4)

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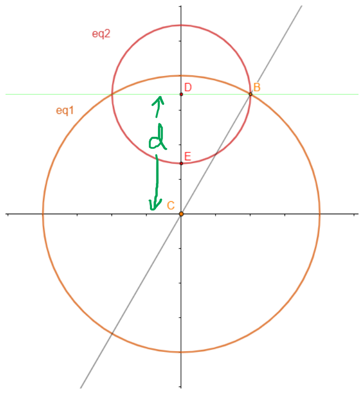


$\overline{CB} = R$

$\overline{DE} = \overline{BD} = r$

Equação do círculo $= (x - x_c)^2 + (y - y_c)^2 = R^2$, onde x_c e y_c são os coordenados do centro

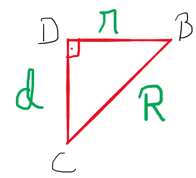
- Círculo maior:

$$x^2 + y^2 = R^2$$

- Círculo menor

$$x^2 + (y - d)^2 = r^2$$

Observando a figura, temos o seguinte:



$$R^2 = r^2 + d^2 \Rightarrow d^2 = R^2 - r^2$$

$$d = \sqrt{R^2 - r^2}$$

Escrevendo as equações do círculo com y em função de x

- Círculo maior:

$$y^2 = R^2 - x^2$$

$$y = \sqrt{R^2 - x^2}$$

- Círculo menor:

$$(y - d)^2 = r^2 - x^2$$

$$y - d = \sqrt{r^2 - x^2}$$

$$y = d + \sqrt{r^2 - x^2}$$

Cálculo da Área

$$A = \int_{-r}^{r} \left(d + \sqrt{r^2 - x^2}\right) - \left(\sqrt{R^2 - x^2}\right) dx$$

$$A = 2\int_{0}^{r} \left(d + \sqrt{r^2 - x^2}\right) - \left(\sqrt{R^2 - x^2}\right) dx$$

$$A = \underbrace{2\int_0^r d\, dx}_{} + \underbrace{2\int_0^r \sqrt{r^2 - x^2}\, dx}_{} - \underbrace{2\int_0^r \sqrt{R^2 - x^2}\, dx}_{}$$

$$2\int_0^r d\, dx = 2d\,(x)\Big|_0^r = 2dr$$

$$2\int_0^r \sqrt{r^2 - x^2}\, dx$$

Substituição: $x = r \cdot \operatorname{sen}\theta$, $-\pi/2 \le \theta \le \pi/2$

$dx = r\cos\theta\, d\theta$

$$\sqrt{r^2 - x^2} = \sqrt{r^2 - r^2\operatorname{sen}^2\theta} = \sqrt{r^2(1 - \operatorname{sen}^2\theta)} = \sqrt{r^2\cos^2\theta} = r\cos\theta$$

$$\int \sqrt{r^2 - x^2} \cdot dx = \int r\cos\theta \cdot r\cos\theta\, d\theta$$

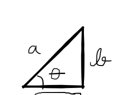
$$= \int r^2\cos^2\theta = r^2\int \frac{(1 + \cos(2\theta))}{2}\, d\theta$$

$$= \frac{r^2}{2}\int 1 + \cos(2\theta)\, d\theta = \frac{r^2}{2}\cdot\left(\theta + \frac{\operatorname{sen}(2\theta)}{2}\right) + C$$

$$= \frac{r^2}{2}\left(\theta + \frac{\cancel{2}\operatorname{sen}\theta\cos\theta}{\cancel{2}}\right) + C = r^2/2\,(\theta + \operatorname{sen}\theta\cos\theta) + C$$

$$= r^2/2\left(\operatorname{sen}^{-1}(x/r) + \frac{x}{r}\cdot\frac{\sqrt{r^2 - x^2}}{r}\right) + C$$

Apoio:



$\operatorname{sen}\theta = b/a \Rightarrow \theta = \operatorname{sen}^{-1}(b/a)$

$\cos\theta = \dfrac{\sqrt{a^2 - b^2}}{a}$

Utilizando o mesmo método e apoio da equação acima, temos:

$$\int \sqrt{R^2 - x^2}\, dx = R^2/2\left(\operatorname{sen}^{-1}(x/R) + \frac{x}{R}\cdot\frac{\sqrt{R^2 - x^2}}{R}\right) + C$$

$$A = 2\int_0^r d\, dx + 2\int_0^r \sqrt{r^2 - x^2}\, dx - 2\int_0^r \sqrt{R^2 - x^2}\, dx$$

$$A = 2dr + \frac{2r^2}{2}\left(\operatorname{sen}^{-1}(x/r) + \frac{x\sqrt{r^2 - x^2}}{r^2}\right)\Bigg|_0^r - \frac{2R^2}{2}\left(\operatorname{sen}^{-1}(x/R) + \frac{x\sqrt{R^2 - x^2}}{R^2}\right)\Bigg|_0^r$$

$$A = 2r\sqrt{R^2 - r^2} + r^2\left[\left(\operatorname{sen}^{-1}(1) + \cancel{\frac{r\sqrt{r^2 - r^2}}{r^2}}\right) - \operatorname{sen}^{-1}(0) + 0\right] - R^2\left[\left(\operatorname{sen}^{-1}\left(\frac{r}{R}\right) + \frac{r\sqrt{R^2 - r^2}}{R^2}\right) - \operatorname{sen}^{-1}(0) - 0\right]$$

$$A = 2r\sqrt{R^2 - r^2} + r^2 - R^2\operatorname{sen}^{-1}(r/R) - r\sqrt{R^2 - r^2}$$