Questao 7 resolvida

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Grupo: Igor Rocha e João Rupp

$$\int_0^1 \frac{1}{(x^2+3x+2)^2}\,dx = \int_0^1 \frac{1}{\left(\left(x+\frac{3}{2}\right)^2-\frac{1}{4}\right)^2}\,dx =$$

$u = x+\frac{3}{2}$

$du = 1\,dx$

$du = dx$

$x^2+3x+2 \;/\; 2u = 3$, $u = \frac{3}{2}$

$x^2+3x+2+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2$

$\underbrace{x^2+3x+\left(\frac{3}{2}\right)^2}+2-\left(\frac{3}{2}\right)^2$

$\left(x+\frac{3}{2}\right)^2+2-\left(\frac{3}{2}\right)^2$

$\left(x+\frac{3}{2}\right)^2+2-\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2+\frac{8-9}{4} = \left(x+\frac{3}{2}\right)^2-\frac{1}{4}$

Mesmo processo feito na questão nº3

$$\int_0^1 \frac{1}{\left(u^2-\frac{1}{4}\right)^2}\,du \longrightarrow \frac{1}{\left(u^2-\frac{1}{4}\right)^2} = \frac{1}{\frac{(4u^2-1)}{(4)^2}} = \frac{1}{(4u^2-1)}\cdot 4^2 = \boxed{\frac{16}{(4u^2-1)^2}}$$

$$\int_0^1 \frac{16}{(4u^2-1)^2}\,du$$

$$16\int_0^1 \frac{du}{(4u^2-1)^2}$$ FATORANDO

$$16\int_0^1 \frac{du}{((2u+1)(2u-1))^2} = 16\int_0^1 \frac{du}{(2u+1)^2(2u-1)^2} =$$

Fração Parcial

$$\frac{1}{(2u+1)^2(2u-1)^2} = \frac{A}{(2u+1)} + \frac{B}{(2u+1)^2} + \frac{C}{2u-1} + \frac{D}{(2u-1)^2}$$

$1 = A(2u+1)(2u-1)^2 + B(2u-1)^2 + C(2u+1)^2(2u-1) + D(2u+1)^2$

$1 = A(2u+1)(4u^2-4u+1) + B(4u^2-4u+1) + C(4u^2+4u+1)(2u-1) + D(4u^2+4u+1)$

$1 = A(8u^3-8u^2+2u+4u^2-4u+1) + B(4u^2-4u+1) + C(8u^3+8u^2+2u-4u^2-4u-1) + D(4u^2+4u+1)$

$1 = 8Au^3-8Au^2+2Au+4Au^2-4Au+A + 4Bu^2-4Bu+B + 8Cu^3+8Cu^2+2Cu-4Cu^2-4Cu-C + 4Du^2+4Du+D$

$1 = (8Au^3+8Cu^3) + (-8Au^2+4Au^2+4Bu^2+8Cu^2-4Cu^2+4Du^2) + (2Au-4Au-4Bu+2Cu-4Cu+4Du) + (A+B-C+D)$

$1 = (8Au^3+8Cu^3) + (-4Au^2+4Bu^2+4Cu^2+4Du^2) + (-2Au-4Bu-2Cu+4Du) + (A+B-C+D)$

$1 = (8A+8C)u^3 + (-4A+4B+4C+4D)u^2 + (-2A-4B-2C+4D)u + (A+B-C+D)$

$$\begin{cases} 8A+8C=0 \\ -4A+4B+4C+4D=0 \\ -2A-4B-2C+4D=0 \\ A+B-C+D=1 \end{cases} \qquad \begin{matrix} 8A+0B+8C+0D=0 \\ -4A+4B+4C+4D=0 \\ -2A-4B-2C+4D=0 \\ A+B-C+D=1 \end{matrix}$$

$$\left[\begin{array}{cccc|c} 8 & 0 & 8 & 0 & 0 \\ -4 & 4 & 4 & 4 & 0 \\ -2 & -4 & -2 & 4 & 0 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right] \frac{1}{2} \Rightarrow \left[\begin{array}{cccc|c} 8 & 0 & 8 & 0 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ -2 & -4 & -2 & 4 & 0 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 8 & 0 & 8 & 0 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & -4 & 0 & 4 & 0 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{cccc|c} 8 & 0 & 8 & 0 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & -4 & 0 & 4 & 0 \\ 0 & 1 & -2 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 8 & 0 & 8 & 0 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 0 & 8 & 8 & 0 \\ 0 & 1 & -2 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 8 & 0 & 8 & 0 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 0 & 8 & 8 & 0 \\ 0 & 0 & -4 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 8 & 0 & 8 & 0 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 0 & 8 & 8 & 0 \\ 0 & 0 & 0 & 4 & 1 \end{array}\right]$$

Resolver o Sistema de equações por escalonamento

$$\boxed{\begin{matrix} A = \frac{1}{4} \\ B = \frac{1}{4} \\ C = -\frac{1}{4} \\ D = \frac{1}{4} \end{matrix}}$$

$8A+8C=0$

$8A+\left(8\cdot-\frac{1}{4}\right)=0$

$8A+(-2)=0$

$8A=0+2$

$\boxed{A=\frac{2}{8}=\frac{1}{4}}$

$4B+8C+4D=0$

$4B+\left(\frac{-8}{4}\right)+\frac{4}{4}=0$

$4B-2+1=0$

$4B-1=0 \;/\; 4B=1$

$\boxed{B=\frac{1}{4}}$

$8C+8D=0$

$8C+2=0$

$8C=-2$

$C=\frac{-2}{4}=\boxed{\frac{-1}{4}}$

$4D=1$

$\boxed{D=\frac{1}{4}}$

$$16\int_0^1 \frac{du}{(2u+1)^2(2u-1)^2} = 16\int_0^1 \left(\frac{\frac{1}{4}}{2u+1} + \frac{\frac{1}{4}}{(2u+1)^2} + \frac{\left(-\frac{1}{4}\right)}{2u-1} + \frac{\frac{1}{4}}{(2u-1)^2}\right)dx =$$

$$16\cdot\Bigg(\underbrace{\int_0^1 \frac{1\,du}{4(2u+1)}}_{I} + \underbrace{\int_0^1 \frac{1\,du}{4(2u+1)^2}}_{II} - \underbrace{\int_0^1 \frac{1\,du}{4(2u-1)}}_{III} + \underbrace{\int_0^1 \frac{1\,du}{4(2u-1)^2}}_{IV}\Bigg)$$

I – $\frac{1}{4}\cdot\frac{1}{2}\ln(2u+1) = \boxed{\frac{1}{8}\ln(2u+1)}$

III – $\frac{1}{4}\cdot\frac{1}{2}\ln(2u-1) = \boxed{\frac{1}{8}\ln(2u-1)}$

II – $\frac{1}{4}\int\frac{du}{(2u+1)^2} = \frac{1}{4}\cdot\frac{1}{2}\int\frac{dt}{t^2} = \frac{1}{8}\int\frac{dt}{t^2} = \frac{1}{8}\int t^{-2}\,dt = \frac{1}{8}\int\frac{t^{-2+1}}{-2+1}\,dt =$

$t = 2u+1$

$\frac{1}{2}dt = 2du\,\frac{1}{2}$

$\frac{1}{8}\left(\frac{t^{-1}}{-1}\right) = \frac{1}{8}\cdot-\frac{1}{t} = -\frac{1}{8t} = \boxed{-\frac{1}{8(2u+1)}}$

IV – $\frac{1}{4}\int\frac{du}{(2u-1)^2} = \frac{1}{4}\cdot\frac{1}{2}\int\frac{dt}{t^2} = \frac{1}{8}\int\frac{dt}{t^2} = \frac{1}{8}\int t^{-2}\,dt = \frac{1}{8}\int\frac{t^{-2+1}}{-2+1}\,dt =$

$t = 2u+1$

$\frac{1}{2}dt = 2du\,\frac{1}{2}$

$\frac{1}{8}\left(\frac{t^{-1}}{-1}\right) = \frac{1}{8}\cdot-\frac{1}{t} = -\frac{1}{8t} = \boxed{-\frac{1}{8(2u-1)}}$

$$16\cdot\left(\frac{1}{4}\cdot\ln(2u+1) - \frac{1}{8(2u+1)} - \frac{1}{8}\ln(2u-1) - \frac{1}{8(2u-1)}\right)$$

$$16\cdot\left(\frac{1}{8}\cdot\ln\left(2\left(x+\frac{3}{2}\right)+1\right) - \frac{1}{8\left(2\left(x+\frac{3}{2}\right)+1\right)} - \frac{1}{8}\ln\left(2\cdot\left(x+\frac{3}{2}\right)-1\right) - \frac{1}{8\left(2\left(x+\frac{3}{2}\right)-1\right)}\right.$$

$\boxed{u = x+\frac{3}{2}}$

$$16\cdot\left(\left(\frac{1}{8}\cdot\ln(2x+4)\right) - \frac{1}{8(2x+4)} - \frac{1}{8}\ln(2x+2) - \frac{1}{8\cdot(2x+2)}\right)$$

$\boxed{U = x + \frac{3}{2}}$

$$16 \cdot \left(\frac{1}{8} \ln(2x+4) - \frac{1}{8(2x+4)} - \frac{1}{8} \ln(2x+2) - \frac{1}{8 \cdot (2x+2)} \right)$$

$$16 \cdot \left(\frac{1}{8} \ln(2x+4) - \frac{1}{8(2x+4)} - \frac{1}{8} \ln(2x+2) - \frac{1}{8(2x+2)} \right) + \cancel{C}$$

$$16 \cdot \left(\frac{1}{8} \ln(2x+4) - \frac{1}{8(2x+4)} - \frac{1}{8} \ln(2x+2) - \frac{1}{8(2x+2)} \right) \Bigg]_0^1$$

$$16 \cdot \left(\frac{1}{8} \ln(6) - \frac{1}{8 \cdot 6} - \frac{1}{8} \ln(4) - \frac{1}{8 \cdot 4} \right) - 16 \cdot \left(\frac{1}{8} \ln(4) - \frac{1}{8 \cdot 4} - \frac{1}{8} \ln(2) - \frac{1}{8 \cdot 2} \right)$$

$$16 \cdot \left(\frac{1}{8} \ln(6) - \frac{1}{48} - \frac{1}{8} \ln(4) - \frac{1}{32} \right) - 16 \left(\frac{1}{8} \ln(4) - \frac{1}{32} - \frac{1}{8} \ln(2) - \frac{1}{16} \right)$$

$$-0{,}02240 - (-0{,}11370) \simeq 0.091$$

$V = \pi \int_a^b [f(x)]^2$ ou seja, o volume será de

$$\boxed{\pi \cdot 0.09130}$$