Automated Verification of Cyber-Physical Systems A.A. 2022/2023

System Level Formal Verification

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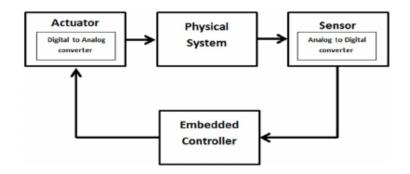


Embedded Systems

- One of the task given to computers from the very start: monitoring and/or controlling some external system
 - where the "system" is anything without computational capabilities
 - 60s: guidance of missiles and Apollo Guidance System
- In the following, we will restrict our attention to control
- Thus, an embedded system is mainly composed by two parts:
 a controller and a plant
 - the plant must accept inputs able to modify its behaviour
 - the plant must also expose some output
- Nowadays, embedded systems are everywhere
 - may control something very little, like an electrical circuit (e.g., buck DC/DC converter)
 - or something very big, like an automobile or aircraft



Embedded Systems: Closed-Loop System







- System level verification has the aim to discover errors to some (embedded) system considered as a whole
 - all components are considered together
 - we assume they have been separately tested before
- Typically done by testing
 - plant is nearly always replaced by a simulator
 - often built in Simulink or Modelica
 - HILS: Hardware-in-the-loop simulation
- System level formal verification: we want to apply Model Checking techniques





- In "standard" Model Checking, we are given
 - a non-deterministic Kripke Structure (KS)
 - an LTL or CTL property to be verified
- We get a PASS/FAIL response
 - possibly with a counterexample
- When we deal with complex embedded systems, having a KS is difficult
 - moreover: most plants are described by real variables, thus they have an infinite number of states
 - approximation may be ok for early verification, but here we want system level verification
 - with actual software involved





- Thus, we want to apply Model Checking to the closed-loop system (SUV, System Under Verification) as:
 - a black-box controller
 - a simulator for the plant
- We are still interested in some property to be verified
 - let us suppose we have a safety property for starting
- How to accomplish such a task?
- The idea is: kind of Statistical Model Checking, but exhaustive
 - that is: perform simulations of the whole system (like in HILS) considering all possible scenarios





- This should be impractible, how can we do this?
- The idea is: if we see the system as a black-box, verification is about
 - (incontrollable) interactions with the external environment
 - (incontrollable) "hardware" (i.e., parts of the plant) failures
 - (incontrollable) changes in the plant simulation parameters
- Interactions between the plant and the controller are inside the system
 - as a consequence of the variations listed above
- We can see all of this as inputs to our closed-loop system
- A system is not expected to withstand any combination of the preceding
 - e.g., if we put an airplane inside a violent windshire, we cannot expect its controller to safely land it

- Requirement 1: we can write a model for the meaningful interactions between the system and the environment
 - "meaningful": those we want to verify
- In the following, we will call such interactions as disturbances
 - because they are deviations from the current behaviour
 - e.g., if we move an inverted pendulum while it is upright and still, we are disturbing it
 - causing its controller to react and return it upright and still
- As in Statistical Model Checking, we consider a bounded verification
 - thus, we are interested in *finite sequences* of possible disturbances
 - e.g., move the inverted pendulum, then move it again before it is returned upright

- Requirement 2: the simulator for the plant accepts the following commands
 - I d: inject disturbance d
 - will modify the plant behaviour
 - that is, the following R commands
 - **R** t: compute the evolution of the plant within t units of time
 - this is the main function for all simulators...
 - **S** / save the current simulator state with id /
 - **F** / free the simulator state with id /
 - L / load (i.e., restore) the simulator state with id /
 - simulator states are saved in some permanent memory, e.g., files on disk
 - S_1 , **S** I, S_2 , **L** I, S_3 , where S_i are command sequences, is equivalent to the command sequences S_1 , S_2 (restart) S_1 ,

- A sequence **R** t_1 , **S** l, **R** t_2 , **L** l, **R** t_3 is equivalent to the following two simulations: **R** $t_1 + t_2$ and **R** $t_1 + t_3$
 - in the middle, the system simulation is restarted from time 0
- A sequence **I** d, **R** t is equivalent to:
 - modify the simulator by changing some plant parameters
 - each disturbance corresponds to a modification of a selection of plant parameters
 - "modification": change the value
 - run a simulation for t units of time with the new plant model
- A sequence **R** t_1 , **I** d, **R** t_2 is equivalent to:
 - modify the simulator so that the d parameters changing happens after t_1 units of time
 - e.g., in Modelica, this could be done with an if inside the main whensample, if any
 - run a simulation for $t_1 + t_2$ units of time



- A sequence **R** t_1 , **I** d, **S** l, **R** t_2 , **L** l, **R** t_3 is equivalent to:
 - modify the simulator for d after t_1 units of time
 - perform simulations \mathbf{R} $t_1 + t_2$ and \mathbf{R} $t_1 + t_3$
- A sequence R t₁, S /, R t₂, I d₁, R t₃, L /, I d₂, R t₄ is equivalent to
 - modify the simulator for d_1 after $t_1 + t_2$ units of time
 - modify the simulator for d_2 after t_1 units of time
 - perform simulations **R** $t_1 + t_2 + t_3$ and **R** $t_1 + t_4$
 - is this correct????





- Simulation campaign: any finite sequence of simulator commands
 - finite because we are performing bounded verification
- We assume that we can write some software which takes as input a simulation campaign and executes it on the simulator
 - we call it driver
 - either within the simulator or with some external script
 - e.g.: in Simulink, we may use Simulink scripts
 - e.g.: in Modelica, we have to use something external
 - we can write model-independent Simulink and Modelica drivers





System Level Formal Verification: Modeling

- Thus, we need two models:
 - disturbance model
 - plant model
- Plus the actual software for the controller
 - which directly interacts with the plant model
 - e.g., using external functions, available both in Modelica an Simulink
 - in the following, we will consider it embedded in the plant model





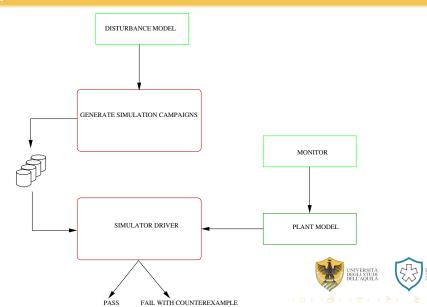
System Level Formal Verification: Modeling

- In embedded systems design a simulation model for the plant is always built
- Thus, the only modeling required is that of the disturbance model
 - we are performing a kind of exhaustive functional testing
 - exhaustive w.r.t. the given disturbance model
- We also need to enlarge the existing plant model with a monitor
 - when an error is found, a boolean variable will become one
 - equivalent to specify a bounded safety property





System Level Formal Verification: Architecture



- Let $d \in \mathbb{N}^+$ be a positive integer
 - total number of disturbances is d+1
 - 0 is a special value for "no disturbance"
- A discrete event sequence is a function $u : \mathbb{R}^{\geq 0} \to [0, d] \cap \mathbb{N}$ s.t., for all $t \in \mathbb{R}^{\geq 0}$, $\operatorname{card}(\{\tilde{t} \mid 0 \leq \tilde{t} \leq t \land d(\tilde{t}) \neq 0\}) < \infty$
 - that is: given a time t, u(t) returns the disturbance at time t
 - thus, we are requiring that it is almost always without disturbances
 - i.e., some disturbance happens only in a finite number of times
- Let $\mathcal{U}_d = \{u \mid u \text{ is a discrete event sequence for } d\}$





- An *event list* is a sequence $(u_0, \tau_0), (u_1, \tau_1), \ldots$ s.t., for all $i \geq 0$, $u_i \in [0, d] \cap \mathbb{N}, \tau_i \in \mathbb{R}^{\geq 0}$
 - not only disturbances, but also their durations
- For each event list there is a unique discrete event sequence u defined as:
 - $u(0) = u_0$
 - $u(t) = u_h$ if $t = \sum_{i=0}^{h-1} \tau_i$ for some $h \ge 1$
 - u(t) = 0 otherwise
- The viceversa also holds (derive the formula by yourself)





- A *Discrete Event System* (DES) is a tuple $\mathcal{H} = \langle S, s_0, d, O, \text{flow}, \text{jump}, \text{output} \rangle$ where:
 - S is a (possibly infinite) set of states; $s_0 \in S$ is the initial state
 - Cartesian product of the domains of the state variables
 - ullet d is the number of disturbances (defines the input space \mathcal{U}_d)
 - O is a (possibly infinite) set of output values
 - useful to define the monitor
 - output : $S \rightarrow O$, i.e., each state defines an output
 - flow : $S \times \mathbb{R}^{\geq 0} \to S$
 - dynamics without disturbances: flow(s, t) is the state reached after t units of time, starting from state s
 - w.r.t. hybrid systems, this may also result in location changes!
 - flow(s,0) = s
 - jump : $S \times [0, d] \rightarrow S$
 - dynamics with disturbances: jump(s,d) is the state reached when disturbance d is applied in state s
 - $\operatorname{jump}(s,0) = s$



- The state function of a DES tells us in which state we go after some simulation time
 - starting from s₀ and considering intervening disturbances in a discrete even sequence
 - our DES are deterministic, thus there is only one such state
- Given a DES $\mathcal{H} = \langle S, s_0, d, O, \text{flow}, \text{jump}, \text{output} \rangle$, the *state function* of \mathcal{H} is $\phi : \mathcal{U}_d \to S$ s.t.:
 - $\phi(u,0) = \text{jump}(s_0, u(0))$
 - i.e., if there is some disturbance at time 0, let us begin from the resulting state
 - otherwise, we begin from s₀
 - for each t > 0, $\phi(u, t) = \text{jump}(\text{flow}(\phi(u, t^*), t t^*), u(t))$
 - $t^* = \max\{\tilde{t} \mid \tilde{t} < t \land u(\tilde{t}) \neq 0\}$
 - with $\max \emptyset = 0$







- We may view the state function in a more computation-like way
- Given a DES $\mathcal{H} = \langle S, s_0, d, O, \text{flow}, \text{jump}, \text{output} \rangle$, a discrete event sequence u and a time t:
 - ① compute the (minimal) event list $(u_0, \tau_0), (u_1, \tau_1), \dots, (u_n, \tau_n)$ corresponding to u
 - must be finite by definition of discrete event sequence
 - ② with $s = s_0$ as initialization, for i = 0, ..., n:
 - ① let s be $jump(s, u_i)$
 - ② let s be flow(s, τ_i)
 - output s



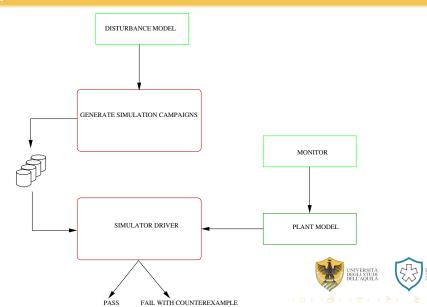


- We also need the *output function* of a DES $\mathcal{H} = \langle S, s_0, d, O, \text{flow}, \text{jump}, \text{output} \rangle$
 - easy when we have the state function
- Namely, $\psi: \mathcal{U}_d \times \mathbb{R}^{\geq 0} \to O$ is defined as $\psi(u,t) = \operatorname{output}(\phi(u,t))$
- Monitor: when the safety property becomes false, the output is false
 - this is the only output we need
 - once is false, it must stay false, otherwise we may not realize it
- A monitored DES is a tuple $\mathcal{H} = \langle S, s_0, d, \text{flow}, \text{jump}, \text{output} \rangle$ s.t.
 - $\langle S, s_0, d, \{0, 1\}, \text{flow}, \text{jump}, \text{output} \rangle$ is a DES
 - for all $u \in \mathcal{U}_d$, $\psi(u,t)$ is non-increasing w.r.





System Level Formal Verification: Architecture



Modeling the Disturbances

- The system part is now ok: a Monitored DES encompasses the closed-loop system and the property monitor
 - let us go with the disturbance model
- The "Generate simulation campaign" part is divided in two parts
 - from a model of disturbances, generate all possible sequences of disturbances (disturbance traces) of length T
 - from sequences of disturbances, generate the optimized simulation campaigns
- Thus, we need some model able to define complex disturbance traces
 - e.g.: in a given trace, d_1 only occurs at most three times but never immediately after d_2

Modeling the Disturbances

- One possible way is using a standard Model Checker
- Here, we will use CMurphi: each rule corresponds to a disturbance
 - by suitably using rule guards, we may implement any wanted logic behind disturbance traces
 - see attached example
- By suitably modifying the CMurphi source code, we may generate disturbance traces as required
- Also a slight modification to the input language is required to introduce final states





Modeling the Disturbances: Definitions

- A disturbance generator (DG) is a tuple $\mathcal{D} = \langle Z, d, \operatorname{dist}, \operatorname{adm}, Z_I, Z_F \rangle$ where:
 - Z is a finite set of states
 - $Z_I, Z_F \subset Z$ are the subsets of initial and final states
 - $d\mathbb{N}^+$ is again the number of disturbances
 - $\operatorname{adm}: Z \times [0,d] \cap \mathbb{N} \to \{0,1\}$ defines the disturbances admitted at a given state
 - dist : $Z \times [0,d] \cap \mathbb{N} \to Z$ defines the deterministic transition relation
 - but CMurphi was nondeterministic!
 - yes, but here we are adding the disturbance, i.e., the rule getting fired...
- Easy to show that this is equivalent to a Kripke Structure



Modeling the Disturbances: Definitions

- Let $\mathcal{D} = \langle Z, d, \operatorname{dist}, \operatorname{adm}, Z_I, Z_F \rangle$ be a DG
- A disturbance path of length h for \mathcal{D} is a sequence $z_0 d_0 \dots z_{h-1} d_{h-1} z_h$ where:
 - $z_0 \in Z_I, z_h \in Z_F$: we start from an initial and end in a final state
 - $\forall i = 0, \ldots, h-1$. $adm(z_i, d_i) = 1$
 - $\forall i = 0, \ldots, h-1$. $dist(z_i, d_i) = z_{i+1}$
 - the DG semantics is preserved
- A disturbance trace is a sequence $\delta = d_0 \dots d_{h-1}$ s.t. there exists a disturbance path $z_0 d_0 \dots z_{h-1} d_{h-1} z_h$ for \mathcal{D}
- We define $\Delta^h_{\mathcal{D}} = \{ \delta \mid \delta \text{ is a disturbance trace for } \mathcal{D} \wedge |\delta| = h \}$







System Level Formal Verification Problem

- We can now formally define the overall problem we want to verify
 - for standard model checking it was: you have a Kripke Structure and a property, tell me if the property holds
 - with suitably defined semantics for the property holding on a Kripke Structure
- \bullet Here things are slightly more complicated: we also need a time step τ
 - not very strange: also simulators use some simulator step to perform simulations
- \bullet au allows us to go from disturbance traces to event lists (and discrete event sequences)
 - from $\delta = d_0, \ldots d_{h-1}$ to $(d_0, \tau) \ldots (d_{h-1}, \tau)$
 - we denote with $u(\delta)$ the discrete event sequence of δ





System Level Formal Verification Problem

- Given an MDES $\mathcal H$ and a DG $\mathcal D$, a System Level Formal Verification Problem (SLFVP) is a tuple $\mathcal P=\langle \mathcal H, \mathcal D, \tau, h \rangle$ where
 - $\tau \in \mathbb{R}^+, h \in \mathbb{N}^+$
 - d is the same both in \mathcal{H} and in \mathcal{D}
- Let ψ be the output function for \mathcal{H} , then the answer to \mathcal{P} is
 - $\langle \text{FAIL}, \delta \rangle$ if $\delta \in \Delta^h_{\mathcal{D}}$ is s.t. $\psi(u_{\tau}(\delta), \tau h) = 0$
 - PASS if such a $\delta \in \Delta^h_{\mathcal{D}}$ does not exist





System Level Formal Verification Problem

- Two main assumptions:
 - \bullet disturbances cannot happen at any time, but only at multiple times of τ
 - disturbances traces are of length h
 - ullet which implies that the total simulation time is T=h au
- The larger h and smaller τ , the closest we are to reality
 - as for h, it is the same of Bounded Model Checking and Statistical Model Checking
- No physical system can withstand arbitrarily (time) close disturbances
 - any operational scenario can be modelled with the desired precision by suitably choosing τ and h





System Level Formal Verification: Algorithms

- To simulate a MDES, we rely on existing simulators
 - Simulink, Modelica, NGSpice...
- As for the "Generate simulation campaign", is divided in two parts
 - ullet from a model of disturbances, generate all disturbance traces of length h
 - from sequences of disturbances, generate the optimized simulation campaigns
- Let us see how this is implemented





Generating all Disturbance Traces: Algorithm

```
function generateByDFS(\mathcal{D}, T):
 1: S_7 \leftarrow \emptyset, S_D \leftarrow \emptyset, DistTraces \leftarrow \emptyset, c \leftarrow 1
 2: Push(S_Z, z_0), Push(S_D, 1), \delta_0 \leftarrow c, c \leftarrow c + 1
 3: while StackIsNotEmpty(S_7) do
       z \leftarrow Top(S_Z), \ \tilde{d} \leftarrow Top(S_D)
         if \tilde{d} \leq d then
 5:
             Top(S_D) \leftarrow \tilde{d} + 1
 6:
            if adm(z, \tilde{d}) then
 7:
                \delta_{|S_z|} \leftarrow (\tilde{d}, c), \ c \leftarrow c + 1
 8:
                if |S_7| < T then
 9:
                    Push(S_7, dist(z, \tilde{d})), Push(S_D, 1)
10:
                else
11:
                    if z \in Z_F then DistTraces \leftarrow DistTraces \cup \delta
12:
13:
         else
             Pop(S_7), Pop(S_D)
14:
```

15: return DistTraces

Generating all Disturbance Traces: Algorithm

- This is for one initial state only, easy to generalize
- Standard non-recursive DFS
 - two stacks, one for states, one for rules
- Main difference 1: no check for already visited states
 - we are interested in transitions, so states may and must be visited multiple times
 - the bound T guarantees termination
- Main difference 2: the disturbance traces also encompass labels
 - simply a growing integer c
- Will be used by the simulation campaign generator





- A DES Simulator is a tuple $S = \langle \mathcal{H}, L, W, m \rangle$ where:
 - $\mathcal{H} = \langle S, s_0, d, O, \text{flow}, \text{jump}, \text{output} \rangle$ is a DES
 - L is a set of labels
 - $m \in \mathbb{N}^+$ is the maximum number of states the simulator can store
 - W is a set of simulator states s.t., for all $w \in W$, w = (s, u, M) and:
 - $s \in S \cup \bot$ (a DES state or a sink state)
 - $u \in \mathcal{U}_d$ (an event list)
 - $M \subseteq L \times S \times \mathcal{U}_d$ s.t., for each $l \in L$, there exist at most one triple $(l, s, u) \in M$
 - $|M| \leq m$
 - the DES simulator initial state is $(s_0, \emptyset, \emptyset)$







- The dynamics of a DES Simulator is simulator is defined on the basis of simulation campaign commands
- That is, we need to define $sim_S: W \times C \rightarrow W$
- Where C is the set of the following commands:
 - load(I) for $I \in L$
 - store(I) for $I \in L$
 - free(I) for $I \in L$
 - $\operatorname{run}(t)$ for $t \in \mathbb{N}^+$
 - inject (\tilde{d}) for $\tilde{d} \in [0,d] \cap \mathbb{N}$
- ullet Thus, we define $\mathrm{sim}_\mathcal{S}$ by cases



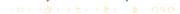


- $sim_{\mathcal{S}}(s, u, M, load(I)) = (s', u', M)$, being $(I, s', u') \in M$
- $\bullet \ \operatorname{sim}_{\mathcal{S}}(s, u, M, \operatorname{free}(I)) = (s, u, M \setminus \{(I, s', u')\})$
- $sim_{\mathcal{S}}(s, u, M, store(I)) = (s, u, M \cup \{(I, s, u)\})$ if |M| < m
- $\bullet \ \operatorname{sim}_{\mathcal{S}}(s, u, M, \operatorname{run}(t)) = (\operatorname{flow}(s, t\tau), u \cdot (0, t), M)$
- $sim_{\mathcal{S}}(s, u, M, inject(\tilde{d})) = (jump(s, \tilde{d}), u \cdot (\tilde{d}, 0), M)$
- Plus error checking, not considered here
 - e.g., trying to free something which was not stored
 - e.g., trying to store when memory is already full
 - e.g., trying to store without freeing first (if already present)





- A simulation campaign is a sequence $\chi = c_0(a_0) \dots c_k(a_k)$ of commands as above
 - note that k and h are independent
- A χ identifies a sequence w_0, \ldots, w_k s.t., for all $i = 0, \ldots, k-1$, $\sin_{\mathcal{S}}(w_i, c_i(a_i)) = w_{i+1}$ and $w_i = (s_i, u_i, M_i)$
 - by construction, u_i leads from s_0 to s_i
- This also defines the *output sequence* $\operatorname{output}(s_0) \dots \operatorname{output}(s_k)$
- ullet Less strightforward: the *event list sequence* associated to χ
 - watch out: a sequence of lists...
 - $U(\chi) = u_{j_1}, \dots, u_{j_\ell}, u_k$ where ℓ is the number of load commands in χ
 - for $r=1,\ldots,\ell,\,j_r$ is the index of the r-th load command in



Generating Simulation Campaigns: Definitions

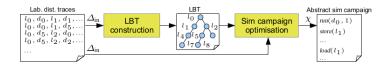
- Let $d \in \mathbb{N}^+$ and L be countably infinite set of labels. A labelling is an injective $\lambda : ([0,d] \cap \mathbb{N})^* \to L$
 - from finite sequence of integers to labels
- The labelling of a disturbance trace $\delta = d_0 \dots d_{h-1}$ is $\lambda(\delta) = l_0 d_0, \dots, h_{h-1} d_{h-1} l_h$
 - for all $i = 0, ..., h, l_i = \lambda(d_0, ..., d_{i-1})$
- Thus, the algorithm for disturbance traces given above returns labelled disturbance traces
- Let us go with the simulation campaign generation







Generating Simulation Campaigns: Idea







- A Labels Branching Tree (LBT) is a DAG where nodes are labels
- There is an edge (I, I') iff $\exists \delta, \delta' \in \Delta_{in}$ s.t.
 - $\delta = l_0, d_0, \ldots, d_{h-1}l_h, \delta' = l'_0, d'_0, \ldots, d'_{h-1}l'_h$
 - $\exists i = 0, \ldots, h-1 : d_i \neq d'_i \forall j = 0, \ldots, i-1. \ l_i = l'_i \land d_i = d'_i$
 - $I = I_{i-1}, I' = I'_i$
 - that is, if there are two traces which differs by (I, I') for the first time, I, I' will be siblings in the LBT
- Branching labels represent simulator states whose storing may save simulation time (by loading them back later)
- The LBT generation keeps into account that memory to store states is limited by m
 - thus, the result is optimal only for at most with the result is optimal.



```
function buildLBT(\Delta^{\lambda})
      LBT \leftarrow empty tree of labels;
      /* for each l \in LBT, LBT[l].lastTrace stores the index of last trace
          where it is known to occur */
      watched \leftarrow empty array [0..h-1] of labels;
24
      let l_0 be the first label common to all traces in \Delta^{\lambda};
25
      set l_0 as the root of LBT with LBT[l_0].lastTrace \leftarrow |\Delta^{\lambda}|;
26
      watched[0] \leftarrow l_0:
27
      i \leftarrow 0:
28
      foreach \delta^{\lambda} = l_0, d_0, \dots, l_{h-1}, d_{h-1}, l_h in \Delta^{\lambda} do
29
        i++; /* \delta^{\lambda} is the i-th trace in \Delta^{\lambda} */
30
        for t \leftarrow 0 to h-1 s.t. l_t \in LBT do LBT[l_t].lastTrace \leftarrow i;
31
        t lbt \leftarrow \max t \text{ s.t. } l_t \in LBT;
32
        t_w \leftarrow \max t \text{ s.t. } l_t \in watched;
33
        if t\_lbt \neq t\_w then
34
          /* label l_{t,w} \not\in LBT: add it */
          t\_child \leftarrow \min t > t\_w \text{ s.t. } watched[t\_child] \in LBT \text{ (if any)};
35
          add l_{t_w} to LBT as child of l_{t_{lbt}} with LBT[l_{t_w}].lastTrace = i;
36
          move l_{t \text{ child}} (if any) as to be child of l_{t \text{ w}} in LBT;
37
        foreach t \leftarrow t_w + 1 to h - 1 do watched[t] \leftarrow l_t;
38
        /* watched now contains labels of the last trace */
```

return LBT:

- Given the LBT \mathcal{L} , the output simulation campaign χ is computed by scanning again Δ_{in}
- For $\delta = l_0, d_0, \ldots, d_{h-1}l_h \in \Delta in$, let r be the higher (i.e., rightmost) index s.t. l_r is in some already generated load command and is in the LBT
- Append to χ first $load(I_r)$ and then one of the following:
 - $\operatorname{inject}(\tilde{d}), \operatorname{run}(t)$ where:
 - in δ there is a subsequence $I_r \tilde{d} I_{r+1} 0 \dots 0 I_{r+t} \hat{d} \hat{I}$
 - $\hat{d} \neq 0$
 - $\operatorname{inject}(\tilde{d}), \operatorname{run}(t), \operatorname{store}(\hat{l})$ where:
 - in δ there is a subsequence $l_r \tilde{d} l_{r+1} 0 \dots 0 l_{r+t} \hat{d} \hat{l}$
 - \hat{l} needs to be stored, i.e., \hat{l} is in the LBT and it will occur again in another $\delta' \in \Delta_{in}$
 - inject(\tilde{d}), run(t), free(\bar{l}), store(\hat{l}) where:
 - if memory is already full, for a suitably chosen





```
Input: \Delta^{\lambda}, a labelled lex-ordered sequence of disturbance traces
    Output: \chi, the computed simulation campaign, initially empty

 LBT ← buildLBT(Δ<sup>λ</sup>);

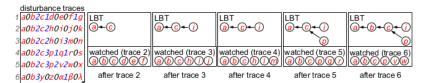
2 let l_0 be the first label common to all traces in \Delta^{\lambda};
 3 stored \leftarrow empty set of labels; /* inv: stored \subseteq LBT and |stored| < h */
 4 append store(l<sub>0</sub>) to χ and add l<sub>0</sub> to stored;
 5 i ← 0:
6 foreach \delta^{\lambda} = l_0, d_0, \dots, l_{h-1}, d_{h-1}, l_h in \Delta^{\lambda} do
      i++; /* \delta^{\lambda} is the i-the trace in \Delta^{\lambda} */
      t \text{ load} \leftarrow \max t \text{ s.t. } l_t \in \text{stored};
      append load(l_{t load}) to \chi;
      foreach label \bar{l} \in stored s.t. LBT[1].lastTrace < i do
10
        append free(l) to \chi;
11
        remove l from stored:
12
      d \leftarrow d_{t \ load}; \quad steps \leftarrow 1;
13
      for t \leftarrow t \ load + 1 \ to \ h - 1 \ do
14
        toBeStored \leftarrow (l_t \in LBT - stored \text{ and } LBT[l_t].lastTrace > i);
15
        if toBeStored or d_t \neq 0 then
16
         append run(\hat{d}, steps) to \chi; \hat{d} \leftarrow d_t; steps \leftarrow 1;
17
         if toBeStored then
18
            append store(l_t) to \chi and add l_t to stored;
19
        else steps++;
20
```

21 return χ ;





Generating Simulation Campaigns: Example



(a)

```
store(a)
load (a) run(0,1) store(b) run(2,1) store(c) run(1,3) run(1,1)
load (c) run(2,2) store(i) run(0,2)
load (i) free(i) run(3,2)
load (c) run(3,1) store(p) run(1,1) run(1,2)
load (p) free(p) free(c) run(2,1) run(2,2)
load (b) free(b) free(a) run(3,3) run(1,2)
```







System Level Formal Verification: Theorem

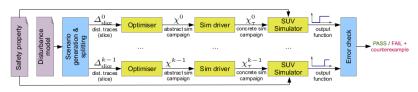
- As a corollary, if an error is present in the specified disturbance traces, our method will find it
- Formally, let $\mathcal{P} = \langle \mathcal{H}, \mathcal{D}, \tau, h \rangle$ be a SLFVP, \mathcal{S} a simulator for \mathcal{H} and $\Delta^h_{\mathcal{D}}$ be the set of all labeled disturbance traces of length h. Let χ be the simulation campaign as computed above.
- Then, the answer to \mathcal{P} is FAIL iff the sequence of simulator states contains (s, u, M) s.t. $\operatorname{output}(s) = 0$
- Thus, our approach is sound (no false positives) and complete (no false negatives)





System Level Formal Verification

For now, suppose k = 1





SUV: Fuel Control System from Simulink; variable fuel_air is never 0 for more than 1s

h	time (h:m:s)	#traces	file size (MB)
50	0:1:35	448,105	195.725
60	0:3:29	805,075	420.743
70	0:6:35	1,314,145	799.584
80	0:11:41	2,002,315	1,390.157
90	0:21:34	2,896,585	2,259.642
100	0:28:39	4,023,955	3,484.489

k	time (h:m:s)	slice size (MB)
2	0:0:14	1,742.244
4	0:0:14	871.122
8	0:0:15	435.561
16		217.78
32	0:0:14	108.89
64	0:0:13	54.445

(a) Disturbance trace generation

(b) Instance h = 100 splitting

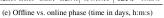
		LBT	n	= 1 m =		100,000	
k	#traces	size	time	#cmds	time	#cmds	%opt
2	2,011,977	670,661	0:3:14	16,040,520	3:47:57	8,047,912	79.42%
4	1,005,988	335,331	0:2:28	8,012,662	1:45:04	4,023,955	83.32%
8	502,994	167,666	0:0:35	4,001,378	0:44:27	2,011,978	86.49%
16	251,497	83,834	0:0:18	1,997,486	0:16:24	1,005,991	88.97%
32	125,748	41,918	0:0:07	996,660	0:4:50	502,996	90.87%
64	62,874	20,959	0:0:03	496,906	0:0:51	251,497	92.47%

(c) Simulation campaign optimisation (h = 100, time in h:m:s)

	m = 1	$m=100,\!000$	
k	time	time	speedup
8	n/a	29, 13:50:12	> 1.7×
16	n/a	14, 6:39:09	>3.5 imes
32	25, 23:07:43	6, 22:32:25	3.8×
64	12 22:58:16	3 9:19:18	3.8×

(d) Simulation (time in days, h:m:s) 'n/a' Simulation aborted after 50 days

		of	fline	online			
k	gener.	split.	optimis.	total	simulation	%offline	%online
8	0:28:39	0:0:15	0:44:27	1:13:21	29, 13:50:12	0.17%	99.83%
					14, 6:39:09		99.78%
					6, 22:32:25		99.66%
64	0:28:39	0:0:13	0:0:51	0:29:43	3, 9:19:18	0.31%	99.69%







Multicore System Level Formal Verification

- If we have multiple processors, we may easily parallelize our computations
 - both with shared (multicore processors) or distributed memory (clusters)
 - also clusters where k nodes have c cores each
 - we will consider K = kc as the overall number of cores available
- To start with, the generation of disturbance traces may be parallelized
 - an "orchestrator" may expand till horizon fT, for some 0 < f < 1
 - ullet and then leave the remaining subtree to a "slave" from the other k-1 cores
- It may be shown that labels are ok
- However, this is not the main part to be improved





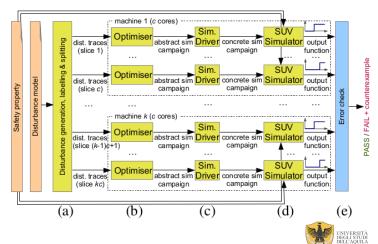
Multicore System Level Formal Verification

- Main advantage is in parallelizing the simulation campaign execution
 - simulation phase dominates the overall verification time
- To this aim, starting from the overall disturbances traces set Δ^h_{calD} , we must generate k simulation campaigns
- The idea is to perform this is 2 steps:
 - ① "slice" Δ_{calD}^h in k equal parts
 - of for each slice, compute the corresponding simulation campaign





System Level Formal Verification





Multicore System Level Formal Verification

- Main advantage is in parallelizing the simulation campaign execution
 - simulation phase dominates the overall verification time
- To this aim, starting from the overall disturbances traces set Δ^h_{CalD} , we must generate k simulation campaigns
- The idea is to perform this is 2 steps:
 - "slice" Δ_{calD}^h in k equal parts
 - all slices have the same length, thus this is easy
 - for each slice, compute the corresponding simulation campaign as before





Multicore System Level Formal Verification

- First slicing and then optimizing is suboptimal
 - optimal would be to detect all maximal prefix of disturbance traces
 - so that they are stored once and then loaded when needed
- If two slices with a common prefix end up in different slices, no way to do this
- However, reading all disturbance traces file requires too computation time
 - easily a file of hundreds of GBs, or even TBs
- Thus, we are happy with a suboptimal solution





Multicore System Level Formal Verification: Results

#slices	#traces per slice	scSLFV optimiser	mcSLFV optimiser	time saving %
1	4,023,955	20:27:26	0:7:16	99.41%
2	2,011,977	3:47:57	0:9:43	95.74%
4	1,005,988	1:45:4	0:9:0	91.43%
8	502,994	0:44:27	0:5:27	87.74%
16	251,497	0:16:24	0:2:8	86.99%
32	125,748	0:4:50	0:0:57	80.34%
64	62,874	0:0:51	0:0:29	43.14%
128	31,437	0:0:35	0:0:17	51.43%
256	15,718	0:0:10	0:0:8	20.00%
512	7,859	0:0:5	0:0:4	20.00%

Table I: Comparison between scSLFV optimiser of [1] and our mcSLFV optimiser (time in h:m:s).

#mach #slices		min	max	avg	$\frac{\text{stddev}}{\text{avg}}\%$	speedup	efficiency
8	64	180:3:0	205:19:57	194:17:52	4.979%	$54.63 \times$	85.35%
16	128	70:6:4	100:17:53	87:49:56	13.772%	$111.56 \times$	87.15%
32	256		57:57:27				
64	512	18:32:36	26:49:4	23:2:19	11.110%	$411.83 \times$	80.43%

Table II: Statistics on the distributed (k = #mach(ines)) multi-core (c = 8) execution of simulation campaigns (time in h:m:s).





Multicore System Level Formal Verification: Results

	sc	scSLFV		SLFV	
#machines	#slices	time	#slices	time	time saving %
8	8	711:3:33	64	205:49:20	71.05%
16	16	343:24:27	128	100:47:4	70.65%
32	32	167:6:9	256	58:26:29	65.03%
64	64	81:49:3	512	27:18:2	66.63%

Table III: Completion time of the parallel simulation (i.e., completion time of the *longest* campaign) with respect to the approach of [1] (time in h:m:s).





Anytime System Level Formal Verification

- Suppose we have the K simulation campaigns and we are performing the verification phase
- Can we do something better than simply wait for it to finish?
 - as an example: in SAT, there are methodologies computing the coverage achieved so far
 - at "anytime" we can get an estimate of such coverage
- Here we are not interested simply in coverage: we want the Omission Probability (OP)
 - i.e., we want an an upper bound to the probability that there is an error in a yet-to-be-simulated scenario
 - to be provided at any time, during the simulation phase







Anytime System Level Formal Verification

- Main difficulty: optimization comes from lexicographically ordered $\Delta^h_{\mathcal{D}}$
- In order to enable some kind of probability on traces, we need random permutations of $\Delta^h_{\mathcal{D}}$
- How to obtain this? see in the following



```
Input: \Delta^{\lambda}, a labelled lex-ordered sequence of disturbance traces
    Output: \chi, the computed simulation campaign, initially empty

 LBT ← buildLBT(Δ<sup>λ</sup>);

2 let l_0 be the first label common to all traces in \Delta^{\lambda};
 3 stored \leftarrow empty set of labels; /* inv: stored \subseteq LBT and |stored| < h */
 4 append store(l<sub>0</sub>) to χ and add l<sub>0</sub> to stored;
 5 i ← 0;
6 foreach \delta^{\lambda} = l_0, d_0, \dots, l_{h-1}, d_{h-1}, l_h in \Delta^{\lambda} do
      i++; /* \delta^{\lambda} is the i-the trace in \Delta^{\lambda} */
      t \text{ load} \leftarrow \max t \text{ s.t. } l_t \in \text{stored};
      append load(l_{t load}) to \chi;
      foreach label \bar{l} \in \text{stored s.t. LBT[1].lastTrace} < i do
10
        append free(l) to \chi;
11
        remove l from stored:
12
      d \leftarrow d_{t \ load}; \quad steps \leftarrow 1;
13
      for t \leftarrow t \ load + 1 \ to \ h - 1 \ do
14
        toBeStored \leftarrow (l_t \in LBT - stored \text{ and } LBT[l_t].lastTrace > i);
15
        if toBeStored or d_t \neq 0 then
16
          append run(\hat{d}, steps) to \chi; \hat{d} \leftarrow d_t; steps \leftarrow 1;
17
          if toBeStored then
18
            append store(l_t) to \chi and add l_t to stored;
19
        else steps++;
20
```

21 return χ ;







Anytime System Level Formal Verification: Algorithm

Algorithm 1: Optimiser pseudo-code

Input: Δ^{λ} , a file holding a labelled lex-ordered sequence of disturbance traces

Output: χ , the computed simulation campaign

```
1 \chi \leftarrow an empty sequence of commands;
```

- 2 $LBT \leftarrow buildLBT(\Delta^{\lambda});$
- 3 $\Delta_{md}^{\lambda} \leftarrow rsg(\Delta^{\lambda});$
- 4 lastTraces ← a map associating to each label l ∈ LBT the index of the last trace in Δ^λ_{md} where l occurs;
- 5 stored \leftarrow empty set of labels; /* invariant: stored \subseteq LBT */
- 6 l_0 ← first label common to all traces:
- 7 append $store(l_0)$ to χ ;
- s stored \leftarrow stored \cup { l_0 };
- 9 foreach δ^{λ} in Δ^{λ}_{rnd} do
- 10 $l_{load} \leftarrow \text{right-most label of } \delta^{\lambda} \text{ in stored};$
- 11 append $load(l_{load})$ to χ ;
- append free(l) to χ for each label $l \in stored$ which will never occur in later traces (according to lastTraces);
- append to χ commands to simulate δ^{λ} (from l_{load}) and to store any intermediate states needed to speed-up simulation of later traces;



Anytime System Level Formal Verification: Example

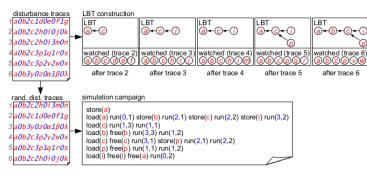


Fig. 5: Simulation campaign optimiser: construction of an LBT from 6 labelled traces in lex order, random sequence generation, and generation of the optimised campaign. Labels are shown as *red letters* and disturbances as *blue numbers*.



- For a finite set $\Delta = \{\delta_0, \dots, \delta_{n-1}\}$, if denote $\operatorname{Perm}(\Delta)$ as the set of all permutations of $\delta \in Delta$
 - i.e., $\operatorname{Perm}(\Delta) = \{(\delta_{\pi(0)}, \dots, \delta_{\pi(n-1)}) \mid \pi : [0, n-1] \cap \mathbb{N} \to [0, n-1] \cap \mathbb{N} \text{ and } \pi \text{ is injective}\}$
 - for a $\hat{\Delta} = (\delta_0, \dots, \delta_{n-1}) \in \operatorname{Perm}(\Delta)$, we write $\hat{\Delta}(i)$ for δ_i
 - \bullet recall that, in our setting, each δ is a disturbance sequence
- A Random Sequence Generator (RSG) for Δ is a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ s.t.:
 - $\Omega = \operatorname{Perm}(\Delta)$ is the space of outcomes
 - ${}_{\bullet}$ ${\cal F}=2^{\Omega}$ is the space of events
 - ullet ${f P}: {\cal F}
 ightarrow [0,1]$ is the probability measure
 - in our setting, **P** is uniform, thus $P(\{\omega\}) = P(\omega) = |Perm(\Delta)|^{-1} = (|\Delta|!)^{-1}$
 - being $|\Omega| < \infty$, $\forall E \in \mathcal{F}$. $\mathbf{P}(E) = \sum_{\omega \in E} \mathbf{P}(\omega)$





- Let $\langle \mathcal{H}, \mathcal{D}, h, \tau \rangle$ be a SLFVP, let Δ be a set of disturbance traces and $(\Omega, \mathcal{F}, \mathbf{P})$ be an RSG for Δ .
- Furthermore, let $0 \le q \le |\Delta|$ be the current progress with the verification.
 - ullet that is, we already simulated q out of $|\Delta|$ disturbance traces
- Then, the Omission Probability for Δ at stage q, denoted as $\mathrm{OP}_{\mathcal{H}}(\Delta,q)$ is defined as $\mathbf{P}(\{\omega\mid A(\omega,q)\wedge B(\omega,q)\})$
 - $A(\omega, q) \equiv [\exists q < j \le |\Delta| : \psi(\omega(j), h\tau)] = 0$
 - $B(\omega, q) \equiv [\forall 0 \le j \le q : \psi(\omega(j), h\tau)] = 1$
 - A stands for "after", B stands for "before"





Anytime System Level Formal Verification: Theorem

- Let $\langle \mathcal{H}, \mathcal{D}, h, \tau \rangle$ be a SLFVP, let Δ be a set of disturbance traces and $(\Omega, \mathcal{F}, \mathbf{P})$ be an RSG for Δ . Furthermore, let $0 \le q \le |\Delta|$ be the current progress with the verification.
- Then, $\mathrm{OP}_{\mathcal{H}}(\Delta,q) \leq 1 \frac{q}{|\Delta|}$
 - ullet at the end of the verification, $q=|\Delta|...$
- ullet The previous definitions and this theorem are generalizable to k slices of Δ
- That is, $\operatorname{OP}_{\mathcal{H}}(\Delta_0,\ldots,\Delta_{k-1},q_0,\ldots,q_{k-1}) \leq 1 \min_{1 \leq i < k} \frac{q_i}{|\Delta_i|}$
 - being the k parallel verifications independent, all q_i may be different
 - taking the minimum means considering the worst case





Anytime System Level Formal Verification: Results

We pay the OP computation in terms of performance degradation

				-				•			
#slices	#traces	dSLFV	rSLFV	#mach.	#slices	min	max	avg	speedup	efficiency	approach
	per slice	optimiser	optimiser	16	128	70:6:4	100:17:53	87:49:56	111.56×	87.15%	dSLFV
1	4,023,955		0:35:35			216:42:13	348:51:47	308:46:18	39.17×	30.60%	rSLFV
2	2,011,977		0:16:33		ĺ	+209.13%	+247.83%	+251.55%	+64.89%	+56.55%	overhead
4	1,005,988	0:9:0	0:8:37						1 .		
8	502,994	0:5:27	0:3:42	32	256	44:0:27	57:57:27	48:34:6	192.38×	75.15%	dSLFV
16	251,497	0:2:8	0:2:51			63:53:54	136:18:14	108:14:19	100.03×	39.08%	rSLFV
32	125,748	0:0:57	0:2:36		i	+45.20%	+135.18%	+122.86%	+48.00%	+36.07%	overhead
64	62,874	0:0:29	0:1:21						1.1		
128	31,437	0:0:17	0:1:44	64	512	18:32:36	26:49:4	23:2:19	411.83×	80.43%	dSLFV
256	15,718	0:0:8	0:0:42			22:9:19	29:23:33	26:43:31	458.01×	89.46%	rSLFV
512	7,859	0:0:4	0:0:13		i	+19.48%	+9.60%	+16.00%	-11.21%	-9.03%	overhead

(a) Computation of simulation campaigns (time in h:m:s)

(b) Parallel execution of simulation campaigns by dSLFV and rSLFV (time in h:m:s)

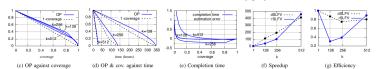


Fig. 6: Experimental results







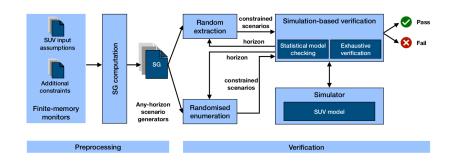
System Level Formal Verification: Enhancements

- Main drawbacks for the method seen so far:
 - need of a huge file holding all disturbance traces
 - to be doubled with slicing
 - CMurphi may be not easily used by testing engineers
 - preprocessing is computationally heavy
- Let us see how we can overcome such points





System Level Formal Verification: New Architecture







- Terminology (system contract): assumptions for inputs, guarantees for outputs
 - if the SUV is fed with inputs satisfying the assumptions...
 - ...then it must provide outputs satisfying the guarantees
- Terminology: monitors for assumptions
 - takes an input sequence, and rejects it if violates assumptions
 - assumptions are typically time-unbounded, but a monitor must be an algorithm with finite memory
 - ullet on the other hand, \mathbb{U}_V is finite
 - that is, we have a finite set of disturbances
 - for continuous disturbances, a discretization is required





- We have a finite set $\mathbb{V} = \{v_1, \dots, v_n\}$
 - each v_i is an input variable
 - may have different domains: values (assignments) for v_i are $u \in \mathbb{U}_{v_i}$
 - for $V \subseteq \mathbb{V}$, $\mathbb{U}_V = \times_{v \in V} \mathbb{U}_v$
 - for $u \in \mathbb{U}_V$ and $V' \subseteq$, $w = u_{V'} \in \mathbb{U}_{V'}$ is s.t. $u_v = w_v$ for $v \in V'$ and $w_v = \bot$ otherwise
- At time t, an assignment is provided for all $v \in \mathbb{V}$ (input time functions)





- A monitor is a Finite State Machine (FSM) $\mathcal{M} = (V, X, x_0, f)$ where:
 - V is the set of input variables as above
 - ullet \mathbb{U}_V is the monitor input space
 - X is a finite set of monitor states, $x_0 \in X$ being the initial one
 - $f: X \times \mathbb{U}_V \to X$ is the monitor transition function
 - possibly partial: if it does not result in an infinite path, it is violating the assumptions
- A *trace* is an infinite sequence $(u_0, u_1, ...)$ s.t.
 - each u_i is an assignment to variables in V (i.e., $u_i \in \mathbb{U}_V$)
 - there is an infinite path $x_0u_0x_1u_1...$ in $\mathcal M$
- $Traces(\mathcal{M})$ is the set of all (infinite) traces
- Traces $|_h(\mathcal{M})$ is the set of all prefixes of length $h \in \mathbb{N}$ of some trace in Traces (\mathcal{M})



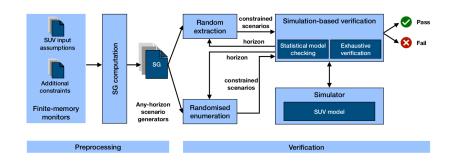
- Systems (and their contracts) may be discrete-time or continuous-time
 - ullet in the former case, we have $\mathbb{T}=\mathbb{N},$ in the latter, $\mathbb{T}=\mathbb{R}$
- Provided that we choose a time-step $\tau \in \mathbb{T}^+$, a may be used for both
 - typically, for discrete-time systems, au >> 1, whilst for continuous-time systems au << 1
- In fact, a trace u_0, u_1, \ldots of a monitor $\mathcal M$ may be translated in an input time function $u(t) = u_{|\tau^{-1}|}$
- For our purposes, monitors may also be black-box: it is sufficient we may repeatedly invoke f
- Note that monitors behave like supervisory controllers



- Suppose we have two monitors $\mathcal{M}_1, \mathcal{M}_2$ with possibly overlapping input variables. The *conjoint monitor* $\mathcal{M} = \mathcal{M}_1 \bowtie \mathcal{M}_2$ is a monitor s.t.
 - $V = V_1 \cup V_2$
 - $X = X_1 \times X_2$, $x_0 = (x_{0,1}, x_{0,2})$
 - $f = f_1 \bowtie f_2$ s.t. $f((x_1, x_2), u) = (f_1(x_1, u|_{V_1}), f_2(x_2, u|_{V_2}))$ if both components are defined
 - the formula holds $\forall x_1 \in X_1, x_2 \in X_2, u \in \mathbb{U}_{V_1 \cup V_2}$
- Note that, for each $(u_0, u_1, \ldots) \in \operatorname{Traces}(\mathcal{M})$, we have that $(u_0|_{V_1}, u_1|_{V_1}, \ldots) \in \operatorname{Traces}(\mathcal{M}_1)$ and $(u_0|_{V_2}, u_1|_{V_2}, \ldots) \in \operatorname{Traces}(\mathcal{M}_2)$
- This allows to define monitors basing on sub-monitors (compositional modeling)
 - e.g., assumptions may be implemented conjoining monitors on separate subsets of variables...
 - ... and then monitors for additional constraints on wider variables subsets



System Level Formal Verification: New Architecture







System Level Formal Verification: Algorithms

- Let us go towards the verification phase: as all black-box approaches, it will be with a finite horizon
- We have monitors which considers disturbance traces of infinite length
- For verification purposes, we need to extract all prefixes with a given length h
 - the verification may be carried out either exhaustively or by statistical model checking
 - thus, extraction must be possible also in a random way
- As usual, a uniform time step for actual verification is added afterwards
- We want to perform this "online", without storing all traces in a file
 - essentially, monitors are a way to compactly the essentially disturbance traces



System Level Formal Verification: Algorithms

- It is sufficient to provide two functions:
 - nb_traces: $\mathbb{N} \to \mathbb{N}$
 - given h, overall number of disturbance traces of length h accepted by the monitor
 - trace: $\mathbb{N} \times \mathbb{N} \to \mathbb{U}_V^*$
 - given h and an index 1 ≤ i ≤ h, the i-th disturbance trace of length h accepted by the monitor
 - lexicographic order: for a random enumeration, simply extract at random i
- We will show an implementation with time:
 - $O(|\mathbb{U}_V| \cdot |X|^2)$ for initialization
 - O(1) for each subsequent nb_traces call
 - $O(h \log \mathbb{U}_V)$ for each subsequent trace call







- The monitor defined by testing engineers may contain finite paths
 - corresponding to non-legal disturbance sequences
 - note that a finite path of length h+1 is not to be considered when performing verification with horizon h...
- This is ok for modeling purposes, but we want to get rid of this for the computation
- Thus, we define a new monitor which discards finite paths
 - retaining infinite ones
 - and not introducing other (spurious) paths, of course





• Let $\mathcal{M} = \langle V, X, x_0, f \rangle$ be a monitor. The safe state function $\Phi_f : X \to \{0, 1\}$ is defined as the greatest fixed point of

$$\Phi_f(x) \equiv [\exists u, x' : x' = f(x, u) \land \Phi_f(x')]$$

- easier if seen backwards: first, all states such that $\forall u. \ f(x,u) = \bot$ are s.t. $\Phi_f(x) = 0$
 - deadlock states
- then, for all other states x, which *only* goes in x' s.t. $\Phi_f(x') = 0$, we have $\Phi_f(x) = 0$ as well
 - that is, if $\forall u$. $\Phi_f(f(x, u)) = 0$, then $\Phi_f(x) = 0$
- for all other states x, $\Phi_f(x) = 1$
- A state $x \in X$ is *safe* for \mathcal{M} iff $\Phi_f(x)$ holds
 - all paths starting from x are of infinite length





- Let $\mathcal{M} = \langle V, X, x_0, f \rangle$ be a monitor. The *Scenario Generator* (SG) of \mathcal{M} is a monitor $\operatorname{Gen}(\mathcal{M}) = \langle V, X, x_0, f_{\operatorname{gen}} \rangle$ s.t. $f_{\operatorname{gen}}(x,u) = f(x_u)$ if $\Phi_f(f(x,u)) = 1$ and $f_{\operatorname{gen}}(x,u) = \bot$ otherwise
 - i.e., we remove transitions towards non-safe states
 - by theorems on fixed points, a SG always exists and it is unique
 - may not contain any transition...
 - ullet using controller theory parlance, the scenario generator is the most liberal supervisory controller for ${\cal M}$
- Given \mathcal{M} , $\operatorname{Gen}(\mathcal{M})$ can be computed in time $O(|\mathbb{U}_V| \cdot |X|^2)$





- Monitors may be accessed as black-box code, provided that they:
 - provide functions to get and set the current internal state
 - as some possibly non-interpretable bytes sequence
 - start from some initial internal state
 - provide a function which, given the current internal state, returns the list of admissible actions
 - provide a function which, given the current internal state and an admissible action, changes its internal state
 - provide a function which, given an action, provide a possibly non-interpretable encoding for such action
- As an example, this is easy to do with Python





- Let $\mathcal{M} = \langle V, X, x_0, f \rangle$ be a monitor and $\operatorname{Gen}(\mathcal{M}) = \langle V, X, x_0, f_{\operatorname{gen}} \rangle$ be its SG. Then:
 - \bullet each finite path in $\operatorname{Gen}(\mathcal{M})$ may be extensible to an infinite path
 - otherwise phrased: the last state of the path always has at least one successor state
 - non-blocking property
 - $\operatorname{traces}(\mathcal{M}) = \operatorname{traces}(\operatorname{Gen}\mathcal{M})$
 - recall that "traces" mean an infinite sequence...
- Such properties follow directly from the definition





- Let $\mathcal{M}_1, \mathcal{M}_2$ be two monitors. Then:
 - $\operatorname{Gen}(\mathcal{M}_1) = \operatorname{Gen}(\operatorname{Gen}(\mathcal{M}_1))$
 - blocking paths only need to be removed once
 - if $V_1 \cap V_2 = \emptyset$, then $\operatorname{Gen}(\mathcal{M}_1 \bowtie \mathcal{M}_2) = \operatorname{Gen}(\mathcal{M}_1) \bowtie \operatorname{Gen}(\mathcal{M}_2)$
 - ullet i.e., if $\mathcal{M}_1, \mathcal{M}_2$ are independent monitors
 - if there is some common variable, then \mathcal{M}_1 could restrict something which is allowed in \mathcal{M}_2 , thus...
 - $\operatorname{Gen}(\mathcal{M}_1 \bowtie \mathcal{M}_2) = \operatorname{Gen}(\operatorname{Gen}(\mathcal{M}_1) \bowtie \operatorname{Gen}(\mathcal{M}_2))$
 - general case
- Such properties allow incremental combination of monitors





- The following is needed to compute nb_traces and trace
- Let $\mathcal{M} = \langle V, X, x_0, f \rangle$ be a monitor and $\operatorname{Gen}(\mathcal{M}) = \langle V, X, x_0, f_{\operatorname{gen}} \rangle$ be its SG. Then:
 - $\operatorname{ext}: X \times \mathbb{N} \to \mathbb{N}$ is s.t.
 - $\operatorname{ext}(x,0) = 1$ for all $x \in X$
 - $\operatorname{ext}(x,k) = \sum_{u \in \mathbb{U}_V} \operatorname{ext}(f_{\operatorname{gen}}(x,u),k-1)$ for all $x \in X, k \in \mathbb{N}^+$
 - of course, $\operatorname{ext}(\bot, k) = 0$ for all $k \in \mathbb{N}$
 - ext(x, k) = #all distinct paths of length k starting from x
 - $\xi: X \times \mathbb{U}_V \times \mathbb{N} \to \mathbb{N}$ is s.t., for all $x \in X$, $u \in \mathbb{U}_V$, $k \in \mathbb{N}$, $\xi(x, u, k) = \sum_{\hat{u} < u} \operatorname{ext}(f_{\text{gen}}(x, \hat{u}), k)$
 - of course, some ordering is required in each \mathbb{U}_{ν} , so we can take the lexicographic one for \mathbb{U}_{V}
 - $\xi(x, u, k) = \#$ distinct paths of length k starting from x with some action preceding u

return $ext(x_0, h)$;

10

```
1 global
   Gen(\mathcal{M}) = (V, X, x_0, f_{gen});
    h_{\max} \in \mathbb{N} \cup \{\text{undef}\}, initially undef;
   ext, a map of the form X \times \mathbb{N} \to \mathbb{N}, initially empty;
5 \xi, a map of the form X \times \mathbb{U}_V \times \mathbb{N} \to \mathbb{N}, init. empty;
    // Invariant: ext(x,h) & \xi(x,u,h) defined iff h \leq h_{max}
6 function nb_traces(h)
    Input: h \in \mathbb{N}
   if h_{max} = undef or h > h_{max} then
      incrementally compute ext and \xi up to h;
8
      h_{max} \leftarrow h;
```



```
11 function trace(i, h)
     Input: i \in \mathbb{N}, h \in \mathbb{N}
     Output: (u_0, u_1, u_2, \dots u_{h-1}), i-th trace of len. h
     if i \ge nb\_traces(h) then error index out of bounds;
12
     x \leftarrow x_0; k \leftarrow h; m \leftarrow i;
13
     for j from 0 to h-1 do
14
       u_i \leftarrow \max \{u \mid \xi(x, u, k-1) \leq m\};
15
       m \leftarrow m - \xi(x, u_i, k - 1);
16
       x \leftarrow f_{\text{gen}}(x, u_i);
17
       k \leftarrow k-1:
18
     return (u_0, u_1, u_2, \dots u_{h-1});
19
```

- The above algorithms are correct, that is the following holds
- Let $\mathcal{M} = \langle V, X, x_0, f \rangle$ be a monitor and $\operatorname{Gen}(\mathcal{M}) = \langle V, X, x_0, f_{\operatorname{gen}} \rangle$ be its SG. Then:
 - for all $h \in \mathbb{N}$, $\operatorname{nb_traces}(h) = \operatorname{card}(\operatorname{traces}(\operatorname{Gen}(\mathcal{M}))|_h)$
 - for all $h \in \mathbb{N}$, $i \in [0, \text{nb_traces}(h) 1] \cap \mathbb{N}$, trace(i, h) returns the i-th element of $\text{traces}(\text{Gen}(\mathcal{M}))|_h$
 - lexicographic order





- Let $\mathcal{M}_1, \mathcal{M}_2$ be two independent monitors. Then, for all $h \in \mathbb{N}$:
 - $\operatorname{nb_traces}_{\mathcal{M}_1 \bowtie \mathcal{M}_2}(h) = \operatorname{nb_traces}_{\mathcal{M}_1}(h) \operatorname{nb_traces}_{\mathcal{M}_2}(h)$
 - for all $i \in [0, \text{nb_traces}_{\mathcal{M}_1 \bowtie \mathcal{M}_2}(h) 1] \cap \mathbb{N}$, $\text{trace}_{\mathcal{M}_1 \bowtie \mathcal{M}_2}(i, h) = \text{trace}_{\mathcal{M}_1}(\text{sel}(i, 1), h) \cdot \text{trace}_{\mathcal{M}_2}(\text{sel}(i, 2), h)$, where:
 - $\operatorname{sel}(i,1) = \left\lfloor \frac{i}{\operatorname{nb_traces}_{\mathcal{M}_2}(h)} \right\rfloor$
 - $\operatorname{sel}(i,2) = i \mod \operatorname{nb_traces}_{\mathcal{M}_2}(h)$

 $((u_{0,1}, u_{0,2}), \ldots, (u_{h-1,1}, u_{h-1,2}))$

- operator \cdot is the pairing of two traces: $(u_{0,1},\ldots,u_{h-1,1})\cdot(u_{0,2},\ldots,u_{h-1,2})=$
- This means that we may compute $\operatorname{nb_traces}_{\mathcal{M}_1\bowtie\mathcal{M}_2}(h)$ and $\operatorname{trace}_{\mathcal{M}_1\bowtie\mathcal{M}_2}(h)$ without computing $\mathcal{M}_1\bowtie\mathcal{M}_2$
 - only the (typically much smaller) $\mathcal{M}_1, \mathcal{M}_2$ are required (separately)



- Fuel control system (FCS): classical example from Simulink distribution
 - also used in papers for Statistical Model Checking
- Controller for a fault tolerant gasoline engine
 - goal: keep the air-fuel ratio close to 14.6
 - that is, a stoichiometric ratio representing a good compromise between power, fuel economy and emissions
 - air-fuel ratio is between the air mass flow rate pumped from the intake manifold and the fuel mass flow rate injected at the valves
- Experiment scenario: a full set of disturbance traces to be verified



FCS Experiment Scenarios

- For FCS, we are interested in its 4 sensors:
 - throttle angle, speed, residual oxygen in exhaust gas (EGO) and manifold absolute pressure (MAP)
- All of them may fail
 - fortunately, they are typically repaired (i.e., restarted) within a few seconds
- FCS is expected to withstand one failure at a time
 - by compensating with internal commands
- From the verification point of view, we want to exercise the system with multiple (non-contemporary) failures and repairs





FCS Experiment Scenarios

- Base assumptions, which are valid for all experiment scenarios:
 - each of the four sensor may fail at any time
 - each sensor, once failed, is repaired within a given time: 3–5 (throttle), 5–7 (speed), 10–15 (EGO), 13–17 (MAP)
 - but for each time, only one sensor may be in "failed" state
 - e.g., if in a disturbance trace throttle fails at step 1 and is repaired at time 4, there cannot be any other failure in [1, 4]
- If we have separate monitors for each sensor, many non-valid traces can be generated
 - to be discarded when computing the SG of the conjoint monitor also considering the above assumptions
- However, here it is easier to implement all such constraints within one monitor
- Experiment scenarios are obtained by adding or more monitors (i.e., constraints) from the following table



FCS Experiment Scenarios

constraint monitor	description
1	Each sensor will fail every 15–20 t.u.
2	Whenever a fault on the throttle sensor occurs, a fault on the speed sensor will occur within 9–11 t.u.
3	Whenever a fault on the throttle sensor occurs, a fault on the speed sensor will occur within 13–15 t.u.
4	Whenever a fault on the throttle sensor occurs, a fault on the speed sensor will occur within 18 or 19 t.u.
5	Whenever a fault on the EGO sensor occurs, a fault on the MAP sensor will occur within 16 or 17 t.u.
6	Whenever a fault on the EGO sensor occurs, a fault on the MAP sensor will occur within 20 or 21 t.u.





- Buck DC/DC Converter: another classical example used in litarature
 - also used in papers for controllers generation
- Mixed-mode analog circuit converting the DC input voltage V_i to a desired DC output voltage V_o
 - e.g., used inside laptop battery
 - ullet to do this, it is equipped with a microcontroller activating a switch u
 - to react to changes in the input voltage and other parameters (e.g., the load R)





Buck DC/DC Converter Experiment Scenarios

- We are interested in the following two parameters: V_i and R
 - disturbances act by modifying the parameter value
 - in an bounded way: it may be modified so as to take values in a *n*-steps discretized interval [m, M], i.e., $\{m+is \mid i=0,\ldots,n-1 \land s=\frac{M-n}{n}\}$
 - we have n = 12 for V_i and n = 6 for R
 - for both V_i and R, [m, M] is the corresponding nominal range: [70, 130]V for V_i and $[70, 130]\Omega$ for R
- Base assumptions: the changes as above and
 - no changes for the first 2 steps
 - once a change is made, do not modify further for the following 6 steps for V_i and 5 steps for R





Buck DC/DC Converter Experiment Scenarios

- Differently from FCS, buck actually has two independent monitors
 - one for V_i and one for R
- As discussed before, they can be computed separately and then conjoined in the "easy" way
- Experiment scenarios are obtained by adding one or more monitors (i.e., constraints) from the following table

Buck DC/DC Converter Experiment Scenarios

constraint monitor	description
1	V_i changes at least every 6 t.u.
2	V_i changes at least every 7 t.u.
3	R changes at least every 5 t.u.
4	R changes at least every 6 t.u.
5	V_i and R do not change simultaneously
6	Whenever V_i changes, R will change after 8 or 9 t.u.
7	Whenever V_i changes, R will change after 2 t.u.

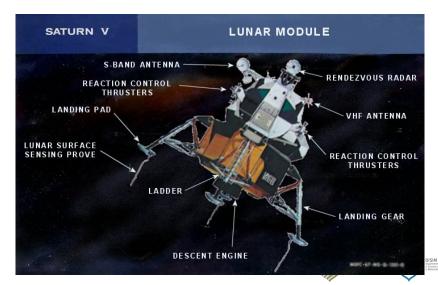


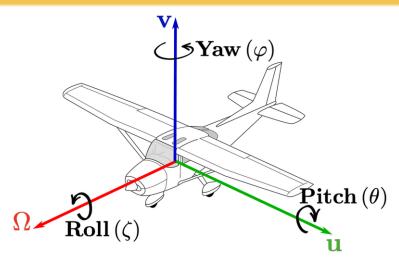


- Apollo: other classical example from Simulink distribution
- Phase-plane controller for the autopilot of the LEM (Lunar Excursion Module) in the Apollo 11 mission
 - goal: given a request to change attitude, actuate jets so as to achieve it
- 3 sensors and 16 jets
 - sensors detect the attitude of the module: yaw, roll and pitch
 - jets to change the attitude













Apollo Experiment Scenarios

- We disturb both the sensors and the jets
- Sensors are disturbed in 6 possible ways
 - for our purposes, a number from 1 to 6
 - such number is then translated at verification time in a one of 6 predefined continuous-time signal noise
- Jets may become unavailable for 2 or 3 time units
 - control will have to compensate...
- External request of change attitude may be any in any of the 3 directions
 - only 3 values: $\{-1, 0, 1\}$
 - no requests undoing the immediately preceding one
- Experiment scenarios are obtained by adding one or more monitors (i.e., constraints) from the following table

Apollo Experiment Scenarios

constraint monitor	description			
1	Only jets number 15 and 16 may be temporarily unavailable			
2	Whenever a jet is actuated for 2 consecutive t.u., it will certainly become unavailable within 3 or 4 t.u.			
3	At most 1 jet is unavailable at any time			
4	Rotation requests regard at most 1 axis each			
5	Rotation requests regard at most 2 axes each			
6	Noise signal changes for at most 1 sensor at any time			
7	Noise signal for each sensor remains stable for at least 5 and at most 10 t.u. and changes by ± 1 position in the given order			

Results for Generating SGs

SUV	SG nb.		\mathcal{M}		Gen(M)
		assumptions monitor	constraint monitors	size of input space	time [s]
FCS	1	\mathcal{A}_{FCS}	-	6	0.1
	2	$\mathcal{A}_{ ext{FCS}}$	1	6	7.99
		$\mathcal{A}_{ ext{FCS}}$	1, 3	6	4.92
	4	$\mathcal{A}_{ ext{FCS}}$	1, 2	6	4.61
	5	$\mathcal{A}_{ ext{FCS}}$	1, 4	6	6.34
	6	$\mathcal{A}_{ extsf{FCS}}$	1, 4, 5	6	5.92
	7	$\mathcal{A}_{ ext{FCS}}$	1, 4, 6	6	6.55
BDC	1	A_i	_	5	0.19
	2	A_R	-	5	0.17
	3	$A_i \bowtie A_R$	-	25	0.36
	4	A_i	1	5	0.12
	5	A_i	2	5 5 5	0.17
	6	\mathcal{A}_R	3	5	0.11
	7	\mathcal{A}_R	4	5	0.16
	8	$A_i \bowtie A_R$	5	25	37.34
	9	$A_i \bowtie A_R$	2, 4, 5	25	29.68
	10	$A_i \bowtie A_R$	2, 4, 5, 6	25	1.94
	11	$A_i \bowtie A_R$	1, 3, 5, 7	25	2.16
ALMA	1	$\mathcal{A}_{\mathrm{rj}}$	-	1769472	0.44
	2	$\mathcal{A}_{ m rj}$	1	108	0.44
		\mathcal{A}_{rj}	1, 2	108	448.88
	4	\mathcal{A}_{rj}	1, 2, 3	108	247.27
	5	$\mathcal{A}_{\mathrm{rj}}$	1, 2, 3, 4	108	55.19
	6	$\mathcal{A}_{\mathrm{rj}}$	1, 2, 3, 5	108	188.3
	7	\mathcal{A}_{s}	-	27	2.94
	8	\mathcal{A}_{s}	6	27	1.33
	9	\mathcal{A}_{s}	6,7	27	782.2
	10	$\mathcal{A}_{ ext{ALMA}}$	1, 2, 3, 4, 6, 7	2916	837.39



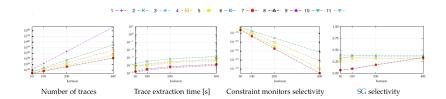


- For each case study, we show, as a function of some meaningful values of the verification horizon h:
 - the number returned by $nb_traces(h)$, i.e., the overall number of traces fulfilling the given monitors
 - trace extraction time: computation time, in seconds, to compute trace(i, h)
 - 1000 values for i are chosen in a uniformly random way in $[0, \text{nb_traces}(h) - 1]$
 - the average value for the computation time is then shown
 - this allows to amortize computation of ext, ξ
 - selectivity of monitors: #traces with all constraints #traces with base assumptions
 - having tiny values shows SGs selects important experiments scenario
 - errors, if any, are discovered first
 - selectivity of SGs: #traces with Gen(M) #traces with M
 - at the denominator, we consider possibly blocking the., non-valid) traces





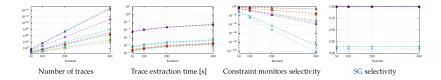
Experimental Results for FCS







Experimental Results for Buck







Experimental Results for Apollo

