

# Software Testing and Validation

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Corso di Laurea in Informatica

## Kripke Structures and Murphi Verification Algorithm(s)

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# Kripke Structures

- Let  $AP$  be a set of “atomic propositions”
  - in the sense of first-order logic: each atomic proposition is either true or false
  - typically identified with lower case letters  $p, q, \dots$
- A *Kripke Structure* (KS) over  $AP$  is a 4-tuple  $\langle S, I, R, L \rangle$ 
  - $S$  is a finite set, its elements are called *states*
  - $I \subseteq S$  is a set of *initial states*
  - $R \subseteq S \times S$  is a *transition relation*
  - $L : S \rightarrow 2^{AP}$  is a *labeling function*



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# Labeled Transition Systems

- A *Labeled Transition System* (LTS) is a 4-tuple  $\langle S, I, \Lambda, \delta \rangle$ 
  - $S$  is a finite set of states as before
  - $I \subseteq S$  is a set of initial states as before (not always included)
  - $\Lambda$  is a finite set of *labels*
  - $\delta \subseteq S \times \Lambda \times S$  is a *labeled transition relation*



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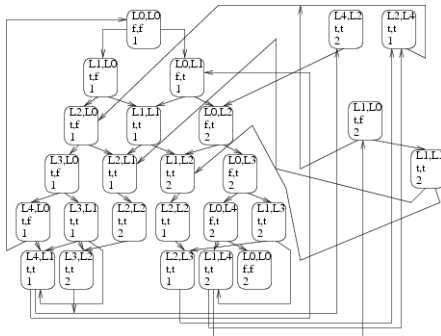
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# Peterson's Mutual Exclusion as a Kripke Structure

- $S = \{(p_1, p_2, q_1, q_2, t) \mid p_1, p_2 \in \{L0, L1, L2, L3, L4\}, q_1, q_2 \in \{0, 1\}, t \in \{1, 2\}\} = \{L0, L1, L2, L3, L4\}^2 \times \{0, 1\}^2 \times \{1, 2\}$
- $I = \{L0\}^2 \times \{0\}^2 \times \{1, 2\}$
- $R$ : see next slide
- $AP = \{(P_1 = v) \mid v \in \{L0, L1, L2, L3, L4\}\} \cup \{(P_2 = v) \mid v \in \{L0, L1, L2, L3, L4\}\} \cup \{(Q_1 = v) \mid v \in \{0, 1\}\} \cup \{(Q_2 = v) \mid v \in \{0, 1\}\} \cup \{(\text{turn} = v) \mid v \in \{1, 2\}\}$ 
  - e.g.:  $L((L0, L0, 0, 0, 1)) = \{(P_1 = L0), (P_2 = L0), (Q_1 = 0), (Q_2 = 0), (\text{turn} = 1)\}$



# Peterson's Mutual Exclusion as a Kripke Structure



E.g.:  $((L0, L0, 0, 0, 1), (L1, L0, 1, 0, 1)) \in R$ , whilst  
 $((L0, L0, 0, 0, 1), (L2, L0, 0, 0, 1)) \notin R$

Transitions in  $R$  corresponds to arrows in the figure above



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# Kripke Structure vs Labeled Transition Systems

- KSs have atomic propositions on states, LTSs have labels on transitions
- In model checking, atomic propositions are mandatory
  - to specify the formula to be verified, as we will see
  - a first example was the invariant in Murphi
- Instead, it is not required to have a label on transitions
  - Murphi allows to do so, but it is optional
  - may be easily added automatically, if needed
- Labels are typically needed when:
  - we deal with macrostates, as in UML state diagrams
  - when we are describing a complex system by specifying its sub-components, so labels are used for synchronization



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# Total Transition Relation

- In many cases, the transition relation  $R$  is required to be *total*
- $\forall s \in S. \exists s' \in S : (s, s') \in R$ 
  - this of course allows also  $s = s'$  (*self loop*)
- In the Peterson's example, the relation is actually total
  - Murphi allows also non-total relations, by using option `-ndl`
  - note however that not giving option `-ndl` is stronger:  
 $\forall s \in S. \exists s' \in S : s \neq s' \wedge (s, s') \in R$
  - otherwise, if  $s$  is s.t.  $\forall s'. s = s' \vee (s, s') \notin R$ , Murphi calls  $s$  a *deadlock* state
  - that is, you cannot go anywhere, except possibly self looping on  $s$
- By deleting any rule, we will obtain a non-total transition relation



# Non-Determinism

- The transition relation is, as the name suggests, a relation
- Thus, starting from a given state, it is possible to go to many different states
  - in a deterministic system,
$$\forall s_1, s_2, s_3 \in S. (s_1, s_2) \in R \wedge (s_1, s_3) \in R \rightarrow s_2 = s_3$$
  - this does not hold for KSs
- This means that, starting from state  $s_1$ , the system may *non-deterministically* go either to  $s_2$  or to  $s_3$ 
  - or many other states
- Motivations for non-determinism: modeling choices!
  - underspecified subsystems
  - unpredictable interleaving
  - interactions with an uncontrollable environment
  - ...



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# Some Useful Notation

- Given a KS  $\mathcal{S} = \langle S, I, R, L \rangle$ , we can define:
  - the *predecessor* function  $\text{Pre}_{\mathcal{S}} : S \rightarrow 2^S$ 
    - defined as  $\text{Pre}_{\mathcal{S}}(s) = \{s' \in S \mid (s', s) \in R\}$
    - we will write simply  $\text{Pre}(s)$  when  $\mathcal{S}$  is understood
  - the *successor* function  $\text{Post} : S \rightarrow 2^S$ 
    - defined as  $\text{Post}(s) = \{s' \in S \mid (s, s') \in R\}$
- Note that, if  $\mathcal{S}$  is deterministic,  $\forall s \in S. |\text{Post}(s)| \leq 1$
- Note that, if  $\mathcal{S}$  is total,  $\forall s \in S. |\text{Post}(s)| \geq 1$



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# Paths in KSs

- A path (or *execution*) on a KS  $\mathcal{S} = \langle S, I, R, L \rangle$  is a sequence  $\pi = s_0 s_1 s_2 \dots$  such that:
  - $\forall i \geq 0. s_i \in S$  (it is composed by states)
  - $\forall i \geq 0. (s_i, s_{i+1}) \in R$  (it only uses valid transitions)
- We will denote  $i$ -th state of a path as  $\pi(i) = s_i$
- Note that paths in LTSs also have actions:  $\pi = s_0 a_0 s_1 a_1 \dots$   
s.t.  $(s_i, a_i, s_{i+1}) \in \delta$



# Paths in KSs

- The *length* of a path  $\pi$  is the number of states in  $\pi$ 
  - paths can be either finite  $\pi = s_0 s_1 \dots s_n$ , in which case  $|\pi| = n + 1$
  - or infinite  $\pi = s_0 s_1 \dots$ , in which case  $|\pi| = \infty$
- We will denote the prefix of a path up to  $i$  as  $\pi|_i = s_0 \dots s_i$ 
  - a prefix of a path is always a finite path
- A path  $\pi$  is *maximal* iff one of the following holds
  - $|\pi| = \infty$
  - $|\pi| = n + 1$  and  $|\text{Post}(\pi(n))| = 0$ 
    - that is,  $\forall s \in S. (\pi(n), s) \notin R$
    - i.e., the last state of the path has no successors
    - often called *terminal state*
- If  $R$  is total, maximal paths are always infinite
  - for many model checking algorithms, this is required



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# Reachability

- The set of paths of  $\mathcal{S}$  starting from  $s \in S$  is denoted by  $\text{Path}(\mathcal{S}, s) = \{\pi \mid \pi \text{ is a path in } \mathcal{S} \wedge \pi(0) = s\}$
- The set of paths of  $\mathcal{S}$  is denoted by  $\text{Path}(\mathcal{S}) = \cup_{s \in I} \text{Path}(\mathcal{S}, s)$ 
  - that is, they must start from an initial state
- A state  $s \in S$  is *reachable* iff  $\exists \pi \in \text{Path}(\mathcal{S}), k < |\pi| : \pi(k) = s$ 
  - i.e., there exists a path from an initial state leading to  $s$  through valid transitions
- The set of reachable states is defined by  $\text{Reach}(\mathcal{S}) = \{\pi(i) \mid \pi \in \text{Path}(\mathcal{S}), i < |\pi|\}$



# Safety Property Verification

- Verification of *invariants*: nothing bad happens
- The property is a formula  $\varphi : S \rightarrow \{0, 1\}$ 
  - built using boolean combinations of atomic propositions in  $p \in AP$
  - i.e., the syntax is

$$\Phi ::= (\Phi) \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \neg \Phi \mid p$$

- The KS  $\mathcal{S}$  satisfies  $\varphi$  iff  $\varphi$  holds on all reachable states
  - $\forall s \in \text{Reach}(\mathcal{S}). \varphi(s) = 1$
- Note that it may happen that  $\varphi(s) = 0$  for some  $s \in S$ : never mind, if  $s \notin \text{Reach}(\mathcal{S})$



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# From Murphi Description $\mathcal{M}$ to KS $\mathcal{S}$

- First, we mathematically define a Murphi description  $\mathcal{M}$
- $V = \langle v_1, \dots, v_n \rangle$  is the set of global variables of  $\mathcal{M}$ , with domains  $\langle D_1, \dots, D_n \rangle$ 
  - all variables are *unfolded* to the Murphi simple types
    - integer subranges
    - enumerations
    - the special “undefined” value should be added to all simple types
  - that is, if a variable is an array with  $q$  elements, then it is actually to be considered as  $q$  different variables
  - the same for records (and any nesting of arrays and records)
  - as an example: `var a : array [1..n] of record begin`  
`b : 1..m; c: 1..k; endrecord`
  - then there will be  $2n$  variables as follows:  
 $a1b, \dots, anb, a1c, \dots, anc$
  - the first  $n$  with type  $1..m$ , the other with type  $1..k$



# From Murphi Description $\mathcal{M}$ to KS $\mathcal{S}$

- $\mathcal{I} = \{I_1, \dots, I_k\}$  is the set of startstate sections in  $\mathcal{M}$ 
  - startstates may be defined inside rulesets; again, all rulesets are *unfolded*
  - thus, if a startstate  $\mathcal{I}$  is inside  $m$  nested rulesets  $\mathcal{R}_1, \dots, \mathcal{R}_m$ ...
  - and each ruleset  $\mathcal{R}_i$  is defined on an index  $j_i$  spanning on a domain  $\mathcal{D}_i$  (note that  $\mathcal{D}_i$  must be a simple type)...
  - then there actually are  $\prod_{i=1}^m |\mathcal{D}_i|$  startstates to be considered, instead of just one
  - of course, in each of these startstates definitions, the tuple  $j_1, \dots, j_m$  takes all possible values of  $\mathcal{R}_1 \times \dots \times \mathcal{R}_m$
- $\mathcal{T} = \{T_1, \dots, T_p\}$  is the set of rule sections in  $\mathcal{M}$ 
  - again, if rulesets are present, they are *unfolded*



# From Murphi Description $\mathcal{M}$ to KS $\mathcal{S}$

- The Kripke structure  $\mathcal{S} = \langle S, I, R, L \rangle$  described by  $\mathcal{M}$  is such that:
  - $S = D_1 \times \dots \times D_n$
  - $s \in I$  iff there is a startstate  $I_i \in \mathcal{I}$  s.t.  $s$  may be obtained by applying the body of  $I_i$
  - $(s, t) \in R$  iff there is a rule  $T_i \in T$  s.t.  $T_i$  guard is true in  $s$  and  $T_i$  body changes  $s$  to  $t$
  - $AP = \{(v = d) \mid v = v_i \in V \wedge d \in D_i\}$
  - $(v = d) \in L(s)$  iff variable  $v$  has value  $d$  in  $s$



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# From Murphi Description $\mathcal{M}$ to KS $\mathcal{S}$

- We also assume to have a function defining the semantics of Murphi (sequence of) statements
  - those in bodies of rules and startstates
- Let  $\mathcal{P}$  be the set of all possible (syntactically legal) Murphi statements
  - including while, if, for, assignments...
- Thus, let  $\eta : \mathcal{P} \times D_1 \times \dots \times D_n \rightarrow D_1 \times \dots \times D_n$  be our evaluation function
  - it takes a Murphi statement  $P \in \mathcal{P}$  and the state  $s$  preceding such statement
  - it returns the new state  $s'$  obtained by executing  $P$  on  $s$
  - e.g.,  $\eta(a := a + 1; b := b - 1, (1, 2, 3)) = (2, 1, 3)$
  - $\eta$  may be defined, e.g., using operational semantics



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# From Murphi Description $\mathcal{M}$ to KS $\mathcal{S}$

- We also assume to have a function defining the semantics of Murphi boolean expression
  - those in guards of rules
  - and in invariants!
- Let  $\mathcal{Q}$  be the set of all possible (syntactically legal) Murphi boolean expressions
  - including forall, exists, equality checks...
- Thus, let  $\zeta : \mathcal{Q} \times D_1 \times \dots \times D_n \rightarrow \{0,1\}$  be our evaluation function
  - it takes a Murphi boolean expression  $Q \in \mathcal{Q}$  and the state  $s$  to be evaluated
  - it returns 1 iff  $Q$  is true in  $s$
  - e.g.,  $\zeta((a = 3 | b = 4), (1, 4, 3)) = 1$
  - $\zeta$  may be defined using atomic propositions



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# From Murphi Description $\mathcal{M}$ to KS $\mathcal{S}$

- Let  $Q \in \mathcal{Q}$  be a Murphi boolean expression
- Flatten  $Q$  w.r.t. Forall and Exists
  - Forall is replaced by ANDs, Exists by ORs
  - e.g., from Exists i1: pid Do Exists i2: pid Do (i1 != i2 & P[i1] = L3 & P[i2] = L3) End End ...
  - ... to (1 != 1 & P[1] = L3 & P[1] = L3) | (2 != 1 & P[2] = L3 & P[1] = L3) | (1 != 2 & P[1] = L3 & P[2] = L3) | (2 != 2 & P[2] = L3 & P[2] = L3)
- If we replace each variable  $v_i \in V$  occurring in  $Q$  with a value  $w_{j_i} \in D_i$ , we obtain a boolean value (true or false)
  - e.g., the former evaluates to true by setting  $P[1] = L3$  and  $P[2] = L3$
- Thus,  $\zeta(Q, s) = 1$  iff  $Q(w_{j_1}, \dots, w_{j_n}) = 1$ 
  - where each  $w_{j_i}$  is such that  $(v_i = w_{j_i}) \in L(s)$
  - $Q(w_{j_1}, \dots, w_{j_n})$  is the result of replacing variable  $v_i$  with value  $w_{j_i}$



# From Murphi Description $\mathcal{M}$ to KS $\mathcal{S}$

- $(s, t) \in R$  iff there is a rule  $T_i \in T$  s.t.  $T_i$  guard is true in  $s$  and  $T_i$  body changes  $s$  to  $t$
- By using  $\eta$  and  $\zeta$ , we can be more precise:
  - “ $T_i$  guard is true” means  $\zeta(G(T_i), s) = 1$ , being  $G(T_i)$  the Murphi expression used as guard of rule  $T_i$
  - “ $T_i$  body changes  $s$  to  $t$ ” means  $\eta(B(T_i), s) = t$ , being  $B(T_i)$  the Murphi statement used as body of rule  $T_i$
- $s \in I$  iff there is a startstate  $I_i \in \mathcal{I}$  s.t.  $s$  may be obtained by applying the body of  $I_i$ 
  - “ $s$  may be obtained by applying the body of  $I_i$ ” means  $\eta(B(I_i), (\perp, \dots, \perp)) = s$ , being  $B(I_i)$  the Murphi statement used as body of startstate  $I_i$



# From Murphi Description $\mathcal{M}$ to KS $\mathcal{S}$

- $(s, t) \in R$  iff there is a rule  $T_i \in T$  s.t.  $T_i$  guard is true in  $s$  and  $T_i$  body changes  $s$  to  $t$ :
  - that is: in the body of  $T_i$ , variables starting values are those of  $s$
  - note that there may be two or more rules defining the same transition from  $s$  to  $t$ ; no problem with this
  - simply, the same transition is described by multiple rules
- A state  $s$  is a deadlock state for two possible reasons:
  - ①  $(s, t) \notin R$  for all  $t \in S$ , i.e., the values for the variables in  $s$  do not satisfy any ruleset guard
  - ②  $(s, t) \in R \rightarrow t = s$ , i.e., there is some ruleset guard which is satisfied by  $s$ , but its body do not change any of the global variables (e.g., the body is empty)



# How to Verify a Murphi Description $\mathcal{M}$

- Theoretically, extract KS  $\mathcal{S}$  and property  $\varphi$  from  $\mathcal{M}$  as described above
  - for a given invariant  $I$  in  $\mathcal{M}$ ,  $\varphi(s) = \zeta(I, s)$  for all  $s \in S$
- Then, KS  $\mathcal{S}$  satisfies  $\varphi$  iff  $\varphi$  holds on all reachable states
  - $\forall s \in \text{Reach}(\mathcal{S}). \varphi(s) = 1$
- Thus, consider KS as a graph and perform a visit
  - states are nodes, transitions are edges
- If a state  $e$  s.t.  $\varphi(e) = 0$  is found, then we have an error
- Otherwise, all is ok



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# How to Verify a Murphi Description $\mathcal{M}$

- From a practical point of view, many optimization may be done, but let us stick to the previous scheme
- The worst case time complexity for a DFS or a BFS is  $O(|V| + |E|)$  (and same for space complexity)
- For KSs, this means  $O(|S| + |R|)$ , thus it is linear in the size of the KS
- Is this good? NO! Because of the *state space explosion problem*
- Assuming that  $B$  bits are needed to encode each state
  - i.e.,  $B = \sum_{i=1}^n b_i$ , being  $b_i$  the number of bits to encode domain  $D_i$
- We have that  $|S| = O(2^B)$



# State Space Explosion

- The “practical” input dimension is  $B$ , rather than  $|S|$  or  $|R|$
- Typically, for a system with  $N$  components, we have  $O(N)$  variables, thus  $O(B)$  encoding bits
- It is very common to verify a system with  $N$  components, and then (if  $N$  is ok) also for  $N + 1$  components
  - verifying a system with a generic number  $N$  of components is a proof checker task...
- This entails an exponential increase in the size of  $|S|$
- Thus we need “clever” versions of BFS/DFS



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# Standard BFS: No Good for Model Checking

- Assumes that all graph nodes are in RAM
- For KSs, graph nodes are states, and we know there are too many
  - state space explosion
- You also need a full representation of the graph, thus also edges must be in RAM
  - using adjacency matrices or lists does not change much
  - for real-world systems, you may easily need TB of RAM
- Even if you have all the needed RAM, there is a huge preprocessing time needed to build the graph from the Murphi specification
- Then, also BFS itself may take a long time



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# Murphi BFS

- We need a definition inbetween the model and the KS: NFSS (Nondeterministic Finite State System)
- $\mathcal{N} = \langle S, I, \text{Post} \rangle$ , plus the invariant  $\varphi$ 
  - $S$  is the set of states,  $I \subseteq S$  the set of initial states
  - $\text{Post} : S \rightarrow 2^S$  is the successor function as defined before
    - given a state  $s$ , it returns  $T$  s.t.  $t \in T \rightarrow (s, t) \in R$
  - no labeling, we already have  $\varphi$



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# Murphi BFS

- KSs and NFSSs differ on having  $\text{Post}$  instead of  $R$
- $\text{Post}$  may easily be defined from the Murphi specification
- Such definition is implicit, as programming code, thus avoiding to store adjacency matrices or lists
  - $t \in \text{Post}(s)$  iff there is a rule  $T_i \in T$  s.t.  $T_i$  guard is true in  $s$  and  $T_i$  body changes  $s$  to  $t$ 
    - see above for using  $\eta$  and  $\zeta$
  - Essentially, if the current state is  $s$ , it is sufficient to inspect all (flattened) rules in the Murphi specification  $\mathcal{M}$ 
    - for all guards which are enabled in  $s$ , execute the body so as to obtain  $t$ , and add  $t$  to  $\text{next}(s)$
  - This is done “on the fly”, only for those states  $s$  which must be explored



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# Murphi Simulation

```
void Make_a_run(NFSS  $\mathcal{N}$ , invariant  $\varphi$ )
{
  let  $\mathcal{N} = \langle S, I, \text{Post} \rangle$ ;
  s_curr = pick_a_state(I);
  if (! $\varphi$ (s_curr))
    return with error message;
  while (1) { /* loop forever */
    s_next = pick_a_state(Post(s_curr));
    if (! $\varphi$ (s_next))
      return with error message;
    s_curr = s_next;
  }
}
```



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# Murphi Simulation

```
void Make_a_run(NFSS  $\mathcal{N}$ , invariant  $\varphi$ )
{
  let  $\mathcal{N} = \langle S, I, \text{Post} \rangle$ ;
  s_curr = pick_a_state(I);
  if ( $\neg \varphi(s_{\text{curr}})$ )
    return with error message;
  while (1) { /* loop forever */
    if ( $\text{Post}(s_{\text{curr}}) = \emptyset$ )
      return with deadlock message;
    s_next = pick_a_state( $\text{Post}(s_{\text{curr}})$ );
    if ( $\neg \varphi(s_{\text{next}})$ )
      return with error message;
    s_curr = s_next;
  }
}
```



# Murphi Simulation

- Similar to testing
- If an error is found, the system is bugged
  - or the model is not faithful
  - actually, Murphi simulation is used to understand if the model itself contains errors
- If an error is not found, we cannot conclude anything
- The error state may lurk somewhere, out of reach for the random choice in `pick_a_state`



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# Standard BFS (Cormen-Leiserson-Rivest)

BFS( $G, s$ )

```
1  for ogni vertice  $u \in V[G] - \{s\}$ 
2      do  $color[u] \leftarrow \text{WHITE}$ 
3           $d[u] \leftarrow \infty$ 
4           $\pi[u] \leftarrow \text{NIL}$ 
5   $color[s] \leftarrow \text{GRAY}$ 
6   $d[s] \leftarrow 0$ 
7   $\pi[s] \leftarrow \text{NIL}$ 
8   $Q \leftarrow \{s\}$ 
9  while  $Q \neq \emptyset$ 
10     do  $u \leftarrow head[Q]$ 
11         for ogni  $v \in Adj[u]$ 
12             do if  $color[v] = \text{WHITE}$ 
13                 then  $color[v] \leftarrow \text{GRAY}$ 
14                      $d[v] \leftarrow d[u] + 1$ 
15                      $\pi[v] \leftarrow u$ 
16                     ENQUEUE( $Q, v$ )
17     DEQUEUE( $Q$ )
18      $color[u] \leftarrow \text{BLACK}$ 
```



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# Murphi BFS

```
FIFO_Queue Q;  
HashTable T;  
  
bool BFS(NFSS  $\mathcal{N}$ , AP  $\varphi$ )  
{  
  let  $\mathcal{N} = (S, I, \text{Post})$ ;  
  foreach s in I {  
    if (! $\varphi(s)$ )  
      return false;  
  }  
  foreach s in I  
    Enqueue(Q, s);  
  foreach s in I  
    HashInsert(T, s);
```



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# Murphi BFS

```
while (Q  $\neq$   $\emptyset$ ) {  
  s = Dequeue(Q);  
  foreach s_next in Post(s) {  
    if (! $\varphi$ (s_next))  
      return false;  
    if (s_next is not in T) {  
      Enqueue(Q, s_next);  
      HashInsert(T, s_next);  
    } /* if */ } /* foreach */ } /* while */  
return true;  
}
```



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# Murphi BFS

- Edges are never stored in memory
  - states are “created” when expanding the current state
  - rules are used to modify the current state so as to obtain the new one
  - at the start, you have an empty state which is modified by startstates
- (Reachable) states are stored in memory only at the end of the visit
  - inside hashtable T
- This is called *on-the-fly* verification
- States are marked as visited by putting them inside an hashtable
  - rather than coloring them as gray or black
  - which needs the graph to be already in memory



# State Space Explosion

- State space explosion hits in the FIFO queue  $Q$  and in the hashtable  $T$ 
  - and of course in running time...
- However,  $Q$  is not really a problem
  - it is accessed *sequentially*
  - always in the front for extraction, always in the rear for insertion
  - can be efficiently stored using disk, much more capable of RAM
- $T$  is the real problem
  - random access, not suitable for a file
  - what to do?
  - before answering, let's have a look at Murphi code



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# Murphi Usage

- As for all *explicit* model checker, a Murphi verification has the following steps:
  - 0 compile Murph source code and write a Murphi model `model.m`
  - 1 invoke Murphi compiler on `model.m`: this generates a file `model.cpp`
    - `mu options model.m`
    - see `mu -h` for available options
  - 2 invoke C++ compiler on `model.cpp`: this generates an executable file
    - `g++ -Ipath_to_include model.cpp -o model`
    - `path_to_include` is the include directory inside Murphi distribution
  - 3 invoke the executable file
    - `./model options`
    - see `./model -h` for available options



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# Murphi compiler

- Executable `mu` is in `src` directory of Murphi distribution
- Obtained by compiling the 25 source files in `src`
  - of course, a Makefile is provided for this
- Standard compiler implementation, with Flex lexical analyzer (`mu.l`) and Yacc parser (`mu.y`)
- The main function which builds `model.cpp` is `program::generate_code` in `cpp_code.cpp` (called by `main`, in `mu.cpp`)
- `program::generate_code` uses the parse tree generated by Yacc to “implement” in C++ the guards and the bodies of the rules
- The result goes in `model.cpp`: model-specific code



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# Organization of model.cpp

- Each Murphi variable  $v$  (local or global) corresponds to a C++ instance  $\text{mu\_}v$  of the class  $\text{mu\_int}$  (possibly through class generalizations)
- Class  $\text{mu\_int}$  is used to handle variables with max value 254 (255 is used for the undefined value)
- For integer subranges with greater values, class  $\text{mu\_long}$  is used; also  $\text{mu\_byte}$  (equal to  $\text{mu\_int}$ ...) and  $\text{mu\_boolean}$  exist
- If  $v$  is a local variable,  $\text{mu\_}v$  directly contains the value (attribute  $\text{cvalue}$ ,  $\text{in\_world}$  is false)
- Otherwise, if  $v$  is global,  $\text{mu\_}v$  retrieves the value from a fixed-address structure containing the current state value ( $\text{workingstate}$ ;  $\text{in\_world}$  is true)



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# Organization of model.cpp

```
class mu__int {  
    enum {undef_value=0xff};  
    bool in_world;           /* local iff false */  
    int lb, ub;              /* bounds */  
    int byteOffset;         /* in bytes */  
    /* points to workingstate->bits[byteOffset]  
       for global variables, to cvalue for  
       local  
    */  
    unsigned char *valptr;  
    unsigned char cvalue;
```



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# Organization of model.cpp

**public:**

```
/* constructor, sets all attributes (the  
   variable is supposed to be local by  
   default, with an undefined value);  
   byteOffset is computed by generate_code  
*/  
mu__int(int lb, int ub, int size, char *n,  
        int byteOffset);  
/* other useful functions */  
int operator= (int val) {  
    if (val <= ub && val >= lb) value(val);  
    else boundary_error(val);  
    return val;  
}
```



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# Organization of model.cpp

```
operator int() const {  
    if (isundefined()) return undef_error();  
    return value();  
};  
const int value() const {return *valptr;};  
int value(int val) {  
    *valptr = val; return val;};  
void to_state(state *thestate) {  
    /* used to make the variable global */  
    in_world = TRUE;  
    valptr = (unsigned char *)&(workingstate->  
        bits[byteOffset]);  
};  
};
```



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# Organization of model.cpp

- As for the `byteOffset` computation, `program::generate_code` simply computes the one for a variable `mu_v` mapping a Murphi variable `v` in the following way
  - Let  $M_1, \dots, M_n$  be the upper bounds of the  $n$  variables preceeding the declaration of `v`
  - Let  $b(x) = \lfloor \log_2(x + 1) \rfloor + 1$  be the number of bits required to represent the maximum value  $x$  (plus the undefined value)
  - Let  $B(x) = 1$  if  $b(x) \leq 8$ , 4 otherwise (i.e. only 1-byte or 4-bytes integers may be used)
  - Then,  $\text{byteOffset}(\text{mu\_v}) = \sum_{i=1}^n B(M_i)$



# Organization of model.cpp: workingstate

- Structure containing the current global state, is an instance of class `state`
- Essentially, it consists of an array of unsigned characters, named `bits`
  - so that any value of any global variable may be casted inside it
  - at a precise location, pointed to by `valptr` from `mu__int`
- Note that `workingstate` has a fixed length, that is  $\text{BLOCKS\_IN\_WORLD} = \sum_{i=1}^N B(M_i)$ 
  - being  $N$  the number of all global variables
  - namely, `bits` has `BLOCKS_IN_WORLD` unsigned chars



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# Translation of Murphi Model Statements

- Straightforward for ifs, whiles and so on: the “difficult” part is assignments (and expressions evaluation)
- Essentially, a `:= b`; in `model.m` becomes `mu_a = (mu_b)`; in `model.cpp`
- The operator `()` is redefined so that `mu_b` retrieves the value for `b`, either from itself (attribute `cvalue`) or from `workingstate` (thanks to `valptr`)
- Then, the redefined operator `=` is called, so that `mu_a` updates the value for `a` to be equal to that of `b`, either from itself (attribute `cvalue`) or from `workingstate`
- If the right side of the assignment has a generic expression, it is evaluated in a similar way (the operator `()` solves the Murphi variable references, the other values will be integer constants or function calls...)
- BTW, functions are mapped as C++ methods...



# Translation of Murphi Rules

- For each rule  $i$  (starting from 0 at the *end* of `model.m`!) there is a class named `RuleBase $i$`
- Such class has `Code` method for the body and `Condition` method for the guard
- Startstates are similar, but they only have the body
- A suitable C++ code flattens rulesets, if present



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# Translation of Murphi Rules: From This...

```
Const VAL_LIM: 5;  
  
Type val_t : 0..VAL_LIM;  
  
Var v : val_t;  
  
Rule "incBy1"  
  v <= VAL_LIM - 1 ==>  
  Var useless : val_t;  
  Begin  
    useless := 1;  
    v := v + useless;  
  End;
```



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# Translation of Murphi Rules: ... To This

```
class RuleBase1 {
public:
  :
  bool Condition(unsigned r) { /* guard */
    return (mu_v) <= (4);
  }
  :
  void Code(unsigned r) { /* body */
    mu_1_val_t mu_useless("useless", 0);
    mu_useless = 1;
    mu_v = (mu_v) + (mu_useless);
  };
  :
}
```



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# Translation of Murphi Rules: From This...

```
ruleset i:  $l_1..u_1$  do
  ruleset j:  $l_2..u_2$  do
    Rule "incBy1"
      i < j ==>
        Begin v := v + i - j; End;
  Endruleset; Endruleset;
```



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## Translation of Murphi Rules: ... To This

```
class RuleBase0 {
public:
    bool Condition(unsigned r) {
        /* called  $(u_1 - l_1 + 1)(u_2 - l_2 + 1)$  with  $r$  ranging
           from 0 to  $(u_1 - l_1 + 1)(u_2 - l_2 + 1) - 1$  */
        static mu__subrange_7 mu_j;
        mu_j.value((r % (u_2 - l_2 + 1)) + l_2);
        r = r / (u_2 - l_2 + 1);
        static mu__subrange_6 mu_i;
        mu_i.value((r % (u_1 - l_1 + 1)) + l_1);
        /* useless, but it is automatically
           generated... */
        r = r / (u_1 - l_1 + 1);
        return (mu_i) < (mu_j);
    }
}
```



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# Translation of Murphi Rules: ... To This

```
void Code(unsigned r) {  
    static mu__subrange_7 mu_j;  
    mu_j.value((r % (u2 - l2 + 1)) + l2);  
    r = r / (u2 - l2 + 1);  
    static mu__subrange_6 mu_i;  
    mu_i.value((r % (u1 - l1 + 1)) + l1);  
    r = r / (u1 - l1 + 1);  
    mu_v = ((mu_v) + (mu_i)) - (mu_j);  
};  
  
:  
};
```



# Murphi Overall Translation

- Note that the first part of `Condition` and `Code` is meant to translate an integer from 0 to  $(u_1 - l_1 + 1)(u_2 - l_2 + 1) - 1$  in 2 values for the rulesets indices
- The interface class for the verification algorithm is `NextStateGenerator`
- Suppose there are  $R$  rules  $r_0, \dots, r_{R-1}$ , and that each  $r_i$  is contained in  $N_i$  nested rulesets having upper bound  $u_{ij}$  and lower bound  $l_{ij}$ , for  $j = 1, \dots, N_i$
- Note that `Condition` simply calls its homonymous method of the `RuleBase` class corresponding the current `r...`



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# Murphi Overall Translation

Let  $P(k) = \sum_{i=0}^{k-1} (\prod_{j=1}^{N_i} (u_{ij} - l_{ij} + 1)) + 1$  be the number of flattened rules preceding the rule  $r_k$ ;

```
class NextStateGenerator {  
    RuleBase0 R0;  
  
    :  
    RuleBase(R - 1) R(R - 1);  
public:  
    void SetNextEnabledRule(unsigned &  
        what_rule);
```



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# Murphi Overall Translation

```
bool Condition(unsigned r) { /* r will  
    range from 0 to P(R) */  
    category = CONDITION;  
    if (what_rule < P(1))  
        return R0.Condition(r - 0);  
    if (what_rule >= P(1) && what_rule < P(2))  
        return R1.Condition(r - P(1));  
    :  
    if (what_rule >= P(R-1) && what_rule <  
        P(R))  
        return R(R-1).Condition(r - P(R-1));  
    return Error;  
}
```



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# Murphi Overall Translation

```
void Code(unsigned r) {  
    if (what_rule < P(1)) {  
        R0.Code(r - 0); return;  
    }  
    if (what_rule >= P(1) && what_rule < P(2)) {  
        R1.Code(r - P(1)); return;  
    }  
    :  
    if (what_rule >= P(R-1) && what_rule <  
        P(R)) {  
        R(R-1).Code(r - P(R-1)); return;  
    } }  
};  
const unsigned numrules = P(R);
```



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## Step 2: What Is Actually Compiled by C++ Compiler

Concatenation of include/*.h
model.cpp
Concatenation of include/*.C



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# Murphi BFS

```
FIFO_Queue Q;  
HashTable T;  
  
bool BFS(NFSS  $\mathcal{N}$ , AP  $\varphi$ )  
{  
  let  $\mathcal{N} = (S, I, \text{Post})$ ;  
  foreach s in I {  
    if (! $\varphi(s)$ )  
      return false;  
  }  
  foreach s in I  
    Enqueue(Q, s);  
  foreach s in I  
    HashInsert(T, s);
```



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# Murphi BFS

```
while (Q  $\neq$   $\emptyset$ ) {  
  s = Dequeue(Q);  
  foreach s_next in Post(s) {  
    if (! $\varphi$ (s_next))  
      return false;  
    if (s_next is not in T) {  
      Enqueue(Q, s_next);  
      HashInsert(T, s_next);  
    } /* if */ } /* foreach */ } /* while */  
return true;  
}
```



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# BFS in Murphi

- $\text{Post}(s)$  is computed using class `NextStateGenerator`
- It is equivalent to a for loop on all flattened rules
- For each flattened rule index  $r$ ,  $\text{Condition}(r)$  tells if the current state `workingstate` enables the guard of  $r$
- If so, the next state is obtained via  $\text{Code}(r)$ , by directly modifying `workingstate`



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# Hashtable in Murphi

- Open addressing ...
  - insert: repeatedly call  $e = h(s, i)$  (for  $i = 1, 2, \dots, m$ ) till  $T[e] = \emptyset$ , then insert  $s$  in  $T[e]$
  - search: repeatedly call  $e = h(s, i)$  (for  $i = 1, 2, \dots, m$ ) till either:
    - $T[e] = \emptyset \rightarrow s$  is not present
    - $T[e] = s \rightarrow s$  is present
- ... with double hashing
  - there are two hash functions  $h_1, h_2$
  - $h(s, i) = (h_1(s) + ih_2(s)) \bmod m$
  - $m$  is the size of  $T$ , and is a prime number
  - $h(s, i_1) = h(s, i_2) \rightarrow i_1 = i_2$



# Reducing Hashtable in Murphi

- States must be stored in T
- For efficiency reasons, T is a fixed-length array, each entry is an instance of state class
  - if T becomes full, the verification is terminated and you have to run it again with more memory
  - option `-m` of `model` executable
- Thus, T stores workingstates
- Two possible ways (also together):
  - 1 use less memory for each state
  - 2 store less states



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# Hash Compaction

- Enabled by compiling the Murphi model with `-c`
- When dealing with hash table insertions and searches, state “signatures” are used instead of the whole states
- The idea is that it is unlikely to happen that two different states have the same signature
- If this happens, some states may be never reached, even if they are indeed reachable
- Thus, there may be “false positives”: the verification terminates with an OK messages, while the system was buggy instead
- However, this is very unlikely to happen, and in every case it is much better than testing, which may miss whole classes of bugs



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# Hash Compaction

- At the beginning of the verification, a vector `hashmatrix` of `24*BLOCKS_IN_WORLD` longs (4 byte per each long) is created and initialized with *random* values (`hashmatrix` will never be modified)
- Then, given a state `s` to be sought/inserted, 3 longs 10, 11 and 12 are computed from `hashmatrix`
- Namely,  $1i$ , for  $i = 0, 1, 2$ , is the bit-to-bit xor of the longs in the set  $H(i) = \{\text{hashmatrix}[3k + i] \mid \text{the } k\text{-th bit of the uncompressed state } s \text{ is } 1\}$ ;
- That is to say, every bit of `s` is used to determine if a given element of `hashmatrix` has or hasn't to be used in the signature computation



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# Hash Compaction

- This is accomplished in the functions of file `include/mu_hash.cpp`, where to avoid to compute  $8 \times \text{BLOCKS\_IN\_WORLD}$  bit-to-bit xor operations, some xor properties allow to use the preceeding computed signature and save some xor computation (`oldvec` variable)
- Then, 10 is used as a hash value (index in the hash table)
- The concatenation of 11 and 12 (truncated to a given number of bits by option `-b`) gives the signature (the value to be sought/inserted in T)
- It should be obvious, now, that a signature cannot be used to generate states, so that's why Q entries do not point to hash table entries any more
- Thus, if current `workingstate` state is found to be new, and so its signature is put inside the hash table, a new memory block is allocated to be assigned to the current front of the queue, and `workingstate` is copied into that



# Bit Compression

- To save some (not much...) space, the Murphi compiler option `-b` may be used to compress states (*bit compression* in SPIN's parlance)
- Whilst hashcompaction is a lossy compression, this is lossless
- But very less efficient
- In this way, `workingstate` contents are not forced to be aligned to byte boundaries, so it occupies less space
- Moreover, effective subranges size is used (remember we store the lower bound...)
- Of course, a more complex handling than the `valptr` and `byteOffset` one has to be used



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# Murphi BFS

**Var**

x : 255..261;

y : 30..53;

**StartState**

x := 256;

y := 53;

**End;**



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# Bit Compression

y

0x0	0x0	0x1	0x0	0x35
-----	-----	-----	-----	------

workingstate→bits  
without -b

x

y

0xc	0x2
-----	-----

workingstate→bits  
with -b



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# Symmetry and Multiset Reductions

- Differently from SPIN's partial order reduction, these techniques are not transparent to the user
- In fact, symmetry reduction are applicable only if some types have been declared using the `scalarset` keyword (for multiset reduction, the keyword is `multiset`)
- Not all systems are symmetric
- However, when it is possible to apply symmetry reduction, only a subset of the state space is (correctly) explored
- To be more precise, symmetry reduction induces a partition of the state space in equivalence classes
- A functions chain (implemented in the model-dependent part in `model.cpp`) is able to return the representative of the equivalence class of a given state



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# Symmetry and Multiset Reductions

- Rules for scalarset:
  - the values are not used in any comparison operation except equality testing
  - the values are not used in any arithmetic operation
  - the result from the for loop with the subrange as index does not depend on the order of the iteration
  - cannot be directly assigned to some value: either it is used on a forall, exists, for, ruleset, or it is used an assignment with some other scalarset value



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# Murphi BFS with Symmetry Reduction

```
FIFO_Queue Q;  
HashTable T;  
  
bool BFS(NFSS  $\mathcal{N}$ , AP  $\varphi$ )  
{  
  let  $\mathcal{N} = (S, I, \text{Post})$ ;  
  foreach ss in I {  
    s = Normalize(ss);  
    if (! $\varphi(s)$ )  
      return false;  
  }  
  foreach s in I  
    Enqueue(Q, s);  
  foreach s in I  
    HashInsert(T, s);
```



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# Murphi BFS with Symmetry Reduction

```
while (Q  $\neq$   $\emptyset$ ) {  
  s = Dequeue(Q);  
  foreach ss_next in Post(s) {  
    s_next = Normalize(ss_next);  
    if ( $\neg \varphi(s\_next)$ )  
      return false;  
    if (s_next is not in T) {  
      Enqueue(Q, s_next);  
      HashInsert(T, s_next);  
    } /* if */ } /* foreach */ } /* while */  
return true;  
}
```



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# Symmetry Reduction

- How is `Normalize` implemented? Here are the main ideas
- Suppose that variable  $v$  is a `scalarset(N)`, and  $v = \tilde{v}$  in a state  $s \in S$
- Then, any *permutation* of the set  $\{1, \dots, N\}$  brings to an *equivalent* state
- Thus, all possible permutations are generated, and the lexicographically smaller state is chosen as the representative
  - apply a permutation means: change the value of  $v$ , and reorder any array or ruleset or for which depends on  $v$
- Could be expensive, heuristics are also used to perform faster but potentially not complete normalizations
  - i.e., two symmetric states may be declared different
  - this does not hinder verification correctness, only its efficiency



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# Modeling the Needham-Schroeder Protocol

- Establish mutual authentication between an *initiator*  $A$  and a *responder*  $B$ 
  - desired outcome:  $A$  knows it is speaking with  $B$  and viceversa
- Public key cryptography:
  - each agent  $\alpha$  has a public key  $K_\alpha$
  - any other agent can get it using a dedicated key server
  - each agent  $\alpha$  has a secret key  $K_\alpha^{-1}$
- Given a message  $m$ , it may be encrypted using some key  $K$ , thus obtaining  $\{m\}_K$ 
  - any agent  $\beta$  may encrypt  $m$  using  $K_\alpha$  for some agent  $\alpha$ , thus obtaining  $\{m\}_{K_\alpha}$
  - only agent  $\alpha$  may decrypt  $\{m\}_{K_\alpha}$ , thus obtaining  $m$
- A random number  $N_\alpha$  (*nonce*) may be generated by any agent  $\alpha$



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# Modeling the Needham-Schroeder Protocol

- We follow the modeling by Lowe, showing an error in the protocol that went undetected for nearly 20 years
- Namely, an agent  $I$  (*intruder*) successfully make an agent  $B$  think that  $I$  is instead  $A$  (impersonation)
- NS protocol for mutual authentication consists on 7 steps, but here we focus on the 3 more important steps
  - in the omitted steps,  $A$  and  $B$  obtain their public keys, let us assume this is ok
  - *assume-guarantee* approach: assume that something works, does the subsequent (dependent) steps work?
  - ubiquously used in verification in its “weakest” form
  - may be formalized, but we skip it



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# Modeling the Needham-Schroeder Protocol

- The three steps are as follows:
  - $A \rightarrow B : \{N_A \cdot A\}_{K_B}$ 
    - $\cdot$  stands for concatenation,  $A$  is identity of  $A$
  - $B \rightarrow A : \{N_A \cdot N_B\}_{K_A}$
  - $A \rightarrow B : \{N_B\}_{K_B}$
- From here onwards,  $B$  should be certain to be talking to  $A$
- The idea is: if only  $A$  can decrypt  $\{N_A \cdot N_B\}_{K_A}$ , then only  $A$  could have sent  $\{N_B\}_{K_B}$  back to me
  - this is the  $B$  viewpoint, of course
- $A$  is the *initiator* and  $B$  the *responder*
  - a bit counter-intuitive, as at the end it is the responder who gets the answer



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# Modeling the Needham-Schroeder Protocol

- Intruder / power:
  - overhear and/or intercept any message between any pair of selected agents
  - reply to any intercepted message
  - know which the (other) intruders are
    - not in the original paper...
  - plus the fact it is itself an agent, thus:
    - may decrypt messages encrypted with its key  $K_I$
    - may create nonces



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# Modeling the Needham-Schroeder Protocol in Murphi

- 4 global variables:
  - at least one initiator
  - at least one responder
  - at least one intruder
  - the network which can contain at least one message
- It is sufficient to have one for each of the above to obtain the error
- Once the error is corrected, you may select higher values to see if it stays correct
  - of course, same number of initiators and responders



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# Modeling the Needham-Schroeder Protocol in Murphi

- Initiator has a “state” and the responder it is talking to
  - “states”: actually modalities or statuses, as in the Peterson protocol
  - SLEEPING: before first message  $A \rightarrow B : \{N_A \cdot A\}_{K_B}$
  - WAIT: after first message and before  $B \rightarrow A : \{N_A \cdot N_B\}_{K_A}$
  - COMMIT: after sending last message  $A \rightarrow B : \{N_B\}_{K_B}$
- Responder has a “state” and the initiator it is talking to
  - SLEEPING: before first message  $A \rightarrow B : \{N_A \cdot A\}_{K_B}$
  - WAIT: after sending  $B \rightarrow A : \{N_A \cdot N_B\}_{K_A}$  and before  $A \rightarrow B : \{N_B\}_{K_B}$
  - COMMIT:  $A$  is authenticated by  $B$



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# Modeling the Needham-Schroeder Protocol in Murphi

- Intruder has two arrays
  - for each agent  $a$  (including itself), the nonce  $N_a$ 
    - modeling choice: it is not important, for this verification purposes, to represent the actual random number
    - otherwise, really too many states
    - instead, only a boolean is stored for each agent: true if the nonce is known, false otherwise
    - to know a nonce, either it is its own or it has been able to intercept and decrypt a message containing it
  - a set of known “full” messages (*knowledge*)
    - set size is finite: it models the intruder “power” of storing messages



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# Modeling the Needham-Schroeder Protocol in Murphi

- The network is a (finite-sized) array of messages
- Each message is a record of:
  - source and destination agents
  - key used for encryption
    - not the actual key: the agent id suffices...
  - the body, which is modeled by its type and single components
  - $N_A \cdot A$ : a nonce and an address
    - both are agent ids...
  - $N_B \cdot N_A$  two nonces
  - $N_B$  one nonce
- Sending a message means setting up all of its parts and then adding it to the network
- Receiving a message means removing it from the network
  - should also check if you are the intended destination, but intruders do not do it...



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# NS Protocol in Murphi: Starting States

- All initiators  $A$  and responders  $B$  are in SLEEP status
- Each intruder only knows its own nonce and has no recorded message
- There are no messages in the network



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# NS Protocol in Murphi: Initiators Behaviour

- Ruleset 1: for all sleeping initiators  $A$  and for all responders/intruders  $B$ 
  - send nonce+address  $\{N_A \cdot A\}_{K_B}$ 
    - this means: set up the message and add it to the network
    - thus a further condition is needed: network must not be full
  - initiator  $A$  goes to WAIT status
  - also records that its responder is  $B$
- Ruleset 2: for all waiting initiators  $A$ ,
  - if there is a message  $m$  on the network which has been sent to  $A$  and was sent by an intruder  $B$ ...
  - ... receive it: it should be  $m = \{N_A \cdot N_B\}_{K_A}$
  - thus, send  $\{N_B\}_{K_B}$  as a response
  - new status for  $A$  is COMMIT



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# NS Protocol in Murphi: Responders Behaviour

- Ruleset 1: for all sleeping responders  $B$ ,
  - if there is a message  $m$  on the network which has been sent to  $B$  and comes from an intruder  $A$ ...
  - ... receive it: it should be  $m = \{N_A \cdot A\}_{K_B}$
  - thus, send  $\{N_A \cdot N_B\}_{K_A}$  as a response
  - new status for  $B$  is WAIT
  - it also records that its initiator is  $A$
- Ruleset 2: for all waiting responders  $B$ ,
  - if there is a message  $m$  on the network which has been sent to  $B$  and comes from an intruder  $A$ ...
  - ... receive it: it should be  $m = \{N_B\}_{K_B}$
  - new status for  $B$  is COMMIT



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# NS Protocol in Murphi: Intruders Behaviour

- Ruleset 1: for all intruders  $I$ ,
  - if there is a message  $m$  on the network which has been sent to  $B$ , and  $B$  is not an intruder...
  - ... receive it: it may be either  $m = \{N_A \cdot A\}_{K_B}$  or  $m = \{N_B\}_{K_B}$  for some  $B$ 
    - that is, any message coming from an initiator
  - there are two possible cases:
    - $B = I$ , then  $m$  may be read and  $N_A$  is now known by  $I$
    - $B \neq I$ , then add  $m$  to knowledge of  $I$
    - provided that there is enough space and it is not already present
- Ruleset 2: for all intruders  $I$  and for all non-intruders  $A$ ,
  - if there is a message  $m$  on the knowledge of  $I$ , send  $m$  to  $A$
  - essentially, this means that ruleset 1 is equivalent to: the intruder sees messages going on the network and actually receives only those which can be decrypted



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# NS Protocol in Murphi: Intruders Behaviour

- Ruleset 3: for all intruders  $I$  and for all non-intruders  $A$ , for all possible messages  $m$ , send  $m$  to  $A$ 
  - “possible messages”: all those which may be composed using the nonces known by  $I$
  - if only one nonce is known, then only  $\{N_B\}_{K_B}$  can be sent
  - if two nonces are known, also  $\{N_A \cdot N_B\}_{K_A}$  can be sent
  - if no nonces are known, this ruleset cannot be fired
  - of course, there must also be room in the network for sending  $m$



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# NS Protocol in Murphi: Invariants

- All responders are correctly authenticated
  - for all initiators  $A$ , if status of  $A$  is COMMIT and its responder is a responder  $B$ , then initiator of  $B$  must be  $A$
  - furthermore,  $B$  must not be sleeping
- All initiators are correctly authenticated
  - for all responders  $B$ , if status of  $B$  is COMMIT and its initiator is an initiator  $A$ , then responder of  $A$  must be  $B$
  - furthermore,  $A$  must be in COMMIT status



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# NS Protocol in Murphi: Conclusions

- Modeler must choose a “category” of attack
  - here, the fact that an intruder may be inbetween an initiator and its responder
  - and may send any message to try to breach the protocol
- The model is deadlocked
  - e.g., initiator sends to intruder, which learns the initiator nonce and sends the answer, then initiator sends final message, which is again taken by the intruder and finally the intruder generates a message with learnt nonce to the initiator
  - initiator is in COMMIT, responder does not see anything for him, network is full thus stop
- For the purposes of this verification, deadlocks are “failed” attacks, thus can be discarded



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# NS Protocol in Murphi: Conclusions

- Corrected protocol:
  - $A \rightarrow B : \{N_A \cdot A\}_{K_B}$
  - $B \rightarrow A : \{N_A \cdot N_B \cdot B\}_{K_A}$ 
    - thus, also  $B$  identity is sent
  - $A \rightarrow B : \{N_B\}_{K_B}$
- A flag in the Murphi model allows to turn this fix on
- It is possible to (manually) prove that, if a bug is still in the protocol for any number of agents, then it should be in the protocol with 3 agents
  - Murphi shows that no attacks exist for 3 agents



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