

# Software Testing and Validation

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Corso di Laurea in Informatica

## Logics in Model Checking

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# Beyond Invariants

- Invariants represent a huge share of properties to be verified on a system
- For many systems, one may be happy with invariants only
  - “nothing bad happens”, that’s all folks
- However, it is not always sufficient: a non-running system of course satisfies invariants
  - no starting states, thus no reachable states...



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# Safety vs. Liveness

- **Safety** properties: something bad must never happen
  - example: in the Peterson's protocol, it must not happen that both processes are accessing the resource (L3 in the Murphi model)
- Invariants are a special case of safety properties
  - there are some safety properties which are not invariants
  - however, they can be expressed with invariants by adding variables to the Kripke Structure
  - in the following, we will consider "invariants" and "safety properties" as synonyms
- **Liveness** properties: something good will eventually happen
  - example: in the Peterson's protocol, both processes will eventually access the resource
  - not at the same time!
  - cannot be expressed with invariants



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# Safety vs. Liveness

- Notation: let  $\mathcal{S}$  be a KS and  $\varphi$  be a formula in any logic
- $\mathcal{S} \models \varphi$  is true iff  $\varphi$  is true in  $\mathcal{S}$ 
  - what this means depends on the logic, as we will see
- For most properties  $\varphi$ , if  $\mathcal{S} \not\models \varphi$  then there exists a path  $\pi \in \text{Path}(\mathcal{S})$  which is a *counterexample*
- For safety properties,  $|\pi| < \infty$ 
  - $\mathcal{S}$  arrives to an *unsafe* state and that's it
- For liveness properties,  $|\pi| = \infty$ 
  - since  $\mathcal{S}$  is finite, this implies that  $\pi$  contains a loop (*lasso*) in its final part



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# Safety vs. Liveness

- Equivalent definition for a safety formula: given a finite counterexample, every extension still contains the error
- There is one formula which is both safety and liveness: the true invariant
  - it cannot have a counterexample...
- There are formulas which are neither safety nor liveness
  - their counterexample is not a path
- For typically used formulas, they are either safety or liveness properties



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# Safety vs. Liveness: Mathematical Definition

- Let a *model*  $\sigma$  be an infinite sequence of truth assignments to all  $p \in AP$ 
  - $\sigma \in (2^{AP})^\omega$
  - could also be seen as a sequence of sets  $P \subseteq AP$
  - given a path  $\pi$  of a KS  $\mathcal{S}$ , we can always obtain a model from  $\pi$  by replacing each  $\pi(i)$  with  $L(\pi(i))$
- It is possible to define if  $\sigma \models \varphi$ , for a given formula  $\varphi$
- $\varphi$  is a safety property if, for all  $\sigma$  s.t.  $\sigma \not\models \varphi$ , there exists  $j$  s.t.  $\forall \sigma'. \sigma|_j = \sigma'|_j \rightarrow \sigma' \not\models \varphi$
- $\varphi$  is a liveness property if, for each prefix  $w_0 \dots w_i$ , there exists  $\sigma$  s.t.  $\sigma|_i = w_0 \dots w_i$  and  $\sigma \models \varphi$



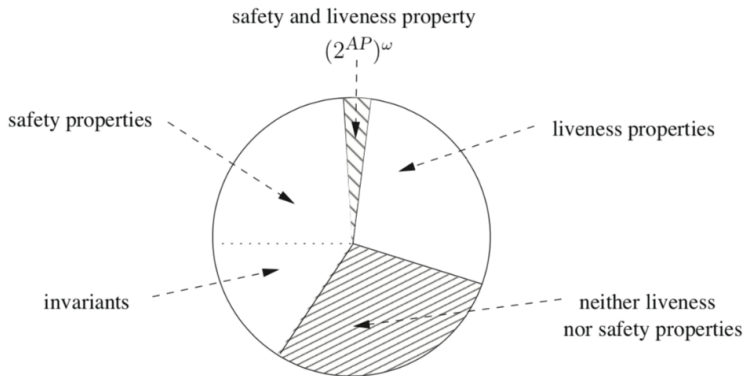
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# Safety vs. Liveness: Mathematical Definition

If we identify a property by the set of its models ( $\varphi = \{\sigma \mid \sigma \models \varphi\}$ )



# Model Checking Logics: Preliminaries

- Model Checking logics are based on the concept of *execution* of a Kripke structure  $\mathcal{S}$ 
  - thus, on  $\pi \in \text{Path}$
- Often, paths are directly viewed as a sequence of atomic propositions, rather than states
  - from  $\pi = s_1, s_2, \dots$  to  $AP(\pi) = L(s_1), L(s_2), \dots$
- Focusing on executions allows to model *time*
  - property on paths, especially useful for liveness properties
  - time in the sense that we have something coming before of something else (in a path...)
- Trade-off between
  - logics expressiveness: interesting properties can be written
  - logics efficiency: there is an efficient model checking algorithm to compute if  $\mathcal{S} \models \varphi$



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# Model Checking Logics: Preliminaries

- We will focus on the two leading Model Checking logics: LTL and CTL
  - with some hints on CTL\*
  - LTL (Linear-time Temporal Logic) established by Pnueli in 1977
  - CTL (Computation Tree Logic) established by Clarke and Emerson in 1981
  - used for IEEE standards:
    - PSL (Property Specification Language, IEEE Standard 1850)
    - SVA (SystemVerilog Assertions, IEEE Standard 1800).
- We will see syntax and semantics of both logics
  - syntax: how a valid formula is written
  - semantics: what a valid formula “means”
  - that is, when  $\mathcal{S} \models \varphi$  holds



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# LTL Syntax

$$\Phi ::= p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{X}\Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

- Other derived operators:
  - of course true, false, OR and other propositional logic connectors
  - future (or eventually):  $\mathbf{F}\Phi = \text{true} \mathbf{U} \Phi$
  - globally:  $\mathbf{G}\Phi = \neg(\text{true} \mathbf{U} \neg\Phi)$
  - release:  $\Phi_1 \mathbf{R} \Phi_2 = \neg(\neg\Phi_1 \mathbf{U} \neg\Phi_2)$
  - weak until:  $\Phi_1 \mathbf{W} \Phi_2 = (\Phi_1 \mathbf{U} \Phi_2) \vee \mathbf{G}\Phi_1$
- Other notations:
  - next:  $\mathbf{X}\Phi = \bigcirc\Phi$
  - $\mathbf{G}\Phi = \square\Phi$
  - $\mathbf{F}\Phi = \diamond\Phi$
- We are dropping *past operators*, thus this is *pure future LTL*



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# LTL Semantics

- Goal: formally defining when  $\mathcal{S} \models \varphi$ , being  $\mathcal{S}$  a KS and  $\varphi$  an LTL formula
  - we say that  $\mathcal{S}$  *satisfies*  $\varphi$ , or  $\varphi$  *holds in*  $\mathcal{S}$
- This is true when, for all paths  $\pi$  of  $\mathcal{S}$ ,  $\pi$  satisfies  $\varphi$ 
  - i.e.,  $\forall \pi \in \text{Path}(\mathcal{S}). \pi \models \varphi$
  - symbol  $\models$  is overloaded...
- For a given  $\pi$ ,  $\pi \models \varphi$  iff  $\pi, 0 \models \varphi$
- Finally, to define when  $\pi, i \models \varphi$ , a recursive definition over the recursive syntax of LTL is provided
  - $\pi \in \text{Path}(\mathcal{S}), i \in \mathbb{N}$



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# LTL Semantics for $\pi, i \models \varphi$

- $\forall \pi \in \text{Path}(\mathcal{S}), i \in \mathbb{N}. \pi, i \models \text{true}$
- $\pi, i \models p$  iff  $p \in L(\pi(i))$
- $\pi, i \models \Phi_1 \wedge \Phi_2$  iff  $\pi, i \models \Phi_1 \wedge \pi, i \models \Phi_2$
- $\pi, i \models \neg \Phi$  iff  $\pi, i \not\models \Phi$
- $\pi, i \models \mathbf{X}\Phi$  iff  $\pi, i + 1 \models \Phi$
- $\pi, i \models \Phi_1 \mathbf{U} \Phi_2$  iff  $\exists k \geq i: \pi, k \models \Phi_2 \wedge \forall i \leq j < k. \pi, j \models \Phi_1$



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# LTL Semantics for Added Operators

- It is easy to prove that:
  - $\pi, i \models \mathbf{G}\Phi$  iff  $\forall j \geq i. \pi, j \models \Phi$
  - $\pi, i \models \mathbf{F}\Phi$  iff  $\exists j \geq i. \pi, j \models \Phi$
  - $\pi, i \models \Phi_1 \mathbf{R} \Phi_2$  iff  $\forall j \geq i. (\forall k < j. \pi, k \models \Phi_1) \rightarrow \pi, j \models \Phi_2$
  - $\pi, i \models \Phi_1 \mathbf{W} \Phi_2$  iff  $(\forall j \geq i. \pi, j \models \Phi_1) \vee (\exists k \geq i : \pi, k \models \Phi_2 \wedge \forall i \leq j < k. \pi, j \models \Phi_1)$
- For many formulas, it is silently required that paths are infinite
- That's why transition relations in KSs must be total



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# Safety and Liveness Properties in LTL

- Given an LTL formula  $\varphi$ ,  $\varphi$  is a safety formula iff
$$\forall \mathcal{S}. (\exists \pi \in \text{Path}(\mathcal{S}) : \pi \not\models \varphi) \rightarrow \exists k : \pi|_k \not\models \varphi$$
- Given an LTL formula  $\varphi$ ,  $\varphi$  is a liveness formula iff
$$\forall \mathcal{S}. (\exists \pi \in \text{Path}(\mathcal{S}) : \pi \not\models \varphi) \rightarrow |\pi| = \infty$$
- All LTL formulas are either safety, liveness, or the AND of a safety and a liveness
  - being defined on paths, the counterexample is always a path
- Safety properties are those involving only **G**, **X**, true and atomic propositions
- Liveness are all those involving an **F**, or a **U** where the first formula is not the constant true
- Some formulas are both safety and liveness, like true, **G** true and so on

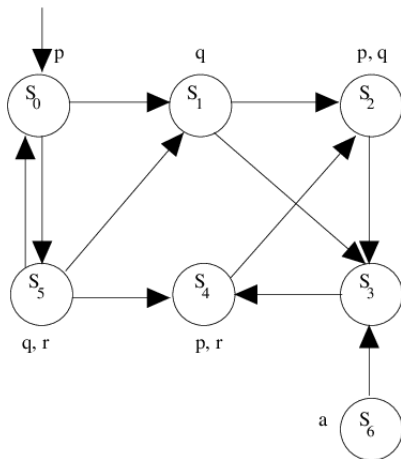


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# LTL Examples



$\mathcal{S} \models \mathbf{F}p$  since  $p$  holds in the first state

For full: let  $\pi \in \text{Path}(\mathcal{S})$

$\pi, 0 \models \mathbf{F}p$  with  $j = 0$

recall:  $\pi, i \models \mathbf{F}\Phi$  iff

$\exists j \geq i. \pi, j \models \Phi$

$\pi, i \models p$  iff  $p \in L(\pi(i))$

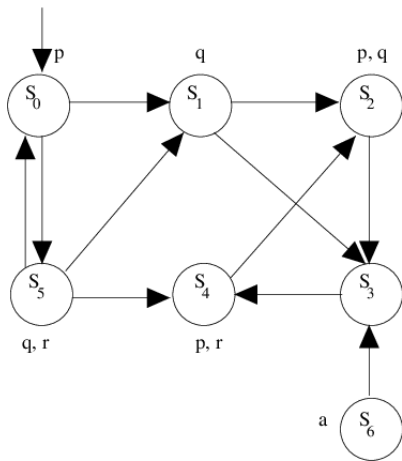


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# LTl Examples



$\mathcal{S} \not\models \mathbf{F}a$  since  $s_6$  is not reachable from  $s_0$

counterexample:  $\pi = s_0 s_5 s_0 s_5 \dots$

For full:  $\pi, 0 \not\models \mathbf{F}a$  as, for all  $j \geq 0$ ,  $a \notin L(\pi(j))$

Counterexample is infinite, thus this is a liveness property  
Any finite prefix of  $\pi$  is not a counterexample



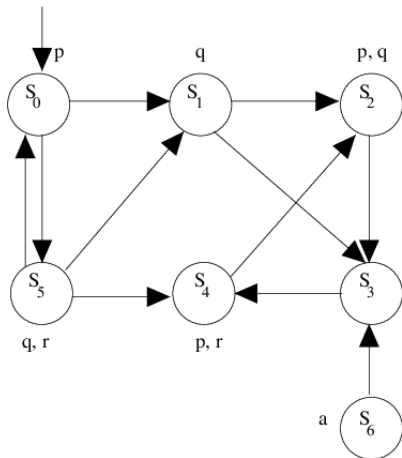
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# LTL Examples



$\mathcal{S} \not\models \mathbf{G}p$  since there are many counterexamples, here is one:

$\pi = s_0 s_5 s_0 s_5 \dots$

For full:  $\pi, 0 \not\models \mathbf{G}p$  with  $j = 1$

recall:  $\pi, i \models \mathbf{G}\Phi$  iff

$\forall j \geq i. \pi, j \models \Phi$

$\pi, i \models p$  iff  $p \in L(\pi(i))$

Safety property, actually  $\pi|_2$  is enough

Every path having  $\pi|_2$  as a prefix is a counterexample

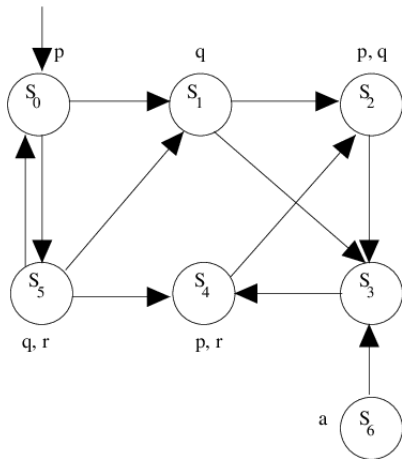


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# LTL Examples



$\mathcal{S} \models \mathbf{G}\neg a$  since  $s_6$  is not reachable from  $s_0$

For full: let  $\pi \in \text{Path}(\mathcal{S})$   
 $\pi, 0 \models \mathbf{G}\neg a$  as the only state  $s$  with  $a \in L(s)$  is  $s_6$ , which is not reachable from  $s_0$

recall:  $\pi \in \text{Path}(\mathcal{S})$  implies  $\pi(0) \in I$ , thus  $\pi(0) = s_0$  here

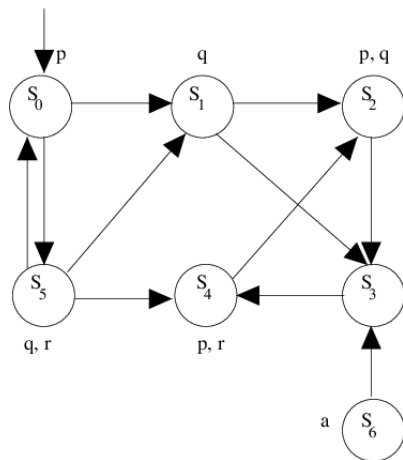


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# LTL Examples



$\mathcal{S} \models p \text{ U } q$  since  $p \in L(s_0)$ ,  
 $\text{next}(s_0) = \{s_1, s_5\}$  and  $q \in L(s_1) \wedge q \in L(s_5)$

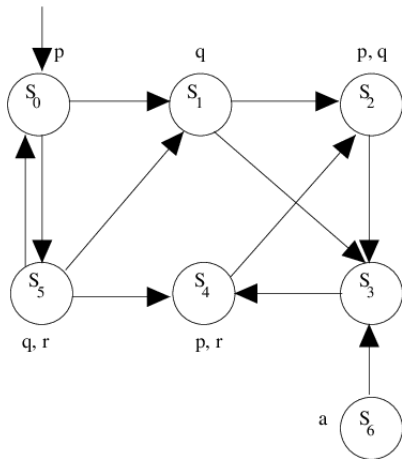


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# LTL Examples



$\mathcal{S} \not\models p \mathbf{U} r$ , a counterexample is  $\pi = s_0 s_1 (s_2 s_3 s_4)$

Again this is a liveness formula, even if  $\pi|_1$  would have been enough

In fact, you have to consider all possible KSs...

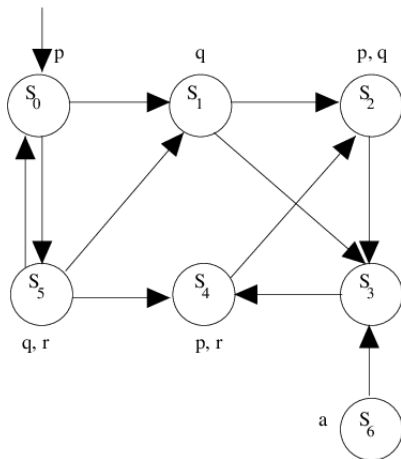


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# LTl Examples



$\mathcal{S} \not\models \neg(p \mathbf{U} r)$ , a counterexample is  $\pi = (s_0 s_5)$

In fact,  $(s_0 s_5), 0 \models p \mathbf{U} r$

Thus it may happen that  $\mathcal{S} \not\models \Phi$  and  $\mathcal{S} \not\models \neg(\Phi)$

Instead, it is impossible that  $\mathcal{S} \models \Phi$  and  $\mathcal{S} \models \neg(\Phi)$

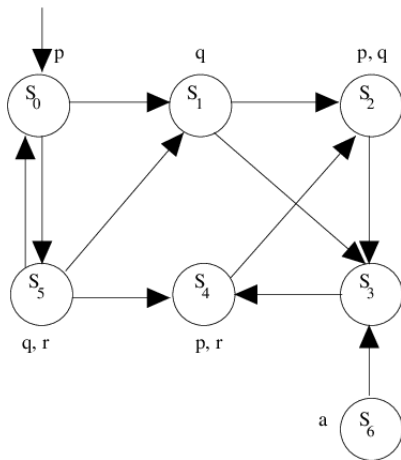


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# LTL Examples



$\mathcal{S} \not\models \mathbf{FG}p$ , a counterexample is  
 $\pi = s_0 s_1 (s_2 s_3 s_4)$   
Again this is a liveness formula

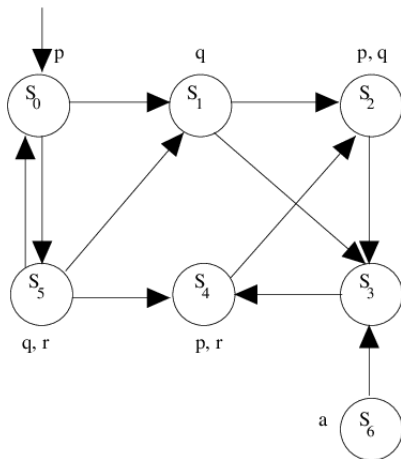


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# LTL Examples



$\mathcal{S} \models \mathbf{GF}p$

All lassos are  $s_0s_5$  or  $s_2s_3s_4$

In both such lassos, there are states in which  $p$  holds

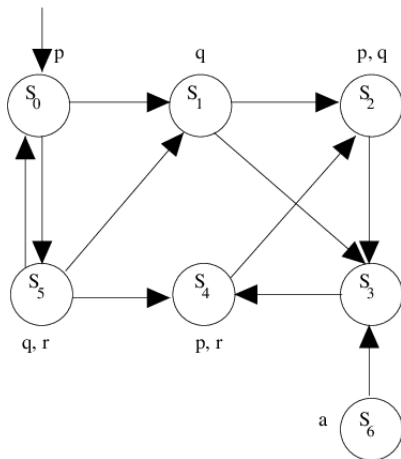


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# LTl Examples



$\mathcal{S} \models \mathbf{GF}p \vee \mathbf{FG}p$

Consequence of the two previous slides



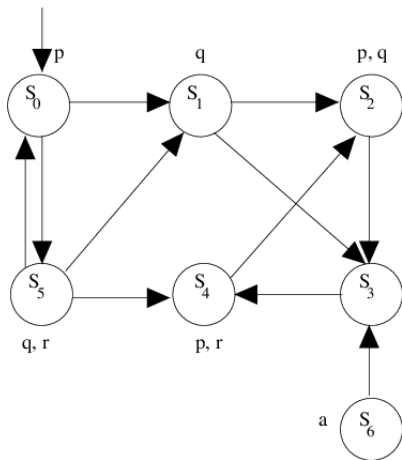
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# LTL Examples



$\mathcal{S} \not\models \mathbf{G}(p \mathbf{U} q)$ , a counterexample is  $\pi = s_0 s_1 (s_2 s_3 s_4)$   
 $(p \mathbf{U} q)$  must hold at any reachable state  
Ok in  $s_0, s_1, s_2$ , but not in  $s_3$



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# LTL Non-Toy Examples

- Recall the Peterson's protocol: checking mutual exclusion is  $\mathbf{G}(\neg(p \wedge q))$ , being  $p = P[1] = L3$ ,  $q = P[2] = L3$ 
  - all invariants are of the form  $\mathbf{G}P$ , where  $P$  does not contain modal operators  $\mathbf{X}$ ,  $\mathbf{U}$  or  $\mathbf{F}$
- Checking that both processes access to the critical section *infinitely often* is  $\mathbf{GF} P[1] = L3 \wedge \mathbf{GF} P[2] = L3$ 
  - liveness property: no process is infinitely banned to access the critical section
- Even better:  $\mathbf{G} (P[1] = L2 \rightarrow \mathbf{F} P[1] = L3)$ 
  - the same for the other process
  - since it is symmetric, this is actually enough



# Equivalence Between LTL Properties

- Definition of equivalence between LTL properties:  
 $\varphi_1 \equiv \varphi_2 \text{ iff } \forall \mathcal{S}. \mathcal{S} \models \varphi_1 \Leftrightarrow \mathcal{S} \models \varphi_2$ 
  - equivalent:  $\forall \sigma \dots$
- Idempotency:
  - $\mathbf{FF}p \equiv \mathbf{F}p$
  - $\mathbf{GG}p \equiv \mathbf{G}p$
  - $p \mathbf{U} (p \mathbf{U} q) \equiv (p \mathbf{U} q) \mathbf{U} q \equiv p \mathbf{U} q$
- Absorption:
  - $\mathbf{GFG}p \equiv \mathbf{FG}p$
  - $\mathbf{FGF}p \equiv \mathbf{GF}p$
- Expansion (used by LTL Model Checking algorithms!):
  - $p \mathbf{U} q \equiv q \vee (p \wedge \mathbf{X}(p \mathbf{U} q))$
  - $\mathbf{F}p \equiv p \vee \mathbf{XF}p$
  - $\mathbf{G}p \equiv p \wedge \mathbf{XG}p$



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# CTL Syntax

$\Phi ::= p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{EX}\Phi \mid \mathbf{EG}\Phi \mid \mathbf{E}\Phi_1 \mathbf{U} \Phi_2$

- Other derived operators (besides true, false, OR, etc):
  - $\mathbf{EF}\Phi = \mathbf{Etrue} \mathbf{U} \Phi$ 
    - cannot be defined using  $\mathbf{E}\neg\mathbf{G}\neg\Phi$ , as this is not a CTL formula
    - actually, it is a CTL\* formula (see later)
  - $\mathbf{AF}\Phi = \neg\mathbf{EG}\neg\Phi$ ,  $\mathbf{AG}\Phi = \neg\mathbf{EF}\neg\Phi$ ,  $\mathbf{AX}\Phi = \neg\mathbf{EX}\neg\Phi$
  - $\mathbf{A}\Phi_1 \mathbf{U} \Phi_2 = (\neg\mathbf{E}\neg\Phi_2 \mathbf{U} (\neg\Phi_1 \wedge \neg\Phi_1)) \wedge \neg\mathbf{EG}\neg\Phi_2$
  - $\Phi_1 \mathbf{AU}\Phi_2 = \mathbf{A}\Phi_1 \mathbf{U}\Phi_2$ ,  $\Phi_1 \mathbf{EU}\Phi_2 = \mathbf{E}\Phi_1 \mathbf{U}\Phi_2$



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# Comparison with LTL Syntax

$$\Phi ::= \text{true} \mid p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{X}\Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

- Essentially, all temporal operators are preceded by either **E** or **G**
  - with some care for **U**



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# CTL Semantics

- Goal: formally defining when  $\mathcal{S} \models \varphi$ , being  $\mathcal{S}$  a KS and  $\varphi$  a CTL formula
- This is true when, for all initial states  $s \in I$  of  $\mathcal{S}$ ,  $s \models \varphi$ 
  - thus, CTL is made of *state* formulas
  - LTL has *path* formulas
- To define when  $s \models \varphi$ , a recursive definition over the recursive syntax of CTL is provided
  - no need of an additional integer as for LTL syntax



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# CTL Semantics for $s, i \models \varphi$

- $\forall s \in S. s, i \models \text{true}$
- $s \models p$  iff  $p \in L(s)$
- $s \models \Phi_1 \wedge \Phi_2$  iff  $s \models \Phi_1 \wedge s \models \Phi_2$
- $s \models \neg\Phi$  iff  $s \not\models \Phi$
- $s \models \mathbf{EX}\Phi$  iff  $\exists \pi \in \text{Path}(\mathcal{S}, s). \pi(1) \models \Phi$
- $s \models \mathbf{EG}\Phi$  iff  $\exists \pi \in \text{Path}(\mathcal{S}, s). \forall j. \pi(j) \models \Phi$
- $s \models \mathbf{E}\Phi_1 \mathbf{U} \Phi_2$  iff  
 $\exists \pi \in \text{Path}(\mathcal{S}, s) \exists k : \pi(k) \models \Phi_2 \wedge \forall j < k. \pi(j) \models \Phi_1$



# CTL Semantics for Added Operators

- It is easy to prove that:
  - $s \models \mathbf{AG}\Phi$  iff  $\forall \pi \in \text{Path}(\mathcal{S}, s). \forall j. \pi(j) \models \Phi$
  - $s \models \mathbf{AF}\Phi$  iff  $\forall \pi \in \text{Path}(\mathcal{S}, s). \exists j. \pi(j) \models \Phi$
  - analogously for **AU**, **AR**, **AW**
  - just replace  $\forall$  with  $\exists$  for **EF**, **ER**, **EW**
- As for CTL, for many formulas, it is silently required that paths are infinite
- So again transition relations in Ks must be total



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# Safety and Liveness Properties in CTL

- Some CTL formulas may be neither safety nor liveness
  - being defined on states, the counterexample may be an entire computation tree
- Safety properties are those involving only **AG**, **AX**, true and atomic propositions
- Some formulas are both safety and liveness, like true, **G** true and so on
- Liveness are formulas like **AF**, **AFAG**, **AU**
- **EF** or **EG** are neither liveness nor safety

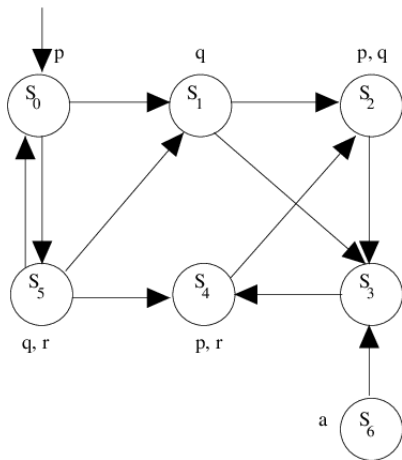


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# CTL Examples



$\mathcal{S} \models \mathbf{AF}p$  since  $p$  holds in the first state

For full:  $s_0 \models \mathbf{F}p$  since  $p \in L(s_0)$ , thus, for all paths starting in  $s_0$ ,  $p$  holds in the first state, so it holds eventually

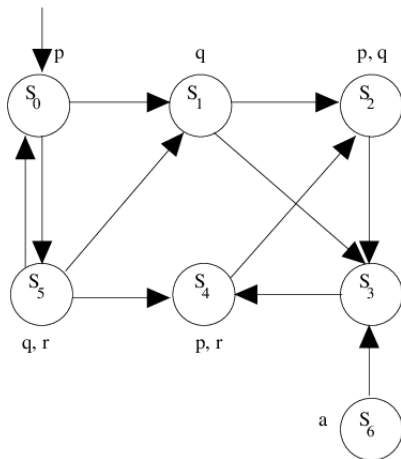


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# CTL Examples



$\mathcal{S} \models \mathbf{EF}p$  for the same reason as above

If it holds for all paths, then it holds for one path

$\mathbf{AF}\phi \rightarrow \mathbf{EF}\phi$

The same holds for the other temporal operators **G**, **U** etc

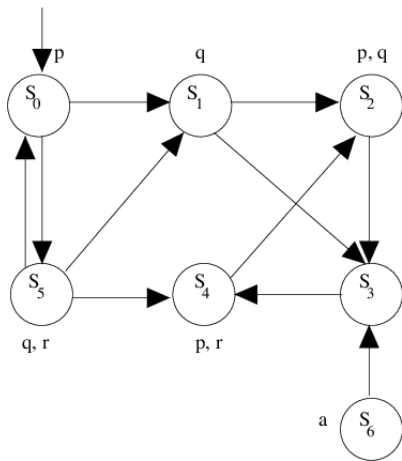


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# CTL Examples



$\mathcal{S} \not\models \mathbf{EF}a$  since  $s_6$  is not reachable

Note that the counterexample cannot be a single path

Since it would not enough to disprove existence

The full reachable graph must be provided

One could also show the tree of all paths

Neither safety nor liveness

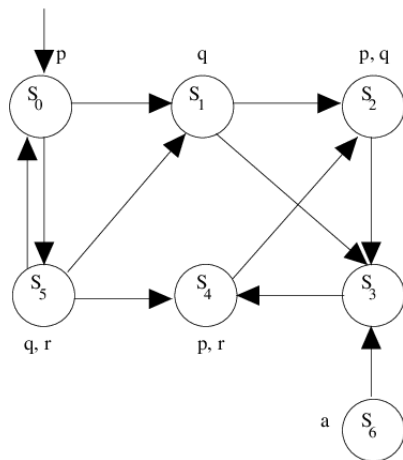


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# CTL Examples



$\mathcal{S} \models \mathbf{A}(p \mathbf{U} q)$  since  $p \in L(s_0)$ ,  
 $\text{next}(s_0) = \{s_1, s_5\}$  and  $q \in L(s_1) \wedge q \in L(s_5)$

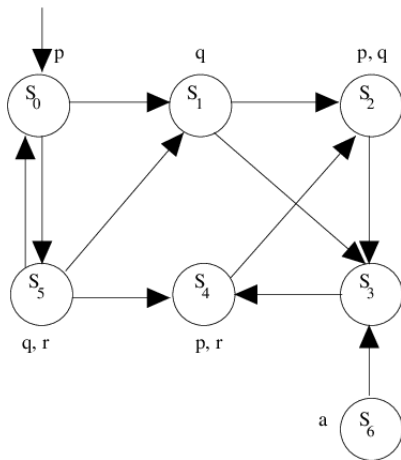


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# CTL Examples



$\mathcal{S} \not\models \mathbf{A}(p \mathbf{U} r)$ , a counterexample is  $\pi = s_0 s_1 (s_2 s_3 s_4)$

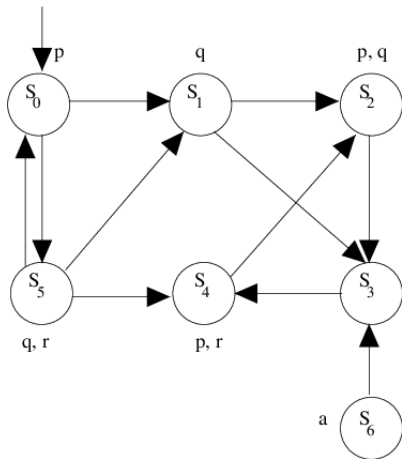


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# CTL Examples



$\mathcal{S} \models \mathbf{E}(p \mathbf{U} r)$ , an example is  
 $\pi = (s_0 s_5)$

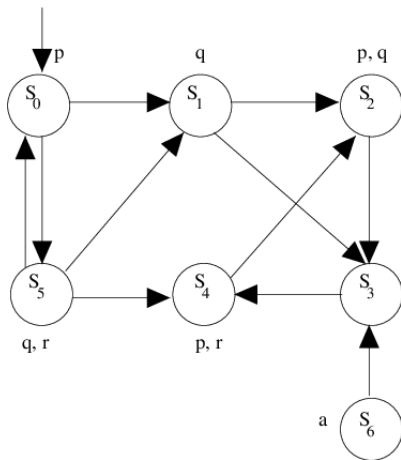


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# CTL Examples



$\mathcal{S} \not\models \neg \mathbf{E}(p \mathbf{U} r)$ , a counterexample is  $\pi = (s_0 s_5)$

In fact,  $\mathcal{S} \not\models \Phi$  iff  $\mathcal{S} \models \neg(\Phi)$

Because here we have a single initial state



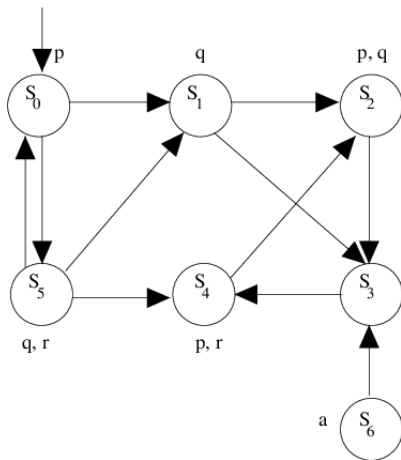
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# CTL Examples



$\mathcal{S} \not\models \mathbf{AFAG}p$ , a counterexample is  $\pi = s_0s_1(s_2s_3s_4)$   
This is a liveness formula

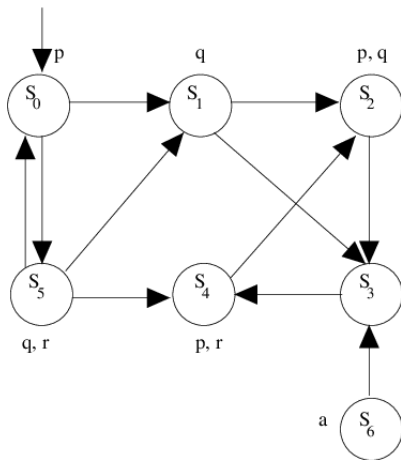


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# CTL Examples



$\mathcal{S} \not\models \mathbf{EFEG}p$ , a counterexample is again a computation tree

All lassos are  $s_0s_5$  or  $s_2s_3s_4$

In both such lassos, there are states in which  $p$  does not hold

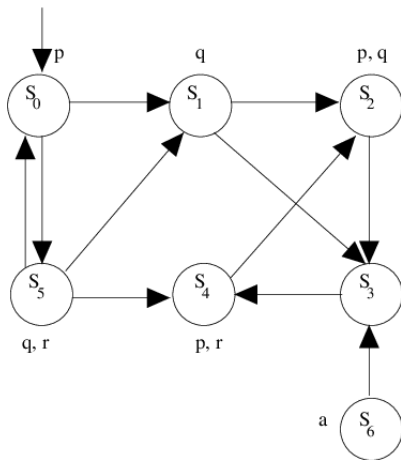


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# CTL Examples



$\mathcal{S} \not\models \mathbf{AFEG}p$ , a counterexample is again a computation tree  
Since  $\mathcal{S} \not\models \mathbf{EFEG}p \dots$

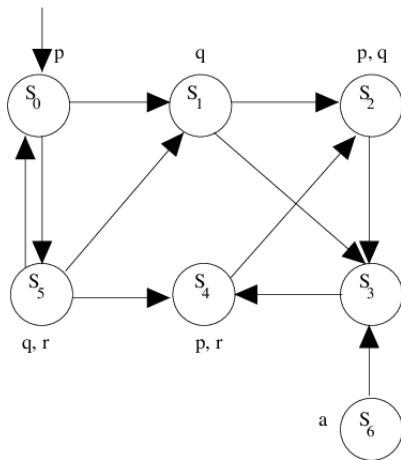


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# CTL Examples



$\mathcal{S} \not\models \mathbf{EFAG}p$ , a counterexample is again a computation tree  
 Since  $\mathcal{S} \not\models \mathbf{EFEG}p$ ...



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# CTL Non-Toy Examples

- Recall the Peterson's protocol: checking mutual exclusion is **AG**( $\neg(p \wedge q)$ ), being  $p = P[1] = L3, q = P[2] = L3$ 
  - equivalent to LTL **G** $p$
- It is always possible to restart:  
**AGEF**  $P[1] = L0 \wedge \mathbf{AGEF} P[2] = L0$



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# CTL vs. LTL: a Comparison

- Recall that  $\varphi_1 \equiv \varphi_2$  iff  $\forall \mathcal{S}. \mathcal{S} \models \varphi_1 \Leftrightarrow \mathcal{S} \models \varphi_2$ 
  - also holds (w.l.g.) when  $\varphi_1$  is LTL and  $\varphi_2$  is CTL
- Of course, some CTL formulas cannot be expressed in LTL
  - it is enough to put an **E**, since LTL always universally quantifies paths
  - so, there is not an LTL  $\varphi$  s.t.  $\varphi \equiv \mathbf{EG}p$ 
    - no,  $\mathbf{F}\neg p$  is not the same, why?
- So, one might think: LTL is contained in CTL
  - simply replace each temporal operator **O** with **AO**, that's it
  - let  $\mathcal{T}$  be a translator doing this
  - for any LTL formula  $\varphi$ ,  $\varphi \equiv \mathcal{T}(\varphi)$
  - actually,  $\mathbf{G}p \equiv \mathcal{T}(\mathbf{G}p) = \mathbf{AG}p$



# CTL vs. LTL: a Comparison

- Theorem. Let  $\varphi$  be an LTL formula. Then, either i)  $\varphi \equiv \mathcal{T}(\varphi)$  or ii) there does not exist a CTL formula  $\psi$  s.t.  $\varphi \equiv \psi$ 
  - idea of proof: replacing with **E** is of course not correct, and temporal operators on paths are the same
- Corollary. There exists an LTL formula  $\varphi$  s.t., for all CTL formulas  $\psi$ ,  $\varphi \not\equiv \psi$
- Proof of corollary:
  - by the theorem above and the definitions, we need to find
    - 1 an LTL formula  $\varphi$
    - 2 a KS  $\mathcal{S}$
  - where  $\mathcal{S} \models \varphi$  and  $\mathcal{S} \not\models \mathcal{T}(\varphi)$ 
    - viceversa is not possible



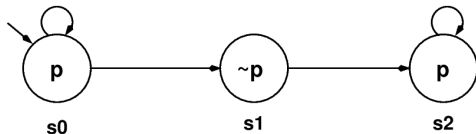
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# CTL vs. LTL: a Comparison

- For example, as for the LTL formula, we may take  $\varphi = \mathbf{FG}p$ 
  - note instead that  $\mathbf{GF}p \equiv \mathbf{AGAF}p$
- For example, as for the KS  $\mathcal{S}$ , we may take

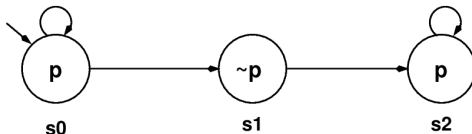


- We have that  $\mathcal{S} \models \mathbf{FG}p$ , but  $\mathcal{S} \not\models \mathbf{AFAG}p$
- Thus, CTL requires “more” than the corresponding LTL





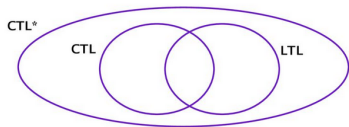
# CTL vs. LTL: a Comparison



- $\mathcal{S} \not\models \mathbf{AFAG}p$  means that
$$\neg(\forall \pi \in \text{Path}(\mathcal{S}). \exists j : \forall \rho \in \text{Path}(\mathcal{S}, \pi(j)). \forall k. p \in \rho(k))$$
$$= \exists \pi \in \text{Path}(\mathcal{S}). \forall j : \exists \rho \in \text{Path}(\mathcal{S}, \pi(j)). \exists k. p \notin \rho(k)$$
  - the path  $\pi$  is a loop on  $s_0 \dots$
- $\mathcal{S} \models \mathbf{FG}p$  means that  $\forall \pi \in \text{Path}(\mathcal{S}). \exists j : \forall k \geq j. p \in \pi(k)$
- Thus, there is not a CTL formula equivalent to  $\mathbf{FG}p$
- Furthermore, there is not an LTL formula equivalent to  $\mathbf{AFAG}p$



# CTL, LTL and CTL\*



- CTL\* introduced in 1986 (Emerson, Halpern) to include both CTL and LTL
- No restrictions on path quantifiers to be 1-1 with temporal operators, as in CTL
- State formulas:  $\Phi ::= \text{true} \mid p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid \mathbf{A}\Psi \mid \mathbf{E}\Psi$
- Path formulas:  $\Psi ::= \Phi \mid \Psi_1 \wedge \Psi_2 \mid \neg \Psi \mid \Psi_1 \mathbf{U} \Psi_2 \mid \mathbf{F}\Psi \mid \mathbf{G}\Psi$

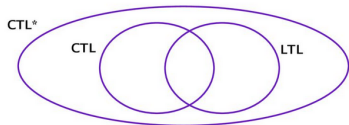


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# CTL, LTL and CTL\*



- The intersection between CTL and LTL is both syntactic and “semantic”
- Some formulas are both CTL and LTL in syntax: all those involving only boolean combinations of atomic propositions
- “Semantic” intersection: some LTL formulas may be expressed in CTL and vice versa, using different syntax
  - **AGAF** $p$  and **GF** $p$
  - **AG** $p$  and **G** $p$
  - etc



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