Multiobjective Optimization

Neighborhood Exploration

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8 Agradecimentos

Section 1

Module: Neighborhood Exploration

Requirements

The requirements for this class are:

- Data Structures and Algorithmic Complexity
    - Graph concepts
- Programming in Python or C/C++
- Module 1 - Fundamentals
- Module 2 - Greedy

Topics

- ...

Section 2

Neighborhood Exploration

Some Definitions (remember from Module 2)

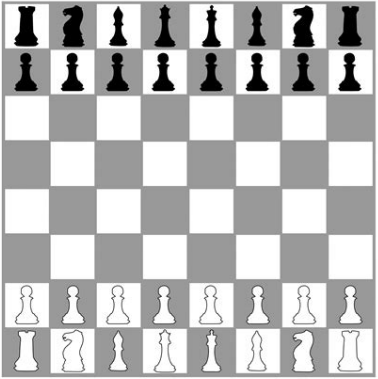
Recall basic definitions for an optimization problem, such as solutions and evaluations, and classic NP-hard problems such as the Knapsack Problem and Traveling Salesman Problem. More precisely:

- The XS denotes a *Solution Space*, where XE is an *Evaluation Space* (or *Objective Space*)
  - The pair $XES = < XS, XE >$ denotes the *XESolution* space
  - A *SO optimization problem* is defined by the triple $< XS, XE, f >$
- The space XE can be partioned into $XE = XFeasible \cup XInfeasible$, where $XFeasible \cap XInfeasible = \emptyset$
- XS denotes all *valid representations of a solution*, that are *structurally correct*
  - it may include *infeasible* solutions s, that are *valid*, but with *infeasible* evaluation $f(s) \in XInfeasible$
  - it depends on how the problem is modeled, but it's not uncommon to have $XInfeasible \neq \emptyset$
- The optimal solution s^* is always feasible $f(s^*) \in XFeasible$, unless the problem is *impossible*

Neighborhood

Given a solution $s \in XS$, we define a *neighbor solution* $s' \in XS$ as:

- a *neighborhood* (or *neighborhood structure*) $\mathcal{N}(s)$ is *a set of solutions* reachable by some *move* function/operator $m : XS \mapsto XS$
- we say that $s' \in \mathcal{N}(s) \iff \exists m \textit{ such that } s' = m(s)$
  - or, typically denoted by operator $\oplus$ notation: $s' = s \oplus m$



Reachability of Solutions and Move Composition

We recall an instance of the Knapsack Problem with 5 items and consider solutions $s_1 = (01001)$ and $s_2 = (11010)$ from XS

- We consider the following move definition $M^{(I)} = \{m_1, m_2, ..., m_i\} =$ `{ change the value of the bit` i`}`
- We can find moves $m_1, m_4, m_5 \in M^{(I)}$ such that
  $s_2 = ((s_1 \oplus m_1) \oplus m_4) \oplus m_5$
  - this changes the values of the first, fourth and fifth bits
  - the following intermediate solutions are visited in this path: $s_1 = (01001) \rightarrow (11001) \rightarrow (11011) \rightarrow (11010) = s_2$
- Alternatively, a composite move $m_{1,4,5}$ could be built with function composition: $m_{1,4,5} = m_5 \circ m_4 \circ m_1$; or $m_{1,4,5} = \bigcirc_{m \in (m_1, m_4, m_5)} m$ or by using $\mapsto$ sequential notation: $m_{1,4,5} = m_1 \mapsto m_4 \mapsto m_5$
  - In other words, $m_{1,4,5}(s) = m_5(m_4(m_1(s)))$
- Now we consider moves $M^{(II)}$ where two bits i and j are simultaneously changed (**Exercise:** What is the size of this neighborhood?)
  - Solution s_2 could never be reachable by s_1 in such neighborhood!

Move Cost

Given a move m, we can compute the *move cost* $\bar{m}$ in the following way:

- given an evaluation function $f : XS \mapsto XE$, and $e = f(s)$ denoting the *evaluation* of a solution s
- given a *neighbor* $s' = m(s)$ the *move cost* $\bar{m}(s)$ is defined by $\bar{m}(s) = f(s') - f(s)$
- naturally, any $e \in XE$ space must support *add* and *subtract* basic arithmetics
- we **do not** require XE to be a *total order*, although this is true for *single objective optimization*, i.e., *minimization* or *maximization*
- we say that moves $M = (m_1, m_2, ...)$ are *independent* if *composite move* $m' = \bigcirc_{m \in M} m$ has a fixed cost $\overline{m'}(s) = \sum_{m \in M} \bar{m}(s)$, $\forall s \in XS$
  - this is an important property for newer neighborhood strategies in literature!

Example for the Traveling Salesman Problem (euclidean)

Let's think of a neighborhood structure for the TSP. What is a move? How much does it cost?

Figure 2: https://simple.wikipedia.org/wiki/Travelling_salesman_problem

Section 3

Heuristics for Neighborhood Exploration

Heuristics for Neighborhood Exploration and Local Optima

Given a neighborhood $\mathcal{N}$ and a solution s, we can explore it, in order to improve solution s by finding a better *neighbor* s'

Some heuristics for neighborhood exploration are classic, mainly three: *random selection* (RS); *first improvement* (FI); and *best improvement* (BI). We have also proposed a *multi improvement* (MI) strategy that will be studied later.

These are also called *refinement heuristics* and are the foundations for several *local search* (LS) algorithms.

Differently from a *global search* (GS) algorithm, that tries to find an *optimal solution*, a *local search* tries to find a *locally optimal solution* regarding some specific neighborhood $\mathcal{N}$.

- So, recalling the basic definitions with XE as a *total order*, we define *local optima* $s^* \in XS$, given neighborhood $\mathcal{N}$ and a solution $s \in XS$:
  - For *minimization*, we have that $f(s^*) \leq f(s'), \forall s' \in \mathcal{N}(s)$
  - For *maximization*, we have that $f(s^*) \geq f(s'), \forall s' \in \mathcal{N}(s)$

Neighborhood Exploration: Four Primitives

Given a neighborhood $\mathcal{N}$ and solution $s \in XS$, we define four neighborhood exploration primitives: `FindAny`, `FindFirst`, `FindNext` and `FindBest`.

- the `FindAny` tries to find *any* solution $s' \in \mathcal{N}(s)$ that *improves* s
  - we assume a more restricted neighborhood $\mathcal{N}_{\leq k} \subseteq \mathcal{N}$, where $|\mathcal{N}_{\leq k}| \leq k$
  - assuming *minimization*, if such $s' \in \mathcal{N}_{\leq k}(s)$ exists, then $f(s') < f(s)$
- the `FindFirst` tries to find *the first* $s_i \in \mathcal{N}(s) = \{s_1, ..., s_i, ...\}$ that *improves* current solution s
  - assuming *minimization*, if such $s_i \in \mathcal{N}(s)$ exists, then i is the *smallest value* such that $f(s_i) < f(s)$
- the `FindNext` tries to find *the next* $s_i \in \mathcal{N}(s) = \{s_j, ..., s_i, ...\}$ that *improves* current solution s
  - assuming *minimization*, if such $s_i \in \mathcal{N}(s)$ exists, then i is the *smallest value* such that $f(s_i) < f(s)$ and $i > j$
- the `FindBest` tries to find *the best* $s^* \in \mathcal{N}(s)$ that *improves* current solution s
  - assuming *minimization*, if such $s^* \in \mathcal{N}(s)$ exists, then $f(s^*) < f(s)$ and $f(s^*) \leq f(s'), \forall s' \in \mathcal{N}(s)$

Neighborhood Exploration: Random Selection

Given a neighborhood $\mathcal{N}$ and a parameter k_{max}, the *Random Selection* (RS) heuristic is an implementation of the primitive `FindAny`:

- RS tries to find *any* solution $s' \in XS$ that *improves* current solution $s \in XS$
- RS is limited to k_{max} tries

Neighborhood Exploration: First Improvement

TODO

Neighborhood Exploration: Best Improvement

TODO

Section 4

Local Search techniques

Creating a Local Search: Hill Climbing

TODO

Creating a Local Search: Random Descent Method

TODO

Section 5

Advanced Topic: Multi Improvement

Exploring the Multi Improvement technique

TODO

Section 6

Practical Exercise

Implementing a Local Search (Step 1/3)

- Choose a language: Python or C/C++
- Consider the following data for a Knapsack Problem with $n = 5$ items and capacity $Q = 10$

```
5
10
1 1 1 5 5
1 2 3 7 8
```

- Save it into a file and read it
  - First load the n and Q
  - Then, for each item, load each profit p_i and weight w_i

Implementing a Local Search (Step 2/3)

- Model the solution representation as an array (or list) of booleans or binary numbers
- Create a neighborhood structure and two neighborhood exploration techniques (example: best improvement and first improvement)

Implementing a Local Search (Step 3/3)

- Choose some Local Search technique, such as Hill Climbing (for BI and FI), or RDM (for RS)
- Generate multiple initial solutions with some randomness (example, 1000)
- Apply each of the two Local Search on them, for each generated solution
- Compute de Average cost and Computational time taken for each of the two local searches
- Generate bigger instances, to make the problem harder!
- Which one is better?

Section 7

Discussions

Short discussion

Current scenario: optimization problems in the university and work

- Do you know of any optimization problem that needs to be solved in the university or your work?
- Can exact methods solve them? Do you need heuristic methods?
- Read the introduction material from prof Marcone (in Portuguese): http://www.decom.ufop.br/prof/marcone/Disciplinas/InteligenciaComput

Section 8

Agradecimentos

Pessoas

Em especial, agradeço aos colegas que elaboraram bons materiais, como o prof. Raphael Machado, Kowada e Viterbo cujos conceitos formam o cerne desses slides.

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Software

Esse material de curso só é possível graças aos inúmeros projetos de código-aberto que são necessários a ele, incluindo:

- pandoc
- LaTeX
- GNU/Linux
- git
- markdown-preview-enhanced (github)
- visual studio code
- atom
- revealjs
- groomit-mpx (screen drawing tool)
- xournal (screen drawing tool)
- ...

Empresas

Agradecimento especial a empresas que suportam projetos livres envolvidos nesse curso:

- github
- gitlab
- microsoft
- google
- ...

Reprodução do material

Esses slides foram escritos utilizando pandoc, segundo o tutorial ilectures:

- https://igormcoelho.github.io/ilectures-pandoc/

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