

Software Testing and Validation

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Corso di Laurea in Informatica

Kripke Structures and Murphi Verification Algorithm(s)

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Kripke Structures

- Let AP be a set of “atomic propositions”
 - in the sense of first-order logic: each atomic proposition is either true or false
 - typically identified with lower case letters p, q, \dots
- A *Kripke Structure* (KS) over AP is a 4-tuple $\langle S, I, R, L \rangle$
 - S is a finite set, its elements are called *states*
 - $I \subseteq S$ is a set of *initial states*
 - $R \subseteq S \times S$ is a *transition relation*
 - $L : S \rightarrow 2^{AP}$ is a *labeling function*



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Labeled Transition Systems

- A *Labeled Transition System* (LTS) is a 4-tuple $\langle S, I, \Lambda, \delta \rangle$
 - S is a finite set of states as before
 - $I \subseteq S$ is a set of initial states as before (not always included)
 - Λ is a finite set of *labels*
 - $\delta \subseteq S \times \Lambda \times S$ is a *labeled transition relation*



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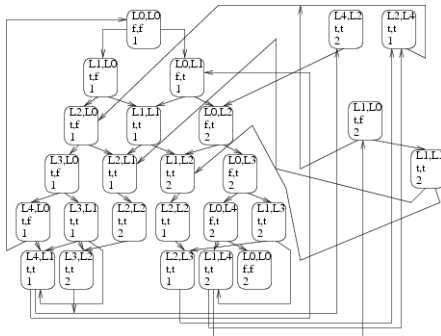
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Peterson's Mutual Exclusion as a Kripke Structure

- $S = \{(p_1, p_2, q_1, q_2, t) \mid p_1, p_2 \in \{L0, L1, L2, L3, L4\}, q_1, q_2 \in \{0, 1\}, t \in \{1, 2\}\} = \{L0, L1, L2, L3, L4\}^2 \times \{0, 1\}^2 \times \{1, 2\}$
- $I = \{L0\}^2 \times \{0\}^2 \times \{1, 2\}$
- R : see next slide
- $AP = \{(P[1] = v) \mid v \in \{L0, L1, L2, L3, L4\}\} \cup \{(P[2] = v) \mid v \in \{L0, L1, L2, L3, L4\}\} \cup \{(Q[1] = v) \mid v \in \{0, 1\}\} \cup \{(Q[2] = v) \mid v \in \{0, 1\}\} \cup \{(\text{turn} = v) \mid v \in \{1, 2\}\}$
 - e.g.: $L((L0, L0, 0, 0, 1)) = \{(P[1] = L0), (P[2] = L0), (Q[1] = 0), (Q[2] = 0), (\text{turn} = 1)\}$



Peterson's Mutual Exclusion as a Kripke Structure



E.g.: $((L0, L0, 0, 0, 1), (L1, L0, 1, 0, 1)) \in R$, whilst
 $((L0, L0, 0, 0, 1), (L2, L0, 0, 0, 1)) \notin R$

Transitions in R corresponds to arrows in the figure above



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Kripke Structure vs Labeled Transition Systems

- KSs have atomic propositions on states, LTSs have labels on transitions
- In model checking, atomic propositions are mandatory
 - to specify the formula to be verified, as we will see
 - a first example was the invariant in Murphi
- Instead, it is not required to have a label on transitions
 - Murphi allows to do so, but it is optional
 - may be easily added automatically, if needed
- Labels are typically needed when:
 - we deal with macrostates, as in UML state diagrams
 - when we are describing a complex system by specifying its sub-components, so labels are used for synchronization



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Total Transition Relation

- In many cases, the transition relation R is required to be *total*
- $\forall s \in S. \exists s' \in S : (s, s') \in R$
 - this of course allows also $s = s'$ (*self loop*)
- In the Peterson's example, the relation is actually total
 - Murphi allows also non-total relations, by using option `-ndl`
 - note however that not giving option `-ndl` is stronger:
 $\forall s \in S. \exists s' \in S : s \neq s' \wedge (s, s') \in R$
 - otherwise, if s is s.t. $\forall s'. s = s' \vee (s, s') \notin R$, Murphi calls s a *deadlock* state
 - that is, you cannot go anywhere, except possibly self looping on s
- By deleting any rule, we will obtain a non-total transition relation



Non-Determinism

- The transition relation is, as the name suggests, a relation
- Thus, starting from a given state, it is possible to go to many different states
 - in a deterministic system,
$$\forall s_1, s_2, s_3 \in S. (s_1, s_2) \in R \wedge (s_1, s_3) \in R \rightarrow s_2 = s_3$$
 - this does not hold for KSs
- This means that, starting from state s_1 , the system may *non-deterministically* go either to s_2 or to s_3
 - or many other states
- Motivations for non-determinism: modeling choices!
 - underspecified subsystems
 - unpredictable interleaving
 - interactions with an uncontrollable environment
 - ...



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Some Useful Notation

- Given a KS $\mathcal{S} = \langle S, I, R, L \rangle$, we can define:
 - the *predecessor* function $\text{Pre}_{\mathcal{S}} : S \rightarrow 2^S$
 - defined as $\text{Pre}_{\mathcal{S}}(s) = \{s' \in S \mid (s', s) \in R\}$
 - we will write simply $\text{Pre}(s)$ when \mathcal{S} is understood
 - the *successor* function $\text{Post} : S \rightarrow 2^S$
 - defined as $\text{Post}(s) = \{s' \in S \mid (s, s') \in R\}$
- Note that, if \mathcal{S} is deterministic, $\forall s \in S. |\text{Post}(s)| \leq 1$
- Note that, if \mathcal{S} is total, $\forall s \in S. |\text{Post}(s)| \geq 1$



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Paths in KSs

- A path (or *execution*) on a KS $\mathcal{S} = \langle S, I, R, L \rangle$ is a sequence $\pi = s_0 s_1 s_2 \dots$ such that:
 - $\forall i \geq 0. s_i \in S$ (it is composed by states)
 - $\forall i \geq 0. (s_i, s_{i+1}) \in R$ (it only uses valid transitions)
- We will denote i -th state of a path as $\pi(i) = s_i$
- Note that paths in LTSs also have actions: $\pi = s_0 a_0 s_1 a_1 \dots$
s.t. $(s_i, a_i, s_{i+1}) \in \delta$



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Paths in KSs

- The *length* of a path π is the number of states in π
 - paths can be either finite $\pi = s_0 s_1 \dots s_n$, in which case $|\pi| = n + 1$
 - or infinite $\pi = s_0 s_1 \dots$, in which case $|\pi| = \infty$
- We will denote the prefix of a path up to i as $\pi|_i = s_0 \dots s_i$
 - a prefix of a path is always a finite path
- A path π is *maximal* iff one of the following holds
 - $|\pi| = \infty$
 - $|\pi| = n + 1$ and $|\text{Post}(\pi(n))| = 0$
 - that is, $\forall s \in S. (\pi(n), s) \notin R$
 - i.e., the last state of the path has no successors
 - often called *terminal state*
- If R is total, maximal paths are always infinite
 - for many model checking algorithms, this is required



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Reachability

- The set of paths of \mathcal{S} starting from $s \in S$ is denoted by $\text{Path}(\mathcal{S}, s) = \{\pi \mid \pi \text{ is a path in } \mathcal{S} \wedge \pi(0) = s\}$
- The set of paths of \mathcal{S} is denoted by $\text{Path}(\mathcal{S}) = \cup_{s \in I} \text{Path}(\mathcal{S}, s)$
 - that is, they must start from an initial state
- A state $s \in S$ is *reachable* iff $\exists \pi \in \text{Path}(\mathcal{S}), k < |\pi| : \pi(k) = s$
 - i.e., there exists a path from an initial state leading to s through valid transitions
- The set of reachable states is defined by $\text{Reach}(\mathcal{S}) = \{\pi(i) \mid \pi \in \text{Path}(\mathcal{S}), i < |\pi|\}$



Safety Property Verification

- Verification of *invariants*: nothing bad happens
- The property is a formula $\varphi : S \rightarrow \{0, 1\}$
 - built using boolean combinations of atomic propositions in $p \in AP$
 - i.e., the syntax is

$$\Phi ::= (\Phi) \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \neg \Phi \mid p$$

- The KS \mathcal{S} satisfies φ iff φ holds on all reachable states
 - $\forall s \in \text{Reach}(\mathcal{S}). \varphi(s) = 1$
- Note that it may happen that $\varphi(s) = 0$ for some $s \in S$: never mind, if $s \notin \text{Reach}(\mathcal{S})$



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From Murphi Description \mathcal{M} to KS \mathcal{S}

- First, we mathematically define a Murphi description \mathcal{M}
- $V = \langle v_1, \dots, v_n \rangle$ is the set of global variables of \mathcal{M} , with domains $\langle D_1, \dots, D_n \rangle$
 - all variables are *unfolded* to the Murphi simple types
 - integer subranges
 - enumerations
 - the special “undefined” value should be added to all simple types
 - that is, if a variable is an array with q elements, then it is actually to be considered as q different variables
 - the same for records (and any nesting of arrays and records)
 - as an example: `var a : array [1..n] of record begin`
`b : 1..m; c: 1..k; endrecord`
 - then there will be $2n$ variables as follows:
 $a_1b, \dots, a_nb, a_1c, \dots, a_nc$
 - the first n with type $1..m$, the other with type $1..k$



From Murphi Description \mathcal{M} to KS \mathcal{S}

- $\mathcal{I} = \{I_1, \dots, I_k\}$ is the set of startstate sections in \mathcal{M}
 - startstates may be defined inside rulesets; again, all rulesets are *unfolded*
 - thus, if a startstate \mathcal{I} is inside m nested rulesets $\mathcal{R}_1, \dots, \mathcal{R}_m$...
 - and each ruleset \mathcal{R}_i is defined on an index j_i spanning on a domain \mathcal{D}_i (note that \mathcal{D}_i must be a simple type)...
 - then there actually are $\prod_{i=1}^m |\mathcal{D}_i|$ startstates to be considered, instead of just one
 - of course, in each of these startstates definitions, the tuple j_1, \dots, j_m takes all possible values of $\mathcal{R}_1 \times \dots \times \mathcal{R}_m$
- $\mathcal{T} = \{T_1, \dots, T_p\}$ is the set of rule sections in \mathcal{M}
 - again, if rulesets are present, they are *unfolded*



From Murphi Description \mathcal{M} to KS \mathcal{S}

- The Kripke structure $\mathcal{S} = \langle S, I, R, L \rangle$ described by \mathcal{M} is such that:
 - $S = D_1 \times \dots \times D_n$
 - $s \in I$ iff there is a startstate $I_i \in \mathcal{I}$ s.t. s may be obtained by applying the body of I_i
 - $(s, t) \in R$ iff there is a rule $T_i \in T$ s.t. T_i guard is true in s and T_i body changes s to t
 - $AP = \{(v = d) \mid v = v_i \in V \wedge d \in D_i\}$
 - $(v = d) \in L(s)$ iff variable v has value d in s



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From Murphi Description \mathcal{M} to KS \mathcal{S}

- We also assume to have a function defining the semantics of Murphi (sequence of) statements
 - those in bodies of rules and startstates
- Let \mathcal{P} be the set of all possible (syntactically legal) Murphi statements
 - including while, if, for, assignments...
- Thus, let $\eta : \mathcal{P} \times D_1 \times \dots \times D_n \rightarrow D_1 \times \dots \times D_n$ be our evaluation function
 - it takes a Murphi statement $P \in \mathcal{P}$ and the state s preceding such statement
 - it returns the new state s' obtained by executing P on s
 - e.g., $\eta(a := a + 1; b := b - 1, (1, 2, 3)) = (2, 1, 3)$
 - η may be defined, e.g., using operational semantics



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From Murphi Description \mathcal{M} to KS \mathcal{S}

- We also assume to have a function defining the semantics of Murphi boolean expression
 - those in guards of rules
 - and in invariants!
- Let \mathcal{Q} be the set of all possible (syntactically legal) Murphi boolean expressions
 - including forall, exists, equality checks...
- Thus, let $\zeta : \mathcal{Q} \times D_1 \times \dots \times D_n \rightarrow \{0,1\}$ be our evaluation function
 - it takes a Murphi boolean expression $Q \in \mathcal{Q}$ and the state s to be evaluated
 - it returns 1 iff Q is true in s
 - e.g., $\zeta((a = 3 | b = 4), (1, 4, 3)) = 1$
 - ζ may be defined using atomic propositions



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From Murphi Description \mathcal{M} to KS \mathcal{S}

- Let $Q \in \mathcal{Q}$ be a Murphi boolean expression
- Flatten Q w.r.t. Forall and Exists
 - Forall is replaced by ANDs, Exists by ORs
 - e.g., from Exists i1: pid Do Exists i2: pid Do (i1 != i2 & P[i1] = L3 & P[i2] = L3) End End ...
 - ... to (1 != 1 & P[1] = L3 & P[1] = L3) | (2 != 1 & P[2] = L3 & P[1] = L3) | (1 != 2 & P[1] = L3 & P[2] = L3) | (2 != 2 & P[2] = L3 & P[2] = L3)
- If we replace each variable $v_i \in V$ occurring in Q with a value $w_{j_i} \in D_i$, we obtain a boolean value (true or false)
 - e.g., the former evaluates to true by setting $P[1] = L3$ and $P[2] = L3$
- Thus, $\zeta(Q, s) = 1$ iff $Q(w_{j_1}, \dots, w_{j_n}) = 1$
 - where each w_{j_i} is such that $(v_i = w_{j_i}) \in L(s)$
 - $Q(w_{j_1}, \dots, w_{j_n})$ is the result of replacing variable v_i with value w_{j_i}



From Murphi Description \mathcal{M} to KS \mathcal{S}

- $(s, t) \in R$ iff there is a rule $T_i \in T$ s.t. T_i guard is true in s and T_i body changes s to t
- By using η and ζ , we can be more precise:
 - “ T_i guard is true” means $\zeta(G(T_i), s) = 1$, being $G(T_i)$ the Murphi expression used as guard of rule T_i
 - “ T_i body changes s to t ” means $\eta(B(T_i), s) = t$, being $B(T_i)$ the Murphi statement used as body of rule T_i
- $s \in I$ iff there is a startstate $I_i \in \mathcal{I}$ s.t. s may be obtained by applying the body of I_i
 - “ s may be obtained by applying the body of I_i ” means $\eta(B(I_i), (\perp, \dots, \perp)) = s$, being $B(I_i)$ the Murphi statement used as body of startstate I_i



From Murphi Description \mathcal{M} to KS \mathcal{S}

- $(s, t) \in R$ iff there is a rule $T_i \in T$ s.t. T_i guard is true in s and T_i body changes s to t :
 - that is: in the body of T_i , variables starting values are those of s
 - note that there may be two or more rules defining the same transition from s to t ; no problem with this
 - simply, the same transition is described by multiple rules
- A state s is a deadlock state for two possible reasons:
 - 1 $(s, t) \notin R$ for all $t \in S$, i.e., the values for the variables in s do not satisfy any ruleset guard
 - 2 $(s, t) \in R \rightarrow t = s$, i.e., there is some ruleset guard which is satisfied by s , but its body do not change any of the global variables (e.g., the body is empty)



How to Verify a Murphi Description \mathcal{M}

- Theoretically, extract KS \mathcal{S} and property φ from \mathcal{M} as described above
 - for a given invariant I in \mathcal{M} , $\varphi(s) = \zeta(I, s)$ for all $s \in S$
- Then, KS \mathcal{S} satisfies φ iff φ holds on all reachable states
 - $\forall s \in \text{Reach}(\mathcal{S}). \varphi(s) = 1$
- Thus, consider KS as a graph and perform a visit
 - states are nodes, transitions are edges
- If a state e s.t. $\varphi(e) = 0$ is found, then we have an error
- Otherwise, all is ok



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How to Verify a Murphi Description \mathcal{M}

- From a practical point of view, many optimization may be done, but let us stick to the previous scheme
- The worst case time complexity for a DFS or a BFS is $O(|V| + |E|)$ (and same for space complexity)
- For KSs, this means $O(|S| + |R|)$, thus it is linear in the size of the KS
- Is this good? NO! Because of the *state space explosion problem*
- Assuming that B bits are needed to encode each state
 - i.e., $B = \sum_{i=1}^n b_i$, being b_i the number of bits to encode domain D_i
- We have that $|S| = O(2^B)$



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State Space Explosion

- The “practical” input dimension is B , rather than $|S|$ or $|R|$
- Typically, for a system with N components, we have $O(N)$ variables, thus $O(B)$ encoding bits
- It is very common to verify a system with N components, and then (if N is ok) also for $N + 1$ components
 - verifying a system with a generic number N of components is a proof checker task...
- This entails an exponential increase in the size of $|S|$
- Thus we need “clever” versions of BFS/DFS



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Standard BFS: No Good for Model Checking

- Assumes that all graph nodes are in RAM
- For KSs, graph nodes are states, and we know there are too many
 - state space explosion
- You also need a full representation of the graph, thus also edges must be in RAM
 - using adjacency matrices or lists does not change much
 - for real-world systems, you may easily need TB of RAM
- Even if you have all the needed RAM, there is a huge preprocessing time needed to build the graph from the Murphi specification
- Then, also BFS itself may take a long time



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Murphi BFS

- We need a definition inbetween the model and the KS: NFSS (Nondeterministic Finite State System)
- $\mathcal{N} = \langle S, I, \text{Post} \rangle$, plus the invariant φ
 - S is the set of states, $I \subseteq S$ the set of initial states
 - $\text{Post} : S \rightarrow 2^S$ is the successor function as defined before
 - given a state s , it returns T s.t. $t \in T \rightarrow (s, t) \in R$
 - no labeling, we already have φ



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Murphi BFS

- KSs and NFSSs differ on having Post instead of R
- Post may easily be defined from the Murphi specification
- Such definition is implicit, as programming code, thus avoiding to store adjacency matrices or lists
 - $t \in \text{Post}(s)$ iff there is a rule $T_i \in T$ s.t. T_i guard is true in s and T_i body changes s to t
 - see above for using η and ζ
 - Essentially, if the current state is s , it is sufficient to inspect all (flattened) rules in the Murphi specification \mathcal{M}
 - for all guards which are enabled in s , execute the body so as to obtain t , and add t to $\text{next}(s)$
 - This is done “on the fly”, only for those states s which must be explored



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Simple Simulation

```
void Make_a_run(NFSS  $\mathcal{N}$ , invariant  $\varphi$ )
{
  let  $\mathcal{N} = \langle S, I, \text{Post} \rangle$ ;
  s_curr = pick_a_state(I);
  if (! $\varphi$ (s_curr))
    return with error message;
  while (1) { /* loop forever */
    s_next = pick_a_state(Post(s_curr));
    if (! $\varphi$ (s_next))
      return with error message;
    s_curr = s_next;
  }
}
```



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Simple Simulation with Deadlock

```
void Make_a_run(NFSS  $\mathcal{N}$ , invariant  $\varphi$ )
{
  let  $\mathcal{N} = \langle S, I, \text{Post} \rangle$ ;
  s_curr = pick_a_state(I);
  if ( $\neg \varphi(s_{\text{curr}})$ )
    return with error message;
  while (1) { /* loop forever */
    if ( $\text{Post}(s_{\text{curr}}) = \emptyset$ )
      return with deadlock message;
    s_next = pick_a_state( $\text{Post}(s_{\text{curr}})$ );
    if ( $\neg \varphi(s_{\text{next}})$ )
      return with error message;
    s_curr = s_next;
  }
}
```



Murphi Simulation

```
void Make_a_run(NFSS  $\mathcal{N}$ , invariant  $\varphi$ )
{
  let  $\mathcal{N} = \langle S, I, \text{Post} \rangle$ ;
  s_curr = pick_a_state(I);
  if ( $\neg \varphi(s_{\text{curr}})$ )
    return with error message;
  while (1) { /* loop forever */
    if ( $\text{Post}(s_{\text{curr}}) = \emptyset \vee \text{Post}(s_{\text{curr}}) = \{s_{\text{curr}}\}$ )
      return with deadlock message;
    s_next = pick_a_state( $\text{Post}(s_{\text{curr}})$ );
    if ( $\neg \varphi(s_{\text{next}})$ )
      return with error message;
    s_curr = s_next;
  }
}
```



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Murphi Simulation

- Similar to testing
- If an error is found, the system is bugged
 - or the model is not faithful
 - actually, Murphi simulation is used to understand if the model itself contains errors
- If an error is not found, we cannot conclude anything
- The error state may lurk somewhere, out of reach for the random choice in `pick_a_state`



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Standard BFS (Cormen-Leiserson-Rivest)

BFS(G, s)

```
1  for ogni vertice  $u \in V[G] - \{s\}$ 
2      do  $color[u] \leftarrow \text{WHITE}$ 
3          $d[u] \leftarrow \infty$ 
4          $\pi[u] \leftarrow \text{NIL}$ 
5   $color[s] \leftarrow \text{GRAY}$ 
6   $d[s] \leftarrow 0$ 
7   $\pi[s] \leftarrow \text{NIL}$ 
8   $Q \leftarrow \{s\}$ 
9  while  $Q \neq \emptyset$ 
10     do  $u \leftarrow head[Q]$ 
11        for ogni  $v \in Adj[u]$ 
12            do if  $color[v] = \text{WHITE}$ 
13                then  $color[v] \leftarrow \text{GRAY}$ 
14                    $d[v] \leftarrow d[u] + 1$ 
15                    $\pi[v] \leftarrow u$ 
16                   ENQUEUE( $Q, v$ )
17     DEQUEUE( $Q$ )
18   $color[u] \leftarrow \text{BLACK}$ 
```



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Murphi BFS

```
FIFO_Queue Q;  
HashTable T;  
  
bool BFS(NFSS  $\mathcal{N}$ , AP  $\varphi$ )  
{  
  let  $\mathcal{N} = (S, I, \text{Post})$ ;  
  foreach s in I {  
    if (! $\varphi(s)$ )  
      return false;  
  }  
  foreach s in I  
    Enqueue(Q, s);  
  foreach s in I  
    HashInsert(T, s);
```



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Murphi BFS

```
while (Q  $\neq$   $\emptyset$ ) {  
  s = Dequeue(Q);  
  foreach s_next in Post(s) {  
    if (! $\varphi$ (s_next))  
      return false;  
    if (s_next is not in T) {  
      Enqueue(Q, s_next);  
      HashInsert(T, s_next);  
    } /* if */ } /* foreach */ } /* while */  
return true;  
}
```



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Murphi BFS

- Edges are never stored in memory
 - states are “created” when expanding the current state
 - rules are used to modify the current state so as to obtain the new one
 - at the start, you have an empty state which is modified by startstates
- (Reachable) states are stored in memory only at the end of the visit
 - inside hashtable T
- This is called *on-the-fly* verification
- States are marked as visited by putting them inside an hashtable
 - rather than coloring them as gray or black
 - which needs the graph to be already in memory



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State Space Explosion

- State space explosion hits in the FIFO queue Q and in the hashtable T
 - and of course in running time...
- However, Q is not really a problem
 - it is accessed *sequentially*
 - always in the front for extraction, always in the rear for insertion
 - can be efficiently stored using disk, much more capable of RAM
- T is the real problem
 - random access, not suitable for a file
 - what to do?
 - before answering, let's have a look at Murphi code



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Murphi Usage

- As for all *explicit* model checker, a Murphi verification has the following steps:
 - 0 compile Murphi source code and write a Murphi model `model.m`
 - 1 invoke Murphi compiler on `model.m`: this generates a file `model.cpp`
 - `mu options model.m`
 - see `mu -h` for available options
 - 2 invoke C++ compiler on `model.cpp`: this generates an executable file
 - `g++ -Ipath_to_include model.cpp -o model`
 - `path_to_include` is the include directory inside Murphi distribution
 - 3 invoke the executable file
 - `./model options`
 - see `./model -h` for available options



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Murphi compiler

- Executable `mu` is in `src` directory of Murphi distribution
- Obtained by compiling the 25 source files in `src`
 - of course, a Makefile is provided for this
- Standard compiler implementation, with Flex lexical analyzer (`mu.l`) and Yacc parser (`mu.y`)
- The main function which builds `model.cpp` is `program::generate_code` in `cpp_code.cpp` (called by `main`, in `mu.cpp`)
- `program::generate_code` uses the parse tree generated by Yacc to “implement” in C++ the guards and the bodies of the rules
- The result goes in `model.cpp`: model-specific code



Organization of model.cpp

- Each Murphi variable v (local or global) corresponds to a C++ instance $\text{mu_}v$ of the class mu_int (possibly through class generalizations)
- Class mu_int is used to handle variables with max value 254 (255 is used for the undefined value)
- For integer subranges with greater values, class mu_long is used; also mu_byte (equal to mu_int ...) and mu_boolean exist
- If v is a local variable, $\text{mu_}v$ directly contains the value (attribute `cvalue`, `in_world` is false)
- Otherwise, if v is global, $\text{mu_}v$ retrieves the value from a fixed-address structure containing the current state value (`workingstate`; `in_world` is true)



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Organization of model.cpp

```
class mu__int {  
    enum {undef_value=0xff};  
    bool in_world;           /* local iff false */  
    int lb, ub;              /* bounds */  
    int byteOffset;         /* in bytes */  
    /* points to workingstate->bits[byteOffset]  
       for global variables, to cvalue for  
       local */  
    unsigned char *valptr;  
    unsigned char cvalue;
```



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Organization of model.cpp

public:

```
/* constructor, sets all attributes (the  
   variable is supposed to be local by  
   default, with an undefined value);  
   byteOffset is computed by generate_code  
*/  
mu__int(int lb, int ub, int size, char *n,  
        int byteOffset);  
/* other useful functions */  
int operator= (int val) {  
    if (val <= ub && val >= lb) value(val);  
    else boundary_error(val);  
    return val;  
}
```



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Organization of model.cpp

```
operator int() const {
    if (isundefined()) return undef_error();
    return value();
};
const int value() const {return *valptr;};
int value(int val) {
    *valptr = val; return val;};
void to_state(state *thestate) {
    /* used to make the variable global */
    in_world = TRUE;
    valptr = (unsigned char *)&(workingstate->
        bits[byteOffset]);
};
};
```



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Organization of model.cpp

- As for the `byteOffset` computation, `program::generate_code` simply computes the one for a variable `mu_v` mapping a Murphi variable `v` in the following way
 - Let M_1, \dots, M_n be the upper bounds of the n variables preceeding the declaration of `v`
 - Let $b(x) = \lfloor \log_2(x + 1) \rfloor + 1$ be the number of bits required to represent the maximum value x (plus the undefined value)
 - Let $B(x) = 1$ if $b(x) \leq 8$, 4 otherwise (i.e. only 1-byte or 4-bytes integers may be used)
 - Then, $\text{byteOffset}(\text{mu_v}) = \sum_{i=1}^n B(M_i)$



Organization of model.cpp: workingstate

- Structure containing the current global state, is an instance of class `state`
- Essentially, it consists of an array of unsigned characters, named `bits`
 - so that any value of any global variable may be casted inside it
 - at a precise location, pointed to by `valptr` from `mu__int`
- Note that `workingstate` has a fixed length, that is $\text{BLOCKS_IN_WORLD} = \sum_{i=1}^N B(M_i)$
 - being N the number of all global variables
 - namely, `bits` has `BLOCKS_IN_WORLD` unsigned chars



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Translation of Murphi Model Statements

- Straightforward for ifs, whiles and so on: the “difficult” part is assignments (and expressions evaluation)
- Essentially, a `:= b`; in `model.m` becomes `mu_a = (mu_b)`; in `model.cpp`
- The operator `()` is redefined so that `mu_b` retrieves the value for `b`, either from itself (attribute `cvalue`) or from `workingstate` (thanks to `valptr`)
- Then, the redefined operator `=` is called, so that `mu_a` updates the value for `a` to be equal to that of `b`, either from itself (attribute `cvalue`) or from `workingstate`
- If the right side of the assignment has a generic expression, it is evaluated in a similar way (the operator `()` solves the Murphi variable references, the other values will be integer constants or function calls...)
- BTW, functions are mapped as C++ methods...



Translation of Murphi Rules

- For each rule i (starting from 0 at the *end* of `model.m`!) there is a class named `RuleBase i`
- Such class has `Code` method for the body and `Condition` method for the guard
- Startstates are similar, but they only have the body
- A suitable C++ code flattens rulesets, if present



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Translation of Murphi Rules: From This...

```
Const VAL_LIM: 5;  
  
Type val_t : 0..VAL_LIM;  
  
Var v : val_t;  
  
Rule "incBy1"  
  v <= VAL_LIM - 1 ==>  
  Var useless : val_t;  
  Begin  
    useless := 1;  
    v := v + useless;  
  End;
```



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Translation of Murphi Rules: ... To This

```
class RuleBase1 {
public:
    :
    bool Condition(unsigned r) { /* guard */
        return (mu_v) <= (4);
    }
    :
    void Code(unsigned r) { /* body */
        mu_1_val_t mu_useless("useless", 0);
        mu_useless = 1;
        mu_v = (mu_v) + (mu_useless);
    };
    :
}
```



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Translation of Murphi Rules: From This...

```
ruleset i:  $l_1..u_1$  do
  ruleset j:  $l_2..u_2$  do
    Rule "incBy1"
      i < j ==>
        Begin v := v + i - j; End;
  Endruleset; Endruleset;
```



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Translation of Murphi Rules: ... To This

```
class RuleBase0 {
public:
    bool Condition(unsigned r) {
        /* called  $(u_1 - l_1 + 1)(u_2 - l_2 + 1)$  with  $r$  ranging
           from 0 to  $(u_1 - l_1 + 1)(u_2 - l_2 + 1) - 1$  */
        static mu__subrange_7 mu_j;
        mu_j.value((r % (u_2 - l_2 + 1)) + l_2);
        r = r / (u_2 - l_2 + 1);
        static mu__subrange_6 mu_i;
        mu_i.value((r % (u_1 - l_1 + 1)) + l_1);
        /* useless, but it is automatically
           generated... */
        r = r / (u_1 - l_1 + 1);
        return (mu_i) < (mu_j);
    }
}
```



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Translation of Murphi Rules: ... To This

```
void Code(unsigned r) {  
    static mu__subrange_7 mu_j;  
    mu_j.value((r % (u2 - l2 + 1)) + l2);  
    r = r / (u2 - l2 + 1);  
    static mu__subrange_6 mu_i;  
    mu_i.value((r % (u1 - l1 + 1)) + l1);  
    r = r / (u1 - l1 + 1);  
    mu_v = ((mu_v) + (mu_i)) - (mu_j);  
};  
  
:  
};
```



Murphi Overall Translation

- Note that the first part of `Condition` and `Code` is meant to translate an integer from 0 to $(u_1 - l_1 + 1)(u_2 - l_2 + 1) - 1$ in 2 values for the rulesets indices
- The interface class for the verification algorithm is `NextStateGenerator`
- Suppose there are R rules r_0, \dots, r_{R-1} , and that each r_i is contained in N_i nested rulesets having upper bound u_{ij} and lower bound l_{ij} , for $j = 1, \dots, N_i$
- Note that `Condition` simply calls its homonymous method of the `RuleBase` class corresponding the current `r...`



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Murphi Overall Translation

Let $P(k) = \sum_{i=0}^{k-1} (\prod_{j=1}^{N_i} (u_{ij} - l_{ij} + 1)) + 1$ be the number of flattened rules preceding the rule r_k ;

```
class NextStateGenerator {
    RuleBase0 R0;

    :
    RuleBase(R - 1) R(R - 1);
public:
    void SetNextEnabledRule(unsigned &
        what_rule);
```



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Murphi Overall Translation

```
bool Condition(unsigned r) { /* r will  
    range from 0 to P(R) */  
    category = CONDITION;  
    if (what_rule < P(1))  
        return R0.Condition(r - 0);  
    if (what_rule >= P(1) && what_rule < P(2))  
        return R1.Condition(r - P(1));  
    :  
    if (what_rule >= P(R-1) && what_rule <  
        P(R))  
        return R(R-1).Condition(r - P(R-1));  
    return Error;  
}
```



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Murphi Overall Translation

```
void Code(unsigned r) {  
    if (what_rule < P(1)) {  
        R0.Code(r - 0); return;  
    }  
    if (what_rule >= P(1) && what_rule < P(2)) {  
        R1.Code(r - P(1)); return;  
    }  
    :  
    if (what_rule >= P(R-1) && what_rule <  
        P(R)) {  
        R(R-1).Code(r - P(R-1)); return;  
    } }  
};  
const unsigned numrules = P(R);
```



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Step 2: What Is Actually Compiled by C++ Compiler

Concatenation of include/*.h
model.cpp
Concatenation of include/*.C



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Murphi BFS

```
FIFO_Queue Q;  
HashTable T;  
  
bool BFS(NFSS  $\mathcal{N}$ , AP  $\varphi$ )  
{  
  let  $\mathcal{N} = (S, I, \text{Post})$ ;  
  foreach s in I {  
    if ( $\neg \varphi(s)$ )  
      return false;  
  }  
  foreach s in I  
    Enqueue(Q, s);  
  foreach s in I  
    HashInsert(T, s);
```



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Murphi BFS

```
while (Q  $\neq$   $\emptyset$ ) {  
  s = Dequeue(Q);  
  foreach s_next in Post(s) {  
    if (! $\varphi$ (s_next))  
      return false;  
    if (s_next is not in T) {  
      Enqueue(Q, s_next);  
      HashInsert(T, s_next);  
    } /* if */ } /* foreach */ } /* while */  
return true;  
}
```



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BFS in Murphi

- $\text{Post}(s)$ is computed using class `NextStateGenerator`
- It is equivalent to a for loop on all flattened rules
- For each flattened rule index r , $\text{Condition}(r)$ tells if the current state `workingstate` enables the guard of r
- If so, the next state is obtained via $\text{Code}(r)$, by directly modifying `workingstate`



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Hashtable in Murphi

- Open addressing ...
 - insert: repeatedly call $e = h(s, i)$ (for $i = 1, 2, \dots, m$) till $T[e] = \emptyset$, then insert s in $T[e]$
 - search: repeatedly call $e = h(s, i)$ (for $i = 1, 2, \dots, m$) till either:
 - $T[e] = \emptyset \rightarrow s$ is not present
 - $T[e] = s \rightarrow s$ is present
- ... with double hashing
 - there are two hash functions h_1, h_2
 - $h(s, i) = (h_1(s) + ih_2(s)) \bmod m$
 - m is the size of T , and is a prime number
 - $h(s, i_1) = h(s, i_2) \rightarrow i_1 = i_2$



Reducing Hashtable in Murphi

- States must be stored in T
- For efficiency reasons, T is a fixed-length array, each entry is an instance of state class
 - if T becomes full, the verification is terminated and you have to run it again with more memory
 - option `-m` of `model` executable
- Thus, T stores workingstates
- Two possible ways (also together):
 - 1 use less memory for each state
 - 2 store less states



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Hash Compaction

- Enabled by compiling the Murphi model with `-c`
- When dealing with hash table insertions and searches, state “signatures” are used instead of the whole states
- The idea is that it is unlikely to happen that two different states have the same signature
- If this happens, some states may be never reached, even if they are indeed reachable
- Thus, there may be “false positives”: the verification terminates with an OK messages, while the system was buggy instead
- However, this is very unlikely to happen, and in every case it is much better than testing, which may miss whole classes of bugs



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Hash Compaction

- At the beginning of the verification, a vector `hashmatrix` of `24*BLOCKS_IN_WORLD` longs (4 byte per each long) is created and initialized with *random* values (`hashmatrix` will never be modified)
- Then, given a state `s` to be sought/inserted, 3 longs 10, 11 and 12 are computed from `hashmatrix`
- Namely, $1i$, for $i = 0, 1, 2$, is the bit-to-bit xor of the longs in the set $H(i) = \{\text{hashmatrix}[3k + i] \mid \text{the } k\text{-th bit of the uncompressed state } s \text{ is } 1\}$;
- That is to say, every bit of `s` is used to determine if a given element of `hashmatrix` has or hasn't to be used in the signature computation



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Hash Compaction

- This is accomplished in the functions of file `include/mu_hash.cpp`, where to avoid to compute $8 \times \text{BLOCKS_IN_WORLD}$ bit-to-bit xor operations, some xor properties allow to use the preceeding computed signature and save some xor computation (`oldvec` variable)
- Then, 10 is used as a hash value (index in the hash table)
- The concatenation of 11 and 12 (truncated to a given number of bits by option `-b`) gives the signature (the value to be sought/inserted in T)
- It should be obvious, now, that a signature cannot be used to generate states, so that's why Q entries do not point to hash table entries any more
- Thus, if current `workingstate` state is found to be new, and so its signature is put inside the hash table, a new memory block is allocated to be assigned to the current front of the queue, and `workingstate` is copied into that



Bit Compression

- To save some (not much...) space, the Murphi compiler option `-b` may be used to compress states (*bit compression* in SPIN's parlance)
- Whilst hashcompaction is a lossy compression, this is lossless
- But very less efficient
- In this way, `workingstate` contents are not forced to be aligned to byte boundaries, so it occupies less space
- Moreover, effective subranges size is used (remember we store the lower bound...)
- Of course, a more complex handling than the `valptr` and `byteOffset` one has to be used



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Murphi BFS

Var

x : 255..261;

y : 30..53;

StartState

x := 256;

y := 53;

End;



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Bit Compression

y

0x0	0x0	0x1	0x0	0x35
-----	-----	-----	-----	------

workingstate→bits
without -b

x

y

0xc	0x2
-----	-----

workingstate→bits
with -b



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Symmetry and Multiset Reductions

- Differently from SPIN's partial order reduction, these techniques are not transparent to the user
- In fact, symmetry reduction are applicable only if some types have been declared using the `scalarset` keyword (for multiset reduction, the keyword is `multiset`)
- Not all systems are symmetric
- However, when it is possible to apply symmetry reduction, only a subset of the state space is (correctly) explored
- To be more precise, symmetry reduction induces a partition of the state space in equivalence classes
- A functions chain (implemented in the model-dependent part in `model.cpp`) is able to return the representative of the equivalence class of a given state



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Symmetry and Multiset Reductions

- Rules for scalarset:
 - the values are not used in any comparison operation except equality testing
 - the values are not used in any arithmetic operation
 - the result from the for loop with the subrange as index does not depend on the order of the iteration
 - cannot be directly assigned to some value: either it is used on a forall, exists, for, ruleset, or it is used an assignment with some other scalarset value



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Murphi BFS with Symmetry Reduction

```
FIFO_Queue Q;  
HashTable T;  
  
bool BFS(NFSS  $\mathcal{N}$ , AP  $\varphi$ )  
{  
  let  $\mathcal{N} = (S, I, \text{Post})$ ;  
  foreach ss in I {  
    s = Normalize(ss);  
    if (! $\varphi(s)$ )  
      return false;  
  }  
  foreach s in I  
    Enqueue(Q, s);  
  foreach s in I  
    HashInsert(T, s);
```



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Murphi BFS with Symmetry Reduction

```
while (Q  $\neq$   $\emptyset$ ) {  
  s = Dequeue(Q);  
  foreach ss_next in Post(s) {  
    s_next = Normalize(ss_next);  
    if ( $\neg \varphi(s\_next)$ )  
      return false;  
    if (s_next is not in T) {  
      Enqueue(Q, s_next);  
      HashInsert(T, s_next);  
    } /* if */ } /* foreach */ } /* while */  
return true;  
}
```



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Symmetry Reduction

- How is `Normalize` implemented? Here are the main ideas
- Suppose that variable v is a `scalarset(N)`, and $v = \tilde{v}$ in a state $s \in S$
- Then, any *permutation* of the set $\{1, \dots, N\}$ brings to an *equivalent* state
- Thus, all possible permutations are generated, and the lexicographically smaller state is chosen as the representative
 - apply a permutation means: change the value of v , and reorder any array or ruleset or for which depends on v
- Could be expensive, heuristics are also used to perform faster but potentially not complete normalizations
 - i.e., two symmetric states may be declared different
 - this does not hinder verification correctness **only its efficiency**



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What About Actual Software States?

- One could think: why not to perform a BFS on a legacy software state space?
 - transition relation: use some debugger to perform one statement at a time
 - e.g., for a C program, gdb may be used
 - if concurrency is important, one machine code statement at a time
- How many states will be there? Let us make an estimate
 - values for all global variables
 - value for the whole current call stack
 - if we have threads, all call stacks...
 - values for allocated memory on heap
 - if some I/O is being used (e.g., open files), also its value must be taken into account



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What About Actual Software States?

- Actually, the whole computer's memory may be used
 - both RAM and disks!
- Suppose we have 2TB of total memory, i.e., 16Tb
- Thus, the number of possible states is $2^{2^{44}} \approx 2^{10^{13}} \approx 10^{3 \cdot 10^{12}}$
 - number of atoms in the universe: 10^{80}
- That's why Murphi does not consider the content of files and heap, and does not allow uncompleted function call
 - functions are called only to determine the next state
 - the full call stack must be empty at the end of each next state computation
- For full software, only simulation (i.e., testing) can be performed



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Modeling the Needham-Schroeder Protocol

- Establish mutual authentication between an *initiator* A and a *responder* B
 - desired outcome: A knows it is speaking with B and viceversa
- Public key cryptography:
 - each agent α has a public key K_α
 - any other agent β can get K_α using a dedicated key server
 - each agent α has a secret key K_α^{-1}
- Given a message m , it may be encrypted using some key K , thus obtaining $\{m\}_K$
 - any agent β may encrypt m using K_α for some agent α , thus obtaining $\{m\}_{K_\alpha}$
 - only agent α may decrypt $\{m\}_{K_\alpha}$, thus obtaining m
- A random number N_α (*nonce*) may be generated by any agent α



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Modeling the Needham-Schroeder Protocol

- We follow the modeling by Lowe, showing an error in the protocol that went undetected for nearly 20 years
- Namely, an agent I (*intruder*) successfully make an agent B think that I is instead A (impersonation)
- NS protocol for mutual authentication consists on 7 steps, but here we focus on the 3 more important steps
 - in the omitted steps, A and B obtain their public keys, let us assume this is ok
 - *assume-guarantee* approach: assuming that something works, does the subsequent (dependent) steps work?
 - ubiquously used in verification in its “weakest” form
 - may be formalized, but we skip it



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Modeling the Needham-Schroeder Protocol

- The three steps are as follows:
 - $A \rightarrow B : \{N_A \cdot A\}_{K_B}$
 - \cdot stands for concatenation, A is identity of A
 - $B \rightarrow A : \{N_A \cdot N_B\}_{K_A}$
 - $A \rightarrow B : \{N_B\}_{K_B}$
- From here onwards, B should be certain to be talking to A
- The idea is: if only A can decrypt $\{N_A \cdot N_B\}_{K_A}$, then only A could have sent $\{N_B\}_{K_B}$ back to me
 - this is the B viewpoint, of course
- A is the *initiator* and B the *responder*
 - a bit counter-intuitive, as at the end it is the responder who gets the answer



Modeling the Needham-Schroeder Protocol

- Intruder / power:
 - overhear and/or intercept any message between any pair of selected agents
 - reply to any intercepted message
 - know which the (other) intruders are
 - not in the original paper...
 - plus the fact it is itself an agent, thus:
 - may decrypt messages encrypted with its key K_I
 - may encrypt messages with some other agents key K_α
 - may create nonces



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Modeling the Needham-Schroeder Protocol in Murphi

- 5 global variables:
 - number of initiators
 - number of responders
 - number of intruders
 - number of messages in the network
 - memory size of the intruder
- To trigger the error, it is sufficient that the first 4 variables are strictly positive
 - the last must be at least 3
- Once the error is corrected, you may select higher values to see if it stays correct
 - of course, same number of initiators and responders



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Modeling the Needham-Schroeder Protocol in Murphi

- Initiator has a “state” and the responder it is talking to
 - “states”: actually modalities or statuses, as in the Peterson protocol
 - SLEEPING: before first message $A \rightarrow B : \{N_A \cdot A\}_{K_B}$
 - WAIT: after first message and before $B \rightarrow A : \{N_A \cdot N_B\}_{K_A}$
 - COMMIT: after sending last message $A \rightarrow B : \{N_B\}_{K_B}$
- Responder has a “state” and the initiator it is talking to
 - SLEEPING: before first message $A \rightarrow B : \{N_A \cdot A\}_{K_B}$
 - WAIT: after sending $B \rightarrow A : \{N_A \cdot N_B\}_{K_A}$ and before $A \rightarrow B : \{N_B\}_{K_B}$
 - COMMIT: A is authenticated by B



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Modeling the Needham-Schroeder Protocol in Murphi

- Intruder has two arrays
 - for each agent a (including itself), the nonce N_a
 - modeling choice: it is not important, for this verification purposes, to represent the actual random number
 - otherwise, too many (unnecessary) states
 - instead, only a boolean is stored for each agent: true if the nonce is known, false otherwise
 - to know a nonce, either it is its own or it has been able to intercept and decrypt a message containing it
 - a set of known “full” messages (*knowledge*)
 - set size is finite: it models the intruder “power” of storing messages



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Modeling the Needham-Schroeder Protocol in Murphi

- The network is a (finite-sized) array of messages
- Each message is a record of:
 - source and destination agents
 - key used for encryption
 - not the actual key: the agent id suffices...
 - the body, which is modeled by its type and single components
 - $N_A \cdot A$: a nonce and an address
 - both are agent ids...
 - $N_B \cdot N_A$ two nonces
 - N_B one nonce
- Sending a message means setting up all of its parts and then adding it to the network
- Receiving a message means removing it from the network
 - should also check if you are the intended destination, but intruders do not do it...



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NS Protocol in Murphi: Starting States

- All initiators A and responders B are in SLEEP status
- Each intruder only knows its own nonce and has no recorded message
- There are no messages in the network



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NS Protocol in Murphi: Initiators Behaviour

- Ruleset 1: for all sleeping initiators A and for all responders/intruders B
 - send nonce+address $\{N_A \cdot A\}_{K_B}$
 - this means: set up the message and add it to the network
 - thus a further condition is needed: network must not be full
 - initiator A goes to WAIT status
 - also records that its responder is B
- Ruleset 2: for all waiting initiators A ,
 - if there is a message m on the network which has been sent to A and was sent by an intruder B ...
 - ... receive it: it should be $m = \{N_A \cdot N_B\}_{K_A}$
 - thus, send $\{N_B\}_{K_B}$ as a response
 - new status for A is COMMIT



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NS Protocol in Murphi: Responders Behaviour

- Ruleset 1: for all sleeping responders B ,
 - if there is a message m on the network which has been sent to B and comes from an intruder A ...
 - ... receive it: it should be $m = \{N_A \cdot A\}_{K_B}$
 - thus, send $\{N_A \cdot N_B\}_{K_A}$ as a response
 - new status for B is WAIT
 - it also records that its initiator is A
- Ruleset 2: for all waiting responders B ,
 - if there is a message m on the network which has been sent to B and comes from an intruder A ...
 - ... receive it: it should be $m = \{N_B\}_{K_B}$
 - new status for B is COMMIT



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NS Protocol in Murphi: Intruders Behaviour

- Ruleset 1: for all intruders I ,
 - if there is a message m on the network which has been sent to B , and B is not an intruder...
 - ... receive it: it may be either $m = \{N_A \cdot A\}_{K_B}$ or $m = \{N_B\}_{K_B}$ for some B
 - that is, any message coming from an initiator
 - there are two possible cases:
 - $B = I$, then m may be read and N_A is now known by I
 - $B \neq I$, then add m to knowledge of I
 - provided that there is enough space and it is not already present
- Ruleset 2: for all intruders I and for all non-intruders A ,
 - if there is a message m on the knowledge of I , send m to A
 - essentially, this means that ruleset 1 is equivalent to: the intruder sees messages going on the network and actually receives only those which can be decrypted



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NS Protocol in Murphi: Intruders Behaviour

- Ruleset 3: for all intruders I and for all non-intruders A , for all possible messages m , send m to A
 - “possible messages”: all those which may be composed using the nonces known by I
 - if only one nonce is known, then only $\{N_B\}_{K_B}$ can be sent
 - if two nonces are known, also $\{N_A \cdot N_B\}_{K_A}$ can be sent
 - if no nonces are known, this ruleset cannot be fired
 - of course, there must also be room in the network for sending m



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NS Protocol in Murphi: Invariants

- All responders are correctly authenticated
 - for all initiators A , if status of A is COMMIT and its responder is a responder B , then initiator of B must be A
 - furthermore, B must not be sleeping
- All initiators are correctly authenticated
 - for all responders B , if status of B is COMMIT and its initiator is an initiator A , then responder of A must be B
 - furthermore, A must be in COMMIT status



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NS Protocol in Murphi: Counterexample

- 1 $A \rightarrow I : \{N_A \cdot A\}_{K_I}$
- 2 $I \rightarrow B : \{N_A \cdot A\}_{K_B}$
- 3 $B \rightarrow A : \{N_A \cdot N_B\}_{K_A}$
- 4 I reads and stores $\{N_A \cdot N_B\}_{K_A}$
- 5 $I \rightarrow A : \{N_A \cdot N_B\}_{K_A}$
- 6 $A \rightarrow I : \{N_B\}_{K_I}$
- 7 $I \rightarrow B : \{N_B\}_{K_B}$



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NS Protocol in Murphi: Conclusions

- Modeler must choose a “category” of attack
 - here, the fact that an intruder may be inbetween an initiator and its responder
 - and may send any message to try to breach the protocol
- The model is deadlocked
 - e.g., initiator sends to intruder, which learns the initiator nonce and sends the answer, then initiator sends final message, which is again taken by the intruder and finally the intruder generates a message with learnt nonce to the initiator
 - initiator is in COMMIT, responder does not see anything for him, network is full thus stop
- For the purposes of this verification, deadlocks are “failed” attacks, thus they can be discarded



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NS Protocol in Murphi: Conclusions

- Corrected protocol:
 - $A \rightarrow B : \{N_A \cdot A\}_{K_B}$
 - $B \rightarrow A : \{N_A \cdot N_B \cdot B\}_{K_A}$
 - thus, also B identity is sent
 - $A \rightarrow B : \{N_B\}_{K_B}$
- A flag in the Murphi model allows to turn this fix on
- It is possible to (manually) prove that, if a bug is still in the protocol for any number of agents, then it should be in the protocol with 3 agents
 - Murphi shows that no attacks exist for 3 agents



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