Cálculo II - Prova II

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1)

$\int sen^4x \cos^4x \, dx$ $\qquad sen^4x \cdot \cos^4x = (senx \cdot \cos x)^4$

$\int (senx \cdot \cos x)^4 \, dx$ $\qquad senx \cdot \cos x = \frac{1}{2} sen(2x)$

$\int \left(\frac{1}{2} \cdot sen(2x)\right)^4 dx$

$\frac{1}{16}\int sen^4(2x)\, dx$ $\qquad u = 2x \quad du = 2\,dx \quad \frac{1}{2} du = dx$

$\frac{1}{16} \cdot \frac{1}{2} \int sen^4 u \, du$

$\frac{1}{32}\left[\frac{1}{4}\left(u - sen(2u) + \frac{u}{2} + \frac{sen(4u)}{8}\right)\right] + C$

$\frac{1}{32}\left[\frac{1}{4}\left(2x - sen(4x) + x + \frac{sen(8x)}{8}\right)\right] + C$

$\frac{1}{32}\left[\frac{3x}{4} - \frac{sen(4x)}{4} + \frac{sen(8x)}{32}\right] + C$

$\int sen^4x \, dx$ $\qquad sen^2x = \frac{1}{2}(1 - \cos(2x))$

$\int (sen^2x)^2 \, dx$ $\qquad \cos^2(2x) = \frac{1}{2}(1 + \cos(4x))$

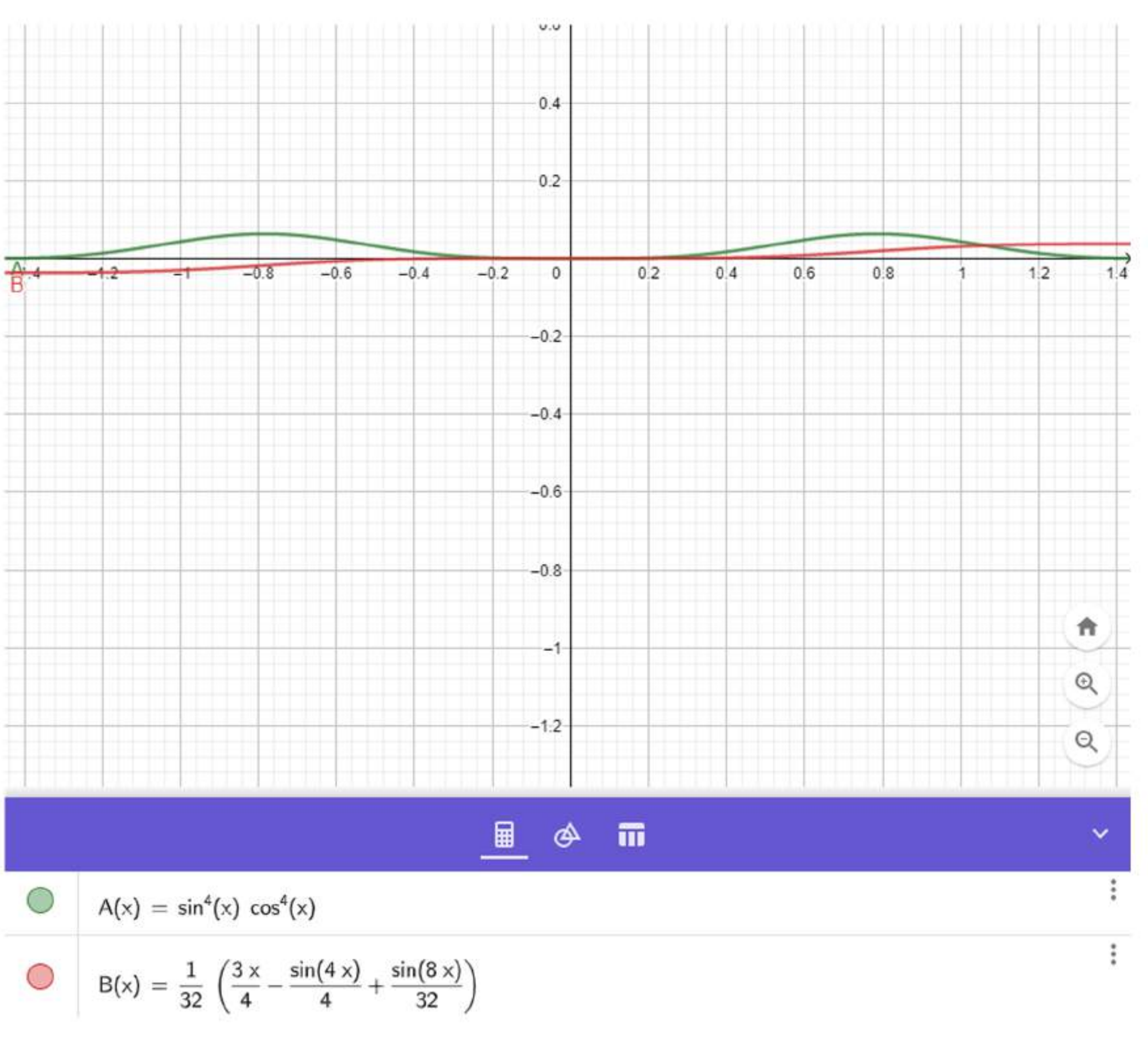
$\int \left(\frac{1}{2}(1 - \cos(2x))\right)^2 dx$

$\frac{1}{4}\int 1 - 2\cos(2x) + \cos^2 2x \, dx$

$\frac{1}{4}\int 1 - 2\cos(2x) + \frac{1}{2}(1 + \cos(4x))\, dx$ $\qquad \int \pi \cos(\pi x)\, dx = sen(\pi x)$

$\frac{1}{4}\int 1 - 2\cos(2x) + \frac{1}{2} + \frac{1}{2} \cdot \cos(4x)\, dx$ $\qquad \frac{4}{8}$

$\frac{1}{4}\left(x - sen(2x) + \frac{x}{2} + \frac{sen(4x)}{8}\right) + C$



2)

$x = \frac{\pi}{2}$

a) $u = \cos x \qquad -du = +senx\, dx \; (-1)$

$\int senx \cos x\, dx = \int \cos x \cdot senx \, dx = -\int u \cdot du$ $\quad x, 0$

$\boxed{-\frac{u^2}{2} + c} \rightarrow \boxed{-\frac{\cos^2 x}{2} + c}$

b) $\int senx \cdot \cos x \, dx = \int u\, du = \frac{u^2}{2} + c = \boxed{\frac{sen^2x}{2} + c}$

$u = senx \rightarrow du = \cos x\, dx$

c) $\int senx \cos x \, dx = \int \frac{1}{2} \cdot sen(2x)\, dx = \frac{1}{2}\int sen2x\, dx$

$\boxed{u = 2x \rightarrow du = 2dx \quad \frac{1}{2}}$ $\quad \frac{1}{2}\int sen(u) \cdot \frac{1}{2} du = \frac{1}{2} \cdot \frac{1}{2}\int senu\, du = \frac{1}{4}(-\cos(u)) = \boxed{-\frac{\cos(2x)}{4} + c}$

d) $\int senx \cos x \, dx$

$u = senx \rightsquigarrow du = \cos x\, dx$

$dv = \cos x \, dx \rightsquigarrow v = senx$

$\int senx \cos x = senx \cdot senx - \int senx \cdot \cos x\, dx$

$2\int senx \cos x\, dx = sen^2x$

$\int senx \cos x \, dx = \frac{sen^2x}{2} + c$

e) Apesar de os resultados terem uma grafia diferente, todos são equivalentes quando submetemos a um intervalo definido:

Ex: Para $\boxed{\int_0^{\frac{\pi}{2}}}$

a) $0 - \left(-\frac{1}{2}\right) = \frac{1}{2}$ $\quad \boxed{-\frac{\cos^2x}{2} + c}$

b) $\frac{1}{2} - 0 = \frac{1}{2}$ $\quad \boxed{\frac{sen^2x}{2} + c}$

c) $\frac{1}{4} - \left(-\frac{1}{4}\right) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$ $\quad \boxed{-\frac{\cos 2x}{4} + c}$

d) $\frac{1}{2} - 0 = \boxed{\frac{1}{2}}$ $\quad \boxed{\frac{sen^2x}{2} + c}$

3)

$\int x^5 e^{x^2} dx = \int x^4 \cdot x \cdot e^{x^2} dx = \int (x^2)^2 \cdot x \cdot e^{x^2} dx$

$\boxed{x^2 = u} \rightarrow du = 2x\,dx \;/\; \frac{du}{2} = x\,dx$

$\int u^2 \cdot e^u \cdot \frac{1}{2} du = \frac{1}{2}\int u^2 \cdot e^u du$ $\qquad$ L I A T E

$\boxed{\begin{array}{l} a = u^2 \rightarrow da = 2u\,du \\ dv = e^u du \rightarrow v = e^u \end{array}}$

$\frac{1}{2}\left(e^u \cdot u^2 - \int e^u \cdot 2u\,du\right) =$ $\qquad b(c - d) = bc - bd$

$\frac{1}{2}e^u \cdot u^2 - \cancel{\frac{1}{2} \cdot 2}\int e^u u\, du =$

$\frac{1}{2}e^u \cdot u^2 - \int e^u u\, du$ $\qquad$ Por Partes novamente: $t = u \rightarrow dt = 1\,du$ $\quad dw = e^u du \rightarrow w = e^u$

$\frac{1}{2}e^u \cdot u^2 - \left(u \cdot e^u - \int e^u \cdot du\right)$

$\frac{1}{2} \cdot e^u \cdot u^2 - (u \cdot e^u - e^u)$

$\boxed{\frac{1}{2} \cdot e^u \cdot u^2 - ue^u + e^u} = e^u\left(\frac{u^2}{2} - u + 1\right)$

$e^{x^2}\left(\frac{(x^2)^2}{2} - x^2 + 1\right) = \boxed{e^{x^2}\left(\frac{x^4}{2} - x^2 + 1\right) + C}$

4)

$\int x \cdot cossec^2 x\, dx =$

L I A T E

$u = x \rightarrow du = 1\,dx$

$dv = \boxed{cossec^2 x} \quad v = \boxed{-\cot g\, x}$

$\int x \cdot \cos^2 x\, dx = x \cdot (-\cot g(x)) - \int -\cot g\, x \cdot dx$

$\int x \cdot cossec^2 x\, dx = x(-\cot g(x)) + \int \cot g\, x\, dx \rightarrow \ln|senx|$

$\int x \cdot cossec^2 x\, dx = x(-\cot g(x)) + \ln|senx|$

$-\cot g(x) \cdot x + \ln|senx| \Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$\left(-\cot g\left(\frac{\pi}{2}\right) \cdot \frac{\pi}{2} + \ln\left|sen\frac{\pi}{2}\right|\right) - \left(-\cot g\left(\frac{\pi}{4}\right) \cdot \frac{\pi}{4} + \ln\left|sen\frac{\pi}{4}\right|\right) =$

$0 \quad -\left(-\frac{\pi}{4} - \frac{1}{2}\ln(2)\right)$

$\boxed{\frac{\pi}{4} + \ln(2) \cdot \frac{1}{2}}$

5)

$\int_1^{\sqrt{3}} arctg\left(\frac{1}{x}\right) dx =$ $\qquad$ L, I → I, A → II, T, E

$\int_1^{\sqrt{3}} \underbrace{arctg\left(\frac{1}{x}\right)}_{\text{Inversa (I)}} \cdot \underbrace{x^0}_{\text{Algébrica (II)}} dx$

$u = arctg\left(\frac{1}{x}\right) \rightarrow du = \boxed{-\frac{1}{x^2 + 1}}$

$dv = dx \rightarrow v = x$

$\frac{d}{dx}(arctg(u)) = \frac{1}{1 + u^2}$

$\frac{d}{dx} arctg\left(\frac{1}{x}\right) = -\left(\frac{1}{1 + \left(\frac{1}{x}\right)^2} \cdot \frac{1}{x^2}\right) = -\frac{1}{1 + \left(\frac{1}{x}\right)^2} \cdot \frac{1}{x^2} = -\left(\frac{1}{1 + \frac{1}{x^2}} \cdot \frac{1}{x^2}\right)$

$-x^{-1-1} = -x^{-2} = \boxed{-\frac{1}{x^2}}$

$\frac{\frac{1}{x^2 + 1}}{x^2} = \frac{1}{1} \cdot \frac{x^2}{x^2 + 1} = \frac{x^2}{x^2 + 1} \;/\; -\left(\frac{\cancel{x^2}}{x^2 + 1} \cdot \frac{1}{\cancel{x^2}}\right) = -\left(\frac{1}{x^2 + 1}\right)$

$-\frac{1}{x^2 + 1}$

$x \cdot arctg\left(\frac{1}{x}\right) - \int x \cdot -\frac{1}{x^2 + 1} dx$

$\int x \cdot \frac{1}{x^2 + 1} \cdot -1\, dx = -\int x \cdot \frac{1}{x^2 + 1} dx = -\int \frac{x}{x^2 + 1} dx = -\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u| = -\frac{1}{2}\ln|x^2 + 1|$

$u = x^2 + 1 \rightarrow du = 2x\,dx \rightarrow \frac{du}{2} = x\,dx$

$x \cdot arctg\left(\frac{1}{x}\right) - \left(-\frac{1}{2}\ln|x^2 + 1|\right)$

$\boxed{x \cdot arctg\left(\frac{1}{x}\right) + \frac{1}{2}\ln|x^2 + 1|}$

$\left[x \cdot arctg\left(\frac{1}{x}\right) + \frac{1}{2}\ln|x^2 + 1|\right]_1^{\sqrt{3}}$

$\left[\frac{1}{2}\left(2x\, arctg\left(\frac{1}{x}\right) + \ln|1 + x^2|\right)\right]_1^{\sqrt{3}}$

$\left[\frac{1}{2}\left(2\sqrt{3}\, arctg\left(\frac{1}{\sqrt{3}}\right) + \ln|4|\right) - \frac{1}{2}\left(2\, arctg(1) + \ln|2|\right)\right]$

$\left[\frac{1}{2}\left(2\sqrt{3} \cdot \frac{\pi}{6} + 2\ln|2|\right) - \frac{1}{2}\left(\frac{2\pi}{4} + \ln|2|\right)\right]$

$\left[\frac{1}{2}\left(\frac{\sqrt{3}\pi}{3} + 2\ln|2|\right) - \frac{1}{2}\left(\frac{\pi}{2} + \ln|2|\right)\right]$

$\left[\frac{\pi\sqrt{3}}{6} + \ln|2| - \frac{\pi}{4} - \frac{\ln|2|}{2}\right]$

$\left[\frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{\ln(2)}{2}\right]$