**Part One: Hypothesis Tests about a Single Population Mean,**

1. **Formulating the Hypotheses**: The three possible forms of hypotheses each correspond to a different question you might want to ask about the true value of the population mean,

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| Hypotheses for Hypothesis Tests about a Single Population Mean, | | |
| **Lower Tail Test** | **Upper Tail Test** | **Two-Tailed Test** |
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| Answers Questions About: | | |
| If the true population mean, is less than a given  number, | If the true population mean, is greater than a given number, | If the true population mean, is different from a given number, |

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| *Example:* Suppose you want to know whether a population mean is more than 3. That matches with the question answered by an upper tail test with a of 3. So, the hypotheses would be: |

Upper Tail and Lower Tail Tests are called *one-tailed* or *directional* tests. Two-tailed tests are called *non-directional* tests.

1. **Calculate the Test Statistic:** In this step, we calculate a test statistic from the sample information. The test statistic standardizes the sample, so we can use it to judge our hypotheses. In the case of testing a population mean, there is an additional preliminary step: we have to choose the correct test statistic to use.

Every hypothesis test incorporates some information about the variability of the underlying population – otherwise it would be impossible to determine how probable a given sample statistic really is. For hypothesis testing about a single population mean, the correct choice of test statistic depends on the sampling distribution of , which in turn depends on what information you have about the variability in the underlying population.

While the population standard deviation, cannot strictly be known without measuring the entire population and calculating it, sometimes we can *assume* we know the true value of The basis for such an assumption would come from prior knowledge, historical information, or past experience with the population in question.

Other times, we might not have knowledge of . In that case, we can still get some idea about the variability of the underlying population by looking at the standard deviation, , of a random sample taken from that population. After all, a random sample is representative of the population it comes from, so the amount of variability in the population ought to be reflected in the amount of variability in the sample. If we cannot assume we know , then is our best *estimate* of the variability in the population. is a measurement on the sample – we calculate it from the sample data.

In summary: The *population standard deviation,*  is sometimes assumed to be known from prior knowledge, historical information, or past experience. On the other hand, if is unknown, then the only information about variability in the underlying population is the standard deviation measured on the sample itself – the *sample standard deviation*, . In hypothesis testing about a single population mean, it will be your task to carefully identify whether the standard deviation in a given problem refers to a sample or to the population. That will determine which test statistic to use.

* + **When is KNOWN:**

Under the assumption that is true as an equality, the sampling distribution of the sample mean, follows the distribution. Therefore, we standardize the sample against the distribution by calculating the following test statistic:

where

NOTE: when calculating this equation, evaluate the numerator and denominator separately, and then divide

* + **When is UNKNOWN, then we use instead:**

Under the assumption that is true as an equality, the sampling distribution of the sample mean, follows the distribution with . Therefore, we standardize the sample against the distribution by calculating the following test statistic:

where

and the degrees of freedom =

NOTE: when calculating this equation, evaluate the numerator and denominator separately, and then divide

1. **Deciding whether to reject or not:**

There are two approaches to deciding whether or not to reject the Null:

* In the ***p-value approach***, we **compare** the **p-value** of our test statistic to the  **significance level** and reject if the p-value is less than or equal to the significance level.
* In the ***critical value approach***, we **compare** the **test statistic** to **a critical value** – this can be done with a diagram in which we use the critical value(s) to construct a rejection region or regions; if the test statistic is in a rejection region, we reject and accept Or, this step can be accomplished by following the mathematical rules given in the tables below.

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| **When to Reject for test statistics:** | | | |
|  | **For a Lower Tail Test:** | **For an Upper Tail Test:** | **For a Two-Tailed Test:** |
| **p-value approach:** | Look up the lower tail of  If the then reject and accept  If the then do not reject is unsupported. | Look up the upper tail of  If the then reject and accept  If the then do not reject is unsupported. | The two-tailed is two times the one tailed p-value of  If the then  reject and accept  If the then do not reject is unsupported. |
| **Critical Value: Approach** | If then reject and accept  If then do not reject is unsupported. | If then reject and accept  If then do not reject is unsupported. | If  , then reject and accept .  If then do not reject is unsupported. |
| NOTES:   1. is the Test Statistic 2) and are Critical Values | | | |

**See the next page for what to do with t test statistics!**

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| **When to Reject for test statistics:** | | | |
|  | **For a Lower Tail Test:** | **For an Upper Tail Test:** | **For a Two-Tailed Test:** |
| **p-value approach:** | Look up the lower tail of  If the then reject and accept  If the then do not reject is unsupported. | Look up the upper tail of  If the then reject and accept  If the then do not reject is unsupported. | The two-tailed is two times the one tailed p-value of  If the then  reject and accept  If the then do not reject is unsupported. |
| **Critical Value: Approach** | If then reject and accept  If then do not reject is unsupported. | If then reject and accept  If then do not reject is unsupported. | If  , then reject and accept .  If then do not reject is unsupported. |
| NOTES:   1. is a Test Statistic 2. and are Critical Values 3. The correct t distribution to use when determining the p-value or the Critical Value depends on the degrees of freedom. | | | |

1. **Interpreting the test:**

(Note: This explanation of interpretation holds for ALL hypothesis tests.) We start every hypothesis test with a question about the parameter of interest, so we must end every hypothesis test with the answer to that question. In other words, we must *interpret* the conclusion of our test in terms of the original question.

Remember: in hypothesis testing **you can never prove the null hypothesis**. You can only prove the alternative hypothesis: that is, when you reject the null and accept the alternative, then at your given level of significance you can conclude that is true. If you do reject then you must conclude that is unsupported by the evidence. This gives us a clear guideline for how to *interpret* hypothesis tests: *always look to the alternative hypothesis!* Restate in words, and say whether or not you can conclude that it is true.

In the table that follows, you would substitute the actual words and numbers from your hypothesis test for the symbols. Notice that each interpretation simply states the alternative hypothesis in words, and says either that we can or cannot conclude it is true.

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| **How to Interpret a Hypothesis Test about a Single Population Mean, :** | | | |
| **When you:** | **For a Lower Tail Test:** | **For an Upper Tail Test:** | **For a Two-Tailed Test:** |
| **Reject** | At the significance level, we can conclude that is less than . | At the significance level, we can conclude that is greater than . | At the significance level, we can conclude that is different than . |
| **Do not reject** | At the significance level, we cannot conclude that is less than . | At the significance level, we cannot conclude that is greater than . | At the significance level, we cannot conclude that is different than . |
| NOTES:   1. is the true value of the population mean 2. is a number that is the hypothesized value of | | | |

**Part Two: Hypothesis Tests about a Single Population Proportion,**

1. **Formulating the Hypotheses:** The three possible forms of hypotheses each correspond to a different question you might want to ask about the true value of a population proportion, It is important to remember that a proportion is a number between 0 and 1. Proportions are often expressed in words as percentages, fractions, or shares. These must be converted into proportions before being using in hypothesis tests (See the Chapter 9 handout on proportions).

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| Hypotheses for Hypothesis Tests about a Single Population Proportion, | | |
| **Lower Tail Test** | **Upper Tail Test** | **Two-Tailed Test** |
|  |  |  |
| Answers Questions About: | | |
| If the true population proportion, is less than a given number, | If the true population proportion, is greater than a given number, | If the true population proportion, is different from a given number, |

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| *Example:* Suppose you want to know whether a population proportion has decreased from 0.25. That matches with the question answered by a lower tail test with a of 0.25. So, the hypotheses would be: |

Once again, Upper Tail and Lower Tail Tests are called *one-tailed* or *directional* tests. Two-tailed tests are called *non-directional* tests.

1. **Calculating the Test Statistic:**

Under the assumption that is true as an equality, the sampling distribution of the sample proportion, is the distribution. Therefore, we have to standardize the sample information against the distribution by calculating the following test statistic:

where

NOTE: when calculating this equation, evaluate the numerator and denominator separately, and then divide

1. **Deciding whether to reject or not:** The rules are the same as the rules given above in the table labeled “**When to Reject for Test Statistics**”
2. **Interpreting the test:**

Remember, in this step: *always look to the alternative hypothesis!*

In the table that follows, you would substitute the actual words and numbers from your hypothesis test for the symbols. Notice that each interpretation simply states the alternative hypothesis in words, and says either that we can or cannot conclude it is true.

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| **How to Interpret a Hypothesis Test about a Single Population Proportion, :** | | | |
| **When you:** | **For a Lower Tail Test:** | **For an Upper Tail Test:** | **For a Two-Tailed Test:** |
| **Reject** | At the significance level, we can conclude that is less than . | At the significance level, we can conclude that is greater than . | At the significance level, we can conclude that is different than . |
| **Do not reject** | At the significance level, we cannot conclude that is less than . | At the significance level, we cannot conclude that is greater than . | At the significance level, we cannot conclude that is different than . |
| NOTES:   1. is the true value of the population proportion 2. is a number that is the hypothesized value of | | | |

**Part Three: Assumptions Underlying These Hypothesis Tests**

All hypothesis tests use sampling distributions to determine the probability of sample statistics. In order for us to be confident that our choice of sampling distribution for any given test really is the way the sample statistic is distributed, certain assumptions must be met. (Note: these are separate from the assumption we make in the null hypothesis. We should only formulate a null hypothesis once these assumptions are met). If the assumptions are not met – that is, if any given assumption is not true – then we cannot rely on the results of the hypothesis tests. They may mislead us, give us the wrong answers, and cause us to draw the wrong conclusions.

* For hypothesis tests about a single population mean,
  + If the population is normally distributed, then the sample size can be small
  + If the population is not normally distributed, or if the distribution is unknown, then the sample size must be greater than or equal to 30
  + When is unknown (that is, when you are using the t distribution), the t test works even with small sample sizes. However, if the population has a skewed distribution or has outliers, then the sample size must be greater than 50
* For hypothesis tests about a single population proportion,
  + The following condition must hold true:

where