Questao 5

quinta-feira, 13 de maio de 2021 16:10

$$\int_1^{\sqrt{3}} \operatorname{arctg}\left(\frac{1}{x}\right) dx =$$

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$$\int_1^{\sqrt{3}} \underbrace{\operatorname{arctg}\left(\frac{1}{x}\right)}_{\text{I Inversa}} \cdot \underbrace{x^0}_{\text{II Algébrica}} \, dx$$

$$u = \operatorname{arctg}\left(\frac{1}{x}\right) \to du = \boxed{-\frac{1}{x^2+1}}$$

$$dv = dx \to v = x$$

$$\frac{d}{dx}(\operatorname{arctg}(u)) = \frac{1}{1+u^2}$$

$$\frac{d}{dx} \operatorname{arctg}\left(\frac{1}{x}\right) = -\left(\frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{1}{x^2}\right) = -\frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{1}{x^2} = -\left(\frac{1}{1+\frac{1}{x^2}} \cdot \frac{1}{x^2}\right)$$

$$-x^{-1-1} = -x^{-2} = \boxed{-\frac{1}{x^2}}$$

$$\frac{\frac{1}{x^2+1}}{x^2} = \frac{1}{1} \cdot \frac{x^2}{x^2+1} = \frac{x^2}{x^2+1} \Big/ -\left(\frac{\cancel{x^2}}{(x^2+1)} \cdot \frac{1}{\cancel{x^2}}\right) = -\left(\frac{1}{(x^2+1)}\right)$$

$$-\frac{1}{x^2+1}$$

$$x \cdot \operatorname{arctg}\left(\frac{1}{x}\right) - \underbrace{\int x \cdot -\frac{1}{x^2+1} \, dx}$$

$$\int x \cdot \frac{1}{x^2+1} \cdot -1 \, dx = -\int x \cdot \frac{1}{x^2+1} \, dx = -\int \frac{x}{x^2+1} \, dx = -\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2} \ln|u| = -\frac{1}{2} \ln|x^2+1|$$

$$u = x^2+1 \to du = 2x\,dx \to \frac{du}{2} = x\,dx$$

$$x \cdot \operatorname{arctg}\left(\frac{1}{x}\right) - \left(-\frac{1}{2}\ln|x^2+1|\right)$$

$$\boxed{x \cdot \operatorname{arctg}\left(\frac{1}{x}\right) + \frac{1}{2}\ln|x^2+1|}$$

$$\left(\sqrt{3} \cdot \frac{\pi}{6} + \frac{\ln|4|}{2}\right) - \left(\frac{\pi}{4} + \ln|2|\right)$$

$$\left(\sqrt{3}\cdot\frac{\pi}{6}+\frac{\ln|4|}{2}\right)-\left(\frac{\pi}{4}+\frac{\ln|2|}{2}\right)$$