

# Automated Verification of Cyber-Physical Systems

A.A. 2022/2023

Corso di Laurea Magistrale in Informatica

## Basic Notions

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# General Info for This Class

- Automated Verification of Cyber-Physical Systems is an elective course for the Master Degree in Computer Science
- Lecturer: Igor Melatti
- Where to find these slides and more:
  - [https://igormelatti.github.io/aut\\_ver\\_cps/20222023/index\\_eng.html](https://igormelatti.github.io/aut_ver_cps/20222023/index_eng.html)
  - also on MS Teams: "DT0759: Automated Verification of Cyber-Physical Systems (2022/23)", code **11xu0gi**
- 2 classes every week, 2 hours per class



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# Rules for Exams

- Each exam has a written part (50% of mark) and a project/paper (50% of mark)
  - each student may choose if making a project or reviewing a paper
  - teams of at most 2 students are allowed for projects
- Written exam will be a mix of open and closed questions on the whole exam program
- Project/paper may be discussed only after having passed the written exam
  - however, pre-evaluation is possible



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# Rules for Exams

- Project: perform verification of a given cyber-physical system
  - each team may choose one among the ones selected by lecturer
  - or may propose one (but wait for lecturer approval!)
  - each team will have to discuss its project with slides
- Paper: read a conference or journal paper and present it with slides
  - each student may choose one among the ones selected by lecturer
  - or may propose one (but wait for lecturer approval!)



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# Model Checking Problem

- Input: a system  $\mathcal{S}$  and (at least) a property  $\varphi$ 
  - more precisely, a *model* of  $\mathcal{S}$  must be provided
  - that is,  $\mathcal{S}$  must be described in some suitable language
- Output:
  - PASS**  $\mathcal{S}$  satisfies  $\varphi$ , i.e.,  $\mathcal{S} \models \varphi$ 
    - the system  $\mathcal{S}$  is correct w.r.t. the property  $\varphi$
    - mathematical certification, much better than, e.g., testing
  - FAIL**  $\mathcal{S}$  does not satisfy  $\varphi$ , i.e.,  $\mathcal{S} \not\models \varphi$ 
    - the system  $\mathcal{S}$  is buggy w.r.t. the property  $\varphi$
    - a *counterexample* providing evidence of the error is also returned



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# Model Checking vs. Other Verification Techniques

- Model checking is fully automatic
  - a model checker only needs the description of  $\mathcal{S}$  and the property  $\varphi$
  - “press button and go”
  - this is not true for other verification tools such as proof checkers, which require human intervention in the process
- Model checking is correct for both PASS and FAIL
  - unless the description of  $\mathcal{S}$ , or the property  $\varphi$ , are wrong
  - this is not true for other verification techniques such as testing, which only guarantees the FAIL result
  - a buggy system may pass all tests, because the error is in some *corner case*



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# Model Checking Shortcomings

- Only works for finite-state systems
  - typical example: you may verify a system with 3, 4 or 5 processes, but not with  $n$  processes, for a generic  $n$
- Requires skilled personnel to write descriptions (and properties)
  - must know both the model checker language and the system
  - however, less skilled than a proof checker user
  - very few exceptions in which the model is automatically extracted from the system
  - also direct translations from digital circuits to NuSMV are available
- Very resource demanding
  - besides PASS and FAIL, also OutOfMem and OutOfTime are expected results...
  - bounded model checking: PASS is limited to execution up to a given number of steps



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# Model Checking Algorithms

Two main categories:

**Explicit** visit the graph induced by the description of  $\mathcal{S}$

- very good for invariants and LTL model checking of communication protocols
- on-the-fly generation of the graph: only the reachable states are stored, the adjacency matrix is implicitly given by the description of  $\mathcal{S}$
- Murphi, SPIN

**Symbolic** represent sets of states and transition relations as OBDDs

- very good for LTL and CTL model checking of hardware-like systems
- all translated into a boolean formula
- also SAT tools may be used (bounded model checking)



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# Cyber-Physical Systems

- A Cyber-Physical System (CPS) is a system where a physical system is controlled and/or monitored by a software
- They are either partially or fully autonomous
  - we will mainly deal with fully autonomous CPSs
- Examples are everywhere:
  - Internet of Things devices
  - Unmanned Autonomous Vehicles
  - Drones
  - Medical Devices
  - Embedded Systems
  - ...

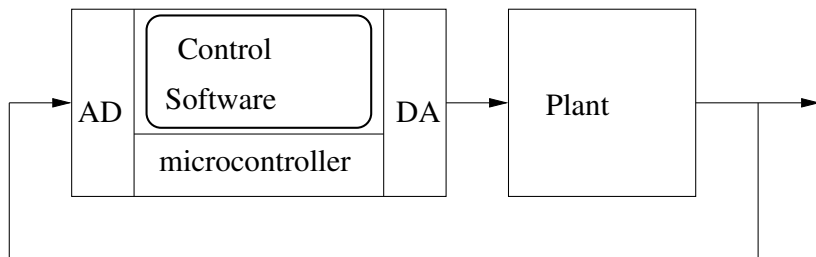


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# Cyber-Physical Systems with Controllers



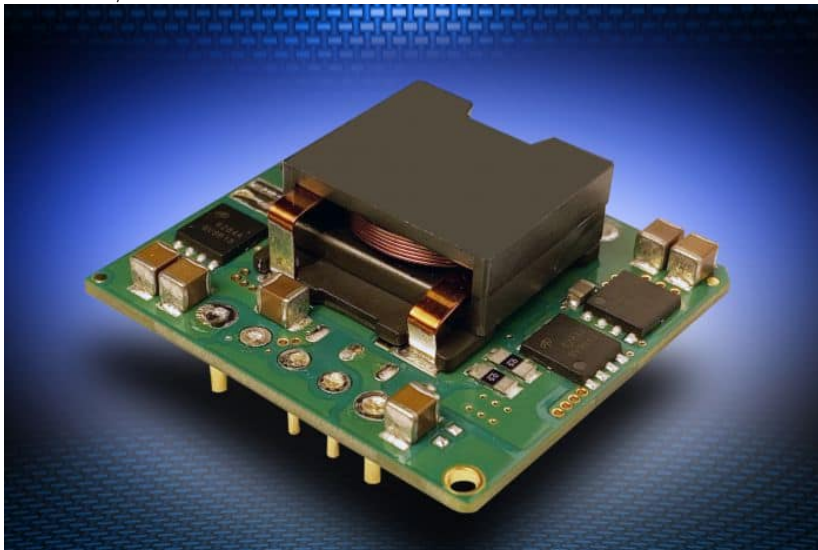
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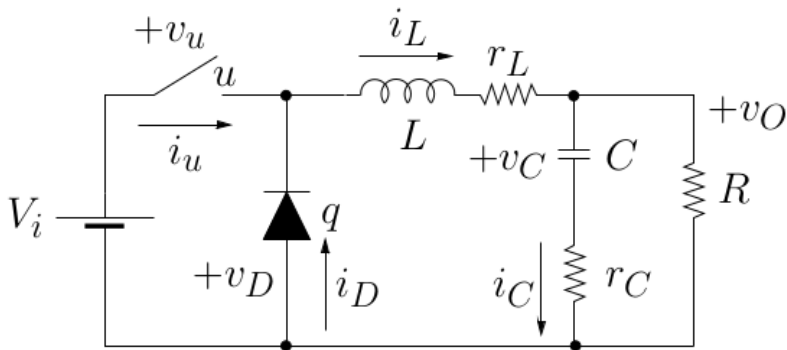
# CPSs with Controllers: Classical Examples

## Buck DC/DC Converter



# CPSs with Controllers: Classical Examples

## Buck DC/DC Converter



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# CPSs with Controllers: Classical Examples

## Continuous time dynamics

$$\dot{i}_L = a_{1,1}i_L + a_{1,2}v_O + a_{1,3}v_D \quad (1)$$

$$\dot{v}_O = a_{2,1}i_L + a_{2,2}v_O + a_{2,3}v_D \quad (2)$$

$$q \rightarrow v_D = R_{\text{on}}i_D \quad (3) \qquad \bar{q} \rightarrow v_D = R_{\text{off}}i_D \quad (7)$$

$$q \rightarrow i_D \geq 0 \quad (4) \qquad \bar{q} \rightarrow v_D \leq 0 \quad (8)$$

$$u \rightarrow v_u = R_{\text{on}}i_u \quad (5) \qquad \bar{u} \rightarrow v_u = R_{\text{off}}i_u \quad (9)$$

$$v_D = v_u - V_{in} \quad (6) \qquad i_D = i_L - i_u \quad (10)$$

where:

- $i_L, v_O$  are state variables
- $u \in \{0, 1\}$  is the action



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# CPSs with Controllers: Classical Examples

Discrete time dynamics with sampling time  $T$

$$i_L' = (1 + Ta_{1,1})i_L + Ta_{1,2}v_O + Ta_{1,3}v_D \quad (11)$$

$$v_O' = Ta_{2,1}i_L + (1 + Ta_{2,2})v_O + Ta_{2,3}v_D. \quad (12)$$

$$q \rightarrow v_D = R_{\text{on}} i_D \quad (13)$$

$$q \rightarrow i_D \geq 0 \quad (14)$$

$$u \rightarrow v_u = R_{\text{on}} i_u \quad (15)$$

$$v_D = v_u - V_{in} \quad (16)$$

$$\bar{q} \rightarrow v_D = R_{\text{off}} i_D \quad (17)$$

$$\bar{q} \rightarrow v_D \leq 0 \quad (18)$$

$$\bar{u} \rightarrow v_u = R_{\text{off}} i_u \quad (19)$$

$$i_D = i_L - i_u \quad (20)$$



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# CPSs with Controllers: Classical Examples

- Goal: keep  $v_O$  in a desired safe interval
  - typically,  $5 - 0.01V \leq v_O \leq 5 + 0.01V$
- Notwithstanding the input voltage  $V_i$  and the resistance  $R$  may vary in some given interval
  - typically,  $R = 5 \pm 25\% \Omega$ ,  $V_i = 15 \pm 25\% V$
- Effectively used in laptops: from battery voltage ( $V_i$ ) to laptop processor voltage ( $v_O$ )



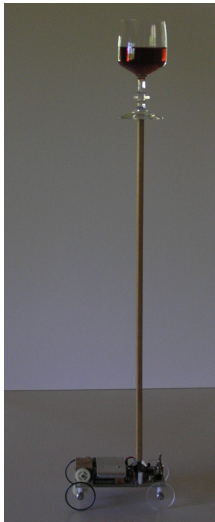
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# CPSs with Controllers: Classical Examples

## Inverted Pendulum



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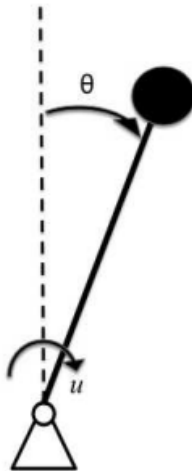


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# CPSs with Controllers: Classical Examples

## Inverted Pendulum



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# CPSs with Controllers: Classical Examples

Continuous time dynamics

$$\ddot{\theta} = \frac{g}{l} \sin \theta + \frac{1}{ml^2} Fu$$

where:

- $\theta$  is the state variable
- $u \in \{0, 1\}$  is the action
- $m, l, F$  are system parameters



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# CPSs with Controllers: Classical Examples

## Continuous time dynamics

$$\dot{x}_1 = x_2 \quad (21)$$

$$\dot{x}_2 = \frac{g}{l} \sin x_1 + \frac{1}{ml^2} Fu \quad (22)$$

## Discrete time dynamics with sampling time $T$

$$x_1' = x_1 + Tx_2 \quad (23)$$

$$x_2' = x_2 + T\frac{g}{l} \sin x_1 + T\frac{1}{ml^2} Fu \quad (24)$$



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# In This Course

To deal with cyber-physical systems:

- Probabilistic Model Checking
  - rather than “are there errors?”, it is “is the error probability low enough?”
  - the system is probabilistic, i.e., a Markov Chain
- System Level Formal Verification
  - directly use a simulator instead of describing the system within the model checker
  - this will also need some background on systems simulation



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# In This Course

To deal with cyber-physical systems:

- Statistical Model Checking
  - rather than “are there errors?”, it is “is the error probability low enough?”
  - the system is a non-probabilistic simulator
  - the answer is given with some statistical confidence
- Automatic Synthesis of Controllers
  - rather than “are there errors in this system?”, it is “generate a controller so that errors are avoided”



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# Formal Verification Methodologies: a Classification

There are two macro-categories:

- *Interactive methods*
  - as the name suggests, not (fully) automatic
  - human intervention is typically required
  - in this course, we do not deal with such techniques
- *Automatic methods*
  - only human intervention is to *model* the system



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- *Automatic methods*
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- There also exist hybridations among the two categories



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# Interactive Methods

- Also called *proof checkers*, *proof assistants* or *high-order theorem provers*
- Tools which helps in building a mathematical proof of correctness for the given system and property
- **Pros**
  - virtually no limitation to the type of system and property to be verified
- **Cons**
  - highly skilled personnel is needed
  - both in mathematical logic and in deductive reasoning
  - needed to “help” tools in building the proof



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# Interactive Methods

- Used for projects with high budgets
- For which the automatic methods limitations are not acceptable
  - used, e.g., to prove correctness of microprocessor circuits or OS microkernels
- Some tools in this category (see [https://en.wikipedia.org/wiki/Proof\\_assistant](https://en.wikipedia.org/wiki/Proof_assistant)):
  - HOL
  - PVS
  - Coq



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# Automatic Methods

- Commonly dubbed *Model Checking*
- Model Checking software tools are called *model checkers*
- There are some tens model checkers developed; the most important ones are listed in [https://en.wikipedia.org/wiki/List\\_of\\_model\\_checking\\_tools](https://en.wikipedia.org/wiki/List_of_model_checking_tools)
- Many are freely downloadable and modifiable for research and study purposes
- Research area with many achievements in over 30 years

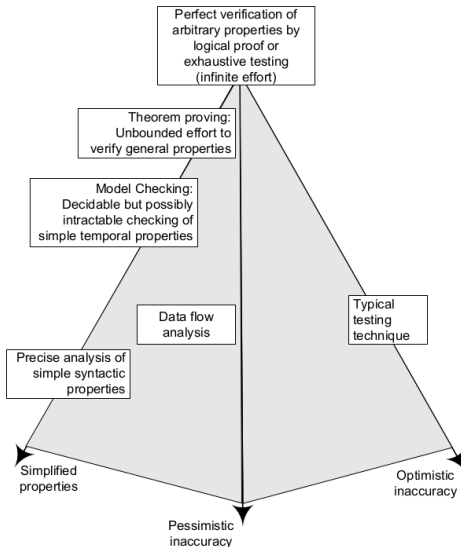


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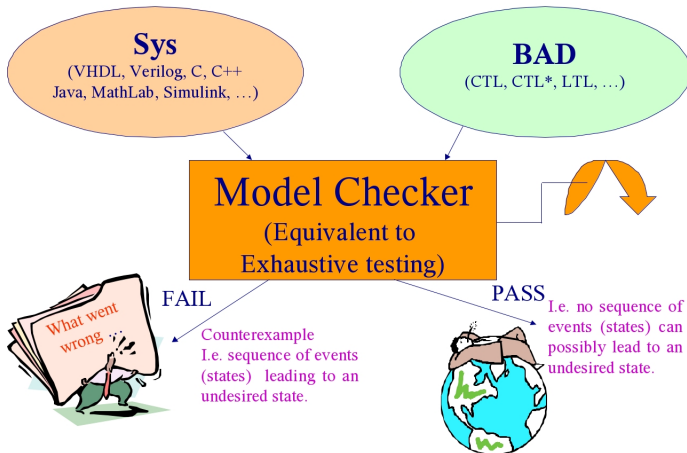


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# Verification Tradeoffs



# The Model Checking Dream

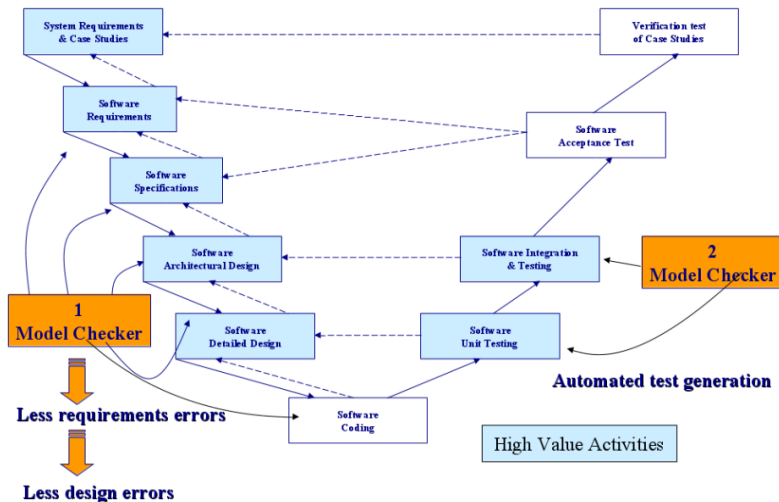


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# The Model Checking Dream



# Actual Model Checking

- In order to have this computationally feasible, we need a strong assumption on the system under verification (SUV)
- I.e., it must have a *finite number of states*
  - *Finite State System* (FSS)
- In this way, model checkers “simply” have to implement reachability-related algorithms on graphs
- Such finite state assumption, though strong, is applicable to many interesting systems
  - that is: many systems are actually FSSs
  - or they may be approximated as such
  - or a part of them may be approximated as such



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# What Is a *State*?

- There are many notions of “state” in computer science
- Model checking states are *not* the ones in UML-like state diagrams
- Model checking states are similar to operational semantics states
- That is: suppose that a system is “described” by  $n$  variables
- Then, a state is an assignment to all  $n$  variables
  - given  $D_1, \dots, D_n$  as our  $n$  variables domains, then a state is  $s \in \times_{i=1}^n D_i$



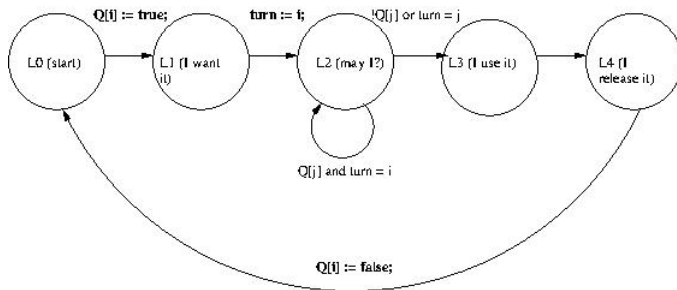
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# What Is a *State*: Example

- We have two identical processes accessing to a shared resource
  - in the figure below,  $i, j$  denote the two processes
  - the well-known Peterson algorithm is used





# What Is a *State*: Example

- The 5 “states” in the preceding figure are actually *modalities*
- From a model checking point of view, they correspond to *multiple* states
- To see which are the actual states, let us model this system with the following variables:
  - $m_i$ , with  $i = 1, 2$ : the modality for process  $i$
  - $Q_i$ , with  $i = 1, 2$ :  $Q_i$  is a boolean which holds iff process  $i$  wants to access the shared resource
  - $\text{turn}$ : shared variable



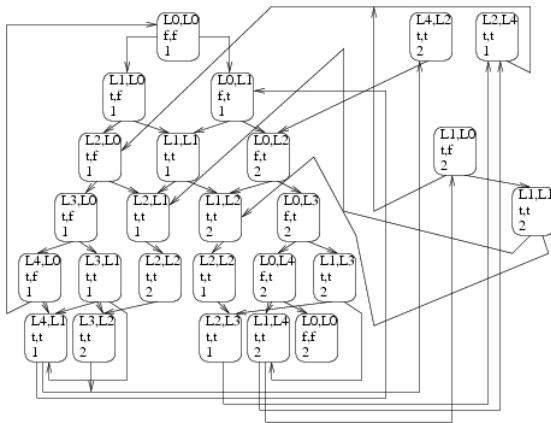
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# What Is a *State*: Example

- Thus, the resulting model checking states are the following:



# What Is a *State*: Example

- There are 25 *reachable states*
  - assuming state  $\langle L0, L0, f, f, 1 \rangle$  as the starting one
- All *possible* states are 200
  - there are 3 variables with two possible values (the 2 variables Q, plus the turn variable) and 2 variables (P) with 5 possible values, thus  $2^3 \times 5^2$  overall assignments
- The L0 modality for the first process encloses 6 (reachable) states

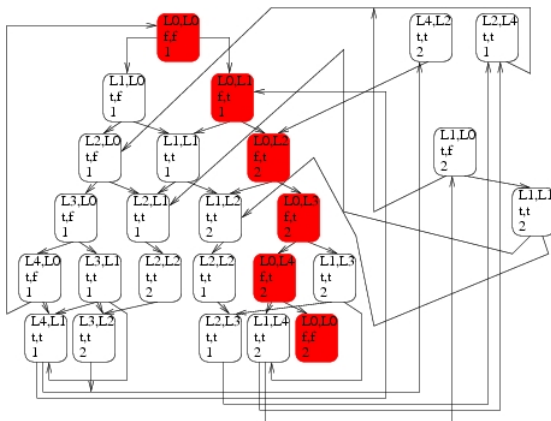


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# What Is a *State*: Example



# What Is a *State*: Example

- There are 25 *reachable states*
  - assuming state  $\langle L0, L0, f, f, 1 \rangle$  as the starting one
- All *possible* states are 200
  - there are 3 variables with two possible values (the 2 variables Q, plus the turn variable) and 2 variables (P) with 5 possible values, thus  $2^3 \times 5^2$  overall assignments
- The L0 modality for the first process encloses 6 (reachable) states
- **No need of guards on transitions!**
  - guards will be needed for systems with external inputs



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# From State Diagrams to Model Checking

- The UML-like state diagram is often useful to write the model
  - as we will see, this will depend on the model checker *input language*
- It is the model checker task to extract the global (reachable) graph as seen before
- And then analyze it



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- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)

```
boolean flag [2];
int turn;
void P0()
{
    while (true) {
        flag [0] = true;
        turn = 1;
        while (flag [1] && turn == 1) /* do nothing */;
        /* critical section */;
        flag [0] = false;
        /* remainder */;
    }
}
void P1()
{
    while (true) {
        flag [1] = true;
        turn = 0;
        while (flag [0] && turn == 0) /* do nothing */;
        /* critical section */;
        flag [1] = false;
        /* remainder */;
    }
}
void main()
{
    flag [0] = false;
    flag [1] = false;
    parbegin (P0, P1);
}
```

## Peterson's Algorithm



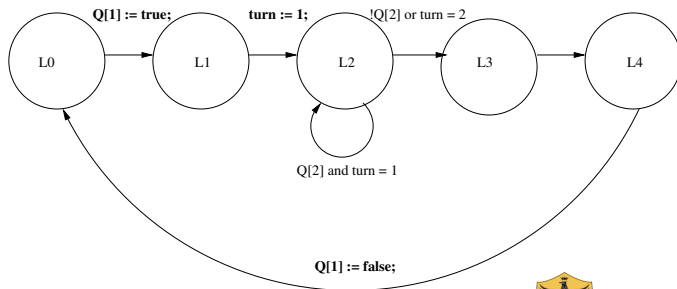
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# Murphi

- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)
- UML-like state diagram: this is the first process; the second may be obtained exchanging 1's with 2's and viceversa





- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)
  - two identical processes
  - each applies Peterson protocol to access to the critical section L3
  - the first issuing the request enters L3
  - $Q$  is a global variable, defined as an array of two integers
    - each process  $i$  may modify  $Q[i]$  and read  $Q[(i + 1) \bmod 2]$
  - $turn$  is another global variable, which may be both read and modified by both processes



# Murphi

- Murphi description for Peterson protocol: let's start with the variables
  - of course turn and Q, but also two variables P for the modality (“states” in the UML-like state diagram)
  - see `01.2_peterson.no_rulesets.no_parametric.m`
  - to this aim, we define constants and types
  - the N constant (number of processes) is here fictitious: only 2 processes, not more
  - this version of Peterson protocol only works for 2 processes
- thus, the state space is
$$S = \text{label\_t}^2 \times \{\text{true}, \text{false}\}^2 \times \{1, 2\}$$



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# Variables for Murphi Model Describing Peterson Protocol

P       $v \in \{L0, L1, L2, L3, L4\}$        $v \in \{L0, L1, L2, L3, L4\}$

Q       $v \in \{true, false\}$        $v \in \{true, false\}$

turn     $v \in \{1..N\}$



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- Hence,  $|S| = 5^2 \times 2^2 \times 2 = 200$  (there are 200 possible states)
  - as a matter of comparison, the “state” L0 in the UML-like state diagram actually contains  $5^1 \times 2^2 \times 2 = 40$  states...
- However, as we will see, *reachable* states are about 10 times less
- 2 initial states: turn may be initialised with any value in its domain
- Note that `01.2_peterson.no_rulesets.no_parametric.m` we have rules repeated 2 times in a nearly equal fashion
- This can be done in this very simple model, but in general descriptions must be *parametric*



# Murphi

- If we want to check Peterson with 3 processi, currently we would have to add one more rule in the description
- Instead, it must be possible to only change the value of  $N$  from 2 to 3
- To write parametric descriptions in Murphi, rules are grouped with *rulesets*
  - an index will allow to describe the behavior of the generic process  $i$
  - see `02.2_peterson.with_rulesets.no_parametric.m`



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# Murphi

- Invariant: of course, at any execution instant, there must be only one state in L3 (mutual exclusion)
- In a first order logic, it would be something like:

$$\forall k \in \{1, \dots, N\}. \forall k' \in \{1, \dots, N\}. (k \neq k' \wedge P[k] = L3) \Rightarrow P[k'] \neq L3$$

- Or, as a reverse:

$$\neg(\exists k \in \{1, \dots, N\}. \exists k' \in \{1, \dots, N\}. k \neq k' \wedge P[k] = L3 \wedge P[k'] = L3)$$

- In the first version, it is stated what is correct to happen
- In the first version, it is stated what is wrong to happen
- In both 00.2\_peterson.with\_rulesets.no\_parametric.m and 02.2\_peterson.no\_rulesets.no\_parametric.m invariant is not parametric
- See 03.2\_peterson.with\_rulesets.parametric.m



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# Murphi Simulation

```
void Make_a_run(NFSS  $\mathcal{N}$ , invariant  $\varphi$ )
{
  let  $\mathcal{N} = \langle S, I, \text{Post} \rangle$ ;
  s_curr = pick_a_state(I);
  if ( $\neg \varphi(s_{\text{curr}})$ )
    return with error message;
  while (1) { /* loop forever */
    if ( $\text{Post}(s_{\text{curr}}) = \emptyset$ )
      return with deadlock message;
    s_next = pick_a_state( $\text{Post}(s_{\text{curr}})$ );
    if ( $\neg \varphi(s_{\text{next}})$ )
      return with error message;
    s_curr = s_next;
  }
}
```



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# Murphi BFS

```
FIFO_Queue Q;  
HashTable T;  
  
bool BFS(NFSS  $\mathcal{N}$ , AP  $\varphi$ )  
{  
  let  $\mathcal{N} = (S, I, \text{Post})$ ;  
  foreach s in I {  
    if (! $\varphi(s)$ )  
      return false;  
  }  
  foreach s in I  
    Enqueue(Q, s);  
  foreach s in I  
    HashInsert(T, s);
```



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# Murphi BFS

```
while (Q  $\neq$   $\emptyset$ ) {  
  s = Dequeue(Q);  
  foreach s_next in Post(s) {  
    if (! $\varphi$ (s_next))  
      return false;  
    if (s_next is not in T) {  
      Enqueue(Q, s_next);  
      HashInsert(T, s_next);  
    } /* if */ } /* foreach */ } /* while */  
return true;  
}
```



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# Murphi BFS

- Edges are never stored in memory
- (Reachable) states are stored in memory only at the end of the visit
  - inside hashtable T
- This is called *on-the-fly* verification
- States are marked as visited by putting them inside an hashtable
  - rather than coloring them as gray or black
  - which needs the graph to be already in memory



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# State Space Explosion

- State space explosion hits in the FIFO queue  $Q$  and in the hashtable  $T$ 
  - and of course in running time...
- However,  $Q$  is not really a problem
  - it is accessed *sequentially*
  - always in the front for extraction, always in the rear for insertion
  - can be efficiently stored using disk, much more capable of RAM
- $T$  is the real problem
  - random access, not suitable for a file
  - what to do?
  - before answering, let's have a look at Murphi code



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# Murphi Usage

- As for all *explicit* model checker, a Murphi verification has the following steps:
  - 0 compile Murph source code and write a Murphi model `model.m`
  - 1 invoke Murphi compiler on `model.m`: this generates a file `model.cpp`
    - `mu options model.m`
    - see `mu -h` for available options
  - 2 invoke C++ compiler on `model.cpp`: this generates an executable file
    - `g++ -Ipath_to_include model.cpp -o model`
    - `path_to_include` is the include directory inside Murphi distribution
  - 3 invoke the executable file
    - `./model options`
    - see `./model -h` for available options



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# LTL Syntax

$$\Phi ::= p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{X}\Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

- Other derived operators:
  - of course true, false, OR and other propositional logic connectors
  - future (or eventually):  $\mathbf{F}\Phi = \text{true} \mathbf{U} \Phi$
  - globally:  $\mathbf{G}\Phi = \neg(\text{true} \mathbf{U} \neg\Phi)$
  - release:  $\Phi_1 \mathbf{R} \Phi_2 = \neg(\neg\Phi_1 \mathbf{U} \neg\Phi_2)$
  - weak until:  $\Phi_1 \mathbf{W} \Phi_2 = (\Phi_1 \mathbf{U} \Phi_2) \vee \mathbf{G}\Phi_1$
- Other notations:
  - next:  $\mathbf{X}\Phi = \bigcirc\Phi$
  - $\mathbf{G}\Phi = \square\Phi$
  - $\mathbf{F}\Phi = \diamond\Phi$
- We are dropping *past operators*, thus this is *pure future LTL*



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# LTL Semantics

- Goal: formally defining when  $\mathcal{S} \models \varphi$ , being  $\mathcal{S}$  a KS and  $\varphi$  an LTL formula
  - we say that  $\mathcal{S}$  *satisfies*  $\varphi$ , or  $\varphi$  *holds in*  $\mathcal{S}$
- This is true when, for all paths  $\pi$  of  $\mathcal{S}$ ,  $\pi$  satisfies  $\varphi$ 
  - i.e.,  $\forall \pi \in \text{Path}(\mathcal{S}). \pi \models \varphi$
  - symbol  $\models$  is overloaded...
- For a given  $\pi$ ,  $\pi \models \varphi$  iff  $\pi, 0 \models \varphi$
- Finally, to define when  $\pi, i \models \varphi$ , a recursive definition over the recursive syntax of LTL is provided
  - $\pi \in \text{Path}(\mathcal{S}), i \in \mathbb{N}$



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# LTL Semantics for $\pi, i \models \varphi$

- $\forall \pi \in \text{Path}(\mathcal{S}), i \in \mathbb{N}. \pi, i \models \text{true}$
- $\pi, i \models p$  iff  $p \in L(\pi(i))$
- $\pi, i \models \Phi_1 \wedge \Phi_2$  iff  $\pi, i \models \Phi_1 \wedge \pi, i \models \Phi_2$
- $\pi, i \models \neg \Phi$  iff  $\pi, i \not\models \Phi$
- $\pi, i \models \mathbf{X}\Phi$  iff  $\pi, i + 1 \models \Phi$
- $\pi, i \models \Phi_1 \mathbf{U} \Phi_2$  iff  $\exists k \geq i: \pi, k \models \Phi_2 \wedge \forall i \leq j < k. \pi, j \models \Phi_1$



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# LTL Semantics for Added Operators

- It is easy to prove that:
  - $\pi, i \models \mathbf{G}\Phi$  iff  $\forall j \geq i. \pi, j \models \Phi$
  - $\pi, i \models \mathbf{F}\Phi$  iff  $\exists j \geq i. \pi, j \models \Phi$
  - $\pi, i \models \Phi_1 \mathbf{R} \Phi_2$  iff  $\forall j \geq i. (\forall k < j. \pi, k \models \Phi_1) \rightarrow \pi, j \models \Phi_2$
  - $\pi, i \models \Phi_1 \mathbf{W} \Phi_2$  iff  $(\forall j \geq i. \pi, j \models \Phi_1) \vee (\exists k \geq i: \pi, k \models \Phi_2 \wedge \forall i \leq j < k. \pi, j \models \Phi_1)$
- For many formulas, it is silently required that paths are infinite
- That's why transition relations in KSs must be total



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# Safety and Liveness Properties in LTL

- Given an LTL formula  $\varphi$ ,  $\varphi$  is a safety formula iff
$$\forall \mathcal{S}. (\exists \pi \in \text{Path}(\mathcal{S}) : \pi \not\models \varphi) \rightarrow \exists k : \pi|_k \not\models \varphi$$
- Given an LTL formula  $\varphi$ ,  $\varphi$  is a liveness formula iff
$$\forall \mathcal{S}. (\exists \pi \in \text{Path}(\mathcal{S}) : \pi \not\models \varphi) \rightarrow |\pi| = \infty$$
- All LTL formulas are either safety, liveness, or the AND of a safety and a liveness
  - being defined on paths, the counterexample is always a path
- Safety properties are those involving only **G**, **X**, true and atomic propositions
- Liveness are all those involving an **F**, or a **U** where the first formula is not the constant true
- Some formulas are both safety and liveness, like true, **G** true and so on

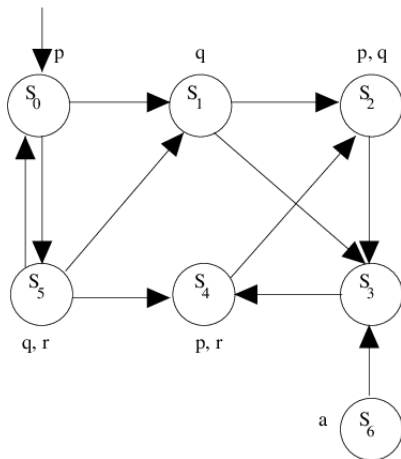


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# LTL Examples



$\mathcal{S} \models \mathbf{F}p$  since  $p$  holds in the first state

For full: let  $\pi \in \text{Path}(\mathcal{S})$

$\pi, 0 \models \mathbf{F}p$  with  $j = 0$

recall:  $\pi, i \models \mathbf{F}\Phi$  iff

$\exists j \geq i. \pi, j \models \Phi$

$\pi, i \models p$  iff  $p \in L(\pi(i))$

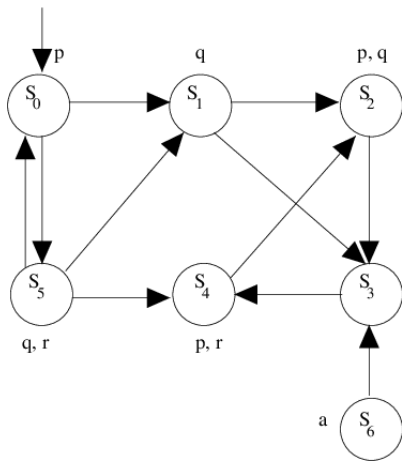


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# LTl Examples



$\mathcal{S} \not\models \mathbf{F}a$  since  $s_6$  is not reachable from  $s_0$

counterexample:  $\pi = s_0 s_5 s_0 s_5 \dots$

For full:  $\pi, 0 \not\models \mathbf{F}a$  as, for all  $j \geq 0$ ,  $a \notin L(\pi(j))$

Counterexample is infinite, thus this is a liveness property  
Any finite prefix of  $\pi$  is not a counterexample

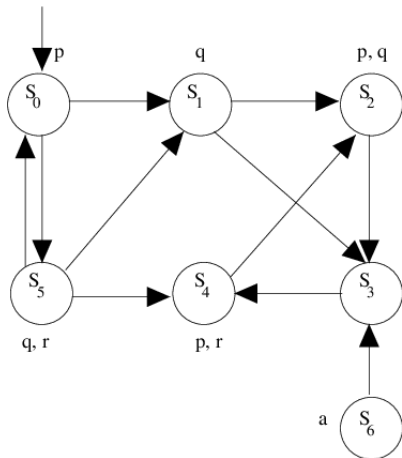


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# LTL Examples



$\mathcal{S} \not\models \mathbf{G}p$  since there are many counterexamples, here is one:

$\pi = s_0 s_5 s_0 s_5 \dots$

For full:  $\pi, 0 \not\models \mathbf{G}p$  with  $j = 1$

recall:  $\pi, i \models \mathbf{G}\Phi$  iff

$\forall j \geq i. \pi, j \models \Phi$

$\pi, i \models p$  iff  $p \in L(\pi(i))$

Safety property, actually  $\pi|_2$  is enough

Every path having  $\pi|_2$  as a prefix is a counterexample

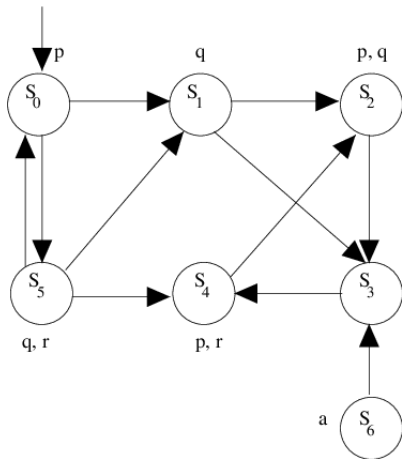


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# LTL Examples



$\mathcal{S} \models \mathbf{G}\neg a$  since  $s_6$  is not reachable from  $s_0$

For full: let  $\pi \in \text{Path}(\mathcal{S})$   
 $\pi, 0 \models \mathbf{G}\neg a$  as the only state  $s$  with  $a \in L(s)$  is  $s_6$ , which is not reachable from  $s_0$

recall:  $\pi \in \text{Path}(\mathcal{S})$  implies  $\pi(0) \in I$ , thus  $\pi(0) = s_0$  here

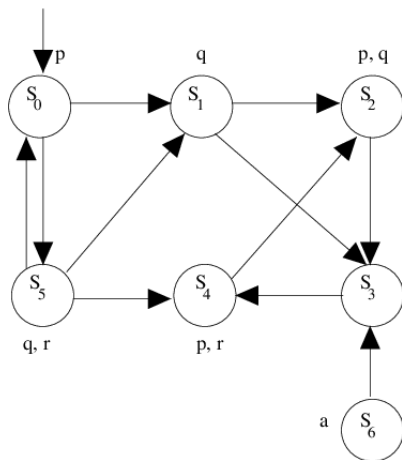


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# LTL Examples



$\mathcal{S} \models p \text{ U } q$  since  $p \in L(s_0)$ ,  
 $\text{next}(s_0) = \{s_1, s_5\}$  and  $q \in L(s_1) \wedge q \in L(s_5)$

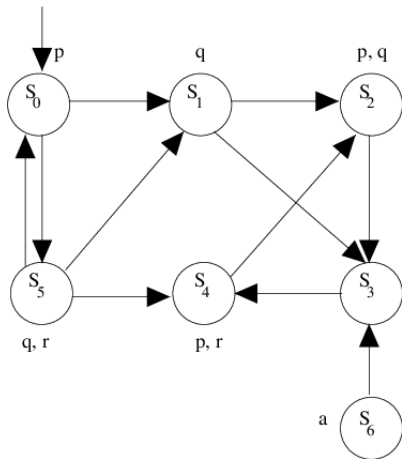


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# LTL Examples



$\mathcal{S} \not\models p \mathbf{U} r$ , a counterexample is  $\pi = s_0 s_1 (s_2 s_3 s_4)$

Again this is a liveness formula, even if  $\pi|_1$  would have been enough

In fact, you have to consider all possible KSs...

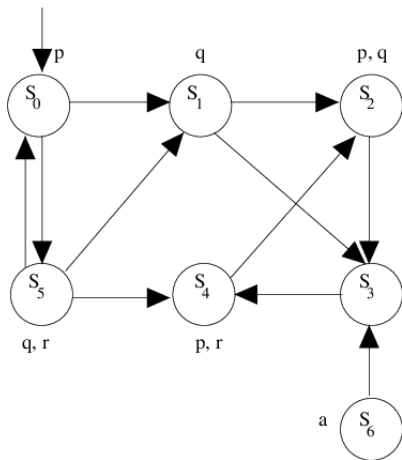


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# LTL Examples



$\mathcal{S} \not\models \neg(p \mathbf{U} r)$ , a counterexample is  $\pi = (s_0 s_5)$

Thus it may happen that  $\mathcal{S} \not\models \Phi$  and  $\mathcal{S} \not\models \neg(\Phi)$

Instead, it is impossible that  $\mathcal{S} \models \Phi$  and  $\mathcal{S} \models \neg(\Phi)$



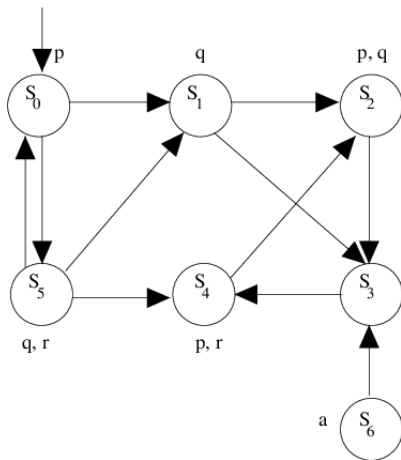
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# LTL Examples



$\mathcal{S} \not\models \mathbf{FG}p$ , a counterexample is  
 $\pi = s_0 s_1 (s_2 s_3 s_4)$   
Again this is a liveness formula

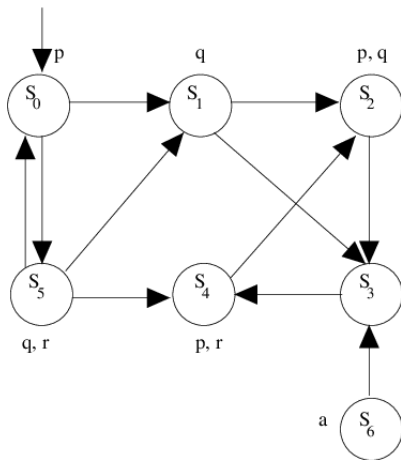


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# LTL Examples



$\mathcal{S} \models \mathbf{GF}p$

All lassos are  $s_0s_5$  or  $s_2s_3s_4$

In both such lassos, there are states in which  $p$  holds

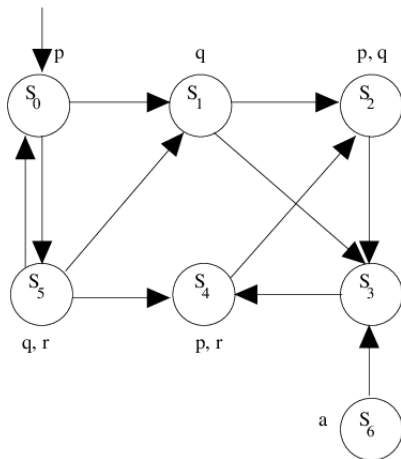


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# LTl Examples



$\mathcal{S} \models \mathbf{GF}p \vee \mathbf{FG}p$

Consequence of the two previous slides

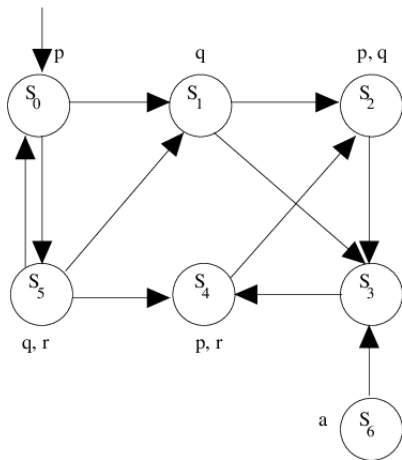


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# LTl Examples



$\mathcal{S} \not\models \mathbf{G}(p \mathbf{U} q)$ , a counterexample is  $\pi = s_0 s_1 (s_2 s_3 s_4)$   
 $(p \mathbf{U} q)$  must hold at any reachable state  
Ok in  $s_0, s_1, s_2$ , but not in  $s_3$



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# LTL Non-Toy Examples

- Recall the Peterson's protocol: checking mutual exclusion is  $\mathbf{G}(p \wedge q)$ , being  $p = P[1] = L3$ ,  $q = P[2] = L3$ 
  - all invariants are of the form  $\mathbf{G}P$ , where  $P$  does not contain modal operators  $\mathbf{X}$ ,  $\mathbf{U}$  or  $\mathbf{F}$
- Checking that both processes access to the critical section *infinitely often* is  $\mathbf{GF} P[1] = L3 \wedge \mathbf{GF} P[2] = L3$ 
  - liveness property: no process is infinitely banned to access the critical section
- Even better:  $\mathbf{G} (P[1] = L2 \rightarrow \mathbf{F} P[1] = L3)$ 
  - the same for the other process
  - since it is symmetric, this is actually enough



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# Equivalence Between LTL Properties

- Definition of equivalence between LTL properties:  
 $\varphi_1 \equiv \varphi_2 \text{ iff } \forall \mathcal{S}. \mathcal{S} \models \varphi_1 \Leftrightarrow \mathcal{S} \models \varphi_2$ 
  - equivalent:  $\forall \sigma \dots$
- Idempotency:
  - $\mathbf{FF}p \equiv \mathbf{F}p$
  - $\mathbf{GG}p \equiv \mathbf{G}p$
  - $p \mathbf{U} (p \mathbf{U} q) \equiv (p \mathbf{U} q) \mathbf{U} q \equiv p \mathbf{U} q$
- Absorption:
  - $\mathbf{GFG}p \equiv \mathbf{FG}p$
  - $\mathbf{FGF}p \equiv \mathbf{GF}p$
- Expansion (used by LTL Model Checking algorithms!):
  - $p \mathbf{U} q \equiv q \vee (p \wedge \mathbf{X}(p \mathbf{U} q))$
  - $\mathbf{F}p \equiv p \vee \mathbf{XF}p$
  - $\mathbf{G}p \equiv p \wedge \mathbf{XG}p$



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# CTL Syntax

$\Phi ::= p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{EX}\Phi \mid \mathbf{EG}\Phi \mid \mathbf{E}\Phi_1 \mathbf{U} \Phi_2$

- Other derived operators (besides true, false, OR, etc):
  - $\mathbf{EF}\Phi = \mathbf{Etrue} \mathbf{U} \Phi$ 
    - cannot be defined using  $\mathbf{E}\neg\mathbf{G}\neg\Phi$ , as this is not a CTL formula
    - actually, it is a CTL\* formula (see later)
  - $\mathbf{AF}\Phi = \neg\mathbf{EG}\neg\Phi$ ,  $\mathbf{AG}\Phi = \neg\mathbf{EF}\neg\Phi$ ,  $\mathbf{AX}\Phi = \neg\mathbf{EX}\neg\Phi$
  - $\mathbf{A}\Phi_1 \mathbf{U} \Phi_2 = (\neg\mathbf{E}\neg\Phi_2 \mathbf{U} (\neg\Phi_1 \wedge \neg\Phi_1)) \wedge \neg\mathbf{EG}\neg\Phi_2$
  - $\Phi_1 \mathbf{AU}\Phi_2 = \mathbf{A}\Phi_1 \mathbf{U}\Phi_2$ ,  $\Phi_1 \mathbf{EU}\Phi_2 = \mathbf{E}\Phi_1 \mathbf{U}\Phi_2$



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# Comparison with LTL Syntax

$$\Phi ::= \text{true} \mid p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{X}\Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

- Essentially, all temporal operators are preceded by either **E** or **G**
  - with some care for **U**



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# CTL Semantics

- Goal: formally defining when  $\mathcal{S} \models \varphi$ , being  $\mathcal{S}$  a KS and  $\varphi$  a CTL formula
- This is true when, for all initial states  $s \in I$  of  $\mathcal{S}$ ,  $s \models \varphi$ 
  - thus, CTL is made of *state* formulas
  - LTL has *path* formulas
- To define when  $s \models \varphi$ , a recursive definition over the recursive syntax of CTL is provided
  - no need of an additional integer as for LTL syntax



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# CTL Semantics for $s, i \models \varphi$

- $\forall s \in S. s, i \models \text{true}$
- $s \models p$  iff  $p \in L(s)$
- $s \models \Phi_1 \wedge \Phi_2$  iff  $s \models \Phi_1 \wedge s \models \Phi_2$
- $s \models \neg\Phi$  iff  $s \not\models \Phi$
- $s \models \mathbf{EX}\Phi$  iff  $\exists \pi \in \text{Path}(\mathcal{S}, s). \pi(1) \models \Phi$
- $s \models \mathbf{EG}\Phi$  iff  $\exists \pi \in \text{Path}(\mathcal{S}, s). \forall j. \pi(j) \models \Phi$
- $s \models \mathbf{E}\Phi_1 \mathbf{U} \Phi_2$  iff  
 $\exists \pi \in \text{Path}(\mathcal{S}, s) \exists k : \pi(k) \models \Phi_2 \wedge \forall j < k. \pi(j) \models \Phi_1$



# CTL Semantics for Added Operators

- It is easy to prove that:
  - $s \models \mathbf{AG}\Phi$  iff  $\forall \pi \in \text{Path}(\mathcal{S}, s). \forall j. \pi(j) \models \Phi$
  - $s \models \mathbf{AF}\Phi$  iff  $\forall \pi \in \text{Path}(\mathcal{S}, s). \exists j. \pi(j) \models \Phi$
  - analogously for **AU**, **AR**, **AW**
  - just replace  $\forall$  with  $\exists$  for **EF**, **ER**, **EW**
- As for CTL, for many formulas, it is silently required that paths are infinite
- So again transition relations in KSs must be total



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# Safety and Liveness Properties in CTL

- Some CTL formulas may be neither safety nor liveness
  - being defined on states, the counterexample may be an entire computation tree
- Safety properties are those involving only **AG**, **AX**, true and atomic propositions
- Some formulas are both safety and liveness, like true, **G** true and so on
- Liveness are formulas like **AF**, **AFAG**, **AU**
- **EF** or **EG** are neither liveness nor safety

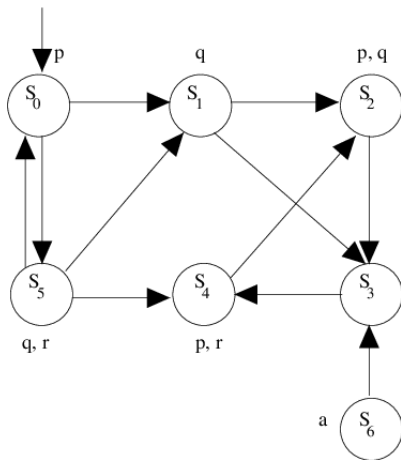


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# CTL Examples



$\mathcal{S} \models \mathbf{AF}p$  since  $p$  holds in the first state

For full:  $s_0 \models \mathbf{F}p$  since  $p \in L(s_0)$ , thus, for all paths starting in  $s_0$ ,  $p$  holds in the first state, so it holds eventually

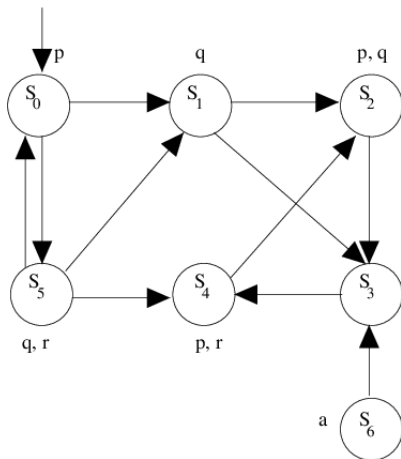


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# CTL Examples



$\mathcal{S} \models \mathbf{EF}p$  for the same reason as above

If it holds for all paths, then it holds for one path

$\mathbf{AF}\phi \rightarrow \mathbf{EF}\phi$

The same holds for the other temporal operators **G**, **U** etc

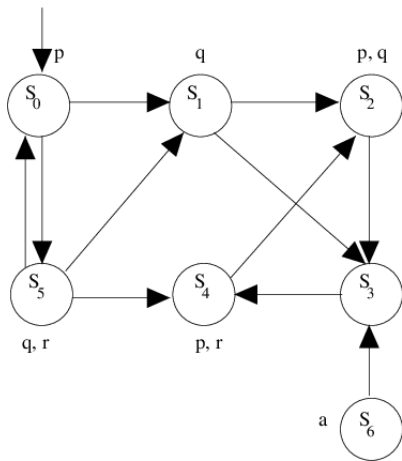


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# CTL Examples



$\mathcal{S} \not\models \mathbf{EF}a$  since  $s_6$  is not reachable

Note that the counterexample cannot be a single path

Since it would not enough to disprove existence

The full reachable graph must be provided

One could also show the tree of all paths

Neither safety nor liveness

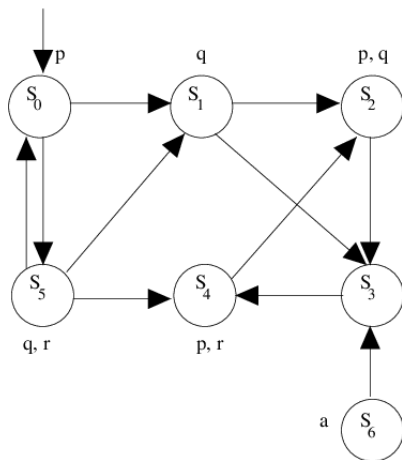


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# CTL Examples



$\mathcal{S} \models \mathbf{A}(p \mathbf{U} q)$  since  $p \in L(s_0)$ ,  
 $\text{next}(s_0) = \{s_1, s_5\}$  and  $q \in L(s_1) \wedge q \in L(s_5)$



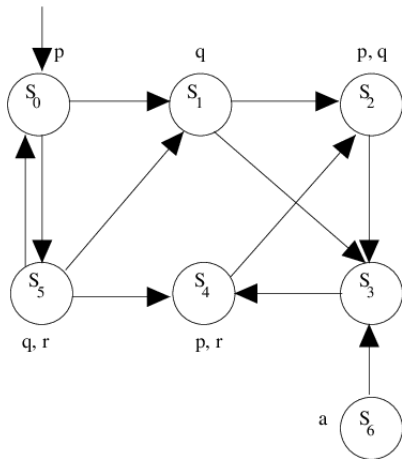
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# CTL Examples



$\mathcal{S} \not\models \mathbf{A}(p \mathbf{U} r)$ , a counterexample is  $\pi = s_0 s_1 (s_2 s_3 s_4)$

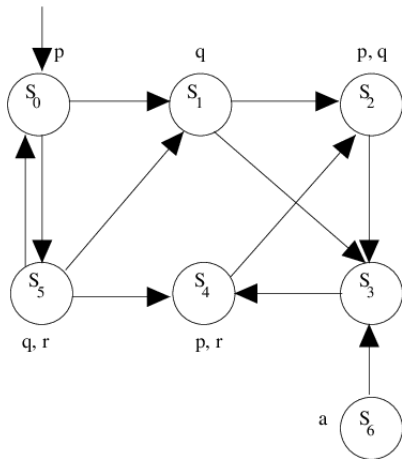


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# CTL Examples



$\mathcal{S} \models \mathbf{E}(p \mathbf{U} r)$ , an example is  
 $\pi = (s_0 s_5)$

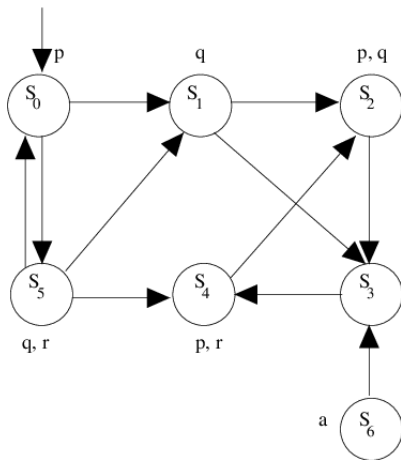


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# CTL Examples



$\mathcal{S} \not\models \neg \mathbf{E}(p \mathbf{U} r)$ , a counterexample is  $\pi = (s_0 s_5)$

In fact,  $\mathcal{S} \not\models \Phi$  iff  $\mathcal{S} \models \neg(\Phi)$

No hidden quantifier...

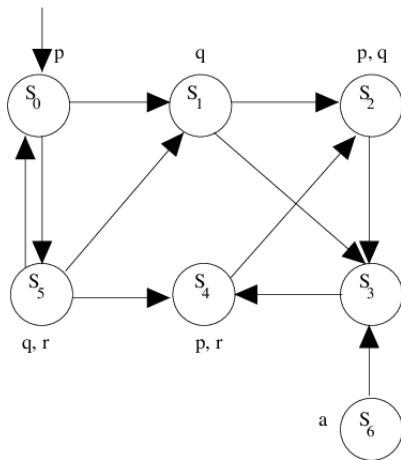


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# CTL Examples



$\mathcal{S} \not\models \mathbf{AFAG}p$ , a counterexample is  $\pi = s_0s_1(s_2s_3s_4)$   
This is a liveness formula

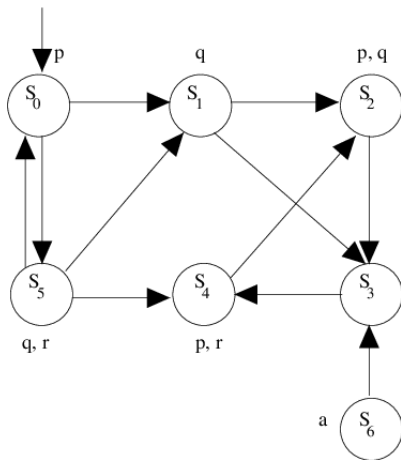


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# CTL Examples



$\mathcal{S} \not\models \mathbf{EFEG}p$ , a counterexample is again a computation tree

All lassos are  $s_0s_5$  or  $s_2s_3s_4$

In both such lassos, there are states in which  $p$  does not hold

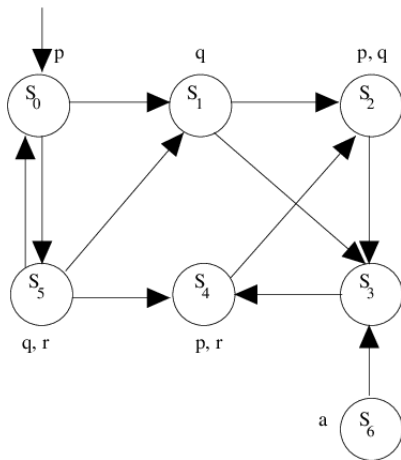


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# CTL Examples



$\mathcal{S} \not\models \mathbf{AFEG}p$ , a counterexample is again a computation tree  
Since  $\mathcal{S} \not\models \mathbf{EFEG}p \dots$

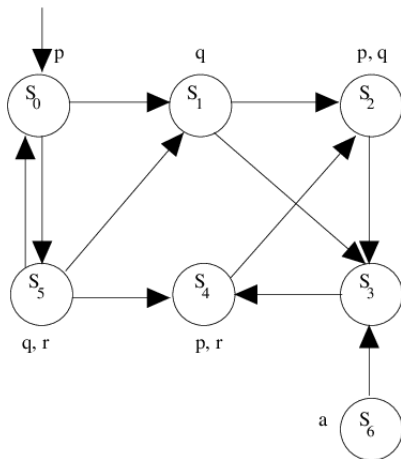


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# CTL Examples



$\mathcal{S} \not\models \mathbf{EFAG}p$ , a counterexample is again a computation tree  
 Since  $\mathcal{S} \not\models \mathbf{EFEG}p$ ...



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# CTL Non-Toy Examples

- Recall the Peterson's protocol: checking mutual exclusion is **AG**( $p \wedge q$ ), being  $p = P[1] = L3$ ,  $q = P[2] = L3$ 
  - equivalent to LTL **G** $p$
- It is always possible to restart:  
**AGEF**  $P[1] = L0 \wedge \mathbf{AGEF} P[2] = L0$



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# CTL vs. LTL: a Comparison

- Recall that  $\varphi_1 \equiv \varphi_2$  iff  $\forall \mathcal{S}. \mathcal{S} \models \varphi_1 \Leftrightarrow \mathcal{S} \models \varphi_2$ 
  - also holds (w.l.g.) when  $\varphi_1$  is LTL and  $\varphi_2$  is CTL
- Of course, some CTL formulas cannot be expressed in LTL
  - it is enough to put an **E**, since LTL always universally quantifies paths
  - so, there is not an LTL  $\varphi$  s.t.  $\varphi \equiv \mathbf{EG}p$ 
    - no,  $\mathbf{F}\neg p$  is not the same, why?
- So, one might think: LTL is contained in CTL
  - simply replace each temporal operator **O** with **AO**, that's it
  - let  $\mathcal{T}$  be a translator doing this
  - for any LTL formula  $\varphi$ ,  $\varphi \equiv \mathcal{T}(\varphi)$
  - actually,  $\mathbf{G}p \equiv \mathcal{T}(\mathbf{G}p) = \mathbf{AG}p$



# CTL vs. LTL: a Comparison

- Theorem. Let  $\varphi$  be an LTL formula. Then, either i)  $\varphi \equiv \mathcal{T}(\varphi)$  or ii) there does not exist a CTL formula  $\psi$  s.t.  $\varphi \equiv \psi$ 
  - idea of proof: replacing with **E** is of course not correct, and temporal operators on paths are the same
- Corollary. There exists an LTL formula  $\varphi$  s.t., for all CTL formulas  $\psi$ ,  $\varphi \not\equiv \psi$
- Proof of corollary:
  - by the theorem above and the definitions, we need to find
    - 1 an LTL formula  $\varphi$
    - 2 a KS  $\mathcal{S}$
  - where  $\mathcal{S} \models \varphi$  and  $\mathcal{S} \not\models \mathcal{T}(\varphi)$ 
    - viceversa is not possible



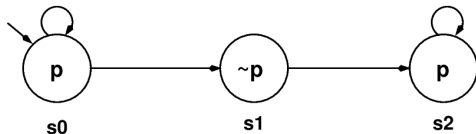
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# CTL vs. LTL: a Comparison

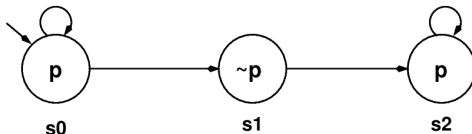
- For example, as for the LTL formula, we may take  $\varphi = \mathbf{FG}p$ 
  - note instead that  $\mathbf{GF}p \equiv \mathbf{AGAF}p$
- For example, as for the KS  $\mathcal{S}$ , we may take



- We have that  $\mathcal{S} \models \mathbf{FG}p$ , but  $\mathcal{S} \not\models \mathbf{AFAG}p$
- Thus, CTL requires “more” than the corresponding LTL



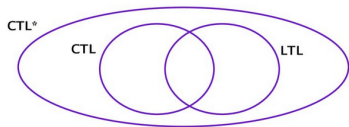
# CTL vs. LTL: a Comparison



- $\mathcal{S} \not\models \mathbf{AFAG}p$  means that
$$\neg(\forall \pi \in \text{Path}(\mathcal{S}). \exists j : \forall \rho \in \text{Path}(\mathcal{S}, \pi(j)). \forall k. p \in \rho(k))$$
$$= \exists \pi \in \text{Path}(\mathcal{S}). \forall j : \exists \rho \in \text{Path}(\mathcal{S}, \pi(j)). \exists k. p \notin \rho(k)$$
  - the path  $\pi$  is a loop on  $s_0 \dots$
- $\mathcal{S} \models \mathbf{FG}p$  means that  $\forall \pi \in \text{Path}(\mathcal{S}). \exists j : \forall k \geq j. p \in \pi(k)$
- Thus, there is not a CTL formula equivalent to  $\mathbf{FG}p$
- Furthermore, there is not an LTL formula equivalent to  $\mathbf{AFAG}p$



# CTL, LTL and CTL\*



- CTL\* introduced in 1986 (Emerson, Halpern) to include both CTL and LTL
- No restrictions on path quantifiers to be 1-1 with temporal operators, as in CTL
- State formulas:  $\Phi ::= \text{true} \mid p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid \mathbf{A}\Psi \mid \mathbf{E}\Psi$
- Path formulas:  $\Psi ::= \Phi \mid \Psi_1 \wedge \Psi_2 \mid \neg \Psi \mid \Psi_1 \mathbf{U} \Psi_2 \mid \mathbf{F}\Psi \mid \mathbf{G}\Psi$

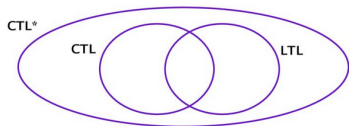


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# CTL, LTL and CTL\*



- The intersection between CTL and LTL is both syntactic and “semantic”
- Some formulas are both CTL and LTL in syntax: all those involving only boolean combinations of atomic propositions
- “Semantic” intersection: some LTL formulas may be expressed in CTL and vice versa, using different syntax
  - **AGAF $p$**  and **GF $p$**
  - **AG $p$**  and **G $p$**
  - etc



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# Peterson Protocol in Promela

```
bool turn, flag[2];
byte ncrit;

active [2] proctype user()
{
  assert(_pid == 0 || _pid == 1);
again:
  flag[_pid] = 1;
  turn = _pid;
  (flag[1 - _pid] == 0 || turn == 1 - _pid);
  ncrit++;
  assert(ncrit == 1); /* critical section */
  ncrit--;
  flag[_pid] = 0;
  goto again
}
```



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# Dijkstra Protocol in Promela

```
#define p 0
#define v 1
chan sema = [0] of { bit }; /* rendez-vous */

proctype dijkstra()
{
    byte count = 1; /* local variable */
    do
        :: (count == 1) -> sema!p; count = 0
        /* send 0 and blocks, unless some other
           proc is already blocked in reception */
        :: (count == 0) -> sema?v; count = 1
        /* receive 1, same as above */
    od
}
```



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# Dijkstra Protocol in Promela

```
proctype user()  
{  
  do  
    :: sema?p;  
      /*      critical section      */  
    sema!v;  
      /* non-critical section */  
  od  
}  
  
init  
{  
  run dijkstra();  
  run user(); run user(); run user()  
}
```



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# SPIN Simulation

Almost equal to Murphi one

```
void Make_a_run(NFSS  $\mathcal{N}$ )
{
  let  $\mathcal{N} = \langle S, \{s_0\}, \text{Post} \rangle$ ;
  s_curr =  $s_0$ ;
  if (some assertion fail in s_curr)
    return with error message;
  while (1) { /* loop forever */
    if (Post(s_curr) =  $\emptyset$ )
      return with deadlock message;
    s_next = pick_a_state(Post(s_curr));
    if (some assertion fail in s_curr)
      return with error message;
    s_curr = s_next;
  }
}
```



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# SPIN Verification

- Able to answer to the following questions:
  - is there a deadlock (invalid end state)?
  - are there reachable assertions which fail (safety)?
  - is a given LTL formula (safety or liveness) ok in the current system?
  - is a given neverclaim (safety or liveness) ok in the current system?
- It is possible to specify some side behaviours:
  - is sending to a full channel blocking, or the message is dropped without blocking?
- It may report unreachable code
  - Promela statements in the model which are never executed



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# SPIN Verification

- Similar to Murphi:
  - 1 the SPIN compiler (`SrcXXX/spin -a`) is invoked on `model.prm` and outputs 5 files:
    - `pan.c`, `pan.h`, `pan.m`, `pan.b`, `pan.t` (unless there are errors...)
  - 2 the 5 files given above are compiled with a C compiler
    - it is sufficient to compile `pan.c`, which includes all other files
    - in this way, an executable file `model` is obtained
  - 3 just execute `model`
    - option `--help` gives an overview of all possible options



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# Standard Recursive DFS

```
HashTable Visited =  $\emptyset$ ;
```

```
DFS(graph  $G = (V, E)$ , node  $v$ )  
{  
    Visited := Visited  $\cup v$ ;  
    foreach  $v' \in V$  t.c.  $(v, v') \in E$  {  
        if ( $v' \notin$  Visited)  
            DFS( $G, v'$ );  
    }  
}
```



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# Iterative DFS

```
DFS(graph  $G = (V, E)$ )
{
     $s := \text{init}$ ;  $i := 1$ ;  $\text{depth} := 0$ ;
    push( $s$ , 1);
Down:
    if ( $s \in \text{Visited}$ )
        goto Up;
    Visited := Visited  $\cup$   $s$ ;
    let  $S' = \{s' \mid (s, s') \in E\}$ ;
    if ( $|S'| \geq i$ ) {
         $s := i\text{-th element in } S'$ ;
        increment  $i$  on the top of the stack;
        push( $s$ , 1);
         $\text{depth} := \text{depth} + 1$ ;
        goto Down;
    }
}
```



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# Iterative DFS

```
Up:
  (s, i) := pop();
  depth := depth - 1;
  if (depth > 0)
    goto Down;
}
```

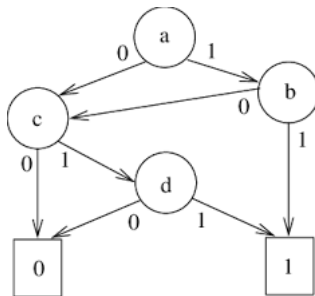


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# Binary Decision Diagrams



Represented function:  $f(a, b, c, d) = ab + \bar{a}cd + \bar{a}bcd$

- recall that  $+$  is OR,  $\cdot$  is AND,  $\bar{\phantom{x}}$  is negation



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# NuSMV Input Language

Taken from `examples/smv-dist/short.smv`

```
MODULE main
```

```
VAR
```

```
  request : {Tr, Fa}; -- same as saying boolean
                      -- (stand for True and False)
```

```
  state : {ready, busy};
```

```
ASSIGN
```

```
  init(state) := ready;
```

```
  next(state) := case
```

```
    state = ready & (request = Tr): busy;
```

```
    1 : {ready, busy};
```

```
  esac;
```

```
SPEC
```

```
  AG((request = Tr) -> AF state = busy)
```

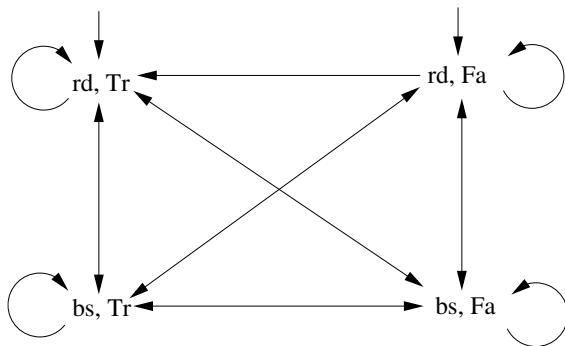


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# Automata for short.smv: $I$ and $R$



# OBDDs for short.smv: $R$

Straight lines are then-edges

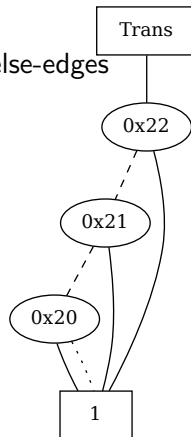
Dashed lines are else-edges

Dotted lines are complemented-else-edges

request.0

state.0

next(state.0)



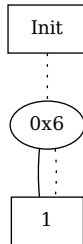
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# OBDDs for short.smv: /

state.0



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# NuSMV Input Language

```
MODULE user(semaphore)
VAR
  state : {idle, entering, critical, exiting};
ASSIGN
  init(state) := idle;
  next(state) :=
    case
      state = idle: entering;
      state = entering & !semaphore: critical;
      state = critical: {critical, exiting};
      state = exiting: idle;
      TRUE : state;
  esac;
```



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# NuSMV Input Language

```
next(semaphore) :=  
  case  
    state = entering: TRUE;  
    state = exiting: FALSE;  
    TRUE: semaphore;  
  esac;
```



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# NuSMV Input Language

```
MODULE main
```

```
VAR
```

```
    semaphore : boolean;
```

```
    proc1 : process user(semaphore);
```

```
    proc2 : process user(semaphore);
```

```
ASSIGN
```

```
    init(semaphore) := FALSE;
```

```
SPEC
```

```
    AG(!(proc1.state = critical & proc2.state = critical))
```

```
LTLSPEC
```

```
    G F proc1.state = critical
```



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# Computation of Least (Minimum) Fixpoint

```
OBDD lfp(MuFormula T) /*  $\mu Z.T(Z)$  */  
{  
  Q =  $\lambda x.0$ ;  
  Q' = T(Q);  
  /* T clearly says where Q must be replaced */  
  /* e.g.: if  $\mu Z.\lambda x.f(x) \vee Z(x)$ , then  
    Q' =  $\lambda x.f(x) \wedge Q(x)$  */  
  while (Q  $\neq$  Q') {  
    Q = Q';  
    Q' = T(Q);  
  }  
  return Q; /* or Q', they are the same... */  
}
```



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# Computation of Greatest (Maximum) Fixpoint

```
OBDD gfp(NuFormula T) /*  $\nu Z.T(Z)$  */  
{  
    Q =  $\lambda x. 1$ ;  
    Q' = T(Q);  
    while (Q  $\neq$  Q') {  
        Q = Q';  
        Q' = T(Q);  
    }  
    return Q;  
}
```



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# CTL Model Checking

```
bool checkCTL(KS S, CTL  $\varphi$ ) {  
  let  $S = \langle S, I, R, L \rangle$ ;  
   $B = \text{Lb1St}(\varphi)$ ;  
  return  $\lambda x. I(x) \wedge \neg B(x) = \lambda x. 0$ ;  
}  
  
OBDD Lb1St(CTL  $\varphi$ ) { /* also  $S = \langle S, I, R, L \rangle$  */  
  if ( $\exists p \in AP. \varphi = p$ ) return  $\lambda x. p(x)$ ;  
  else if ( $\varphi = \neg\phi$ ) return  $\lambda x. \neg \text{Lb1St}(\phi)(x)$ ;  
  else if ( $\varphi = \phi_1 \wedge \phi_2$ )  
    return  $\lambda x. \text{Lb1St}(\phi_1)(x) \wedge \text{Lb1St}(\phi_2)(x)$ ;  
  else if ( $\varphi = \mathbf{EX}\phi$ )  
    return  $\lambda x. \exists y : R(x, y) \wedge \text{Lb1St}(\phi)(y)$ ;  
  else if ( $\varphi = \mathbf{EG}\phi$ )  
    return  $\text{gfp}(\nu Z. \lambda x. \text{Lb1St}(\phi)(x) \wedge (\exists y : R(x, y) \wedge Z(y)))$ ;  
  else if ( $\varphi = \phi_1 \mathbf{EU} \phi_2$ )  
    return  $\text{lfp}(\mu Z. \lambda x. \text{Lb1St}(\phi_2)(x) \vee$   
       $(\text{Lb1St}(\phi_1)(x) \wedge (\exists y : R(x, y) \wedge Z(y))))$ ;  
}
```

