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Statistical evaluation of Data Envelopment Analysis versus COLS Cobb–Douglas benchmarking models for the 2011 Brazilian tariff revision

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ABSTRACT

In 2011, the Brazilian Electricity Regulator (ANEEL) implemented a benchmarking model to evaluate the operational efficiency of power distribution utilities. The model is based on two benchmarking methods: Data Envelopment Analysis (DEA) and Corrected Ordinary Least Squares (COLS) with a Cobb Douglas production function. Although the estimated scores are highly correlated, differences between the scores are as high as 41%. For some companies differences between the efficiency scores result in substantial reduction in regulatory operational costs. We provide a detailed statistical comparison which indicates that the COLS Cobb Douglas model has major deficiencies in terms of estimating efficiency scores.

1. Introduction

The use of frontier-based methods in the incentive regulation of power companies has grown in recent years. With an international survey performed in 40 countries on a total of 43 regulators between June and October 2008, Haney and Pollitt [1] found that 51% of the companies were applying benchmarking techniques. The frontier techniques can be divided into three major groups: Data Envelopment Analysis (DEA), Corrected Least Squares (COLS) analysis and Stochastic Frontier Analysis (SFA). The authors found that in 2008, 34.8%, 13% and 8.7% of the power distribution regulation models were applying DEA, COLS analysis and SFA, respectively. They also found that 40%, 15% and 5% were applying DEA, COLS and SFA, respectively, in power transmission regulation models. Haney and Pollitt [1] report that Austria applies DEA and COLS for both power transmission and distribution utilities; Belgium applies DEA for power transmission and distribution and applies SFA and COLS in power transmission regulation models; Denmark applies COLS analysis; Portugal applies SFA; Slovenia applies DEA; Iceland, Netherlands and Norway apply DEA in power distribution and transmission models; Argentina and Brazil applied DEA for power transmission in 2008, but Brazil applied DEA and COLS models for power distribution companies in 2011. Colômbia began to apply DEA for power transmission in 2000 and for power distribution in 2002.

Several studies have indicated the suitability of DEA in the analysis of efficiency in power-regulated sectors. For example, Edvardsen et al. [2] describe the Norwegian Electricity regulation model. The Norwegian Water Resources and Energy Directorate (NVE) was one of the first European regulators to use DEA. Bogetoft and Otto [3] describe the DEA-based incentive model applied in Germany; Hu and Wang [4] evaluate the efficiency of electric utilities and their effects on consumer prices; Souza et al. [5] compare DEA and SFA in the measurement of the efficiency of 40 Brazilian energy distribution companies; Sarıca and Or [6] assess the efficiency of Turkish power plants using DEA; Vaninsky [7] uses DEA to measure the efficiency of electric power generation in the United States; Hu and Wang [4] analyze the energy efficiencies of 19 administrative regions in China for the period of 1995–2002 using DEA. A summary of the use of DEA in energy and environmental studies can be found in Ref. [8].

In September 10, 2010, the Brazilian National Electric Energy Agency began a debate with the Brazilian society regarding the rules and methodologies for defining the revenues of electricity distribution utilities for the 3rd Periodic Tariff Review Cycle (3PTRC) through public hearing 040/2010 (AP040). Through

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technical note 265/2010, the national regulator proposed a full review of the model that calculates regulatory operational costs. The definition of efficient operating cost is a central point in incentive regulation because it was chosen for the regulation of natural monopolies in the Brazilian electricity sector after their privatization in the nineteen-nineties. Incentive regulation requires the definition of a revenue or rate level for a fixed period of time, which is defined in a formal contract. Given the rate or revenue level defined by the regulator, companies are encouraged to reduce their costs to achieve higher financial returns. After the concession period defined in the agreement contract, which is 4 or 5 years, the companies' costs are revised; i.e., the regulator defines new levels that are considered more efficient. In this case, the new efficiency levels are proposed by the regulator for the benefit of consumers. Therefore, a consistent methodology for the definition of operational costs is important for both the regulated company and consumers.

This paper evaluates the benchmarking models proposed by ANEEL, which are the DEA and COLS models, and discusses major inconsistencies of the COLS methodology. Specifically, we aim to identify the causes of the discrepancy between the DEA and COLS methodologies. We provide a detailed statistical analysis of the Cobb Douglas regression model and the efficiency score estimates. We apply statistical hypothesis testing, linear regression theory, and Monte Carlo simulations. The results show that the differences between the COLS Cobb–Douglas and the DEA efficiency scores are statistically significant. We show that the main causes of the differences are related to sample size. We also explore missing variables in the Cobb–Douglas regression model.

The paper is organized as follows: Section 2 presents the historical background of the benchmarking models proposed by ANEEL in 2011, reviews the DEA model and the Cobb–Douglas production function, and presents the statistical methods used to evaluate the efficiency scores. Section 3 presents the results. Section 4 discusses the main findings. Section 5 presents the conclusion.

2. Material and methods

2.1. Background

During an initial stage of public hearing 040/2010, ANEEL proposed two DEA models organized into two stages, hereafter named model 1 and model 2. In the first stage, both models use operational cost as the input variable and network extension (km – kilometers) as the output variable. Model 1 uses number of customers as the second output variable, whereas model 2 uses energy consumption (MWh – Mega Watt-hour) as the second output variable. Both models assume non-decreasing returns to scale (NDRS).

The Data Envelopment Analysis (DEA) benchmarking method was first considered by ANEEL because it has been successfully applied by other regulatory agencies from Austria, Britain, Belgium, Finland, Netherlands, among others. Therefore, it is of interest of ANEEL to replace previous reference company method by a concise and robust benchmarking approach, like DEA. By applying DEA, the operating cost of each company is compared to the costs/results of the remaining companies and, by means of a linear system of equations, an efficiency frontier is calculated. The final results of this model are efficiency scores that indicate the efficiency of each company in transforming inputs (costs) in outputs (electricity consumption, number of customers and network extension), when compared with similar companies.

The parameters of the models were estimated using a database covering the years 2003–2009. However, first, the power distribution companies were split into two groups. Group A is composed of companies with a 2003 annual energy consumption greater than 1 Tera Watt-hour (TWh), whereas group B is composed of companies with a 2003 annual energy consumption smaller than 1 TWh. The DEA method was applied separately to each group to estimate the efficiency scores. Later, the estimated scores were further adjusted using a second-stage model with environmental variables.

After public contributions, the benchmarking methodology was reviewed by ANEEL. The new proposed methodology consists of two benchmarking models: one DEA model in the first stage that aggregates all previous output variables and a second benchmarking model known as Corrected Ordinary Least Squares (COLS), presented in Technical Note 101/2011. COLS is a parametric model that fits a linear regression model using ordinary least squares. In sequence, the regression model is shifted toward the smallest observed value among the residuals of the regression model. Thus, the lower bound or the efficiency frontier of the operational cost is determined. In this case, the regression model equation is the Cobb–Douglas production function [9].

The COLS method is currently used by two regulators (Denmark and Great Britain), and it is known to be more restrictive; i.e., it strongly penalizes companies that are not on the frontier [3]. To overcome this limitation, ANEEL applies the mean value of the DEA and COLS efficiency score as the first stage outcome.

In ANEEL's final decision, released in November 2011, the energy consumption (MWh) output variable was replaced by a weighted energy consumption variable that aggregates high-, medium- and lower-voltage energy consumption. The weights were chosen to be proportional to the amount of consumption in the high-, medium-, and lower-voltage markets of each company. Again, the DEA and COLS methods were applied separately to groups A and B to estimate the final efficiency scores. A summary of the proposed models is presented in Table 1.

Although the methodology proposed by ANEEL was based on the experience of leading European regulatory agencies, it was subject to criticisms and suggestions from Brazilian community and power distribution companies. One of the major concerns was the use of the DEA model with non-decreasing returns to scale (NDRS) as a replacement for the most commonly used model, the DEA with variable returns to scale (VRS) [10]. A technical report was submitted to ANEEL [11] providing evidence that the VRS model is the most appropriate model. Banker [11] states that "*even if economic theory argues that non-decreasing returns to scale prevails in situations of natural monopoly, empirical evidence strongly suggests that*

Table 1
Proposed benchmarking models by ANEEL.

Method	Input variables	Output variables
Technical note 265/2010 (first proposal)		
DEA-NDRS	Operational Cost (R$)	Network length (km), Number of customers
DEA-NDRS	Operational Cost (R$)	Network length (km), Power consumption (MWh)
Technical note 101/2011 (second proposal)		
DEA-NDRS	Operational Cost (R$)	Network length (km), Number of customers, Power consumption (MWh)
COLS	Operational Cost (R$)	Network length (km), Number of customers, Power consumption (MWh)
Technical note 294/2011 (final proposal)		
DEA-NDRS	Operational Cost (R$)	Network length (km), Number of customers, Weighted power consumption (MWh)
COLS	Operational Cost (R$)	Network length (km), Number of customers, Weighted power consumption (MWh)

VRS achieves better fit". A mathematical model is an abstract representation, and without the proper production function, outputs and environmental variables, such a model should not hold on strong assumptions, which is the case with NDRS.

A separate analysis of the estimated efficiency scores achieved with DEA and COLS, presented by ANEEL, shows that the efficiency scores present major inconsistencies. The efficiency scores achieved using the COLS model are exceptionally smaller than DEA estimates for all companies except one. Differences between COLS and DEA results are as high as 21% in Group A and 41% in Group B. Appendix 1 and Appendix 2 compare the DEA and COLS efficiency scores for power companies in groups A and B for 2009. Although both models were adjusted using data from years 2003–2009, only the most recent efficiency scores, those from 2009, were used to define the regulatory operational costs for the next period. Appendix 1 shows the efficiency scores for group A in year 2009, sorted in decreasing order by the differences between the DEA and COLS efficiency scores, as shown in column 5. The CEMAT power company exhibits the largest difference between the efficiency scores. In this case, the DEA efficiency score is 39.57% higher than the COLS efficiency score. The CEEE power company exhibits the smallest difference. In this case, the COLS efficiency score is 0.65% higher than the DEA efficiency score. Overall, only 6 COLS efficiency scores were higher than the DEA efficiency scores in the database, which includes data from 2003 to 2009 for both the A and B groups. Among these 6 values, the smallest difference between the scores is 0.000056 and the largest difference is 0.003280. Section 3.1.1 provides further statistical analysis of the differences between DEA and COLS. Nonetheless, DEA efficiency scores are usually higher than COLS scores for both the A and B groups. Furthermore, 6 shows that for some of the power companies, the expected reduction in regulatory operational costs is more than half a billion reais (R$) if the COLS efficiency score is applied. For instance, CEMIG, one of the largest power companies in Brazil, must reduce its operational cost by R$ 1.140 billion (or U$ 571 million, considering an average exchange rate of R$ 1.998 per U$ 1.00 in year 2009) if the COLS efficiency score is applied. This amount represents a reduction of approximately 60% of CEMIG's operational costs in 2009. Note that 4 of the power companies, COELBA, COSERN, PIRATININGA and RGE, achieved a DEA efficiency score of 100% and COLS efficiency scores higher than 90%. COELBA has a COLS efficiency score of 91.97%, which represents a reduction of R$ 35.7 million in operational costs.

Appendix 2 shows the efficiency scores for group B in 2009, sorted in decreasing order by the differences between the DEA and COLS efficiency scores, as shown in column 5. Note that JOAOCESA power company exhibits the largest difference between the efficiency scores. In this case, the DEA efficiency score is 194.5% higher than the COLS efficiency score. The UHENPAL power company exhibits the smallest difference. In this case, the COLS efficiency score is also lower than the DEA efficiency score, in contrast to the smallest difference in group A (see Appendix 2), which is negative. Among the power companies in group B, BOA VISTA ENERGIA exhibits the most significant loss in operational costs, R$ 50 million (or U$ 25 million) if COLS is applied. In this case, the expected reduction in BOA VISTA ENERGIA's operational costs is 80% based on the COLS efficiency score. In general, the lowest COLS efficiency score values are found in group B. Therefore, major relative operational cost reductions are expected for power companies in group B if COLS efficiency scores are applied.

Despite the major inconsistencies between DEA and COLS efficiency scores, ANEEL claims that the correlation coefficient of DEA-NDRS and COLS Cobb–Douglas efficiency scores is high, at 0.94 (94%) [12], paragraph 68, page13], and therefore, that the efficiency scores are similar.

2.2. The database

The database provided by ANEEL (aneel.gov.br, accessed on February 22, 2013) reports yearly operational costs, number of customers, network length, and weighted power consumption for the years 2003–2009. The database consists of 59 electricity distribution companies, 29 of which demonstrated total electricity consumption greater than 1 TWh in year 2003 and were classified in group A. The remaining 30 companies were classified in group B. The total number of observations is 413.

Mathematically, we can represent the database with one input variable x_{it} and three output variables, y_{1it}, y_{2it} and y_{3it}. x_{it} is the total operation cost of company i at time t; y_{1it}, y_{2it} and y_{3it} are the number of customers, network length, and weighted energy consumption, respectively, for company i at time t, $i = 1,\ldots,59$, and $t = \{2003,2004,\ldots,2009\}$. Alternatively, one index that represents both company and time can be applied, $j_{(it)}$, which leads to the following variables: x_j, y_{1j}, y_{2j}, and y_{3j}, where $j = 1,\ldots,413$.

Fig. 1 provides scatter plots of the input and output variables on the logarithm scale.

2.3. The DEA model

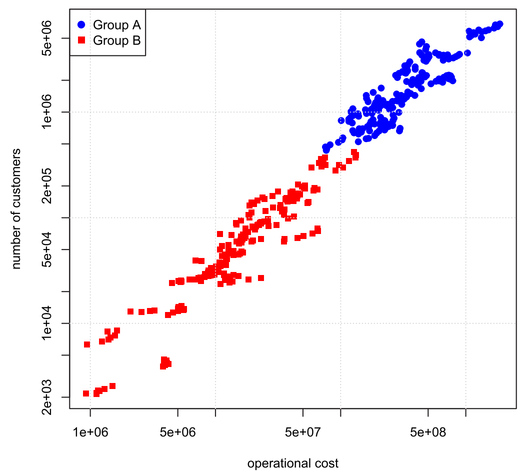
The Data Envelopment Analysis methodology was first introduced by Charnes et al. [13] and then extended by Banker et al. [10]. This method has been widely used to estimate technical efficiencies of Decision-Making Units (DMU). DEA is a mathematical programming method that provides a single measure of efficiency. The method calculates the best practice frontier with the use of multiple inputs and multiple outputs. The relative efficiency of each DMU is measured based on its distance from the efficiency frontier. For each DMU, DEA calculates an efficiency score, which typically ranges between zero and 1. Briefly, the farther the DMU is from the frontier, the more inefficient the DMU is. Nevertheless, the DEA efficiency frontier can be used as a guideline so that inefficient companies can improve their inputs and outputs and reach the frontier.

The DEA models calculate the efficiency score of each DMU j by imposing different returns to scale. Let θ be the efficiency score of DMU j. Let y_{rk} be the output variables ($r = 1,\ldots,R$) and x_{ik} be the input variables ($i = 1,\ldots,I$) for each DMU k. R is the total number of outputs, I is the total number of inputs, and N is the total number of DMUs. Let λ_k be the parameters or weights of the DEA benchmarking model. Below, we present the variable returns to scale model (VRS) and the non-decreasing return to scale model (NDRS).

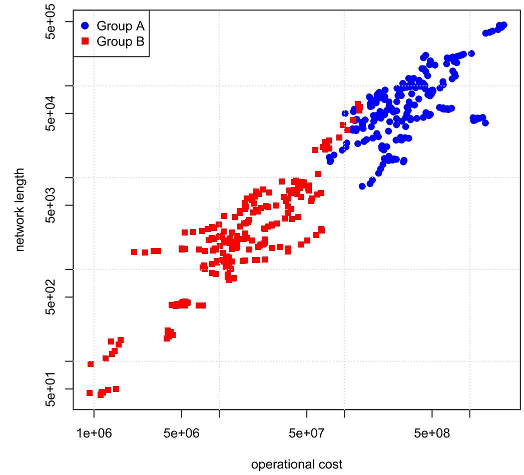
$$\begin{array}{ll} \theta_j^{\mathrm{VRS}} = \min\theta & \\ \text{Subject to}: & \\ \sum_{k=1}^{N}\lambda_k y_{rk} \geq y_{rj}, & \forall r = 1,\ldots,R; \\ \sum_{k=1}^{N}\lambda_k x_{ik} \leq \theta x_{ij}, & \forall i = 1,\ldots,I; \\ \sum_{k=1}^{N}\lambda_k = 1, & \\ \lambda_k \geq 0, & \forall k = 1,\ldots,N \end{array} \quad (1)$$

$$\begin{array}{ll} \theta_j^{\mathrm{NDRS}} = \min\theta & \\ \text{Subject to}: & \\ \sum_{k=1}^{N}\lambda_k y_{rk} \geq y_{rj}, & \forall r = 1,\ldots,R; \\ \sum_{k=1}^{N}\lambda_k x_{ik} \leq \theta x_{ij}, & \forall i = 1,\ldots,I; \\ \sum_{k=1}^{N}\lambda_k \geq 1, & \\ \lambda_k \geq 0, & \forall k = 1,\ldots,N \end{array} \quad (2)$$

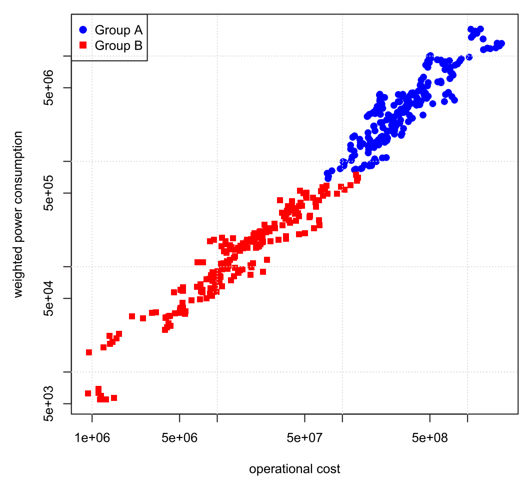
As shown in Equations (1) and (2), to estimate the DEA efficiency score for each company, j, one linear programming model must be



(a) number of customers



(b) network length



(c) weighted energy consumption

Fig. 1. Scatter plots on the log scale of total operational cost (x-axis), versus each output variable: number of customers, network length, and weighted energy consumption.

solved. Therefore, there are N linear programming problems. Fig. 2 illustrates the differences between the efficiency frontier of the DEA-VRS and DEA-NDRS models. In this example, the number of customers was chosen as the output and the operational cost as the input. By doing so, the efficiency frontier can be visualized in a simple plot. As observed, the DEA-NDRS frontier provides a better fit to companies with a small number of customers and low operational costs. These companies achieve better efficiency scores than companies with more customers and higher operational costs, which are farther from the frontier. The DEA-VRS model achieves a better fit for both companies with a small number of customers and low operational costs and those with many customers and high operational costs.

2.4. The Cobb–Douglas regression model

The Cobb–Douglas production function was developed by Paul Douglas and Charles Cobb [9] as a mathematical model to represent a functional form of production. The initial hypothesis was to represent total output as a log linear function. Mathematician Charles Cobb proposed the following equation: $P = b \cdot L^k C^{1-k}$, where b and k are parameters, P is total output in manufacturing, L is total number of workers, and C is fixed capital.

In our case, the Cobb–Douglas regression model assumes a log-linear regression equation in the following form:

$$x_j = \beta_0 \cdot y_{1j}^{\beta_1} \cdot y_{2j}^{\beta_2} \cdot y_{3j}^{\beta_3} \tag{3}$$

By applying the logarithm function to Equation (3), a linear model is obtained.

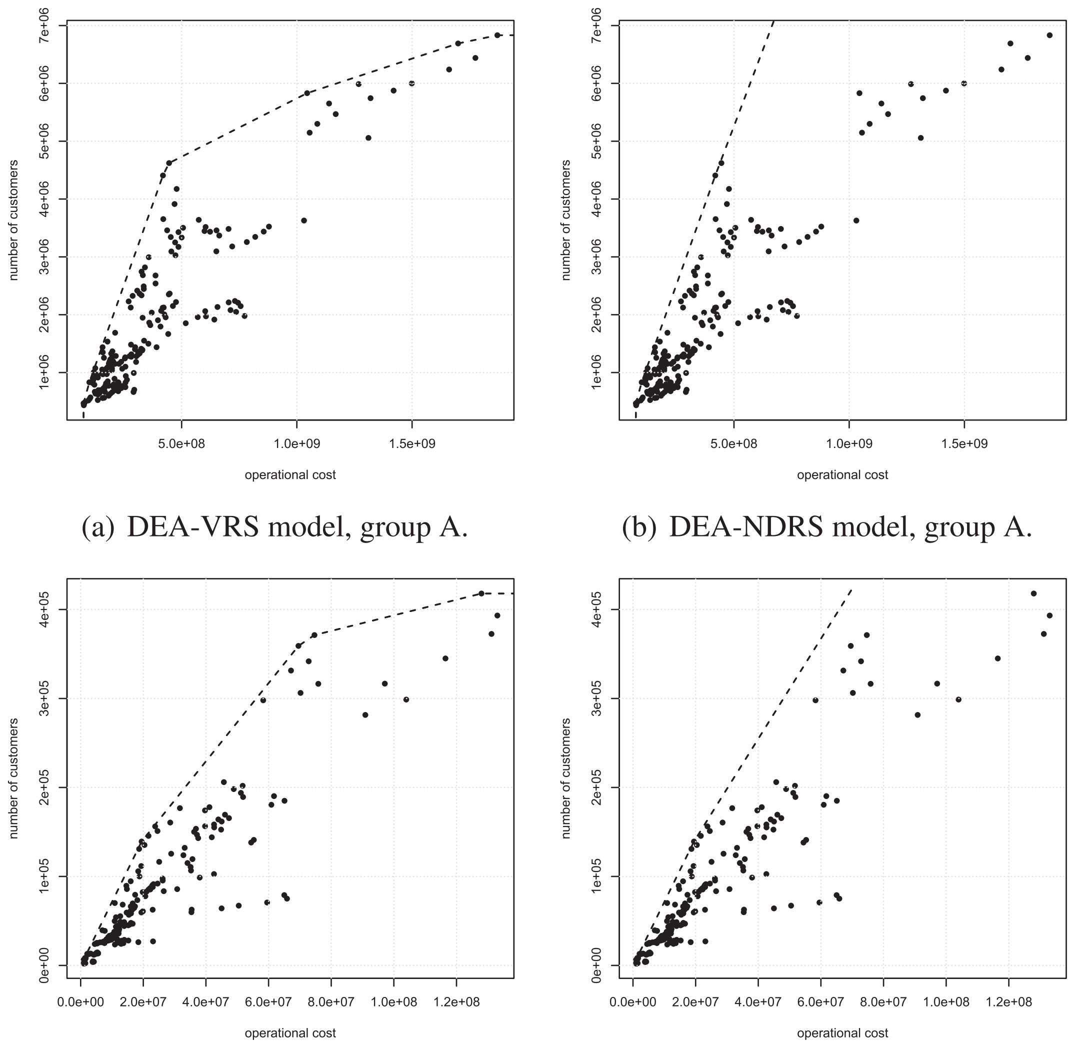
$$\log x_j = log\ \beta_0 + \beta_1 \log y_{1j} + \beta_2 \log y_{2j} + \beta_3 \log y_{3j}$$

or

$$x_j^* = \beta_0^* + \beta_1 y_{1j}^* + \beta_2 y_{2j}^* + \beta_3 y_{3j}^* \tag{4}$$

where $x_j^* = \log x_j$, $\beta_0^* = log \beta_0$, and $y_{kj}^* = \log y_{kj}$, k=1,2,3.

Equation (4) is a linear regression model, and the estimates of the parameters can be achieved with ordinary least squares [14,15]. Interestingly, Fig. 1 provides evidence of linear correlations



(a) DEA-VRS model, group A.

(b) DEA-NDRS model, group A.

(c) DEA-VRS model, group B.

(d) DEA-NDRS model, group B.

Fig. 2. Comparison of the DEA model with variable returns to scale (VRS) and non-decreasing returns to scale (NDRS), using number of customers as the output variable and operational cost as the input variable for groups A and B.

between the output and the input variables on the log scale. Therefore, Equation (4) appears to be an appropriate model for data fit.

2.5. Corrected Ordinary Least Squares (COLS)

The Corrected Ordinary Least Squares method estimates the parameters of the regression equation using ordinary least squares (OLS) and then shifts the regression model toward the smallest residual by adding the minimum residual of the OLS estimate to the intercept of the regression model. Briefly, the COLS estimate is the adjustment made to the OLS constant term so that all estimated residuals become positive except one, which is crossed by the efficiency frontier, and therefore is the most efficient DMU.

In this paper, ANEEL's COLS Cobb–Douglas efficiency frontier is written as follows:

$$f\left(y_{1j}, y_{2j}, y_{3j}\right) = e^{\min_j \varepsilon_j} \cdot \beta_0 \cdot y_{1j}^{\beta_1} \cdot y_{2j}^{\beta_2} \cdot y_{3j}^{\beta_3} \cdot \varepsilon_j \tag{5}$$

where ε_j is the error component.

2.6. Statistical properties of the COLS Cobb–Douglas efficiency frontier

In linear regression models, the ordinary least squares (OLS) method is a standard method for estimating unknown parameters. OLS minimizes the sum of squared distances between the observed response and the response predicted by the linear model [16]. OLS also represents the maximum likelihood estimator of a Gaussian random variable. In this case, the response variable is a linear function of the predictors plus a vector of independent and identically distributed Gaussian random variables. Therefore, the complete linear equation of the OLS Cobb–Douglas production function can be written as

$$x_j^* = \beta_0^* + \beta_1 y_{1j}^* + \beta_2 y_{2j}^* + \beta_3 y_{3j}^* + \varepsilon_j \tag{6}$$

where ε_j is an independent and identically distributed Gaussian random variable with mean of zero and variance of σ^2, $\varepsilon_i \sim Normal(\mu = 0; \sigma^2)$. In the original domain, i.e., before applying the logarithm transformation, the Cobb–Douglas model accounting for the random variable is written as

$$x_j = \beta_0 \cdot y_{1j}^{\beta_1} \cdot y_{2j}^{\beta_2} \cdot y_{3j}^{\beta_3} \cdot e^{\varepsilon_j} \tag{7}$$

Let $\zeta_j = e^{\varepsilon_j}$ be a log-Normal random variable, $\zeta_j \sim logNormal(\mu = 0; \sigma^2)$. The random variable ζ_j has the following mean and variance: $E[\zeta_j] = e^{\sigma^2/2}$ and $Var[\zeta_j] = (e^{\sigma^2/2} - 1)e^{\sigma^2/2}$ [17,18].

Let $\eta_j = \beta_0 \cdot y_{1j}^{\beta_1} \cdot y_{2j}^{\beta_2} \cdot y_{3j}^{\beta_3}$ be the Cobb Douglas regression equation. If we assume that the variance parameter σ^2 is known then the mean and variance of x_j, given the output variables, is written as

$$E\left[x_j \middle| y_{1j}, y_{2j}, y_{3j}\right] = \eta_j \cdot e^{\sigma^2/2} \tag{8}$$

$$\mathrm{Var}\left[x_j \middle| y_{1j}, y_{2j}, y_{3j}\right] = \eta_j^2 \left(e^{\sigma^2/2} - 1\right) e^{\sigma^2/2} \tag{9}$$

In particular, Equation (9) shows that the variance of x_j is proportional to the squared regression equation, $\mathrm{Var}[x_j | y_{1j}, y_{2j}, y_{3j}] \propto \eta_j^2$. In practice, the variability of the data is expected to be proportional to the squared mean.

2.6.1. The efficiency frontier

To calculate the COLS Cobb Douglas efficiency frontier, we first calculate the residuals of the fitted multiple linear model in the log-domain:

$$\widehat{\varepsilon}_j = \log x_j - \left(\widehat{\beta}_0^* + \widehat{\beta}_1 y_{1j}^* + \widehat{\beta}_2 y_{2j}^* + \widehat{\beta}_3 y_{3j}^*\right)$$

where $\widehat{\beta}_0^*$, $\widehat{\beta}_1$, $\widehat{\beta}_2$, and $\widehat{\beta}_3$ are the OLS estimates of the regression model. The COLS Cobb Douglas efficiency frontier in the log-domain is the fitted regression model shifted towards the observation with the minimum residual, or

$$\begin{aligned} \log f(y_1, y_2, y_3) &= \widehat{\beta}_0^* + \widehat{\beta}_1 y_1^* + \widehat{\beta}_2 y_2^* + \widehat{\beta}_3 y_3^* + \min_j \widehat{\varepsilon}_j \\ &= \left(\widehat{\beta}_0^* + \min_j \widehat{\varepsilon}_j\right) + \widehat{\beta}_1 y_1^* + \widehat{\beta}_2 y_2^* + \widehat{\beta}_3 y_3^* \end{aligned} \tag{10}$$

where $f(y_1, y_2, y_3)$ is the efficiency frontier function. Equation (10) shows that

$$f(y_1, y_2, y_3) = e^{\min_j \widehat{\varepsilon}_j} \cdot \widehat{\beta}_0 \cdot y_1^{\widehat{\beta}_1} \cdot y_2^{\widehat{\beta}_2} \cdot y_3^{\widehat{\beta}_3} \tag{11}$$

In fact, Equation (11) represents a lower bound confidence interval for the random variable x_j. For example, if $\varepsilon_i \sim Normal(\mu; \sigma^2)$ then the lower bound of a two-tailed α-level confidence interval is $\widehat{\mu} - z_{\alpha/2} \cdot \sigma$, where $z_{\alpha/2}$ is the standard score statistic [19], $z_{\alpha/2} > 0$. Alternatively, the confidence interval can be written as $\widehat{E}[\varepsilon_i] - z_{\alpha/2} \cdot \sqrt{Var[\varepsilon_i]}$. Thus, it can be shown that the COLS Cobb–Douglas efficiency frontier equation is a special case of a confidence interval:

$$\begin{aligned} f(y_1, y_2, y_3) &= \widehat{E}\left[x | y_1, y_2, y_3\right] - z \cdot \sqrt{\widehat{\mathrm{Var}}\left[x | y_1, y_2, y_3\right]} \\ &= \left[e^{\sigma^2/2} - z\sqrt{\left(e^{\sigma^2/2} - 1\right) e^{\sigma^2/2}}\right] \cdot \widehat{\beta}_0 \cdot y_1^{\widehat{\beta}_1} \cdot y_2^{\widehat{\beta}_2} \cdot y_3^{\widehat{\beta}_3} \end{aligned} \tag{12}$$

Equations (12) and (11) are equivalent. In this case, the value of the score statistic is chosen so that.

$$e^{\sigma^2/2} - z\sqrt{\left(e^{\sigma^2/2} - 1\right) e^{\sigma^2/2}} = e^{\min_j \widehat{\varepsilon}_j}$$

Therefore, the COLS Cobb Douglas efficiency frontier represents a particular case of a lower bound of a confidence interval in which the α-level is so that all observations are above the boundary with the exception of one that intersects the frontier.

Furthermore, if the efficiency frontier represents a confidence interval of the random variable x_j then different assumptions can be explored for the random variable, and consequently, for the regression model. For example, we can retain the mean structure of the standard Cobb Douglas regression model, $E[x_j | y_{1j}, y_{2j}, y_{3j}] = \eta_j$, and change the variance structure to

$$\mathrm{Var}\left[x_j \middle| y_{1j}, y_{2j}, y_{3j}\right] \propto \eta_j$$

In this case the final equation of the efficiency frontier is

$$f(y_1, y_2, y_3) = \beta_0 \cdot y_1^{\beta_1} \cdot y_2^{\beta_2} \cdot y_3^{\beta_3} - z\sqrt{\beta_0 \cdot y_1^{\beta_1} \cdot y_2^{\beta_2} \cdot y_3^{\beta_3}} \tag{13}$$

2.7. Assessing the distribution of the COLS Cobb–Douglas efficiency scores with Monte Carlo simulations

The COLS Cobb–Douglas efficiency score, hereafter named θ_j^{COLS}, for each DMU, j, is the ratio of the estimated operational cost at the estimated efficiency frontier, $\widehat{f}(y_{1j}, y_{2j}, y_{3j})$, and the observed operational cost, x_j, or

$$\begin{aligned} \theta_j^{\mathrm{COLS}} &= \frac{\widehat{f}\left(y_{1j}, y_{2j}, y_{3j}\right)}{x_j} \\ &= \frac{e^{\min_j \widehat{\varepsilon}_j} \cdot \widehat{\beta}_0 \cdot y_{1j}^{\widehat{\beta}_1} \cdot y_{2j}^{\widehat{\beta}_2} \cdot y_{3j}^{\widehat{\beta}_3}}{x_j} \end{aligned} \tag{14}$$

where $\widehat{\varepsilon}_j$ is the residual of the Cobb–Douglas regression model, or

$$\widehat{\varepsilon}_j = x_j^* - \left(\widehat{\beta}_0^* + \widehat{\beta}_1 y_{1j}^* + \widehat{\beta}_2 y_{2j}^* + \widehat{\beta}_3 y_{3j}^*\right)$$

Let T be a random variable defined as the minimum of n Gaussian random variables, $T = \min_j \varepsilon_j$, where ε_j are independent and identically distributed Gaussian random variables with mean of zero and variance of σ^2, $\varepsilon_i \sim Normal(\mu = 0; \sigma^2)$. It is known that the cumulative distribution function (cdf) of T is

$$F_T(t) = 1 - \left[1 - \Phi\left(\frac{t}{\sigma}\right)\right]^n \tag{15}$$

where $\Phi(x)$ is the cdf of the standard normal distribution. Equation (15) shows that the cdf of T is also a function of the number of random variables, n, or the sample size. Fig. 3 shows the probability density function (pdf) of T for different values of n, assuming $\sigma^2 = 1$. The figure shows that the distribution of T shifts toward smaller values as the sample size increases. Therefore, the distribution of the COLS Cobb–Douglas efficiency scores, θ_j^{COLS}, is expected to also be affected by the sample size.

However, there is no explicit formula for the distribution of the COLS Cobb–Douglas efficiency scores defined. In fact, there is no explicit formula for $\Phi(x)$. Alternatively, a detailed analysis of the distribution of the efficiency scores can be provided with a Monte Carlo simulation study [20].

In this case, we aim at providing an empirical distribution of the COLS Cobb–Douglas efficiency scores, conditioned on the available database. To do so, we apply a Monte Carlo simulation study that combines resampling techniques [21] and simulated envelops [22], using the following steps:

Step 1: We estimate the parameters of the Cobb–Douglas production function and the variance parameter, σ^2, for groups A

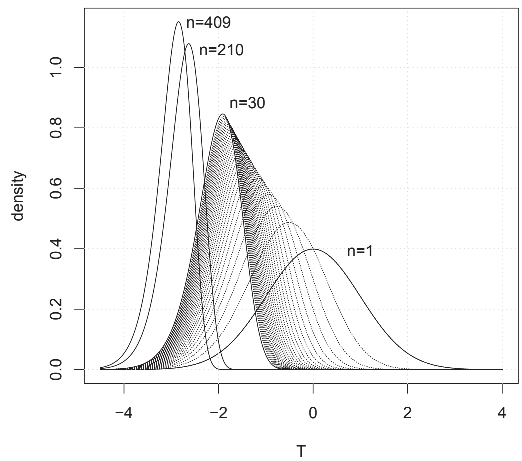


Fig. 3. Probability density function of a minimum of n Gaussian random variables with means of zero and $\sigma^2 = 1$ for different values of n.

and B. We use the estimated parameters as if they were the true unknown parameters of the Cobb–Douglas production function.
Step 2: By applying resampling with replacement, we generate n values of the output variables: y_1, y_2 and y_3 from the original database.
Step 3: New values for the input variable are generated using a Gaussian random number generator in which the mean value is the estimated Cobb–Douglas regression equation, $\log y_i \sim \text{Normal}(\widehat{\mu} = \log\widehat{\eta}_i,\ \widehat{\sigma}^2)$, $i = 1,\ldots,n$.
Step 4: We fit the COLS Cobb–Douglas model to the simulated database and calculate the efficiency scores.
Step 5: We repeat steps 2–4 s times. In our simulation study, $s = 10{,}000$.

The data base statistical analyses and the simulations were executed using the R software [23].

3. Results

Standard statistical methods were applied to compare both DEA and COLS efficiency scores. Statistical hypothesis testing, for instance, were developed specifically to deal with certain conditions. Therefore, we explore more than one statistical method in order to provide appropriate comparisons between DEA and COLS Cobb Douglas efficiency scores. These theories are presented briefly and individually with supporting data and results for each one. This structure has been adopted for clarity and comprehensibility of the reader.

Table 2
Estimates of the parameters of the Cobb–Douglas model proposed by ANEEL for groups A and B.

Coefficient	Estimate	Std. Error	*P*-value	95% confidence interval
Group A				
β_0^*	6.6838	0.4059	0.0000	(5.8834; 7.4842)
β_1	0.1121	0.0348	0.0015	(0.0434; 0.1807)
β_2	0.5374	0.0650	0.0000	(0.4092; 0.6656)
β_3	0.2521	0.0872	0.0043	(0.0800; 0.4241)
$R^2_{Adj} = 0.8467$				
Group B				
β_0^*	5.8573	0.3201	0.0000	(5.2262; 6.4883)
β_1	0.1162	0.0395	0.0036	(0.0383; 0.1941)
β_2	0.7211	0.0699	0.0000	(0.5834; 0.8589)
β_3	0.1220	0.0751	0.1057	(-0.0260; 0.2701)
$R^2_{Adj} = 0.9223$				

Table 2 presents the estimates of the multiple linear regression model presented in Equation (4) for groups A and B, as reported by ANEEL. The DEA efficiency scores reported by ANEEL, hereafter named θ_j^{DEA}, for each DMU, j, is the solution of the linear programming model shown in Equation (2).

3.1. COLS Cobb–Douglas efficiency score estimates with adjusted variance

The estimates of the parameters of the Cobb–Douglas regression models with adjusted variance, as shown in Equation (13), can be obtained with Quasi-likelihood methods [24]. Figs. 4 and 5 illustrate the differences between the standard Cobb Douglas regression model and the Cobb Douglas regression model with adjusted variance structure for groups A and B. The example applies the *number of customers* as the sole output variable and the *operational cost* as the input variable for the sake of visual representation. Figs. 4 and 5(a) show that the adjusted variance provides a better fit for DMUs with large numbers of customers in each group. A similar behavior is shown in Figs. 4 and 5(b): the efficiency frontier using the standard COLS Cobb–Douglas model does not fit DMUs with large numbers of customers. In this case, the frontier best fits DMUs with smaller numbers of customers. It is worth noting that the most efficient DMUs are different in both models. The standard Cobb–Douglas efficiency frontier crosses a DMU with a small number of customers, whereas the adjusted Cobb–Douglas efficiency frontier crosses a DMU with a large number of customers. However, the adjusted efficiency frontier is farther from DMUs with smaller number of customers than the standard efficiency frontier, and it produces negative operational cost values.

3.1.1. Fair comparison of COLS Cobb–Douglas and DEA efficiency scores

ANEEL reports that the DEA and COLS estimated efficiency scores are highly correlated, with a correlation coefficient of 0.9317 for group A and 0.7092 for group B. In this section, we show that despite the high correlation between the efficiency scores, they are not similar. In fact, we provide statistical evidence that the COLS efficiency scores are, on average, lower than the DEA efficiency scores.

A linear correlation between the DEA and COLS efficiency scores assumes that, on average,

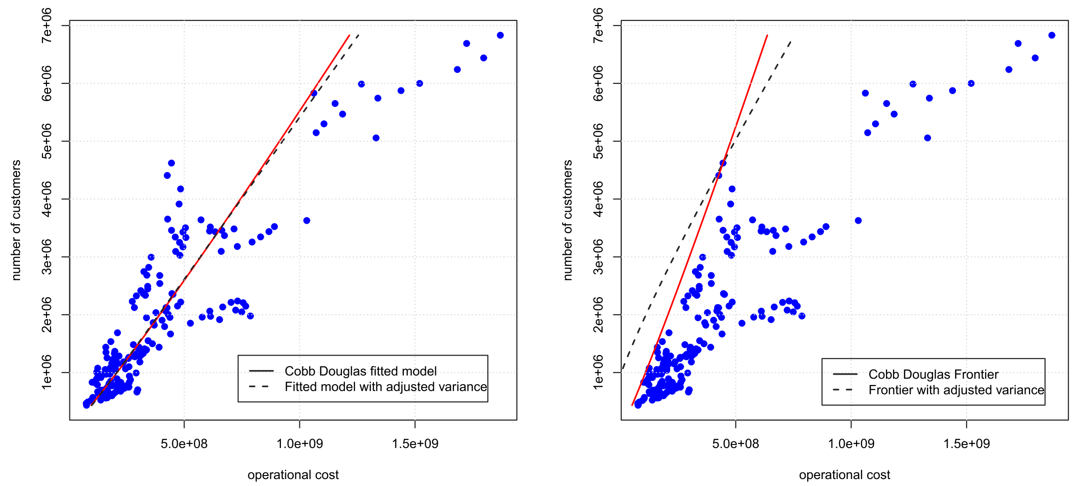
$$\theta^{COLS} = \beta_0 + \beta_1 \cdot \theta^{DEA} \tag{16}$$

Equation (16) cannot be estimated using standard linear regression analysis. In this case, the assumption of additive Gaussian errors does not apply, as the scores are within the 0–1 range. Furthermore, because the DEA and COLS efficiency scores are calculated using the same database and with non-parametric and parametric benchmarking techniques, it can be assumed that, although the scores are correlated, distinct uncertainties are associated with each θ^{DEA} and θ^{COLS} score. Therefore, the liner model shown in Equation (16) can be said to have different errors related to each dependent and independent variable.

Nevertheless, statistical hypothesis testing can be applied to β_0 and β_1 for specific assumptions. For instance, a high correlation coefficient suggests that $\beta_1 \neq 0$. Nonetheless, if we want to test whether θ^{DEA} and θ^{COLS} are similar, then our hypothesis testing can be written as H_0: $\beta_0 = 0$ and $\beta_1 = 1$, or similarly,

$$H_0 : \theta^{DEA} = \theta^{COLS} \tag{17}$$

Because θ^{DEA} and θ^{COLS} are correlated, the hypothesis testing described in Equation (17) can be evaluated using a paired t-test [25]. The paired t-test first calculates the mean of the differences between



(a) Fitted regression equations (b) Efficiency frontiers

Fig. 4. Fitted regression equations and efficiency frontiers using the standard Cobb Douglas regression model and the model with adjusted variance for group A.

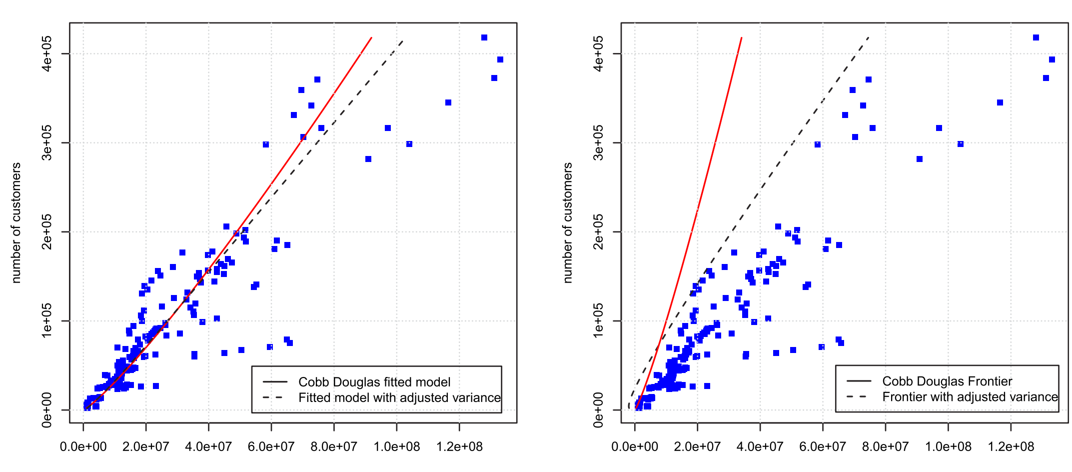
the COLS and DEA efficiency scores of each company and then tests whether the true mean of the differences is zero. Fig. 6 shows the distribution of the differences between the efficiency scores for groups A and B and the paired *t*-test results. The *p*-value of the *t*-test is very low for both groups, which provides statistical evidence that the DEA and the COLS efficiency scores are different. In fact, the estimated means of the differences between the scores are negative for both groups, which provides further evidence that the COLS efficiency scores are, on average, lower than the DEA efficiency scores. Furthermore, Fig. 6 shows that both histograms are highly asymmetric to the left and that most of the scores are negative. It is worth noting that differences between the efficiency scores reach extreme values of -0.25 for group A and -0.6 for group B, which indicate large discrepancies between the DEA and COLS efficiency scores.

Point estimates for the parameters of Equation (16) can be achieved with total least squares (TLS) methods [26]. The TLS method minimizes the sum of the squared orthogonal distances from the data points to the fitting line. This method provides weakly consistent estimator of the parameters and does not depend on the distributions of the errors [27].

Table 3 shows the estimates of the linear model between the DEA and COLS efficiency scores. Empirical confidence intervals were generated using bootstrap techniques [28]. In this case, pairs of DEA and COLS efficiency scores are sampled with replacement until reaching a sample size equal to the original data. The new data set is used to estimate the parameters of the linear model. This procedure is repeated 10,000 times. The 10,000 values of the estimated parameters are used to build empirical confidence intervals.

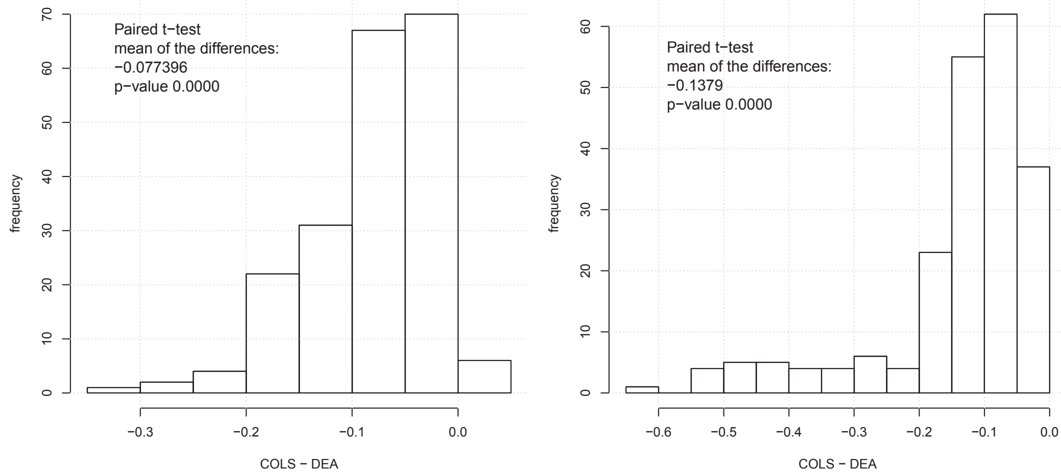
Results show that the DEA efficiency scores are consistently smaller for group A by a value of -0.0914 on average. It is worth noting that the confidence interval of β_1 includes $\beta_1 = 1$, and that the lower and upper bounds for β_0 are both negatives. Therefore, there is statistical evidence that for group A, $\beta_1 = 1$ and $\beta_0 < 0$. Similarly, there is statistical evidence that for group B, $\beta_0 = 0$ and $\beta_1 < 1$. Such is the case because the confidence interval of β_0 includes $\beta_0 = 0$, and both the lower and upper bounds for β_1 are positive. Therefore, for group B, the higher the DEA efficiency scores, the lower the COLS scores, by a factor of 0.7133, are on average.

ANEEL applies the mean of the COLS and DEA efficiency scores as the final efficiency score. As will be shown next, the proposed



(a) Fitted regression equations (b) Efficiency frontiers

Fig. 5. Fitted regression equations and efficiency frontiers using the standard Cobb Douglas regression model and the model with adjusted variance for group B.



(a) group A (b) group B

Fig. 6. Histograms of the differences between COLS and DEA efficiency scores and the *t*-test results for groups A and B.

COLS equation has major deficiencies. The equation generates very low efficiency scores, and consequently, the final efficiency scores are driven primarily by the lower values of the COLS efficiency scores.

3.1.2. The empirical distribution of COLS Cobb–Douglas efficiency scores

Equation (14) shows that the COLS Cobb–Douglas efficiency score is a function of random variables and therefore is also a random variable. We are particularly interested in pursuing the score's statistical distribution, as described in Section 2.7.

Using the Monte Carlo simulation algorithm presented in Section 2.7, we generated $n \times 10{,}000$ simulated efficiency scores, where n is the number of DMUs. Among the simulated efficiency scores, 10,000 values are exactly 1, because in each of the 10,000 simulations, one DMU is the most efficient or the reference DMU. Scores of 1 were eliminated because our main interest is the study of the distribution of efficiency scores smaller than 1. Therefore, we evaluate $(n-1) \times 10{,}000$ simulated efficiency scores.

The empirical distributions of the efficiency scores for different values of the sample size, n, are shown in Fig. 7 for groups A and B. The results for group A are shown in the first column, and the results for group B are shown in the second column. In general, the efficiency score distribution shifts toward smaller values as the sample size increases. Table 4 presents quantile confidence intervals of 95%. It can be observed that the larger the sample, the lower the upper bound of the confidence interval is. Furthermore, the larger the sample, the more asymmetric to the left the distribution is. In conclusion, the larger the sample, the lower the efficiency scores are and the closer most of the efficiency scores are to the lower bound of the distribution.

Table 3
Estimates of the linear model for the DEA and COLS efficiency scores for groups A and B.

Parameters	TLS estimates	Bootstrap confidence interval of 95%
Group A		
β_0	−0.0914	(−0.1277; −0.0545)
β_1	1.0199	(0.9648; 1.0716)
Group B		
β_0	0.0313	(−0.0318; 0.1014)
β_1	0.7133	(0.5806; 0.8286)

Finally, Fig. 8 compares the simulated distribution of the efficiency scores for groups A and B with the histograms of the estimated values.

3.2. The Cobb Douglas regression model's lack of fit

Fig. 8(a) shows that the estimated efficiency scores for group A are slightly different from the simulated distribution This issue may result from the Cobb Douglas regression model's lack of fit to group A. Briefly, statistical regression analysis aims to fit a model to observed data to quantify the relationship between a dependent variable and the independent variables as accurately as possible. In regression analysis, the error estimates, or residuals, represent the information on the dependent variable that cannot be predicted by independent variables. However, if the regression equation is incomplete, then the residuals account for both errors and the model's lack of fit. Thus, the Cobb Douglas regression equation's lack of fit with regard to group A may generate the efficiency scores shown in Fig. 8(a).

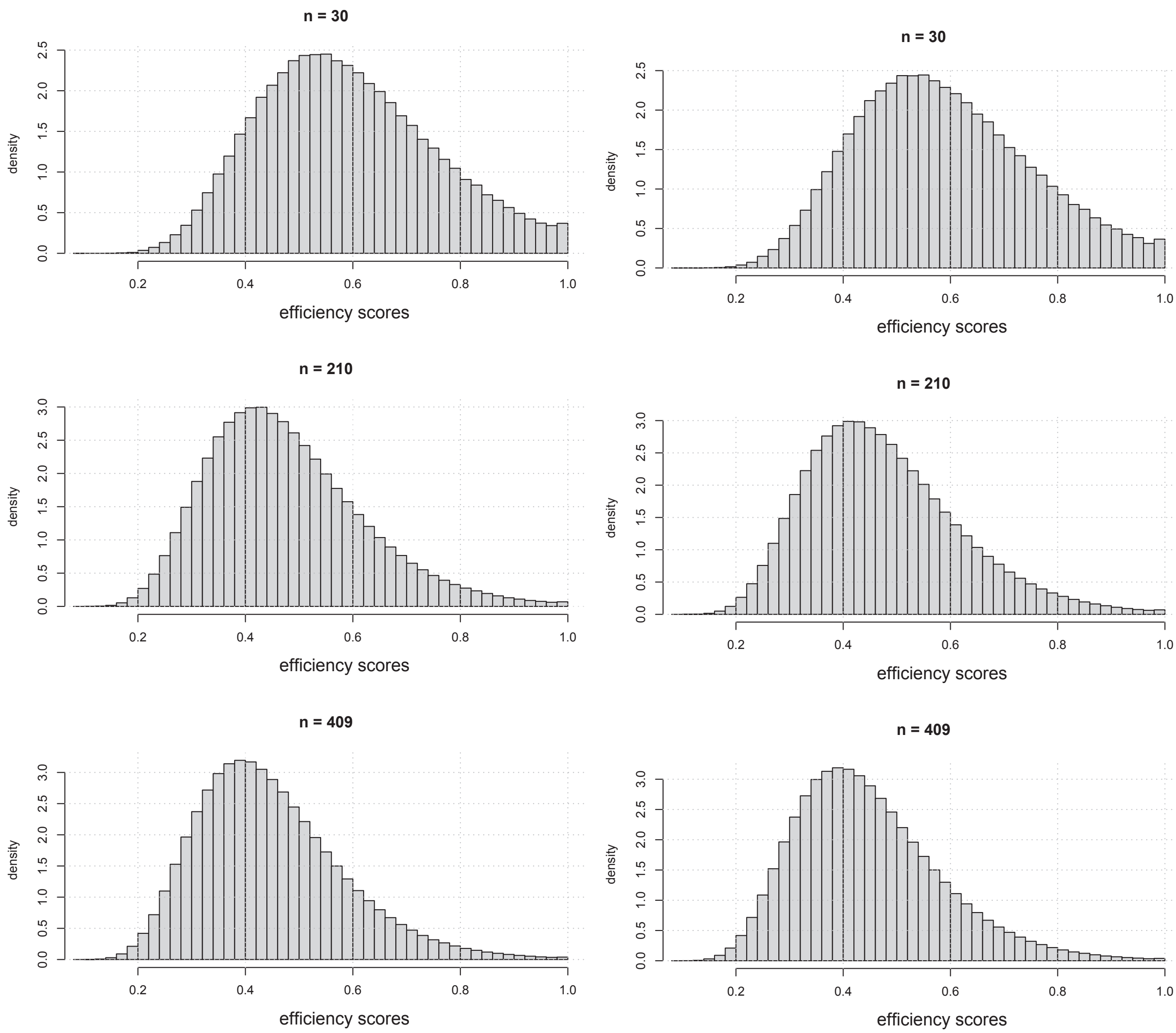
To test whether the Cobb Douglas model's lack of fit is statistically significant, we evaluate the Cobb–Douglas regression equation with interaction between the output variables, as shown in Equation (18).

$$x_j^* = \beta_0^* + \beta_1 y_{1j}^* + \beta_2 y_{2j}^* + \beta_3 y_{3j}^* + \beta_4 y_{1j}^* y_{2j}^* + \beta_5 y_{1j}^* y_{3j}^* \tag{18}$$

Table 5 shows the analysis of variance (ANOVA) of the Cobb–Douglas regression model with interaction. The analysis of variance provides information on the contribution of each variable to the regression model. The variables were sorted based on their conditional contribution to the model. That is, y_3^* provides the best univariate fit, y_2^* achieves the best fit with y_3^*, and so on. $y_3^* \cdot y_2^*$ is the most relevant iteration term with y_1^*, y_2^* and y_3^*. Results show that the interaction estimate of $y_3^* \cdot y_2^*$ is statistically significant (P-value <0.05). Therefore, there is evidence that the standard Cobb–Douglas model lacks fit to group A. Both $y_3^* \cdot y_1^*$ and $y_3^* \cdot y_1^*$ iteration terms are not statistically significant (P-value >0.05).

4. Discussion

The main criticism of the COLS Cobb–Douglas benchmarking model in ANEEL's proposal is that it represents a production function and not a cost function. In this study, we further explore



(a) Group A (b) Group B

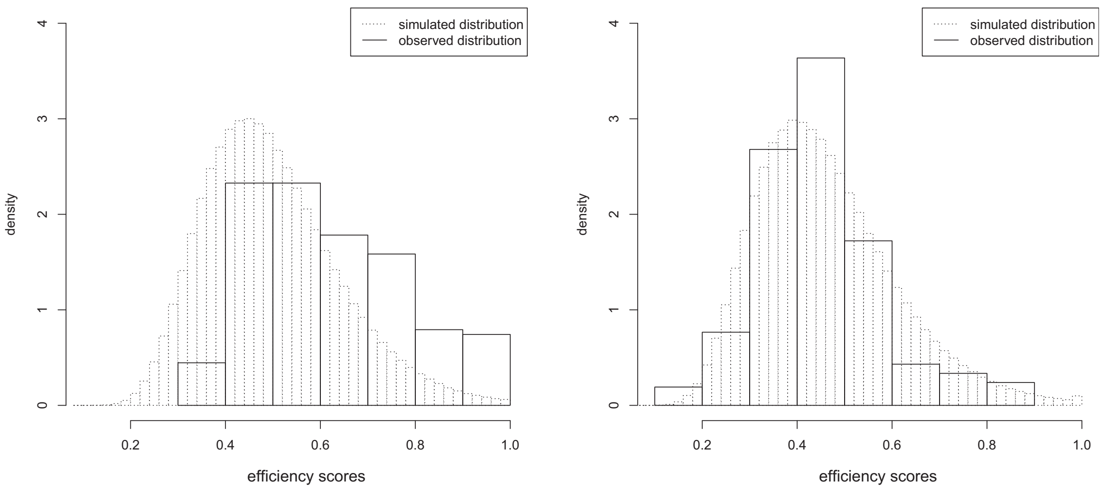
Fig. 7. Empirical distribution of the COLS Cobb–Douglas efficiency scores for groups A and B for the following sample sizes: $n = 30$, $n = 210$, and $n = 409$.

major deficiencies of the COLS Cobb–Douglas method as a benchmarking model. We found that in contrast to previous studies using the COLS Cobb–Douglas model, ANEEL's database is composed of yearly data obtained from 59 power distribution utilities for a 7-year period. Therefore, the database contains 413 records. The model considers each record an independent DMU, resulting in the consideration of an unusually large number of entities. The literature on this subject usually compares fewer than 50 independent utilities on average [29,30,10,31,13,32,1,33,34].

Table 4
Empirical confidence intervals for distribution of COLS Cobb–Douglas efficiency scores.

Sample size	Group A		Group B	
	2.5% Quantile	97.5% Quantile	2.5% Quantile	97.5% Quantile
$n = 30$	0.3164	0.9318	0.3142	0.9311
$n = 210$	0.2485	0.8104	0.2493	0.8107
$n = 409$	0.2346	0.7669	0.2122	0.7534

As mentioned, although the standard COLS Cobb–Douglas regression model can be fitted using Ordinary Least Squares, it relies on a very strong statistical assumption regarding the variance of the regression model. The statistical analysis in the original domain, i.e., the domain before the application of the logarithm transformation, shows that the variance structure imposed by the Cobb–Douglas production function is proportional to the square conditional mean, η^2. Therefore, the higher the conditional mean, the higher the conditional variance is, which penalizes DMUs with high input and output values. Essentially, The Cobb–Douglas regression model can be fitted using successive weighted least squares without the log-transformation [24]. This regression provides exact estimates. In this procedure, the weights assigned to each observation are proportional to the inverse of the variance structure. Therefore, observations with high mean values are penalized much more than observations with smaller mean values. That is, the procedure gives less weight to DMUs with high inputs and output values. As a consequence, the efficiency frontier is driven primarily by DMUs with smaller input and output values. By choosing a different variance structure, the weights of the DMUs



(a) Group A (n=202) (b) Group B (n=207)

Fig. 8. Comparison of simulated and observed distributions of efficiency scores for groups A and B.

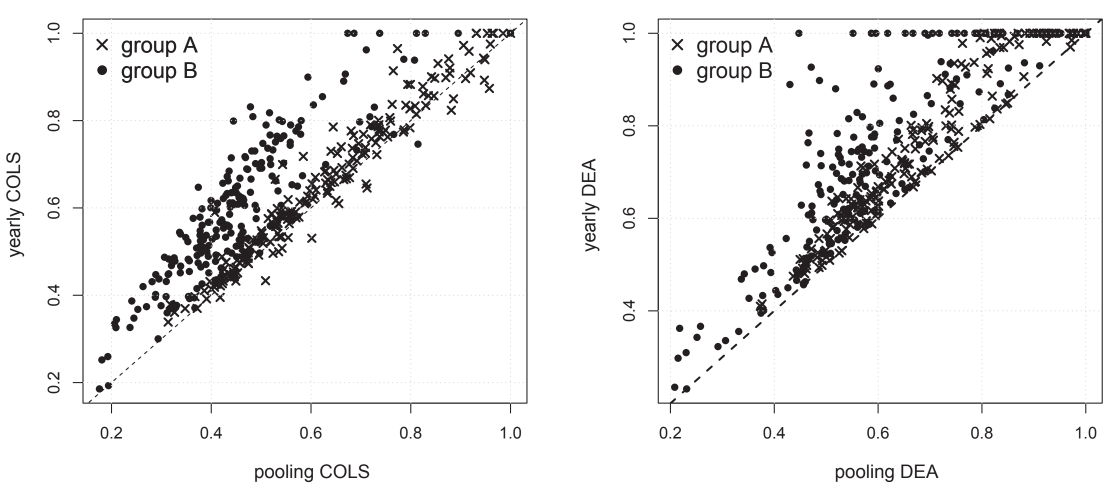
Table 5
Analysis of variance for Cobb–Douglas regression model with adjusted lack of fit for group A.

Variable	df	Sum Sq	Contribution	Mean Sq	*F* value	*P*-value
y_3^*	1	78.41	81.62%	78.41	1199.28	0.0000
y_2^*	1	2.54	2.64%	2.54	38.83	0.0000
y_1^*	1	0.61	0.63%	0.61	9.31	0.0026
$y_3^* \cdot y_2^*$	1	1.56	1.62%	1.56	23.81	0.0000
$y_3^* \cdot y_1^*$	1	0.10	0.10%	0.10	1.57	0.2114
$y_2^* \cdot y_1^*$	1	0.03	0.03%	0.03	0.46	0.4995
Residuals	196	12.82	13.36%	0.06		

can be balanced appropriately. In conclusion, in its current form, the Cobb–Douglas regression model imposes a variance structure that is more complex than the data's observed variance structure. Therefore, the model exhibits *underdispersion* [35].

According to basic statistical theory, the larger the sample, the more reliable statistics are. In fact, the Central Limit Theorem states that the variance of the sample mean estimator is proportional to the inverse of the square-root of the sample size; i.e., the larger the sample, the lower the variance of the mean is. Based on this statement, a larger sample size would be expected to provide better efficiency estimates.

However, a theoretical analysis of the COLS Cobb–Douglas model shows that the distribution of the efficiency scores is changed by the sample size. We proved that the larger the sample, the farther the distribution of the efficiency scores is toward smaller values. Briefly, the larger the sample, the lower the efficiency scores are, which is a major deficiency of ANEEL's model. This deficiency is partially corrected by splitting the database into two subsets and calculating the efficiency scores separately for each subset. Fig. 9 (a) and (b) illustrate the impact of sample size on the COLS and DEA efficiency score estimates. On the *y*-axis, the efficiency scores are calculated separately for each year, whereas the *x*-axis exhibits the efficiency scores calculated using the full data set (data (*pooling*). Dashed lines represent the equation $\theta^{\text{yearly}} = \theta^{\text{pooling}}$, in which the yearly efficient scores and the pooled efficient scores are identical. The results show that the efficiency



(a) COLS efficiency scores (b) DEA efficiency scores

Fig. 9. Comparison of DEA and COLS efficiency scores obtained if the efficiency scores are calculated separately for each year and if the efficiency scores are calculated using the full data set.

scores estimated separately for each year are much higher than the efficiency scores estimated using the full data set. Furthermore, Fig. 9(a) shows that the COLS efficiency scores calculated separately for each year provide much higher efficiency scores for group B than for group A. That is, the yearly COLS efficiency scores for group B are grouped farther above the dashed line than those for group A. In other words, using yearly data, the reduction in operational costs achieved when using COLS efficiency scores is much smaller than that achieved when using the 2003 to 2009 data. Similar behavior is observed in 9 (b) for the DEA efficiency scores.

Furthermore, the COLS Cobb–Douglas efficiency frontier assumes that the residuals of the regression equation represent inefficiencies. Thus, the regression equation must be carefully chosen to avoid lack of fit. That is, if important terms or variables are missing from the regression equation, then residuals account for both inefficiencies and lack of fit. Consequently, the estimated efficiency scores are unreliable. We show that although one deficiency is related to sample size, the proposed model also exhibits a lack of fit. Briefly, iteration terms were found to be statistically significant.

5. Conclusion

We provided statistical evidence that the estimated efficiency scores obtained using the DEA and COLS benchmarking models proposed by the Brazilian Electricity Regulator (ANEEL) in 2011 are statistically different. In fact, the COLS efficiency scores are statistically lower than the DEA efficiency scores. Consequently, averaging of the scores renders the final estimates unreliable. It is worth noting that this paper does not propose a new benchmarking model. Rather, we indicate major statistical issues of the COLS Cobb Douglas benchmarking model that have produced major controversies among ANEEL and power distribution utilities in Brazil. Ongoing work aims to propose benchmarking models that properly represent the inefficiencies of power distribution utilities in Brazil. One alternative is to improve the COLS Cobb Douglas efficiency scores by considering a piecewise linear model [36].

Acknowledgments

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Appendix 1. Differences between DEA and COLS efficiency scores for group A in year 2009, which defines regulatory operational costs for subsequent years.

Company	Operational cost (OPEX)	Efficiency score		Difference	Operational cost loss	
		DEA	COLS	DEA-COLS	DEA OPEX loss	COLS OPEX loss
CEMAT	R$ 291,621,296.88	0.7410	0.5309	0.2101	-R$ 75,517,368.37	-R$ 136,790,396.60
CELG	R$ 702,870,364.85	0.5774	0.4006	0.1768	-R$ 297,065,339.01	-R$ 421,323,243.07
ENERSUL	R$ 208,022,494.82	0.7294	0.5575	0.1719	-R$ 56,292,277.88	-R$ 92,053,701.72
ELETROPAULO	R$ 1,267,776,860.56	0.6678	0.5088	0.1590	-R$ 421,093,925.65	-R$ 622,721,928.09
ESE	R$ 104,110,842.21	0.8243	0.6658	0.1585	-R$ 18,294,884.42	-R$ 34,795,939.87
EPB	R$ 178,790,110.72	0.7410	0.6038	0.1372	-R$ 46,311,967.16	-R$ 70,839,486.63
CEMAR	R$ 210,814,354.99	0.8716	0.7538	0.1178	-R$ 27,076,662.25	-R$ 51,904,465.73
CEMIG	R$ 1,869,287,127.70	0.5066	0.3907	0.1160	-R$ 922,246,506.60	-R$ 1,139,025,443.49
LIGHT	R$ 572,753,299.45	0.8151	0.7123	0.1028	-R$ 105,876,230.06	-R$ 164,755,551.37
CPFLPAULISTA	R$ 505,472,806.64	0.9702	0.8813	0.0889	-R$ 15,043,899.65	-R$ 59,974,431.71
CEPISA	R$ 208,616,759.49	0.5110	0.4239	0.0871	-R$ 102,013,968.36	-R$ 120,180,518.26
CEAL	R$ 224,198,789.64	0.4557	0.3718	0.0840	-R$ 122,029,137.19	-R$ 140,851,902.29
COELBA	R$ 444,756,767.14	1.0000	0.9197	0.0803	R$ **0.00**	-R$ 35,703,557.75
AMAZONAS	R$ 224,558,917.62	0.4861	0.4178	0.0683	-R$ 115,396,335.91	-R$ 130,736,816.26
CEB	R$ 225,676,552.84	0.5972	0.5351	0.0621	-R$ 90,913,321.37	-R$ 104,926,164.46
COELCE	R$ 325,234,142.71	0.8459	0.7867	0.0592	-R$ 50,107,054.84	-R$ 69,357,090.90
COPEL	R$ 1,030,811,331.91	0.5300	0.4745	0.0555	-R$ 484,436,610.74	-R$ 541,673,014.18
COSERN	R$ 125,143,623.04	1.0000	0.9486	0.0514	R$ **0.00**	-R$ 6,429,890.67
CELESC	R$ 731,230,800.18	0.5251	0.4738	0.0513	-R$ 347,289,507.70	-R$ 384,791,592.47
CELPE	R$ 356,893,144.57	0.8495	0.7983	0.0512	-R$ 53,706,128.76	-R$ 71,971,025.42
AMPLA	R$ 446,854,953.01	0.5880	0.5420	0.0460	-R$ 184,109,147.58	-R$ 204,674,657.62
PIRATININGA	R$ 199,249,337.37	1.0000	0.9576	0.0424	R$ **0.00**	-R$ 8,446,864.17
ELEKTRO	R$ 421,453,770.14	0.7026	0.6772	0.0254	-R$ 125,329,217.94	-R$ 136,029,165.20
BANDEIRANTE	R$ 279,779,538.92	0.7357	0.7103	0.0254	-R$ 73,947,216.97	-R$ 81,040,083.26
CELPA	R$ 440,559,844.52	0.4366	0.4184	0.0182	-R$ 248,221,565.32	-R$ 256,247,015.44
RGE	R$ 189,905,561.15	1.0000	0.9850	0.0150	R$ **0.00**	-R$ 2,839,555.96
AES SUL	R$ 210,034,697.24	0.8554	0.8464	0.0090	-R$ 30,371,886.81	-R$ 32,256,129.23
ESCELSA	R$ 237,059,063.23	0.6718	0.6711	0.0006	-R$ 77,807,422.09	-R$ 77,959,141.65
CEEE	R$ 391,288,328.80	0.4919	0.4951	−0.0032	-R$ 198,815,221.01	-R$ 197,565,879.45

Appendix 2. Differences between DEA and COLS efficiency scores for group B in year 2009, which defines regulatory operational costs for subsequent years.

Company	Operational cost (OPEX)	Efficiency score		Difference	Operational cost loss	
		DEA	COLS	DEA-COLS	DEA OPEX loss	COLS OPEX loss
JOAOCESA	R$ 1,500,275.63	0.6183	0.2099	0.4084	-R$ 572,625.24	-R$ 1,185,390.86
SULGIPE	R$ 25,073,746.49	0.6797	0.3145	0.3652	-R$ 8,031,834.88	-R$ 17,188,963.44
CELTINS	R$ 127,969,357.67	0.6671	0.3563	0.3108	-R$ 42,597,485.38	-R$ 82,370,719.92

(*continued*)

Company	Operational cost (OPEX)	Efficiency score		Difference	Operational cost loss	
		DEA	COLS	DEA-COLS	DEA OPEX loss	COLS OPEX loss
EBO	R$ 28,602,803.34	0.7946	0.4880	0.3066	-R$ 5,876,276.57	-R$ 14,645,151.60
EMG	R$ 74,652,712.68	0.7386	0.4561	0.2825	-R$ 19,514,302.90	-R$ 40,602,457.34
ENF	R$ 23,257,103.73	0.5664	0.3791	0.1873	-R$ 10,083,449.31	-R$ 14,440,193.84
CJE	R$ 8,777,772.13	1.0000	0.8144	0.1856	R$ **0.00**	-R$ 1,628,847.93
SANTACRUZ	R$ 31,686,065.88	0.8507	0.6920	0.1587	-R$ 4,731,181.31	-R$ 9,760,707.77
CAIUA	R$ 45,720,850.54	0.6893	0.5616	0.1277	-R$ 14,205,669.78	-R$ 20,043,909.32
ELETROACRE	R$ 61,732,238.72	0.4558	0.3306	0.1252	-R$ 33,597,354.08	-R$ 41,325,497.03
MUXFELDT	R$ 1,636,107.33	0.8321	0.7108	0.1213	-R$ 274,680.94	-R$ 473,176.98
NACIONAL	R$ 26,162,288.20	0.5603	0.4410	0.1193	-R$ 11,503,384.42	-R$ 14,625,017.11
DEMEI	R$ 7,344,188.07	0.5546	0.4395	0.1151	-R$ 3,271,051.66	-R$ 4,116,548.54
EVP	R$ 39,720,743.68	0.6000	0.4889	0.1111	-R$ 15,888,877.33	-R$ 20,300,878.69
CSPE	R$ 10,907,512.83	1.0000	0.8952	0.1048	R$ **0.00**	-R$ 1,143,425.15
MOCOCA	R$ 6,947,285.18	0.9112	0.8076	0.1036	-R$ 617,132.85	-R$ 1,337,001.74
EFLUL	R$ 3,876,046.19	0.4629	0.3671	0.0958	-R$ 2,081,932.07	-R$ 2,453,169.66
CHESP	R$ 9,858,068.25	0.4650	0.3706	0.0944	-R$ 5,274,116.89	-R$ 6,205,137.55
DME-PC	R$ 22,996,102.66	0.4564	0.3659	0.0905	-R$ 12,500,657.55	-R$ 14,582,141.62
BRAGANTINA	R$ 32,807,983.07	0.5835	0.5007	0.0828	-R$ 13,665,377.00	-R$ 16,382,059.50
CPEE	R$ 10,852,306.87	0.8034	0.7213	0.0821	-R$ 2,133,676.75	-R$ 3,024,329.90
CFLO	R$ 13,904,983.10	0.5361	0.4599	0.0762	-R$ 6,450,851.81	-R$ 7,510,766.79
COOPERALIANï¿½A	R$ 10,439,451.39	0.4440	0.3737	0.0704	-R$ 5,803,987.73	-R$ 6,538,564.04
SANTAMARIA	R$ 22,233,712.37	0.6032	0.5399	0.0633	-R$ 8,823,401.00	-R$ 10,230,291.92
HIDROPAN	R$ 5,335,519.96	0.4423	0.3912	0.0511	-R$ 2,975,387.22	-R$ 3,248,287.18
ELETROCAR	R$ 10,573,431.60	0.4720	0.4287	0.0433	-R$ 5,582,518.37	-R$ 6,040,495.55
COCEL	R$ 12,163,593.15	0.5061	0.4637	0.0424	-R$ 6,007,655.80	-R$ 6,523,566.30
BOA VISTA ENERGIA	R$ 65,012,675.07	0.2313	0.1939	0.0373	-R$ 49,976,945.04	-R$ 52,403,696.95
IENERGIA	R$ 13,920,302.58	0.3934	0.3713	0.0222	-R$ 8,443,623.98	-R$ 8,752,169.00
UHENPAL	R$ 5,025,208.48	0.4712	0.4669	0.0042	-R$ 2,657,428.95	-R$ 2,678,706.76

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