Software Testing and Validation

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Corso di Laurea in Informatica

The NuSMV Model Checker

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CTL (and LTL) Model Checking

- We saw the theoretical algorithm for CTL model checking
  - we said it was not effective, as it required S and R to be in RAM
- Actually, there are methodologies which are able to fit S and R in RAM, also for industrial-sized models
- The "father" of the model checkers using such technologies is SMV
  - Symbolic Model Verifier
  - it has then been refactored as NuSMV
- This set of techniques is referred to as *symbolic model checking*
  - Murphi and SPIN style is dubbed *explicit model checking*

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CTL (and LTL) Model Checking

- In order to understand how symbolic model checking works, we need some preliminaries
- ROBDDs
  - needed to actually fit S and R in RAM
- μ-calculus
  - together with fixpoint computation
  - extension of λ-calculus
  - needed to efficiently implement CTL and LTL model checking using ROBDDs

ROBDD

- Reduced Ordered (Complemented Edges) Binary Decision Diagrams
  - sometimes called simply OBDDs, and even BDDs
  - here we stick to the precise notation, by also outlining the differences
- Let us start with the basis: BDD
- A BDD is a data structure representing a boolean function
  - of course, OBDDs and ROBDDs are data structures as well
  - we will define them in the following

Boolean Functions

- In our setting a boolean function is $f : \mathbb{B}^n \to \mathbb{B}$
  - where $\mathbb{B} = \{0, 1\}$ is the set of boolean values
  - 0 stands for false, 1 for true
  - thus, our boolean functions have n boolean variables as arguments
  - and return a single boolean value
- Examples:
  - 0 and 1 are boolean functions with $n = 0$
  - complementation ($f(x) = \neg x$) and identity ($f(x) = x$) are boolean functions with $n = 1$
  - AND ($f(x, y) = x \wedge y$), OR ($f(x, y) = x \vee y$) are boolean functions with $n = 2$
  - generally speaking, there are 2^{2^n} different boolean functions of n boolean variables

All Boolean Functions of 2 Variables

p	*q*	F^0	NOR^1	$\nleftarrow^2$	$\neg p^3$	$\nrightarrow^4$	$\neg q^5$	XOR^6	$NAND^7$	AND^8	$XNOR^9$	q^{10}	$\rightarrow^{11}$	p^{12}	$\leftarrow^{13}$	OR^{14}	T^{15}
T	**T**	F	F	F	F	F	F	F	F	T	T	T	T	T	T	T	T
T	**F**	F	F	F	F	T	T	T	T	F	F	F	F	T	T	T	T
F	**T**	F	F	T	T	F	F	T	T	F	F	T	T	F	F	T	T
F	**F**	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T

Boolean Functions Representation

- Roughly speaking, if you have $f(x) = x + 1$ with $x \in \mathbb{R}$, you can only represent f through its computation
  - rules s.t., given x, you compute $x + 1$
- For boolean functions, the explicit tabular representation is also possible (*truth table*)
  - a table with $n + 1$ columns
  - first n columns are for variables values
  - last column is for function value
  - of course, you need 2^n rows
  - actually, only one column, thus $\lceil 2^{n-3} \rceil$ bytes
    - thus, $O(2^n)$

Boolean Functions Representation

- A truth table must take into account all possible values for all its n arguments
- Which leads to a $O(2^n)$ RAM required
  - even with optimizations (e.g., only 1 column is actually needed)
- This also implies $O(2^n)$ time to compute composition of functions
  - e.g., $f \wedge g$
  - worst case time is also best case...
- One very good thing about truth tables: they are *canonical*
  - for a function f, given the (standard) order of the lines, there is only one truth table

Boolean Functions Representation

- What about CNF or DNF?
  - CNF: $(x_1 + x_2)(\bar{x}_3 + x_4)$
  - DNF: $x_1\bar{x}_3 + x_1x_4 + x_2\bar{x}_3 + x_2x_4$
  - recall that $+$ is OR, $\cdot$ is AND, $\bar{\cdot}$ is negation
- Approx ok to compute function compositions
- Difficult to obtain a minimal representation
- Above all, not canonical: there may be multiple CNFs or DNFs for the same function
  - also if you consider the minimal one

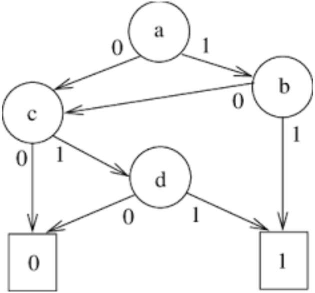
Boolean Functions for Model Checking

- In Model Checking algorithms, the following operations are needed:
  - compute the returned value for a given tuple of values $b_1, \ldots, b_n$
    - could be ok for truth tables and DNF/CNF
  - test of equivalence between boolean functions $f_1 \equiv f_2$
    - not ok neither for truth tables nor for CNF/DNF
  - compute the representation of a logical combination of boolean functions
    - e.g.: given the representation of f_1, f_2, compute the representation of $f_1 \wedge f_2$
    - not ok for truth tables
    - slightly better for CNF/DNF
- Goal: find a representation able to fulfill such requirements
  - while possibly requiring less than $O(2^n)$ memory

Binary Decision Diagrams

- Roughly speaking, it is a connected DAG (Directed Acyclic Graph), i.e., a tree
  - only one root
  - each internal node has two successors
  - nodes are labeled by boolean variables
  - edges are labeled by boolean values
  - only two leaves, labeled with boolean values

Binary Decision Diagrams



Represented function: $f(a, b, c, d) = ab + \bar{a}cd + a\bar{b}cd$

BDDs: Formal Definition

- A BDD is a tuple $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high} \rangle$ where:
  - V is a finite set of nodes containing two special nodes $\mathbf{0}$ and $\mathbf{1}$
  - $E \subseteq V \times V$ is a set of edges s.t.:
    - there are no cycles, i.e., for all path $\pi = v_0, \ldots, v_n$, where $\forall i = 0, \ldots, n.\ v_i \in V$ and $\forall i = 0, \ldots, n-1.\ (v_i, v_{i+1}) \in E$, we have that $i \neq j$ implies $v_i \neq v_j$
    - let $S(v) = \{w \in V \mid (v, w) \in E\}$ be the set of successors of v
    - each internal node has exactly two successors, i.e., $\forall v \in V \setminus \{\mathbf{0}, \mathbf{1}\}.\ |S(v)| = 2$
    - $\mathbf{0}$ and $\mathbf{1}$ are *terminal nodes*, i.e., $\forall v \in \{\mathbf{0}, \mathbf{1}\}.\ |S(v)| = 0$
  - $r \in V$ is the root (i.e., $\forall v \in V.\ (v, r) \notin E$)
  - $\mathrm{low}, \mathrm{high} : V \to V$ is the labeling of edges
    - the labeling must be consistent with E, i.e., $\forall v \in V.\ \mathrm{low}(v), \mathrm{high}(v) \in S(v)$

BDDs: Formal Definition

- A BDD is a tuple $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high} \rangle$ where:
  - $\mathcal{V}$ is a finite set of boolean variables
    - thus, the boolean function represented by $\mathcal{B}$ will depend on variables in $\mathcal{V}$
    - it may be a subset of $\mathcal{V}$
  - $\mathrm{var} : V \to \mathcal{V}$ is the labeling of nodes
- A *maximal path* in $\mathcal{B}$ starts from r and ends up either in $\mathbf{0}$ or $\mathbf{1}$
- The *semantics* of $\mathcal{B}$ is the boolean function represented by $\mathcal{B}$
  - intuitively, we follow all maximal paths which end up in $\mathbf{1}$
  - formally: next slide

BDDs: Semantics

- Given a BDD $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high} \rangle$, we recursively define the semantics of each node $v \in V$
  - each node may be seen as the root of a subtree...
  - notation: $[\![v]\!]_{\mathcal{B}}$, or simply $[\![v]\!]$ when $\mathcal{B}$ is understood
- Terminal nodes denote the boolean constants: $[\![\mathbf{0}]\!] = \mathrm{false}, [\![\mathbf{1}]\!] = \mathrm{true}$
- For internal nodes $v \in V \setminus \{\mathbf{0}, \mathbf{1}\}$, semantics is defined as $[\![v]\!] = \mathrm{var}(v)[\![\mathrm{high}(v)]\!] + \overline{\mathrm{var}(v)}[\![\mathrm{low}(v)]\!]$
  - this is called Shannon expansion
  - recall that $+$ is OR, $\cdot$ is AND, $\bar{\cdot}$ is negation
- The semantics of $\mathcal{B}$ is of course $[\![r]\!]$

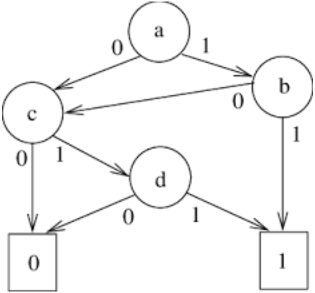
Canonicity of BDDs

- For a given BDD $\mathcal{B}$, we have a unique represented boolean function
- Given a boolean function f, there is a BDD $\mathcal{B}$ representing f, i.e., $[\![r]\!]_{\mathcal{B}} = f$
- However, there may be a BDD $\mathcal{B}' \neq \mathcal{B}$ s.t. $[\![r']\!]_{\mathcal{B}'} = f$ as well
  - thus, BDDs are not canonical
- Thus, ROBDDs are introduced: by setting limitations, they achieve canonicity
  - for a boolean function f, there exists a *unique* ROBDD representing f
- Furthermore, for increasing efficiency, complemented edges are introduced
  - number of nodes is reduced

OBDDs

- An OBDD (Ordered BDD) $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high}, \mathrm{ord} \rangle$, is a BDD with an additional ord function
- Namely, $\mathrm{ord} : \mathcal{V} \to \{1, \ldots, |\mathcal{V}|\}$
- The following properties must hold
  - ord is injective, i.e., $\forall v, w \in \mathcal{V}.\ \mathrm{ord}(v) = \mathrm{ord}(w) \to v = w$
  - note that this implies that ord is indeed bijective...
  - defines an *ordering* on variables in $\mathcal{V}$, e.g., if $\mathrm{ord}(v) = 10$ then v is the tenth variable
  - given a path π on $\mathcal{B}$, variables on nodes follow ord
  - i.e., $\forall \pi = v_0, \ldots, v_n$ s.t. $\forall i = 0, \ldots, n.\ v_i \in V$ and $\forall i = 0, \ldots, n-1.\ (v_i, v_{i+1}) \in E$ and $v_n \notin \{\mathbf{0}, \mathbf{1}\}$, we have that $i < j$ implies $\mathrm{ord}(\mathrm{var}(v_i)) < \mathrm{ord}(\mathrm{var}(v_j))$

OBDDs

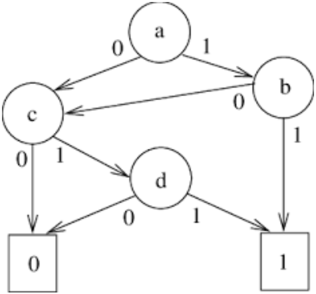


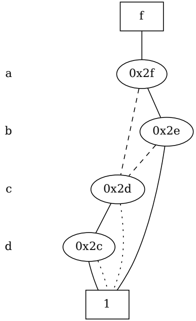
Supposing that $V = \mathcal{V}$, a possible ordering is:
$\mathrm{ord}(a) = 1, \mathrm{ord}(b) = 2, \mathrm{ord}(c) = 3, \mathrm{ord}(d) = 4$
If b were connected to d instead of c, also:
$\mathrm{ord}(a) = 1, \mathrm{ord}(b) = 3, \mathrm{ord}(c) = 2, \mathrm{ord}(d) = 4$

COBDDs

- A COBDD (Complemented edges OBDD) $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high}, \mathrm{ord}, \mathrm{flip}\rangle$, is an OBDD with an additional $\mathrm{flip} : V \setminus \{\mathbf{1}\} \to \{0, 1\}$
- For an internal node v, if $\mathrm{flip}(v)$ holds then the *else edge* of v is complemented
- There is now only one terminal node $\mathbf{1}$
  - $\mathbf{0}$ is not needed because of complementation
- Semantics changes, also a flipping bit $b \in \{0, 1\}$ is necessary
- Terminal node denote the boolean constants: $[\![\mathbf{1}, b]\!] = \bar{b}$
- For internal nodes $v \in V \setminus \{\mathbf{1}\}$, semantics is defined as $[\![v, b]\!] = \mathrm{var}(v)[\![\mathrm{high}(v), b]\!] + \overline{\mathrm{var}(v)}[\![\mathrm{low}(v), b \oplus \mathrm{flip}(v)]\!]$
- Semantics of $\mathcal{B}$ is $[\![r, \mathrm{flip}(r)]\!]$

COBDDs





Represented function: $f(a,b,c,d) = ab + \bar{a}cd + a\bar{b}cd$

straight: then, dashed: else, dotted: complemented else

ROBDDs

- A ROBDD (Reduced OBDD) $\mathcal{B}$ is a COBDD with the least number of nodes
  - among the ones representing the same boolean function
- From now on, as usual in the literature, we will use OBDD as synonym for ROBDD
- Efficient algorithms ($O(n)$, being n the number of nodes) exist to compute the AND and the OR of two OBDDs
  - negation is $O(1)$: just complement $\mathrm{flip}(r)$!
- Typically implemented with hash tables of already computed ROBDDs
  - speedup computations, make it easier to find shared subtrees
  - equality check is $O(1)$: just compare r and r'
- Furthermore: multi-rooted DAG can be used to represent multiple functions, sharing some nodes

Other Important OBDD Operations

- Application: given the OBDD for $f(x_1, \dots, x_i, \dots, x_n)$, compute the OBDD for $f(x_1, \dots, 0, \dots, x_n)$ or $f(x_1, \dots, 1, \dots, x_n)$
    - sometimes also written $f(x_1, \dots, x_n)|_{x_i=0}$ or $f(x_1, \dots, x_n)|_{x_i=1}$
    - Shannon expansion: for every boolean function f, $f(x_1, \dots, x_n) = \bar{x}_i f(x_1, \dots, x_n)|_{x_i=0} + x_i f(x_1, \dots, x_n)|_{x_i=1}$
- Given $f(x, y)$, compute the OBDD for:
    - existentialization: $\exists x : \ f(x, y) \equiv f(0, y) + f(1, y)$
    - universalization: $\forall x. \ f(x, y) \equiv f(0, y) \cdot f(1, y)$
    - both generalized to multiple variables $x_1, \dots, x_n$
- Given $f(x), g(x), h(x)$, compute the OBDD for $ITE(f, g, h)$
    - ITE stands for if-then-else
    - thus, $ITE(f, g, h) = fg + \bar{f}h$

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OBDD and Model Checking

- OBDDs extremely good in representing *characteristic functions* of finite sets
  - the characteristic function $\chi : U \to \{0, 1\}$ of a set $X \subseteq U$ is defined as

$$\chi(x) = \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{otherwise} \end{cases}$$

- If U is finite, then each element $x \in U$ may be encoded using $n = \lceil \log(|U|) \rceil$ boolean variables $x_1, \ldots, x_n$
- Thus, χ may be represented by an OBDD on $x_1, \ldots, x_n$
  - as for Model Checking, we may represent S, $\mathrm{Reach}(\mathcal{S})$, R, ...
  - R will need $2n$ variables!
  - CTL Model Checking algorithm becomes feasible!
    - for many interesting real-sized systems, S, $\mathrm{Reach}(\mathcal{S})$, R will now fit in RAM

OBDD and Model Checking

- The most difficult part is to derive the OBDD for R directly from the model specification
  - i.e., from the model checker input language
  - it would be rather difficult to do it with SPIN
    - especially because it has a dynamic state space
  - also the one for Murphi would require some effort
  - S is easy, you only have to look at global variables
    - not in SPIN...
- NuSMV input language is tailored to be easily translated into OBDDs
  - also into CNF, as we will see...

NuSMV

- SMV (Symbolic Model Verifier): McMillan implementation of the ideas in the famous paper "Symbolic model checking: 10^{20} states and beyond"
  - McMillan PhD dissertation about SMV is one of the most important dissertations in Computer Science
- SMV has been then re-written and standardized by the research group in Trento (also Genova and CMU collaborated), thus becoming NuSMV
  - the engine is still McMillan's work
  - code has been nearly entirely commented, and made more readable
  - some features has been added: interactive mode, bounded model checking
  - OBDDs are handled via the CUDD library (by F. Somenzi at Colorado University)

NuSMV Input Language

Taken from `examples/smv-dist/short.smv`

```
MODULE main
VAR
  request : {Tr, Fa}; -- same as saying boolean
                      -- (stand for True and False)
  state : {ready, busy};
ASSIGN
  init(state) := ready;
  next(state) := case
                   state = ready & (request = Tr): busy;
                   TRUE : {ready,busy};
                 esac;
SPEC
  AG((request = Tr) -> AF state = busy)
```

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NuSMV Input Language

- One *module*, there may be more, but one of them must be named `main`
- Module variables are those declared with `VAR`
- Base types are like Murphi ones: enumerations and integer subranges, plus the `word` type (i.e., an array of bits)
- Arrays are possible, but can be indexed only with constants
- Structures are modeled through modules
  - that is, each module has its variables (fields of a structure) and may be instantiated many times

NuSMV Input Language

- `ASSIGN` section specifies the set I (via `init`) and the relation R (via `next`)
  - as in Murphi, there expressions which are essentially guard/action
  - differently from Murphi, each action deals with *one variable only*
    - the guard may be defined on any other variable (and it is typically the case)
  - if something is not specified, then it is understood to be non-deterministic
  - indirect specification; also direct specification is allowed, as we will see

- E.g., in `short.smv` initial states are those in which `state` is `ready` and `request` may be either `Tr` or `Fa`
- Thus, there are 2 initial states $I = \{\langle \mathtt{ready}, \mathtt{Tr}\rangle, \langle \mathtt{ready}, \mathtt{Fa}\rangle\}$, which may be represented with $\langle \mathtt{ready}, \perp\rangle$
- Also `next(request)` is not specified; before analyzing what does this mean, let us see `next(state)`
- The `case` expression works as follows: the first condition C which is evaluated to true is fired, other true guards possibly following C are ignored

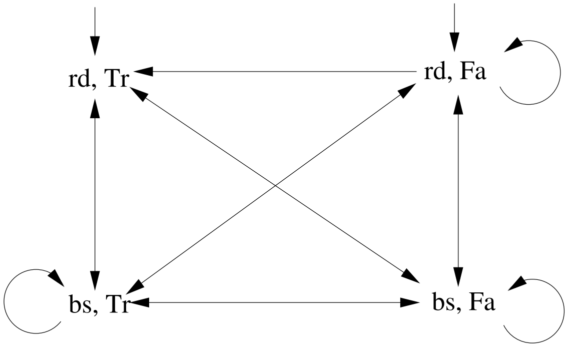
NuSMV Input Language: `ASSIGN`

- This allows to put 1 (i.e., true) as the last guard, representing the "default" case
- NuSMV also checks if a `case` expression is exhaustive in its conditions, as this allows it to assume that R is total
- Note that the last condition on `state` leads to a non-deterministic transition: if the first guard is false, then `state` may take any value between `ready` e `busy`, that is any value in its domain
- In general, any subset of the variable domain may be used

NuSMV Input Language: `ASSIGN`

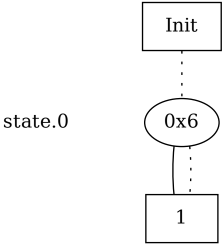
- `request` is completely non-deterministic, as it does not occur in any `next`
- I.e., if other rules tells that the system may go from s to t and $(\texttt{request} = \texttt{Fa}) \in L(t)$, then there exists a transition from s to t' with $(\texttt{request} = \texttt{Tr}) \in L(t')$ and $L(t) \setminus \{(\texttt{request} = \texttt{Fa})\} = L(t') \setminus \{(\texttt{request} = \texttt{Tr})\}$
- Simply stated, if the system may go from s to t and `request` has a value v in t, then the system may also go from s to t' s.t. t and t' only differ in the value of `request`, which is different from v
- By combining all non-determinism in this example, the Kripke structure defined here excludes just two transitions

Automata for `short.smv`: *I* and *R*



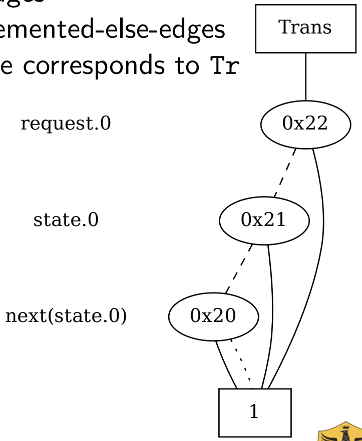
OBDDs for `short.smv`: I

Straight lines are then-edges
Dashed lines are else-edges
Dotted lines are complemented-else-edges



OBDDs for `short.smv`: R

Straight lines are then-edges
Dashed lines are else-edges
Dotted lines are complemented-else-edges
`request.0` "false" edge corresponds to `Tr`



NuSMV Input Language

```
MODULE main
VAR
  request : {Tr, Fa};
  state : {ready, busy};
ASSIGN
  init(state) := ready;
  next(state) := case
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                    TRUE : {ready,busy};
                 esac;
SPEC
  AG((request = Tr) -> AF state = busy)
```

NuSMV Input Language

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                 esac;
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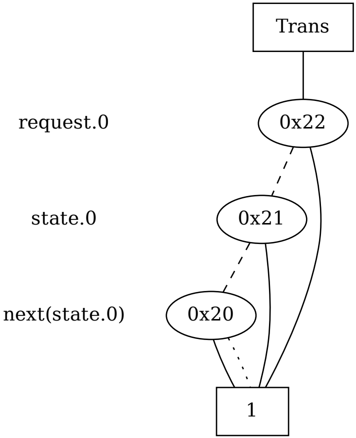
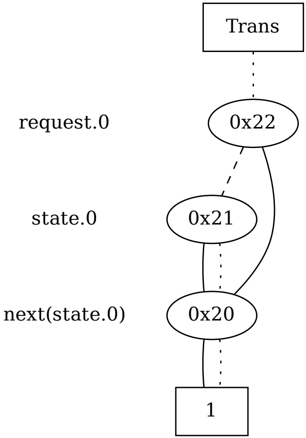
```

OBDDs for `short.smv`: R

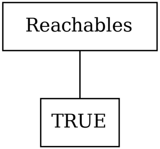


OBDDs for `short.soloready.smv`: R



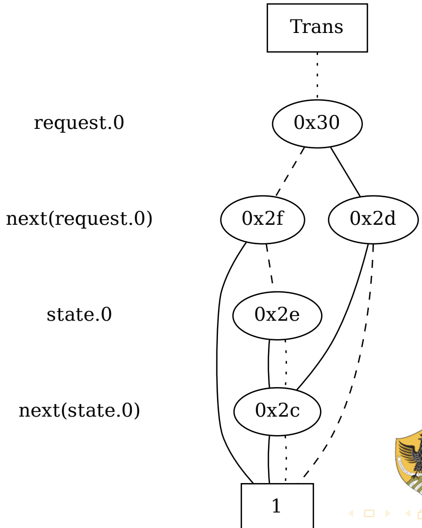
OBDDs for `short.smv`: Reach

The one for soloready is the same

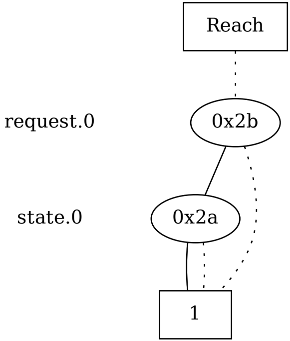


```
MODULE main
VAR
  request : {Tr, Fa};
  state : {ready, busy};
ASSIGN
  init(state) := ready;
  next(state) := case
                   state = ready & (request = Tr): busy;
                   TRUE : ready;
                 esac;
  next(request) := request;
SPEC
  AG((request = Tr) -> AF state = busy)
```

OBDDs for `short.soloready.req_const.smv`: R



OBDDs for `short.soloready.req_const.smv`: Reach



OBDDs Pros and Cons

```
MODULE main
VAR
  m1 : 0..15; -- m1.0 is MSB!
  m2 : 0..15;
  m3 : 0..30;
ASSIGN
  next(m3) := m1 + m2;

SPEC
  AG(m3 <= 30);
```

OBDDs Pros and Cons

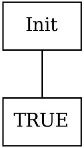
```
MODULE main
VAR
  m1 : 0..15;
  m2 : 0..15;
  m3 : 0..30;
ASSIGN
  next(m3) := case
    m1*m2 <= 30: m1*m2;
    TRUE: m3;
  esac;

SPEC
  AG(m3 <= 30);
```

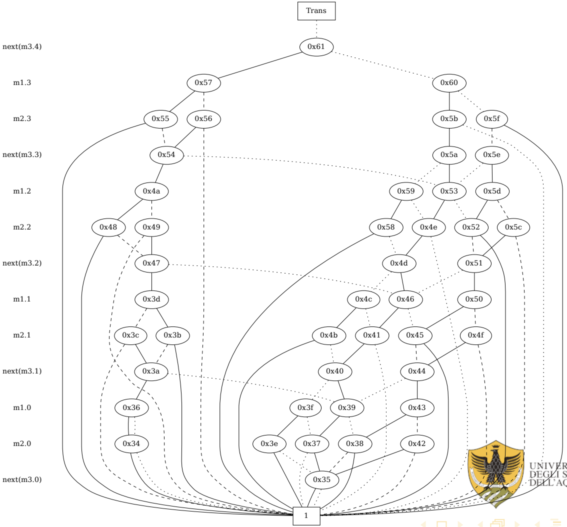
OBDDs for Adder and Multiplier: *I*

This is a set with $16 \cdot 16 \cdot 31 = 7936$ elements

Just one node to represent it...



OBDDs for Adder: R



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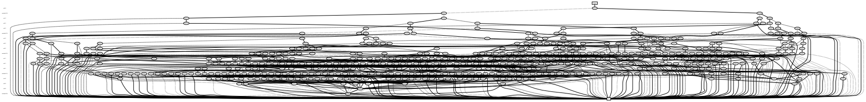
OBDDs for Multiplier: R



OBDDs Pros and Cons

- Number of variables is 13 for both models
  - 4 each for `m1` and `m2`, plus 5 for `m3`
- Number of BDD nodes:
  - adder: 47
  - multiplier: 538

OBDDs Pros and Cons

- No magic: SAT could be solved using OBDDs
  - just represent the instance with an OBDD and check if it is different from 0
  - very roughly speaking: if it were possible to solve it "efficiently" in this way, P=NP...
- Thus, there are boolean functions for which OBDDs representation is exponential, regardless of variable ordering
  - one example is the multiplier seen above
- It is not possible to say if OBDDs will be a good way to represent a problem, before trying it
  - for the adder, it is much more efficient
- Furthermore, finding a variable order in order to minimize the OBDD representation for a given function is an NP-complete problem

OBDDs Pros and Cons

- This also holds for Model Checking in general
- Not possible to say a-priori if a system will fit in the available resources when using a model checker
  - RAM and computation time
- Also, it is not possible to decide which model checker is better
  - explicit (Murphi-or-SPIN like) or symbolic (NuSMV like)?
- However, we are going to see some guidelines
  - as for OBDDs: a good ordering is to interleave present and future variables
  - variable ordering: if OBDDs grow, the model checker can try a different variable ordering

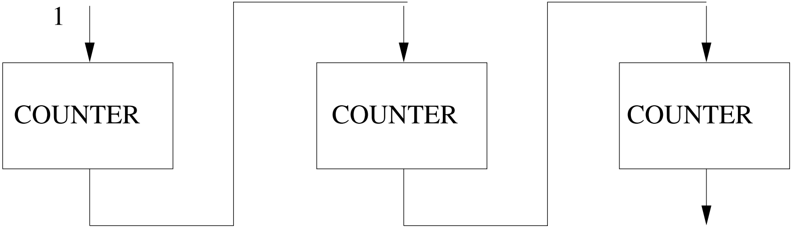
NuSMV Input Language

```
MODULE counter_cell(carry_in)
VAR value : boolean;
ASSIGN
  init(value) := 0;
  next(value) := (value + carry_in) mod 2;
DEFINE carry_out := value & carry_in;

MODULE main
VAR
  bit0 : counter_cell(1);
  bit1 : counter_cell(bit0.carry_out);
  bit2 : counter_cell(bit1.carry_out);

SPEC AG(!bit3.carry_out)
```

Counter Cell



NuSMV Input Language

- 2 modules, `main` and `counter_cell`
- Main *instantiates* the module `counter_cell` for 3 times
- This is an hardware-like instantiation: the `main` module contains 3 equal copies of the `counter_cell` module, the only difference being the lines in input
- Note that this means the module `main` will have 3 copies of variable `value`

- Note that `carry_out` (being inside a `DEFINE` section) is *not* a variable, as it is only a shortcut for the expression it defines
  - i.e., there will not be a corresponding variable in the OBDD
  - and indeed, it is not declared as a variable...
- Hence, `bit0` will always sum 1 to its internal variable, and `bit1` will sum 1 only if `bit0` will generate a carry
- The `main` module defines a counter from 0 to 7

```
MODULE user(semaphore)
VAR
  state : {idle, entering, critical, exiting};
ASSIGN
  init(state) := idle;
  next(state) :=
    case
      state = idle: entering;
      state = entering & !semaphore: critical;
      state = critical: {critical, exiting};
      state = exiting: idle;
      TRUE : state;
    esac;
```

NuSMV Input Language

```
next(semaphore) :=
  case
   state = entering: TRUE;
   state = exiting: FALSE;
   TRUE: semaphore;
  esac;
```

NuSMV Input Language

```
MODULE main
VAR
  semaphore : boolean;
  proc1 : process user(semaphore);
  proc2 : process user(semaphore);
ASSIGN
  init(semaphore) := FALSE;

SPEC
  AG(!(proc1.state = critical & proc2.state = critical))

LTLSPEC
  G F proc1.state = critical
```

NuSMV Input Language

- In the previous examples, all variables were evolving at the same time
- There is a global clock as in a synchronous digital circuit: given the current value for all variables in the current clock tick, in the next clock tick all variables may change their variables at the same time (synchronously: hardware parallel execution)
- In this example, instead, instantations are *processes*
- I.e., just one variable at a time may change; other variables are forced to stay fixed
    - this entails that only variables inside the selected process may change
    - other "free" (non-process) variables may change as well, as `sempahore`
    - try this example without processes (and without `RUNNING`)
- No dynamic process spawning as in SPIN: the number of processes is known from the beginning

NuSMV Input Language

- *Synchronous* vs. *asynchronous* systems
- In asynchronous systems, there is essentially one (implicit) additional module, which acts as a scheduler
- This is indeed what the verification algorithm does
- Each process is automatically provided with an additional variable `running` which is true iff that process is currently running

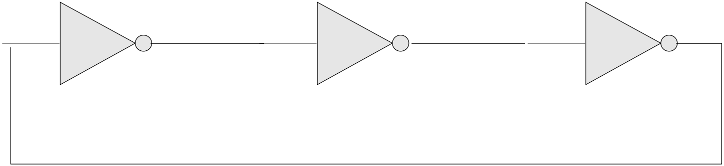
NuSMV Input Language

```
MODULE inverter(input)
VAR
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := !input;

MODULE main
VAR
  gate1 : process inverter(gate3.output);
  gate2 : process inverter(gate1.output);
  gate3 : process inverter(gate2.output);

SPEC
  AG(!gate2.output | !gate3.output)
```

Inverter Cell



NuSMV Input Language

Using direct specification it is possible to define non-total transition relations or empty initial states set

```
MODULE inverter(input)
VAR
  output : boolean;
INIT
  output = 0
TRANS
  next(output) = !input
```

NuSMV Input Language

Without processes, is it equivalent?

```
MODULE inverter(input)
VAR
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := !input union output;
               -- or {!input, output}

MODULE main
VAR
  gate1 : inverter(gate3.output);
  gate2 : inverter(gate1.output);
  gate3 : inverter(gate2.output);
```

NuSMV As A Tool

- NuSMV is provided with an interactive shell, as there are many tasks it may accomplish (simulation, many verification options); see user manual from chapter 3, especially Figure 3.1 at page 87
- Differently from explicit model checkers, no need to give separate commands to generate a file to be compiled and executed: all is represented as OBDDs, you only have to use them properly
- Executing a non-interactive verification in NuSMV is the same as giving the following list of interactive commands
- 1. `read_model` reads and stores the syntactic structure of the input model
  - no OBDDs here: tree-like structure, but representing the syntactic structure of the input (abstract syntax tree)

NuSMV As A Tool

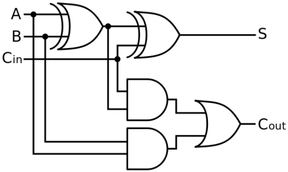
- 2. `flatten_hierarchy` (recursively) brings inside `main` all modules instantiated by `main`
  - very similar to the unfolding we mentioned for Murphi and SPIN: for such explicit model checkers, this was only needed for theoretical purposes, in order to define the Kriepke structure of an input model
  - here, it must be actually performed in the source code of NuSMV, in order to then be able to encode R and I as OBDDs
  - to this aim, there must be only one module, the `main`, containing all variables coming from the modules it instantiates (to be applied recursively)
  - note that, again, this resembles digital circuits, where such a flattening is a natural operation
  - this could entail adding a scheduler module if processes are used

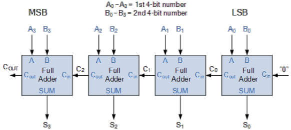
NuSMV As A Tool

- 3. `encode_variables` for each variable x with domain D s.t. $|D| > 2$, NuSMV defines $x_1 \ldots, x_m$ *boolean* variables with $m = \lfloor \log_2 |D| \rfloor + 1$; it also defines the encoding for constants used in the input models
- 4. `build_flat_model` combines the result of the preceding operations to obtain the flattenized and boolenized syntactic structure which represents the Kriepke structure defined by the input model
- 5. `build_model` from the syntactic structure to OBDDs for R ed I (plus other ones)
- 6. `check_ctlspec` (or `check_ltlspec`, or both, depending on what you have to verify); it starts the actual verification
  - we will be back soon on these last 2 steps

From Syntactic Structure to OBDDs

- How does `build_model` work?
- All operations must be implemented bitwise (*bit-vector*)
  - this means that we have to build the corresponding digital circuit, remember the Digital Systems Design course?
  - if we have to implement a sum between two variables encoded with maximum 4 bits (note that result is on 5 bits):





From Syntactic Structure to OBDDs

- Analogously, you can represent other arithmetic operations (subtract, multiply, divide)
- With other simple digital circuits, also equality and ordering can be easily implemented
  - e.g., `next(a) = b + c` is translated in this way:
  - multiple OBDDs are used to sum all bits of `b` and `c`
  - an OBDD B is created which is true iff all variables of `next(a)` are equal to such OBDDs
  - e.g., `next(a) = case a < b:  b + c; TRUE : a` is translated in this way:
  - again we have B as before, plus an OBDD C which is true if `a < b`
  - then, NuSMV computes the OBDD $ITE(C, B, a)$

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From a NuSMV Description to KS

- From a NuSMV model $\mathcal{M}$ (defined with the `ASSIGN` section) to the corresponding Kriepke structure $\mathcal{S} = (S, I, R, L)$
  - $V = \langle v_1, \ldots, v_n \rangle$ is the set of variables defined inside the `main` module of $\mathcal{M}$, with domains $\langle D_1, \ldots, D_n \rangle$
    - note that each D_i may be the instantiation of other modules
    - in which case, again, all variables must be considered as *unfolded*
    - that is, if a variable v is the instantiation of a module with k variables, then v counts as k variables instead of one
    - if one of such k variables is another instantiation, this procedure must be recursively repeated
    - NuSMV calls this operation *hierarchy flattening*
    - essentially, it is the same as for records in Murphi
    - simple types are the recursion base step

From a NuSMV Description to KS

- $S = D_1 \times \ldots \times D_n$ (as in Murphi)
- I is defined by looking at `init` predicates
  - $s \in I$ iff, for all variables $v \in V$, $s(v) \in \texttt{init}(v)$
    - note that, by NuSMV syntax, each $\texttt{init}(v)$ is actually a set (possibly a singleton)
  - if $\texttt{init}(v)$ is not specified in $\mathcal{M}$, then any value for v is ok: in this case, formally, if $s \in I$, then also $s' \in I$ being $s'(v') = s(v') \forall v' \neq v$

From a NuSMV Description to KS

- R is defined by looking at `next` predicates
  - we assume all `next` predicates to be defined by the `case` construct (if not, simply assume it is the `case` construct with just one `TRUE` condition)
  - for each (flattened) variable v, we name $g_1(v), \ldots g_{k_v}(v)$ the conditions (guards) of the `case` for `next(`v`)`, and $a_1(v), \ldots a_{k_v}(v)$ the resulting values (actions) of the `case` for `next(`v`)`
  - note that, by NuSMV syntax, each $a_i(v)$ is actually a set (possibly a singleton)
  - $(s, s') \in R$ iff, for all variables $v \in V$, if $g_i(s(v)) \wedge \forall j < i \neg g_j(s(v))$ then $s'(v) \in a_i(v)$
  - that is, s may go in s' iff, for all variables v, if the values of v in s satisfy the guard g_i (and none of the preceding guards for the same variable), then the value of v in s' is one of the values specified by the `case` for guard g_i
  - note that, in doing this, you also have to resolve inputs for modules

From a NuSMV Description to KS

- $AP = \{(v = d) \mid v = v_i \in V \wedge d \in D_i\}$
- $(v = d) \in L(s)$ iff variable v has value d in s
- If, instead, the NuSMV model $\mathcal{M}$ is defined with the `TRANS` section, then
  - $V = \langle v_1, \ldots, v_n \rangle$ is the set of variables as above and $S = D_1 \times \ldots \times D_n$
  - I is defined by looking at `INIT` section
    - $s \in I$ iff, for all variables $v \in V$ and for all `INIT` sections $\mathcal{I}$, $\mathcal{I}(s(v))$ holds
  - R is defined by looking at `TRANS` section
    - $(s, s') \in R$ iff, for all variables $v \in V$ and `TRANS` sections T, $T(s(v), s'(v))$ holds

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λ-calculus: Representing Functions

- In a nutshell: using $f(x)$ has some drawbacks
  - you are forced to name a function (f in the example above)
  - it is not always clear if a letter is a parameter or an argument
  - it is not computationally clear what happens for multiple inputs
    - $f(x, y)$: do you have to provide both x, y, otherwise you get an error?
    - as an alternative, you may provide just one argument, and obtain a new function
    - e.g. $f(x, y) = x + y$, we have that $f(x, 4)$ is a function on x

λ-calculus: Representing Functions

- Instead of writing $f(x) = E(x)$, for some expression $E(x)$, we write $\lambda x.E(x)$
  - if you *want*, you can name a function $f(x) = \lambda x.E(x)$
- $\lambda(x, y).x + y$: both arguments must be given, otherwise it is an error
- $\lambda x \lambda y.x + y$: if you provide $x = 4$ only, you get a function $\lambda y.4 + y$
- If an OBDD contains variables $x_1, \ldots, x_n$, then it represent some function $\lambda x_1 \ldots \lambda x_n.\ E(x_1, \ldots, x_n)$

μ-calculus: Fixpoints

- In a nutshell: we have a set L with an ordering $\leq$
  - $\leq$ could be partial, i.e., not defined on some pair $(l_1, l_2) \in L \times L$
  - $L, \leq$ is a *complete lattice* if any subset $A \subseteq L$ has a greatest lower bound and a least upper bound in L
  - $\sup A = \min\{\xi \in L \mid \forall \alpha \in A.\ \alpha \leq \xi\} \rightarrow \sup A \in L$
  - $\inf A = \max\{\xi \in L \mid \forall \alpha \in A.\ \xi \leq \alpha\} \rightarrow \inf A \in L$

μ-calculus: Fixpoints

- Let $I = \{0, \ldots, 10\}$, then $L = (2^I, \subseteq)$ is a complete lattice
  - e.g., $\{0, 1, 2\} \leq \{0, 1, 2, 3\}$, whilst $\{0, 1, 2\}, \{0, 1, 3\}$ cannot be compared
  - $\sup\{\{0, 1, 2\}, \{0, 1, 3\}\} = \min\{\xi \in 2^I \mid \forall \alpha \in \{0, 1, 2\}, \{0, 1, 3\}. \alpha \subseteq \xi\} = \min\{\{0, 1, 2, 3\}, \ldots, I\} = \{0, 1, 2, 3\}$
  - $\inf\{\{0, 1, 2\}, \{0, 1, 3\}\} = \max\{\xi \in 2^I \mid \forall \alpha \in \{0, 1, 2\}, \{0, 1, 3\}. \xi \subseteq \alpha\} = \max\{\{0, 1\}, \ldots, \varnothing\} = \{0, 1\}$
- $2^I, \subseteq$ is always a complete lattice, if I is a finite set
  - $\sup J = \cup_{\xi \in J}\, \xi$, $\inf J = \cap_{\xi \in J}\, \xi$
  - at the worst, $\sup J = I$ and $\inf J = \varnothing$

μ-calculus: Fixpoints

- Suppose you have a function $T : L \to L$. An element $\xi \in L$ is a *fixpoint of T* iff $T(\xi) = \xi$
- Given a T, there may be several fixpoints: we are interested in the maximum or the minimum of such fixpoints
  - notation μT and νT
  - where typically T is expressed with a λ notation
  - $\mu T \equiv \xi$ s.t. $T(\xi) = \xi \wedge \forall \rho \in L. T(\rho) = \rho \to \xi \leq \rho$
  - $\nu T \equiv \xi$ s.t. $T(\xi) = \xi \wedge \forall \rho \in L. T(\rho) = \rho \to \rho \leq \xi$

μ-calculus: Fixpoints

- Let again $I = \{0, \ldots, 10\}$
- Let $T : 2^I \to 2^I$ be defined as $T(\xi) = \xi$, or better $T \equiv \lambda\xi.\xi$
  - we have $\mu T = \varnothing$, $\nu T = I$
- Let $T \equiv \lambda\xi.\varnothing$
  - we have $\mu T = \nu T = \varnothing$
- Let $T \equiv \lambda\xi.\ \xi \cup \{10\}$
  - we have $\mu T = \{10\}, \nu T = I$
- Let $T \equiv \lambda\xi.\ \xi \setminus \{10\}$
  - we have $\nu T = \{0, \ldots, 9\}, \mu T = \varnothing$

μ-calculus: Fixpoints

- We define sets by their characteristic function, thus let us rewrite the previous examples
  - thus the ξ in $\lambda\xi$ is a function $\xi : I \to \{0, 1\}$
  - it represents a set X, thus $\xi(x) = 1$ iff $x \in X$
- $T \equiv \lambda\xi.\xi$ is ok also if ξ is a characteristic function
  - or, more explicit: $T \equiv \lambda\xi.\lambda x.\xi(x)$
- $T \equiv \lambda\xi.\varnothing$ could be rewritten as $T \equiv \lambda\xi.\lambda x.0$
- $T \equiv \lambda\xi.\xi \cup \{10\}$ could be rewritten as $T \equiv \lambda\xi.\lambda x.[x = 10 \to 1] \wedge [x \neq 10 \to \xi(x)]$
  - $\mu T \equiv \lambda x.x = 10, \nu T \equiv \lambda x.1$
- $T \equiv \lambda\xi.\xi \setminus \{10\}$ could be rewritten as $T \equiv \lambda\xi.\lambda x.[x = 10 \to 0] \wedge [x \neq 10 \to \xi(x)]$
  - $\nu T \equiv \lambda x.x \neq 10, \mu T \equiv \lambda x.0$

μ-calculus: Fixpoints

- We deal with monotonic (i.e., increasing or decreasing) T, thus fixpoints always exists
  - $\xi \leq \rho \rightarrow T(\xi) \leq T(\rho)$, T monotonically increasing
  - $\xi \leq \rho \rightarrow T(\rho) \leq T(\xi)$, T monotonically decreasing
- Previous examples are all monotonic
- By (weak) Knaster-Tarski theorem, $\mu T = \inf\{\xi \mid T(\xi) \leq \xi\}$
  - analogously, $\nu T = \sup\{\xi \mid T(\xi) \geq \xi\}$

μ-calculus: Fixpoints Computation

- Consequence of Knaster-Tarski: computing μT and νT may be done as follows
- For $k \geq 1$, let $T^k(\xi) = T(T^{k-1}(\xi))$, with $T^1 = T$
- For least fixpoints (μT), start with $\varnothing$, and apply T since $T^k(\varnothing) = T^{k-1}(\varnothing)$
  - of course, $\varnothing = \lambda x.0$
- For greatest fixpoints (νT), start with U, and apply T since $T^k(U) = T^{k-1}(U)$
  - of course, $U = \lambda x.1$
- At most, $k = |U|$

Computation of Fixpoints in CTL Model Checking

- Given a KS $\mathcal{S} = (S, I, R, L)$, we want to label states, i.e., to identify subsets of S
  - those for which a given labeling holds
  - labels are CTL/LTL subformulas
- Thus, $L = 2^S$, $\leq$ is $\subseteq$ and $T : 2^S \to 2^S$
  - in the following, $x = x_1, \ldots, x_n$ with $n = \lceil \log |S| \rceil$
  - characteristic functions of subsets of S
  - thus, each subset of S (member of 2^S) is an OBDD
  - hence, a T takes an OBDD and returns another (possibly modified) OBDD
- At most, $k = |S|$
  - usually, much less than that

μ-calculus: Fixpoints

- The "really interesting" fixpoints are those which are recursively defined
  - typically, basing on some other already defined sets, i.e., characteristic functions
  - e.g., $T \equiv \lambda\xi.\lambda x.f(x) \vee \xi(x)$, where $f : S \to \{0,1\}$ is known
  - the compactly-written least and greatest fixpoints are $\mu Q.\lambda x.f(x) \vee Q(x)$ and $\nu Q.\lambda x.f(x) \vee Q(x)$
  - e.g., $T \equiv \lambda\xi.\lambda x.f(x) \wedge \xi(x)$
  - e.g., $T \equiv \lambda\xi.\xi(x)$
- By the Knaster-Tarski theorem and the previous reasoning, we may apply the following algorithms
  - least fixpoints μ are computed for increasing T
  - greatest fixpoints ν are computed for decreasing T
  - viceversa are trivial: μT is $\lambda x.0$ for decreasing T and νT is $\lambda x.1$ for increasing T

Computation of Least (Minimum) Fixpoint

```
OBDD lfp(MuFormula T) /* μZ.T(Z) */
{
  Q = λx. 0;
  Q' = T(Q);
  /* T clearly says where Q must be replaced */
  /* e.g.: if μZ. λx. f(x) ∨ Z(x), then
       Q' = λx. f(x) ∨ Q(x) */
  while (Q ≠ Q') {
    Q = Q';
    Q' = T(Q);
  }
  return Q; /* or Q', they are the same... */
}
```

Computation of Greatest (Maximum) Fixpoint

```
OBDD gfp(NuFormula T) /* νZ.T(Z) */
{
  Q = λx. 1;
  Q' = T(Q);
  while (Q ≠ Q') {
    Q = Q';
    Q' = T(Q);
  }
  return Q;
}
```

Symbolic Model Checking of **AG**p

- The idea is to compute the set of reachable states, and check if for all of them p holds
- $\mathrm{Reach} = \mu Z.\ \lambda x.\ [I(x) \vee \exists y :\ (Z(y) \wedge R(y, x))]$
  - of course, we get an OBDD on x as a result
  - recall that x (and y) is a vector of all boolean variables
- $\forall x \in S.\ \mathrm{Reach}(x) \rightarrow p(x)$
  - computationally easier: check that $\mathrm{Reach}(x) \wedge \neg p(x) = 0$
  - otherwise, we have a reachable state for which p does not hold...

Symbolic CTL Model Checking

- All CTL formulas can be reduced to 3: $\mathbf{EX}f$, $f\ \mathbf{EU}\ g$, $\mathbf{EG}f$
  - all other formulas may be reduced to these three, using negation and other boolean combinations
  - with OBDDs, we can do all such things!
- Given OBDDs for f (and g), we compute the OBDD representing $\mathbf{EX}f$, $f\ \mathbf{EU}\ g$, $\mathbf{EG}f$
  - that is, the OBDD for the set $X = \{s \in S \mid \mathcal{S}, s \models \mathbf{EX}f\}$ etc
- Let it be B: then, simply check $\neg B(x) \wedge I(x) = 0$
  - recall that $\mathcal{S} \models \Phi$ iff $\forall s \in I.\ \mathcal{S}, s \models \Phi$
- $\mathbf{EX}f$ does not require a fixpoint computation: it is equivalent to (the OBDD representing) $\lambda x.\ \exists y:\ R(x, y) \wedge f(y)$

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Symbolic CTL Model Checking

- For f **EU** g, recall that it is equivalent to the CTL formula $g \vee (f \wedge \mathbf{EX}(f\ \mathbf{EU}\ g))$
- Thus, $f\ \mathbf{EU}\ g = \mu Z.\ \lambda x.\ g(x) \vee (f(x) \wedge \mathbf{EX}Z(x)) = \mu Z.\ \lambda x.\ g(x) \vee (f(x) \wedge (\exists y:\ R(x,y) \wedge Z(y)))$
  - note that $g(x) \vee (f(x) \wedge \mathbf{EX}Z(x))$ is increasing, i.e. for $Z_1 \subseteq Z_2$ we have that $(g(x) \vee (f(x) \wedge \mathbf{EX}Z_1(x)) \rightarrow (g(x) \vee (f(x) \wedge \mathbf{EX}Z_2(x))$
- Analogously: $\mathbf{EG}f = f \wedge \mathbf{EX}(\mathbf{EG}f)$, thus $\mathbf{EG}f = \nu Z.\ \lambda x.\ f(x) \wedge \mathbf{EX}Z(x) = \nu Z.\ \lambda x.\ f(x) \wedge (\exists y:\ R(x,y) \wedge Z(y))$
  - note that $f(x) \wedge \mathbf{EX}Z(x)$ is decreasing, i.e. for $Z_1 \subseteq Z_2$ we have that $(f(x) \wedge \mathbf{EX}Z_2(x)) \rightarrow (f(x) \wedge \mathbf{EX}Z_1(x))$

```
bool checkCTL(KS 𝒮, CTL φ) {
   let 𝒮 = ⟨S, I, R, L⟩;
   B = LblSt(φ);
   return λx. I(x) ∧ ¬B(x) = λx. 0;
}
OBDD LblSt(CTL φ) { /* also 𝒮 = ⟨S, I, R, L⟩ */
  if (∃p ∈ AP. φ = p) return λx. p(x);
  else if (φ = ¬ϕ) return λx. ¬LblSt(ϕ)(x);
  else if (φ = ϕ1 ∧ ϕ2)
   return λx.LblSt(ϕ1)(x)∧LblSt(ϕ2)(x);
  else if (φ = EXϕ)
   return λx. ∃y : R(x, y)∧LblSt(ϕ)(y);
  else if (φ = EGϕ)
   return gfp(νZ. λx. LblSt(ϕ)(x) ∧ (∃y : R(x, y) ∧ Z(y)));
  else if (φ = ϕ1 EU ϕ2)
   return lfp(μZ. λx. LblSt(ϕ2)(x)∨
     (LblSt(ϕ1)(x) ∧ (∃y : R(x, y) ∧ Z(y))));
}
```