[ \frac{\sum e^{2\mu_i + \sigma_i^2} (e^{\sigma_i^2} + 1)}{(\sum e^{\mu_i + \sigma_i^2/2})^2} + 1 \right]$$

$$\mu_X = \log(\sum e^{\sigma_i^2}) - \frac{\sigma_X^2}{2}.$$

To simplify the notation, we write  $\sigma = \sigma_X$  and  $\mu = \mu_X$  and we use the notation  $X \in \log N(\mu, \sigma)$ .

#### 4.1. The probability distribution of path-loss

A quick look at (1) reveals that the path-loss  $L(X)$  is the convolution of a random variable:  $Y = 40 \log X$  and a constant value  $-(10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$ .

**Lemma 2.** Assuming that  $X \in \log N(\mu, \sigma)$ , the random variable  $L(X) = 40 \log X - (10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$  is normally distributed.

The proof of the lemma is routine and is, therefore, omitted; we note that since  $L(X)$  is normally distributed, its probability density function (pdf) reads  $l(z) \in \frac{1}{40} N(\frac{\mu}{40} + b, \sigma)$  where  $b = -(10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$ . To simplify the notation, we write  $L(X) = Z \in aN(\mu_z, \sigma_z^2)$ , where  $a = \frac{1}{40}$ ,  $\mu_z = a\mu + b$  and  $\sigma_z^2 = \sigma^2$ .

##### 4.1.1. The probability distribution of the existence of a link

The existence of a communication link between a source vehicle and a destination vehicle depends on the path-loss at the receiver's side. For a link between these vehicles, the path-loss between them needs to be smaller than a given threshold  $PL_{thr}$ . Thus, the probability distribution  $F(z)$  of the existence of a link between two vehicles separated by a distance of  $z$ :

$$F(z) = P\{L(X) \leq z\}$$

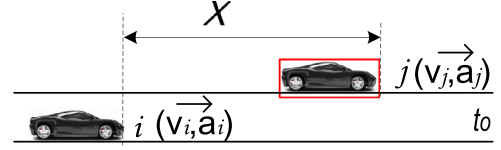


Fig. 3. Illustrating our basic scenario. Vehicle  $i$  and  $j$  are apart with distance  $X$ . Each of them has velocity and acceleration  $(v_i, a_i)$  and  $(v_j, a_j)$  respectively.

$$\begin{aligned} &= \int_{-\infty}^z \frac{a}{\sigma_z \sqrt{2\pi}} \exp\left(-\frac{(t - \mu_z)^2}{2\sigma_z^2}\right) dt \\ &= \frac{C_1 a}{2} \left[ 1 + \operatorname{erf}\left(\frac{z - \mu_z}{\sigma_z \sqrt{2}}\right) \right] \end{aligned} \quad (1)$$

where  $C_1$  is a normalization coefficient. Since  $\lim_{z \rightarrow \infty} F(z) = 1$ , it follows that  $C_1 = \frac{1}{a}$  and, thus, we write

$$F(z) = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{z - \mu_z}{\sigma_z \sqrt{2}}\right). \quad (2)$$

#### 4.2. The link duration model

Referring to Fig. 3, assume that at time  $t_0 = 0$  a link is established between co-directional vehicles  $i$  and  $j$  with  $j$  ahead of  $i$ . Let the random variable  $X$  denote the distance separating the two vehicles at link setup time. Mindful of the 300 m DSRC transmission range constraint, we have

$$0 \leq X < 300. \quad (3)$$

Recall that  $X$  is the convolution of  $m$  independent headway distances with a common log-normal distribution [30] and that  $X$  is approximately log-normal with parameters  $\mu$  and  $\sigma$ . The assumption of log-normal distribution has been verified in the previous work [30]. We assume that the speed limit on the roadway is  $v_m$  and that no vehicle will travel faster than  $v_m$ . For  $t \geq 0$ , we define  $a(t)$ , the acceleration of the vehicle at time  $t$  as follows:

- if  $a(0) = 0$ , then  $a(t) = 0$  for all  $t \geq 0$ ;
- if  $a(0) > 0$ , then

$$a(t) = \begin{cases} a(0) & \text{for } t \leq \frac{v_m - v(0)}{a(0)} \\ 0 & \text{otherwise;} \end{cases} \quad (4)$$

- if  $a(0) < 0$ , then

$$a(t) = \begin{cases} a(0) & \text{for } t \leq \frac{-v(0)}{a(0)} \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

In other words, (4) and (5) indicate that as long as the vehicle has not reached the speed limit  $v_m$  or has not stopped (in case  $a(0) < 0$ ), its acceleration remains  $a(0)$ . However, once the vehicle reaches the speed limit (or has stopped), its acceleration becomes 0.

Given a generic vehicle with initial speed  $v(0)$ , the instantaneous speed  $v(t)$  at time  $t$  is defined as

$$v(t) = v(0) + \int_0^t a(u)du, \quad (6)$$

where for all  $u \in [0, t]$ ,  $a(u)$  is the instantaneous acceleration at time  $u$  defined above.

Now, (4) and (5) and (6), combined imply that

- if  $a(0) = 0$ , then  $v(t) = v(0)$  for all  $t \geq 0$ ;
- if  $a(0) > 0$ , then

$$v(t) = \begin{cases} v(0) + a(0)t & \text{for } t \leq \frac{v_m - v(0)}{a(0)} \\ v_m & \text{otherwise;} \end{cases} \quad (7)$$

- if  $a(0) < 0$ , then

$$v(t) = \begin{cases} v(0) + a(0)t & \text{for } t \leq \frac{-v(0)}{a(0)} \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

Similarly, with  $v(x)$  defined above, the distance that our generic vehicle travels in the time interval  $[0, t]$  is defined as

$$S(t) = \int_0^t v(x)dx. \quad (9)$$

We now return to our vehicles  $i$  and  $j$ . To simplify the notation, we write  $v_i = v_i(0)$ ,  $a_i = a_i(0)$  and  $v_j = v_j(0)$ ,  $a_j = a_j(0)$ . The instantaneous speeds and accelerations  $v_i(t)$  and  $a_i(t)$ , respectively,  $v_j(t)$  and  $a_j(t)$  are obtained by suitably instantiating (4)–(8). Now, (9) guarantees that the distances traversed in the time interval  $[0, t]$  by vehicles  $i$  and  $j$  are, respectively,

$$S_i(t) = \int_0^t v_i(x)dx$$

and

$$S_j(t) = \int_0^t v_j(x)dx.$$

Assuming that at connection setup (i.e., at time 0) the distance between the two vehicles was  $x$ , it follows that the distance between  $i$  and  $j$  at time  $t$  can be written as

$$S_j(t) - S_i(t) + X. \quad (10)$$

It is important to note that (10) defines a *signed* distance: indeed, if at time  $t$ ,  $S_j(t) - S_i(t) + X > 0$ , then vehicle  $j$  is ahead of  $i$ ; otherwise, vehicle  $i$  is ahead of  $j$ .

We find it convenient to define the indicator function  $I(i, j)$  intended to capture information about which of the two vehicles is ahead when the communication link between them breaks

$$I(i, j) = \begin{cases} 1 & \text{if } S_j(t) - S_i(t) + X > 0 \\ -1 & \text{otherwise} \end{cases}$$

Given that DSRC links break at 300 m, it follows that when the link breaks the following relation holds:

$$S_j(t) - S_i(t) + X = 300 \cdot I(i, j). \quad (11)$$

We distinguish special time moments  $t_\alpha$ ,  $t_\beta$ ,  $t_\gamma$  where

- $t_\alpha$  is the time when two vehicles have same speed,
- $t_\beta$  is the time when exactly one vehicle has stopped,
- $t_\gamma$  is the time when both vehicles have stopped.
- $t_\varepsilon$  is the time when one vehicle have stopped and the other vehicle reached the speed limit  $v_m$ .
- $t_\zeta$  is the time when both vehicles have reached the speed limit  $v_m$ .

Due to the existence of a speed limit, we have  $t_\varepsilon$  and  $t_\zeta$

- $t_\varepsilon$  is the time when one vehicle have stopped and the other vehicle reached the speed limit  $v_m$ .
- $t_\zeta$  is the time when both vehicles have reached the speed limit  $v_m$ .

The reason for discussing these special time instances is that they affect in a crucial way the link duration. We now define a number of time instances that will be used in our analysis:

- provided that  $\frac{v_j - v_i}{a_j - a_i} > 0$ , we write

$$t_\alpha = \frac{v_j - v_i}{a_j - a_i};$$

- define  $t_\beta$  as follows

$$t_\beta = \begin{cases} \frac{-v_i}{a_i} & \text{if } \frac{-v_i}{a_i} > 0 \text{ and } \frac{-v_j}{a_j} < 0 \\ \frac{-v_j}{a_j} & \text{if } \frac{-v_j}{a_j} > 0 \text{ and } \frac{-v_i}{a_i} < 0 \\ \min \left\{ \frac{-v_i}{a_i}, \frac{-v_j}{a_j} \right\} & \text{if } \frac{-v_i}{a_i} > 0 \text{ and } \frac{-v_j}{a_j} > 0 \\ \text{undefined} & \text{otherwise.} \end{cases}$$

- similarly, define  $t_\gamma$  as follows

$$t_\gamma = \begin{cases} \max \left\{ \frac{-v_i}{a_i}, \frac{-v_j}{a_j} \right\} & \text{if } \frac{-v_i}{a_i} > 0 \text{ and } \frac{-v_j}{a_j} > 0 \\ \text{undefined} & \text{otherwise.} \end{cases}$$

- define  $t_\varepsilon$  as follows

$$t_\varepsilon = \begin{cases} \frac{v_m - v_i}{a_i} & \text{if } \frac{-v_i}{a_i} > 0 \text{ and } \frac{v_m - v_j}{a_j} < 0 \\ \frac{v_m - v_j}{a_j} & \text{if } \frac{-v_j}{a_j} > 0 \text{ and } \frac{v_m - v_i}{a_i} < 0 \\ \min \left\{ \frac{v_m - v_i}{a_i}, \frac{v_m - v_j}{a_j} \right\} & \text{if } \frac{v_m - v_i}{a_i} > 0 \text{ and } \frac{v_m - v_j}{a_j} > 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

- define  $t_\zeta$  as follows

$$t_\zeta = \begin{cases} \max \left\{ \frac{v_m - v_i}{a_i}, \frac{v_m - v_j}{a_j} \right\} & \text{if } \frac{v_m - v_i}{a_i} > 0 \text{ and } \frac{v_m - v_j}{a_j} > 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

It is important to note that  $\{t_\alpha \leq t_\beta \leq t_\gamma\}$  and  $\{t_\varepsilon, t_\zeta\}$  only depend on the speeds and acceleration of the two vehicles at connection setup time and on the value of the speed limit  $v_m$ .

### 4.3. Deriving the link duration

Because of obvious similarities, we only discuss the scenario that both vehicle move in the same direction and have positive accelerations. Sooner or later they will reach the speed limit and will, thereafter, cruise at the maximum speed.

#### 4.3.1. $0 \leq t \leq t_\alpha$

In this simulation, when the link breaks, vehicle  $j$  must be ahead of  $i$  and, thus,  $I(i, j) = 1$ . Recalling that in DSRC the link breaks when the distance between  $i$  and  $j$  is 300 m, we can write (11) as

$$S_j(t) - S_i(t) + X = 300. \quad (12)$$

On the other hand, by (9),  $S_j(t) - S_i(t) = \frac{1}{2}a_r t^2 + v_r t$ , where  $a_r = a_j - a_i$ ,  $v_r = v_j - v_i$ . All that remains, is to substitute  $S_j(t) - S_i(t)$  in (12) and to solve for  $t$ . Since  $t < t_\alpha$ , we obtain

$$t = \frac{-v_r + \sqrt{v_r^2 + 2a_r(300 - X)}}{a_r}.$$



4.3.2.  $t_\alpha < t \leq t_\varepsilon$ 

In this case, when the link breaks vehicle  $i$  must be ahead of vehicle  $j$ ; thus  $l(i, j) = -1$  and by (11) we can write

$$S_j(t) - S_i(t) + X = -300.$$

We know that  $S_j(t) - S_i(t) = \frac{1}{2}a_r t^2 + v_r t$ . By substituting the value of  $S_j(t) - S_i(t)$  in (IV-C2) and by solving for  $t$  we obtain

$$t = \frac{-v_r - \sqrt{v_r^2 - 2a_r(300 + X)}}{a_r}.$$

4.3.3.  $t_\varepsilon < t \leq t_\zeta$ 

In this case, when the link breaks, vehicle  $i$  must be ahead of vehicle  $j$  and so  $l(i, j) = -1$ . Thus, we can write

$$S_j(t) - S_i(t) + X = -300.$$

Observe that by (9),  $S_j(t) = \frac{1}{2}a_j t^2 + v_j t$  and  $S_i(t) = v_m t - \frac{v_m - v_i}{2} t_\varepsilon$ . After substituting the value of  $S_j(t) - S_i(t)$  in (IV-C3) and after solving for  $t$  we obtain

$$t = \frac{-(v_j - v_m) - \sqrt{(v_j - v_m)^2 - 2a_j(300 + x + \frac{v_m - v_i}{2} t_\varepsilon)}}{a_j}.$$

4.3.4.  $t_\zeta < t$ 

In this case, the link will not break because the two vehicles move at the same speed (alternatively, the link breaks at  $+\infty$ ).

The scenario shows that vehicle  $i$  catches up with vehicle  $j$ , passes  $j$  and, finally, breaks the link with vehicle  $j$ . Experience tells us that this is a very frequent occurrence in highway traffic.

## 4.4. The distribution function of link duration

The duration of a link is the lifetime of an established communication link between two vehicles. The main goal of this section is to derive analytical expressions for the probability distribution and density function of the duration of a link.

With the preamble of the previous section out of the way, we are now ready to state and prove the following important result. The proofs of Lemmas can be referred in Appendices.

**Lemma 3.** Assuming  $X \in \log N(\mu, \sigma)$ , the random variable  $T = \sqrt{aX + b} + c$  is log-normally distributed, where  $a, b, c \in \mathbb{R}$ ,  $a, b, c \neq 0$  and  $aX + b \geq 0$ .

**Lemma 4.** Assuming that  $X$  is log-normal with parameters  $\mu$  and  $\sigma$ , the random variable  $T = aX + b$  is log-normally distributed, where  $a, b, c \in \mathbb{R}$  and  $a, b, c \neq 0$ .

**Lemma 5.** Suppose that the communication link between two vehicles  $i$  and  $j$  breaks at time  $t$ . The link duration time is either a linear function of  $X$  or a square root function of  $X$ .

Then we come up with Theorem 6 as below. The proof can be referred in appendices.

**Theorem 6.** The duration  $T$  of the link between vehicles  $i$  and  $j$  is log-normally distributed.

Let  $\Phi$  be a set of all real combination of  $v_i, v_j, a_i, a_j$  on roads and  $\phi$  be the size of  $\Phi$ . Let  $P_k$  be the probability of the case  $k \in \Phi$  and  $T_k$  be the link duration time of case  $k$ . By the law of total expectation, we can obtain the overall expected duration of a link  $E[\text{link}]$ ,

$$E[\text{link}] = \sum_{k=1}^{\phi} P_k T_k. \quad (13)$$

Theoretically,  $E[\text{link}]$  can be computed by (13). However, to the best of our knowledge, there are no analytical results or field-test data on  $P_k$  and  $T_k$  in literature. We will leave the computation of the expected link duration for future investigations.

## 5. Analysis for successful data exchange

Let  $A$ ,  $T$  and  $L$  be, respectively, association time<sup>3</sup>, data exchange time (time left after successful association) and total available time for given transmission range in V2V communication. For successful association and data exchange,  $A + T \leq L$  condition must be satisfied. Comparison of ZigBee, Bluetooth and Wi-Fi technology is presented in Table 1. Probability of successful association and data exchange between vehicles can be expressed as  $P_s = \Pr\{A + T \leq L\}$ . Then  $1 - P_s$  gives the probability of unsuccessful data exchange which represents, in our case, partial or no messages being exchanged between vehicles. Value of total available time for V2V communication can be computed as

$$L = \begin{cases} \frac{D_{i,j}}{v_{rel}}, & D_{i,j} \leq \min\{R_i, R_j\} \quad \forall i, j \\ 0, & \text{Otherwise} \end{cases} \quad (14)$$

where  $D_{i,j}$  is the distance between vehicles  $i$  and  $j$ . Assuming that the two vehicles are located at  $(x_i, y_i)$  and  $(x_j, y_j)$ , we can compute  $D_{i,j}$  as

$$D_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (15)$$

Obviously, for the vehicles to be able to communicate, we must have  $D_{i,j} \leq \min\{R_i, R_j\}$ . From equation (14), it is clear that when  $v_{rel} \rightarrow 0$ , the value of  $L \rightarrow \infty$ . This implies that when the vehicles move at constant speed in the same direction while exchanging their information, they will have very long time (theoretically  $\infty$ ) to get associated and exchange the messages.

Based on the analysis above, we present the algorithm for time calculation as Algorithm 1:

**Algorithm 1** : V2V communication time ( $L$ ).

```

1: Input: geographic locations of vehicles ( $i$  and  $j$ ) and their
   transmission ranges ( $R_i$  and  $R_j$ ).
2: for each vehicle pair,  $i$  and  $j$  do
3:   for  $v_{rel} = 1$  to 140 miles/hour do
4:     Compute  $D_{i,j}$  using equation (15)
5:     if  $D_{i,j} \leq \min\{R_i, R_j\}$  then
6:       Compute  $L = \frac{D_{i,j}}{v_{rel}}$ 
7:     else
8:        $L = 0$ 
9:     endif
10:   end for
11: end for
12: Output: time ( $L$ ) for different relative speeds

```

Once we have the total time for V2V communication, we can compute the time for message exchange (that is the time left after successful association of devices) and amount of data that could be exchanged between vehicles using Algorithm 2.

Furthermore, without transmitting a complete message, the message would have no meaning in VANETs. Note that when partial message is exchanged in V2V communication, receiving vehicle may not be able to understand the actual semantic/meaning of the message unless it gets a copy of the message from other vehicles. Thus, we consider that the complete message of size  $S$  has to be transmitted with given data rate  $D_r$  for successful V2V communication. The time  $T$  needed to transmit given message of size  $S$  is given by

$$T = \frac{S}{D_r} \quad (16)$$

<sup>3</sup> Note that the association time depends on technologies such as typical association time for ZigBee is about 30 ms, Wi-Fi is about 600 ms and so on [35].

**Table 1**  
Comparison of The Bluetooth, ZigBee, And Wi-Fi protocols.

Standard	Bluetooth	ZigBee	WiFi
IEEE spec.	802.15.1	802.15.4	802.11 a/b/g
Frequency band	2.4 GHz	868/915 MHz; 2.4 GHz	2.4 GHz; 5 GHz
Max signal rate	1 Mb/s	250 Kb/s	54 Mb/s
Nominal range	10 m	10 – 100 m	100 m
Nominal TX power	0 – 10 dBm	(–25) – 0 dBm	15 – 20 dBm
Number of RF channels	79	1/10; 16	14 (2.4 GHz)
Channel bandwidth	1 MHz	0.3/0.6 MHz; 2 MHz	22 MHz
Modulation type	GFSK	BPSK (+ ASK), O-QPSK	BPSK, QPSK, COFDM, CCK, M-QAM
Spreading	FHSS	DSSS	DSSS, CCK, OFDM
Coexistence mechanism	Adaptive freq. hopping	Dynamic freq. selection	Dynamic freq. selection, power control (802.11 h)
Basic cell	Piconet	Star	BSS
Extension of the basic cell	Scatternet	Cluster tree, Mesh	ESS
Max number of cell nodes	8	> 65,000	2007
Encryption	EQ stream cipher	AES block cipher (CTR, CTR mode)	RC4 stream cipher (WEP), AES block cipher
Authentication	Shared secret	CBC-MAC (ext. of CCM)	WPA2 (802.11i)
Data protection	16-bit CRC	16-bit CRC	32-bit CRC
Approx. association time	45 ms	650 ms	4 s

**Algorithm 2** : Message size calculation.

```

1: Input: time ( $L$ ) from Algorithm 1, association time ( $A$ ) and data
   rate ( $D_r$ ).
2: if  $0 < L$  then
3:   Compute  $T = \max\{0, L - A\}$ 
4:   Compute message size with header  $MSH = D_r \times T$ 
5:   Actual message size  $MS = MSH - \text{Header} - \text{Size}$ 
6: endif
7: Output: Message size ( $MS$ )

```

Because of dynamic nature of VANET topology, it is important to estimate the time (available for data exchange) after successful association of wireless devices. Note that this value depends on various parameters such as relative speed of vehicle, transmission range, association time of technology being used, etc.

**Estimation of expected time for data exchange:** We assume that communication range overlap between vehicles is random process and it does not depend on the time and range overlap of the previous vehicles. We consider distribution of vehicles being within a communication range of each other as Poisson distribution with parameter  $\beta$  [36, Ch. 8] to analyze how association time can influence the data exchange in V2V communications. Note that  $L \geq 0$  is the random variable representing the total time/duration available for device association and data exchange between vehicles for V2V communications which follows the Poisson distribution with parameter  $\beta$ . Thus the cumulative distribution function (cdf) for a random variable  $L$  is defined as [36]

$$F_L(L) = 1 - e^{-\beta L}, \quad L \geq 0 \quad (17)$$

Then we can find probability distribution function (pdf) as

$$f_L(L) = \begin{cases} \beta e^{-\beta L}, & L \geq 0 \\ 0, & \text{Otherwise} \end{cases} \quad (18)$$

Using total time available  $L$  and association time  $A$ , we can write time for data exchange (after connection setup) as

$$T = [L - A]^+ \quad (19)$$

Where  $[b]^+ = \max\{0, b\}$ . Note that  $T$  is also a random variable and should be positive to be able to transmit messages using V2V communication. Then we can write probability of successful message exchange for given time  $T$  as

$$F_T(T) = P\{T \leq T\} = P\{L - A \leq T\} = F_L(T + A) \quad (20)$$

Now, we can write cdf as

$$F_T(T) = 1 - e^{-(T+A)\beta} \quad (21)$$

and then pdf as

$$f_T(T) = \beta e^{-(T+A)\beta} \quad (22)$$

Then the expected value of time  $T$  can be computed as [27]

$$\begin{aligned} \bar{T} = E[T] &= \int_0^\infty T \beta e^{-(T+A)\beta} dx \\ &= \frac{1}{\beta} e^{-A\beta} \end{aligned} \quad (23)$$

Note that (23) gives the expected value of usable time as a function of successful association time when we consider  $T$  as a random variable and there is no boundary in the transmission range. In our case, transmission range is given by (15). Considering factors such as distance between vehicles, data rate, data size, etc, we compute ratio of time,  $P_r$ , as

$$P_r = \min \left\{ 1, \left[ \frac{\left( \frac{D_{i,j}}{v_{rel}} \right)}{\left( A + \frac{S}{D_r} \right)} \right] \right\} \quad (24)$$

It is worth noting that ratio  $P_r$  values is bounded by  $[0, 1]$  and its value is 1 when relative speed is zero (this happens when two vehicles travel with equal speed in the same direction). Then, the expected time  $\bar{T}$  becomes

$$\bar{T} = \frac{1}{\beta} e^{-A\beta} \times P_r \quad (25)$$

Note that the expected time  $\bar{T}$  for data exchange for given association time and data rate of the technology, and overlapping of transmission range and relative speed of communicating vehicles. From (25), it follows that when relative speed is zero,  $P_r$  becomes 1 and the expected value  $\bar{T}$  depends only on association time  $A$  for a given  $\beta$ .

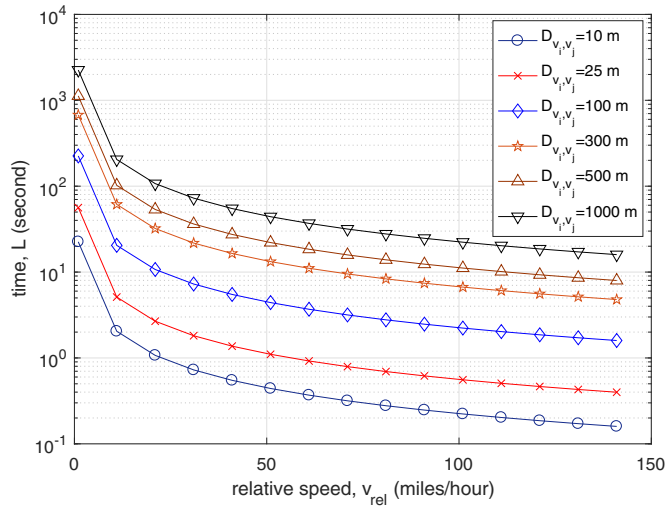
Using (14) and (25), the probability of successful data exchange is expressed as

$$P_s = \Pr\{A + \bar{T} \leq L\} \quad (26)$$

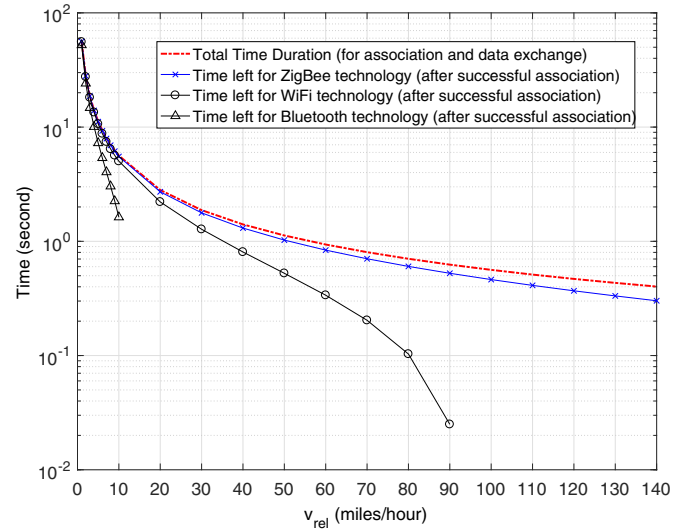
This probability depends on association time and data rate of a given technology, relative speed of communicating vehicles and size of the message to be transmitted.

## 6. Simulation and results

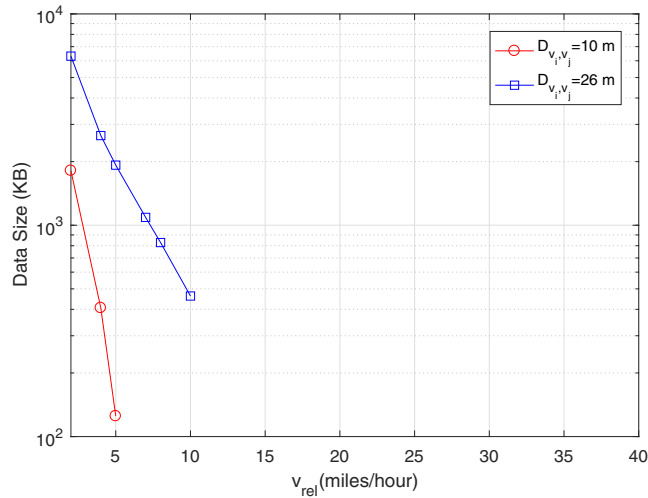
We considered that the overlap of communication range between vehicles is Poisson distribution process with the parameter



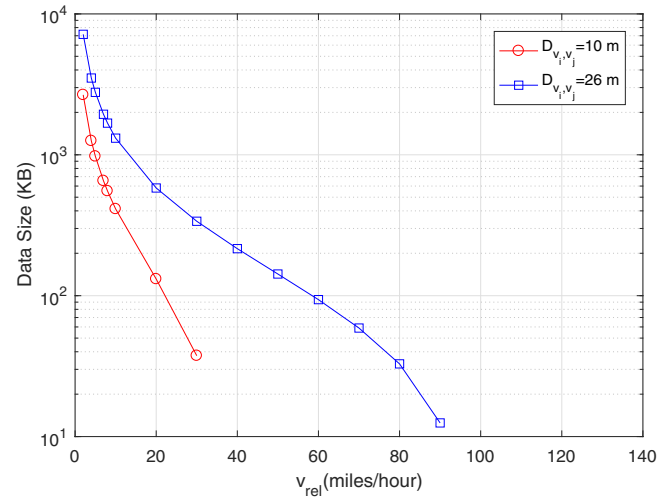
**Fig. 4.** Upper limit of total communication time ( $L$ ) for vehicles  $v_i$  and  $v_j$  for different relative speeds (starting from 1 miles/h through 140 miles/h) and communication ranges of  $D_{v_i, v_j}$  (starting from 10 m through 1000 m - DSRC recommended maximum range).



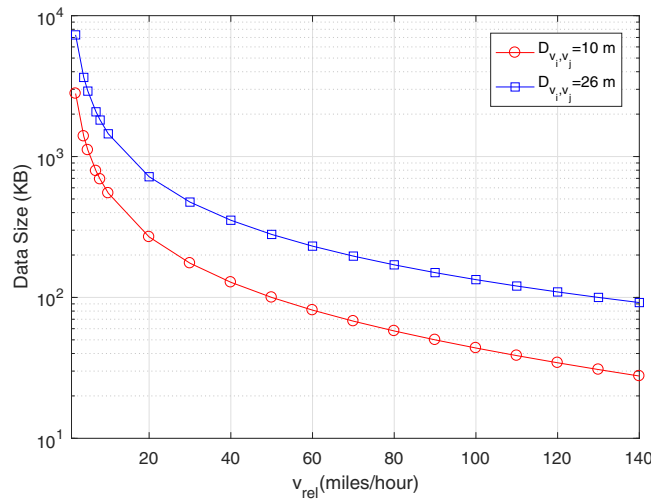
**Fig. 5.** Total time and time left after successful association between vehicles  $v_i$  and  $v_j$  for different relative speeds and communication range equal to  $D_{v_i, v_j} = 26$  m.



(a) Bluetooth Technology



(b) Wi-Fi Technology



(c) ZigBee/XBee Technology

**Fig. 6.** Upper bound of message size that could be transmitted with a given data rate of 250 Kbps for different relative speed of vehicles and communication range  $D_{v_i, v_j} \in \{10, 26\}$  meter.

$\beta = 1$  vehicle/s, and overlap in range does not depend on the time and range overlap of the previous vehicles. Relative speed of vehicles varied from 0 mile/h to 140 miles/h.

In the first experiment, using Algorithm 1, we plotted the variation in available time for given transmission range (varying from 10 m through 1000 m) and different relative speeds of vehicles (varying from 1 mile/h through 140 miles/h<sup>4</sup>) as shown in Fig. 4. As expected for a given transmission range, when relative speed of vehicles increases, available time for communication decreases. For a given relative speed (e.g. 40 miles/h), time increases when transmission range increases. Observe that the sum of association time and the time needed for message/data exchange cannot exceed the line in Fig. 4 for a given transmission range and relative speed.

In the second experiment, we have considered three scenarios where vehicles are assumed to be equipped with Bluetooth, Wi-Fi, ZigBee/xBee devices<sup>5</sup> for short range V2V communications. We calculated the total available time for given transmission ranges and relative speeds of vehicles and plotted both total time and time remaining after successful association in Fig 5. Note that Bluetooth, Wi-Fi and ZigBee devices take, respectively, about 4 s, 600 ms and 30 ms for association or connection setup [35]. After successful association, devices exchange actual information. Fig 5 shows that the vehicles with Bluetooth devices had no time left when their relative speed is greater than 10 miles/h since Bluetooth devices could not be associated or they spent all of the available time for association. As a result, vehicles could not exchange messages when they traveled with a relative speed greater than 10 miles/h as shown in Fig 6(a). Note that the size of the message transmitted is about 4000 KB when relative speed is about 10 miles/h.

Next, we assumed that each vehicle was equipped with a Wi-Fi device. When the relative speed was not greater than 90 miles/h, Wi-Fi devices exchanged messages using V2V communications as shown in Fig 6(b). For a relative speed of 90 miles/h, vehicles exchanged 6 KB of messages. Note that the size of a typical one-page message with 568 words is about 5 Kb.

Finally, we assumed that individual vehicles were equipped with ZigBee devices. ZigBee devices were able to transmit messages of size 90 KB even when their relative speed was 140 miles/h.

It is noted that for a given transmission range of 26 m and relative speed not greater than 90 miles/h, Wi-Fi devices could be used for successful V2V communications. Similarly, Bluetooth devices could be used only when relative speed is not greater than 10 miles/h. However, ZigBee devices could be used to exchange messages even when relative speed is about 140 miles/h or even higher.

To see the effect in message exchange, we then changed the transmission range from 26 m to 10 m for all three cases and plotted the data exchanged for given relative speeds as shown in Fig 6. We noted that vehicles could not exchange data when they move with relative speed higher than 5 miles/h and 30 miles/h using Bluetooth and Wi-Fi devices respectively. Furthermore, message size decreased significantly when we decreased the transmission range to 10 m in all cases as shown in Fig 6(a)–(c). It is noted that tradeoff between transmission range and message should be considered.

Finally, we computed probability of successful data exchange in V2V communication using Algorithm 3 and plotted it for given message size of 5 KB (typical size of one page message of 568

### Algorithm 3 : $P_s$ calculation.

```

1: Input: geographic coordinates of vehicles ( $i$  and  $j$ ), their transmission ranges ( $R_i$  and  $R_j$ ),  $A$ ,  $S$ ,  $D_r$ ,  $\beta$ , simulation time and success = 0.
2: repeat
3:   for each vehicle pair,  $i$  and  $j$  do
4:     for  $v_{rel} = \text{random}(0, 70)$  do
5:       Compute  $D_{i,j}$  using equation (15)
6:       if  $D_{i,j} \leq \min\{R_i, R_j\}$  then
7:         Compute  $L = \frac{D_{i,j}}{v_{rel}}$ 
8:       else
9:          $L = 0$ 
10:      endif
11:      Compute  $\bar{T}$  using (25).
12:      if  $A + \bar{T} \leq L$  then
13:        success = success + 1
14:      endif
15:    end for
16:  end for
17: until end of trials or simulation time
18:  $P_s = \frac{\text{success}}{\text{trials}}$ 
19: Output:  $P_s$ 

```

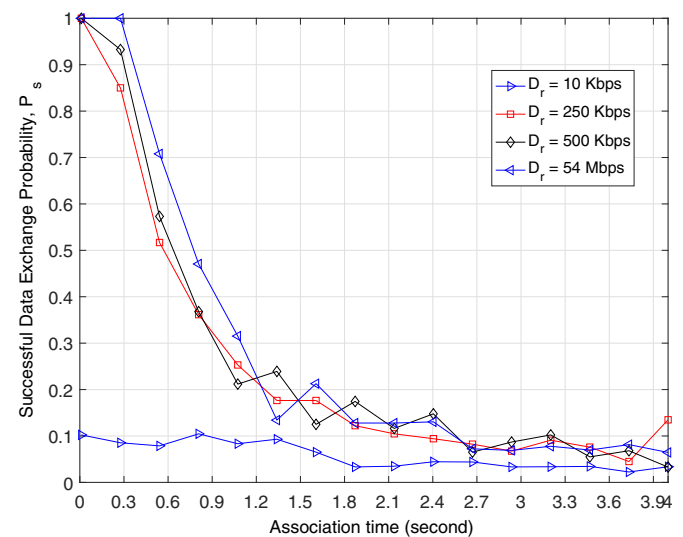


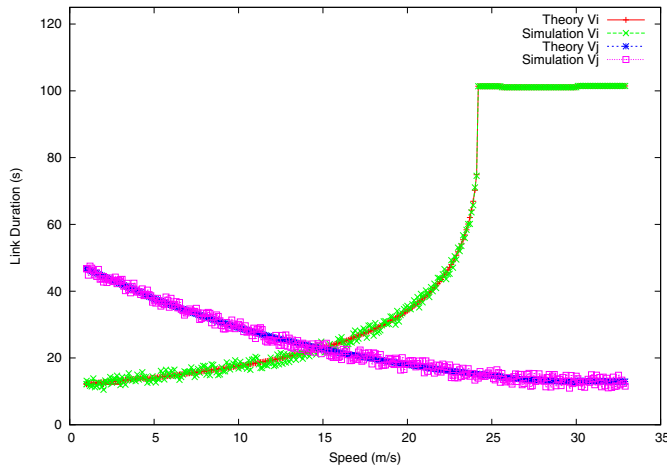
Fig. 7. Effect of Association Time for given message size (e.g. the size of a typical one page message with 568 words is about 5 KB) and data rate on successful data exchange probability for V2V based communication when the communication range equal to  $D_{v,v_j} = 26$  m.

words) using different data rates and association time by averaging 1000 trials. Relative speeds of vehicles were generated randomly in the range  $70 \pm 70$  miles/h. This represents both low and high relative speeds ranging from 0 miles/h to 140 miles/h. From Fig 7, note that as expected, when association time increases, probability of successful data exchange decreases. It is worth noting that for given transmission range and relative speed, there is a fixed time during where vehicles remain within communication range of each other. The overlap time is used for both association and data exchange, when association time increases, time left for data exchange decreases resulting in unsuccessful data exchange. Again, note that unsuccessful data exchange in our case is partial or no information is exchanged at all. Furthermore, to exchange typical one page message (i.e., 5 KB), there is no significant difference in probability  $P_s$  value when data rate is higher than or equal to 100 Kbps.

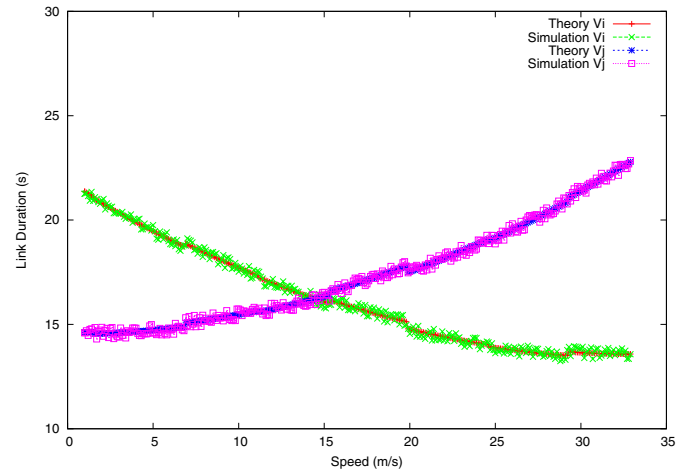
<sup>4</sup> Posted speed of the US roads/highways is not greater than 70 miles/h. Thus the relative speed of 140 miles/h is for vehicles moving with 70 miles/h in opposite direction.

<sup>5</sup> Note that to make typical wireless technologies such as Bluetooth, Wi-Fi, ZigBee/xBee, etc. suitable for V2V communications, specifications such as transmission range/power could have to be changed to meet the needs for V2V communications.





(a) Speed impact in case I.



(b) Speed impact in case II.

Fig. 8. Speed has significant impact on link duration.

We conclude that V2V communication requires fast association between wireless devices for successful message exchange between communicating vehicles to forward messages.

We varied the speed (both  $v_i$  and  $v_j$ ) in cases I and II and recorded the link duration. Fig. 8 illustrated the results we obtained. We assume that the link duration 100 s represent infinite link duration. Fig. 8(a) shows that larger speed of vehicle  $i$  will cause longer link duration and larger speed of vehicle  $j$  will cause shorter link duration because both  $i$  and  $j$  are accelerating in case I. When  $v_i$  increases, it increases the likelihood of reaching  $v_m$ . Therefore both  $i$  and  $j$  reach the speed limit  $v_m$  before the link breaks. Thus, in this case, the link tends to be stable. As we expected, the simulation results match well the theoretically-predicted values, as shown in Fig. 8(b). The higher speed of vehicle  $j$  will cause a shorter link duration, and the higher speed of vehicle  $i$  will cause a longer link duration. If vehicle  $j$  has higher speed, the relative speed between  $j$  and  $i$ , i.e.  $v_j - v_i$  will be higher. Therefore, the link duration will be shorter.

## 7. Conclusion

In this paper, we presented the analysis for information exchange success rate and probability of constructing communication link using V2V communication for vehicular network. Both analysis models considered mobility factors such as headway distance, relative speed, acceleration, and time. As a conclusion remark, some common wireless devices can be used to exchange the information using V2V communication. For a given transmission range, ZigBee devices could be used when relative speed is high (e.g. 140 miles/h), Wi-Fi devices could be used when relative speed is not greater than 90 miles/h. Bluetooth could be used when relative speed is not greater than 10 miles/h.

In many applications especially emergency related applications, V2V communication are required to provide fast association and low communication latency between communicating vehicles. Thus short range based V2V communication is applicable. However, other communication factors such as handshake time of communication, throughput, response time, etc. can significantly impact V2V communication as well. Therefore, the selection of wireless technology really depends on the services and applications that the V2V communication is served.

In the future work, we plan to compare the proposed model with other models that are comparable. In addition, we will apply the model to develop a more reliable routing algorithm.

## Acknowledgments

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## Appendix A. Proofs of Lemma and Theorem

**Lemma 1.** Assuming  $X \in \log N(\mu, \sigma)$ , the random variable  $T = \sqrt{aX + b} + c$  is log-normally distributed, where  $a, b, c \in \mathbb{R}$ ,  $a, b, c \neq 0$  and  $aX + b \geq 0$ .

**Proof.** Let  $G_T$  be the probability distribution function of  $T$ . For every positive  $t$ , we write

$$G_T(t) = \Pr\{T \leq t\}. \quad (1)$$

Since  $T$  is obviously continuous, (1) allows us to write

$$\begin{aligned} G_T(t) &= \Pr\{T \leq t\} \\ &= \Pr\{\sqrt{aX + b} + c \leq t\} \\ &= \Pr\{aX \leq (t - c)^2 - b\} \\ &= \begin{cases} F_X\left(\frac{(t-c)^2 - b}{a}\right) & \text{for } a > 0 \\ 1 - F_X\left(\frac{(t-c)^2 - b}{a}\right) & \text{for } a < 0 \end{cases} \end{aligned}$$

where  $F_X$  is the probability distribution function of  $X$ . When  $a > 0$ , it is clear that  $T$  is log-normally distributed.

Next, we propose to show that  $T$  is also log-normally distributed when  $a < 0$ . For this purpose, letting  $z = \frac{(t-c)^2 - b}{a}$  and using (2), we write

$$\begin{aligned} 1 - F_X\left(\frac{(t-c)^2 - b}{a}\right) &= \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{\ln z - \mu(X)}{\sigma(X)\sqrt{2}}\right) \\ &= F_Y\left(\frac{a}{(t-c)^2 - b}\right) \end{aligned}$$

where  $Y$  is a log-normal random variable with parameters  $-\mu(X)$  and  $\sigma(X)$ ,  $z = \frac{(t-c)^2 - b}{a}$ . Note that we use the fact  $-\operatorname{erf}(x) = \operatorname{erf}(-x)$ . Thus, in all cases,  $T$  obeys a log-normal distribution, completing the proof.  $\square$

**Lemma 2.** Assuming that  $X$  is log-normal with parameters  $\mu$  and  $\sigma$ , the random variable  $T = aX + b$  is log-normally distributed, where  $a, b, c \in \mathbb{R}$  and  $a, b, c \neq 0$ .

**Proof.** This lemma can be easily proved using the arguments employed in the proof of Lemma 3.  $\square$

**Lemma 3.** Suppose that the communication link between two vehicles  $i$  and  $j$  breaks at time  $t$ . The link duration time is either a linear function of  $X$  or a square root function of  $X$ .

**Proof.** Recall that when the link breaks,  $t$  satisfies (11). By the definition of  $S_i(t)$ , we know that  $S_i(t) = \int_0^t v_i(x)dx$  is a linear function of  $t$  when the speed  $v_i$  is constant, i.e.,  $v_i(t) = v_m$ . Let  $S_i(t) = at + b$ . Similarly,  $S_j(t)$  is a linear function of  $t$  when  $v_j$  is constant, i.e.,  $v_j(t) = v_m$ . Let  $S_j(t) = ct + d$ . When both  $S_i(t)$  and  $S_j(t)$  are linear function of  $t$ , substituting the corresponding values of  $S_j(t)$  and  $S_i(t)$  in (11), we obtain

$$(a - c)t + b - d + X = 300I(i, j)$$

$$t = 300 \frac{I(i, j) - X - b + d}{a - c}.$$

Clearly, the link duration  $t$  is a linear function when both  $v_i(t)$  and  $v_j(t)$  are constant. If any of  $v_i(t)$  and  $v_j(t)$  are not a constant, the distance function will be a quadratic polynomial. Without loss of generality, we let  $v_i(t) = v_i(0) + a_it$ , by definition, the distance function

$$S_i(t) = \int_0^t v_i(0) + a_ix dx \\ = v_i(0)t + \frac{1}{2}a_it^2.$$

Therefore,  $S_j(t) - S_i(t)$  will be a quadratic polynomial. Suppose  $S_j(t) - S_i(t) = at^2 + bt + c$  and  $a \neq 0$ . Substitute  $S_j(t) - S_i(t)$  in (11), to get the quadratic equation

$$at^2 + bt + c + X - 300I(i, j) = 0.$$

Clearly,  $t$  exists because the link breaks at time  $t$ . Therefore, the solution must be a square root function of  $X$ . This completes the proof.  $\square$

**Theorem 4.** The duration  $T$  of the link between vehicles  $i$  and  $j$  is log-normally distributed.

**Proof.** By Lemma 5, the link duration can be expressed as either  $aX + b$  or  $\sqrt{aX + b} + c$ . By Lemma 3, the expression  $\sqrt{aX + b} + c$  is log-normally distributed. By Lemma 4,  $aX + b$  is log-normally distributed. Thus, in all cases, the duration time of the link has a log-normal distribution. This completes the proof of the theorem.  $\square$

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