## CHAPTER 2

# ELECTRIC POWER DEFINITIONS: BACKGROUND

There are several basic concepts that must be established before the analysis of electric power systems involving power electronic devices can begin. The calculation of electric system variables, for instance, voltage, current, power factor, and active and reactive power, under nonsinusoidal conditions is perhaps the cornerstone of this analysis. The concepts and definitions of electric power for sinusoidal ac systems are well established and accepted worldwide. However, under nonsinusoidal conditions, several and different power definitions are still in use. For instance, the conventional concepts of reactive and apparent power lose their usefulness in nonsinusoidal cases [1,2]. This problem has existed for many years and is still with us. Unfortunately, no agreement on a universally applicable power theory has been achieved yet.

At the beginning, two important approaches to power definitions under nonsinusoidal conditions were introduced by Budeanu in 1927 [4,5], and by Fryze in 1932 [3]. Budeanu worked in the frequency domain, whereas Fryze defined the power in the time domain. Unfortunately, those power definitions are dubious, and may lead to misinterpretation in some cases. No other relevant contributions were made until the 1970s, because power systems were satisfactorily well represented as balanced and sinusoidal ac sources and loads. However, the problems related to nonlinear loads became increasingly significant at the beginning of advances in power electronics devices. These modern devices behave as nonlinear loads and most of them draw a significant amount of harmonic current from the power system. An increasing number of nonlinear loads are being connected to the network. Hence, this has spurred analysis of power systems under nonsinusoidal conditions. It is imperative to establish a consistent set of power definitions that are valid also under transient and nonsinusoidal conditions.

Many different approaches to power definitions can be found in the literature. They are based on the frequency or time domains. Although system engineers used to deal with power systems in the frequency domain, the authors believe that power definitions in the time domain are more appropriate for the analysis of power systems if they are operating under nonsinusoidal conditions.

Power definitions in the frequency domain will be summarized in this chapter, just to give a background for the time-domain approach to be presented in the next chapter. The electric power definitions in the frequency domain will be presented just to make it evident why they should be complemented to become more coherent and useful in the analysis and design of modern power systems involving nonlinear loads.

## 2.1. POWER DEFINITIONS UNDER SINUSOIDAL CONDITIONS

The definitions of electric power for single-phase, sinusoidal systems have been well established. Nowadays, there are no divergences between the results obtained in the time and frequency domains.

An ideal single-phase system with a sinusoidal voltage source and a linear (resistive–inductive) load has voltage and current that are analytically represented by

$$v(t) = \sqrt{2}V \sin(\omega t) \qquad i(t) = \sqrt{2}I \sin(\omega t - \phi) \tag{2.1}$$

where $V$ and $I$ represent the root-mean-square (rms) values of the voltage and current, respectively, and $\omega$ is the angular line frequency. The instantaneous (active) power is given by the product of the instantaneous voltage and current, that is,

$$\begin{aligned} p(t) = v(t)i(t) &= 2VI \sin(\omega t)\sin(\omega t - \phi) \\ &= VI \cos\phi - VI\cos(2\omega t - \phi) \end{aligned} \tag{2.2}$$

Equation (2.2) shows that the instantaneous power of the single-phase system is not constant. It has an oscillating component at twice the line frequency added to a dc level (average value) given by $VI \cos\phi$. Decomposing the oscillating component and rearranging (2.2) yields the following equation with two terms, which derives the traditional concept of active and reactive power:

$$p(t) = \underbrace{VI\cos\phi[1 - \cos(2\omega t)]}_{\text{(I)}} - \underbrace{VI \sin\phi \sin(2\omega t)}_{\text{(II)}} \tag{2.3}$$

The decomposition given by (2.3) shows two parts of the instantaneous power that can be interpreted as:

**Part I** has an average value equal to $VI \cos\phi$ and has an oscillating component on it, pulsing at twice the line frequency. This part never becomes negative (provided that $-90° \leq \phi \leq 90°$) and, therefore, represents an unidirectional power flow from the source to the load.

**Part II** has a pure oscillating component at the double frequency ($2\omega$), and has a peak value equal to $VI \sin \phi$. Clearly, it has a zero average value.

Conventionally the instantaneous power given in (2.3) is represented by three "constant" powers: the *active power,* the *reactive power* and the *apparent power.* These powers are discussed below.

**Active power** $P$. The average value of part I is defined as the *active* (*average*) *power* $P$:

$$P = VI \cos \phi \tag{2.4}$$

The unit for the measurement of the active power in the International System is the Watt (W).

**Reactive power** $Q$. The conventional *reactive power* $Q$ is just defined as the peak value of part II.

$$Q = VI \sin \phi \tag{2.5}$$

The unit for the measurement of the reactive power in the International System is the var (volt-ampere reactive).

A signal for the displacement angle $\phi$ should be adopted to characterize the inductive or capacitive nature of the load. In this book, the most common convention will be adopted, assigning *positive* values for the reactive power of *inductive* loads. Capacitive loads have reactive power with the negative sign.

Several authors refer to the reactive power as "*the portion of power that does not realize work,*" or "*oscillating power*" [6]. The reactive power as conventionally defined represents a power component with zero average value. However, this assertion is not complete, as will be shown later. This physical meaning for the reactive power was established when the basic reactive devices were only capacitors and inductors. At that time, there were no power electronics devices. These devices are capable of creating reactive power without energy storage elements. This subject will be discussed in the next chapter.

Now, the instantaneous power $p(t)$ can be rewritten as

$$p(t) = \underbrace{P[1 - \cos(2\omega t)]}_{\text{(I)}} - \underbrace{Q \sin(2\omega t)}_{\text{(II)}} \tag{2.6}$$

Figure 2-1 illustrates the above power components for a given voltage and current. In this figure, the ac current lags the ac voltage by a displacement angle $\phi$, which is equal to $\pi/3$. From (2.6) and Fig. 2-1, it is easy to understand that the energy flow (instantaneous power) in the ac single-phase system is not unidirectional and not constant. During the time interval corresponding to area "$A$," the

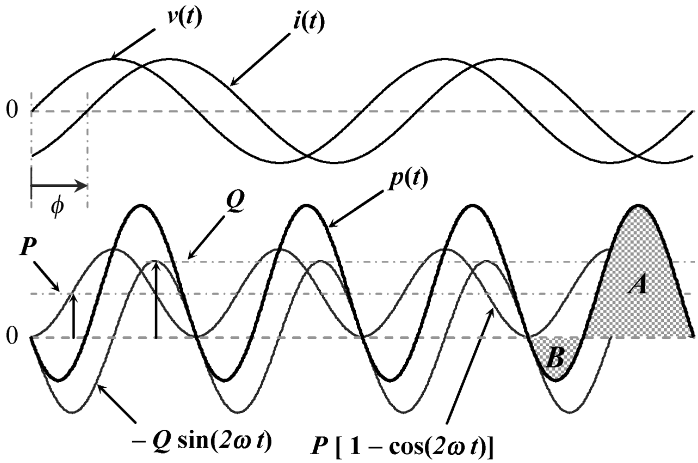


**Figure 2-1.** Conventional concepts of active and reactive power.

source is delivering energy to the load, whereas during the time interval corresponding to area "*B*," a percentage of this amount of energy is being returned back to the source.

Another power quantity that is commonly used to define the power rating of electrical equipment is the *apparent power S.*

**Apparent power** $S$ is defined as

$$S = V \cdot I \tag{2.7}$$

The unit for the apparent power in the International System is VA (Volt-Ampere). This power is usually understood to represent the "*maximum reachable active power at unity power factor*" [7]. The definition of power factor is given in the next section.

## 2.2. VOLTAGE AND CURRENT PHASORS AND COMPLEX IMPEDANCE

Sometimes, the analysis of power systems can be significantly simplified by using the phasor notation instead of using sinusoidal voltages and currents as time functions given in (2.1). Therefore, a short review of phasor definitions will be presented here.

A sinusoidal time function $f(t)$, with a given angular frequency of $\omega$, can be represented as the imaginary part of a complex number as follows:

$$f(t) = \sqrt{2}A \sin(\omega t + \phi) = \mathrm{Im}\{\dot{F} \cdot e^{j\omega t}\} \tag{2.8}$$

Here, $\dot{F}$ is a complex number with a magnitude of $\sqrt{2}A$ and phase $\phi$, and is defined as the phasor related to the sinusoidal function $f(t)$. Thus,

$$\dot{F} = \sqrt{2}A \angle \phi \tag{2.9}$$

The representation of a sinusoidal waveform by a phasor is only possible if it is purely sinusoidal with an angular frequency of $\omega$, and "frozen" for a complete cycle. Therefore, phasor representation, although practical, is valid only for steady-state conditions.

A voltage phasor $\dot{V}$ and a current phasor $\dot{I}$ can be represented by complex numbers in its polar or Cartesian notation:

$$\left|\begin{array}{ll} \dot{V} = V \angle \theta_V = V_{\Re} + jV_{\Im} & \\ V_{\Re} = V \cos \theta_V & V_{\Im} = V \sin \theta_V \end{array}\right| \begin{array}{ll} \dot{I} = I \angle \theta_I = I_{\Re} + jI_{\Im} & \\ I_{\Re} = I \cos \theta_I & I_{\Im} = I \sin \theta_I \end{array} \tag{2.10}$$

where $V$ and $I$ are the rms values of the sinusoidal voltage and current time functions, and $\theta_V$ and $\theta_I$ are the phase angles at a given time instant. Positive phase angles are conventionally measured in the counterclockwise direction. Figure 2-2 shows the sinusoidal voltage and current time functions and the corresponding phasor representation at an instant when $\omega t + \theta_V = 0$. The displacement angle $\phi$ between $\dot{V}$ and $\dot{I}$ is given by $\phi = \theta_V - \theta_I$.

If the voltages and currents of an ac power system are sinusoidal, it is possible to define the concept of impedance. For instance, on a series *RLC* impedance branch, the ratio between the terminal voltage phasor and the current phasor is equal to a complex number, defined as *complex impedance* **Z**, given by:

$$\mathbf{Z} = \frac{\dot{V}}{\dot{I}} = \frac{V \angle \theta_V}{I \angle \theta_I} = \frac{V}{I} \angle (\theta_V - \theta_I) = V \angle \theta_Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{2.11}$$

It is known that the current through an inductive load lags its terminal voltage, and the current through a capacitive load leads its terminal voltage. Therefore, the above convention of complex impedance produces a positive impedance angle (positive reactance) for inductive loads.

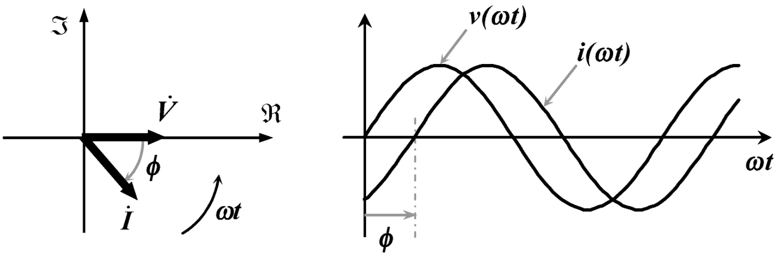


**Figure 2-2.** Relation between phasors and sinusoidal time functions.

## 2.3. COMPLEX POWER AND POWER FACTOR

A complex power, **S**, can be defined as the product of the voltage and current phasors. In order to be coherent with the sign convention of the reactive power given in (2.5) that is related to a "positive reactive power for inductive loads," the following definition of complex power should use the conjugate value ($\dot{I}^* = I \angle -\theta_I$) of the current phasor.

**Complex power S** is defined as

$$\mathbf{S} = \dot{V}\dot{I}^* = (V \angle \theta_V)(I \angle -\theta_I) = \overbrace{VI\cos\underbrace{(\theta_V - \theta_I)}_{\phi}}^{P} + \overbrace{jVI\sin\underbrace{(\theta_V - \theta_I)}_{\phi}}^{Q} \tag{2.12}$$

Note that the absolute value of the complex power is equal to the apparent power defined in (2.7), that is,

$$|\mathbf{S}| = \sqrt{[VI\cos(\theta_V - \theta_I)]^2 + [VI\sin(\theta_V - \theta_I)]^2} = S = VI \tag{2.13}$$

The term $\cos\phi$ is equal to the ratio between the active power $P$ and the apparent power $S$, and is referred to as *power factor* $\lambda$. In the modern literature, it is common to find the symbol "PF" to denote power factor.

**Power factor** $\lambda$ (PF) is defined as

$$\lambda = \text{PF} = \cos\phi = \frac{P}{S} \tag{2.14}$$

The concept of complex power and power factor can be graphically represented in the well-known triangle of powers, as shown in Fig. 2-3. This figure, together with the above set of power and power factor definitions can be summarized as follows. If the load is not purely resistive, the reactive power $Q$ is not zero, and the ac-

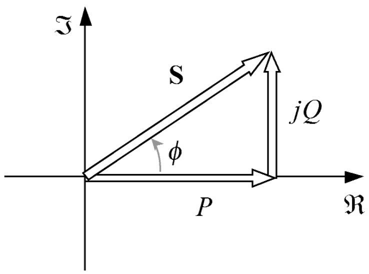


**Figure 2-3.** Graphical representation of the complex power—the triangle of powers.

tive power $P$ is smaller than the apparent power $S$. Thus, the power factor $\lambda$ is smaller than unity. These are the traditional meanings of the above electric powers defined under pure sinusoidal conditions [6]. They are widely used in industry to characterize electric equipment like transformers, machines, and so on. Unfortunately, these concepts of power are not valid, or lead to misinterpretations, under nonsinusoidal conditions [16]. This point will be deeply discussed in the next sections of this chapter.

## 2.4. CONCEPTS OF POWER UNDER NONSINUSOIDAL CONDITIONS—CONVENTIONAL APPROACHES

The concepts of power under nonsinusoidal conditions are not unique; they are divergent and lead to different results in some aspects. Two distinct sets of power definitions are commonly used; one is established in the frequency domain and the other in the time domain. They are presented below, to highlight their inconsistencies and to show why they are inadequate for use in controllers of power-line conditioners.

### 2.4.1. Power Definitions by Budeanu

A set of power definitions established by Budeanu [4,5] in 1927 is still very important for the analysis of power systems in the frequency domain. He introduced definitions that are valid for generic waveforms of voltage and current. However, since they are defined in the frequency domain, they can be applied only in steady-state analysis. In other words, they are limited to periodic waveforms of voltage and current.

If a single-phase ac circuit with a generic load and a source is in steady state, its voltage and current waveforms can be decomposed in Fourier series. Then, the corresponding phasor for each harmonic component can be determined, and the following definitions of powers can be derived.

**Apparent power** $S$ is defined as

$$S = VI \tag{2.15}$$

The apparent power in (2.15) is, in principle, identical to that given in (2.7). The difference is that $V$ and $I$ are the rms values of generic, periodic voltage and current waveforms, which are calculated as

$$V = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\sum_{n=1}^{\infty} V_n^2} \qquad I = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\sum_{n=1}^{\infty} I_n^2} \tag{2.16}$$

Here, $V_n$ and $I_n$ correspond to the rms value of the $n$th order harmonic components of the Fourier series, and $T$ is the period of the fundamental component. No direct current (dc) component is being considered in this analysis. The displacement angle of each pair of $n$th order harmonic voltage and current components is represented by $\phi_n$. Budeanu defined the active and reactive powers as follows.

**Active power** $P$:

$$P = \sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} V_n I_n \cos \varphi_n \tag{2.17}$$

**Reactive power** $Q$:

$$Q = \sum_{n=1}^{\infty} Q_n = \sum_{n=1}^{\infty} V_n I_n \sin \varphi_n \tag{2.18}$$

The definitions of the apparent power and the reactive power seem to come from the necessity of quantifying in ac systems "*the portion of power that does not realize work.*" In other words, it might be interesting to have indexes for quantifying the quality of the power being supplied by a power-generating system. However, under nonsinusoidal conditions, both reactive power and apparent power cannot characterize satisfactorily the issues of power quality or the efficiency of the transmission system. For instance, the above-defined reactive power does not include cross products between voltage and current harmonics at different frequencies. Note that neither the active power in (2.17), nor the reactive power in (2.18), includes the products of harmonic components at different frequencies. Furthermore, (2.18) comprises the algebraic sum of "harmonic reactive power" components that are positive or negative or even cancel each other, depending on the several harmonic displacement angles $\phi_n$.

The loss of power quality under nonsinusoidal conditions is better characterized by another power definition, the *distortion power D,* that was introduced by Budeanu. This distortion power complements the above set of power definitions.

**Distortion power** $D$ is defined as

$$D^2 = S^2 - P^2 - Q^2 \tag{2.19}$$

The powers defined in (2.15) to (2.19) are well known and widely used in the analysis of circuit systems operating under nonsinusoidal conditions. However, only the active power $P$, as defined in (2.17), has a clear physical meaning, not only in the sinusoidal case, but also under nonsinusoidal conditions [8,9]. The active power represents the *average* value of the instantaneous active power. In other words, it represents the average ratio of energy transfer between two electric subsystems. In contrast, the reactive power and the apparent power as introduced by Budeanu [4,5] are just mathematical formulations, as an extension of the definitions for the sinusoidal case, without clear physical meanings. Another drawback is that a common instrument for power measurement based on the power definitions in the frequency domain cannot indicate easily a loss of power quality in practical cases. This point is clarified in the following example.

***Example #1***

Consider an ideal sinusoidal voltage source $v(t) = \sin(2\pi 50t)$ [pu] supplying:

a) $i(t) = \sin(2\pi 50t - \pi/4)$ [pu] for a linear load
b) $i(t) = \sin(2\pi 50t - \pi/4) + 0.1\sin(14\pi 50t - \pi/4)$ [pu] for a nonlinear load

The fundamental components of voltage and current in both cases are identical. Therefore, the active and reactive powers calculated from (2.17) and (2.18) are the same in both cases:

$$P = \sum_{n=1}^{\infty} V_n I_n \cos\varphi_n = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{4}\right) = 0.3536$$

$$Q = \sum_{n=1}^{\infty} V_n I_n \sin\varphi_n = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{4}\right) = 0.3536$$

Case (a) is purely sinusoidal, whereas case (b) is not. The rms value of the voltage ($V$) and the current ($I$) in case (a) is equal to $1/\sqrt{2} = 0.7071$, whereas the rms of the current in case (b) is

$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\sum_{n=1}^{\infty} I_n^2} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{0.1}{\sqrt{2}}\right)^2} = 0.7106$$

Although a 10% harmonic current with respect to the fundamental current was added in case (b), an increase of only 0.5% was verified in the rms current with respect to that in case (a). This causes an increase of only 0.5% in the apparent power $S$, which would be very difficult to be detected by conventional instruments of power measurement. This example is the evidence that instruments for power measurements based on rms values of voltage and current in the frequency domain may be inadequate to deal with power systems under nonsinusoidal conditions. Note that case (b) is undesirable in terms of harmonics and power quality. Moreover, the measurement of a distorted current is not so easy. Figure 2-4 shows

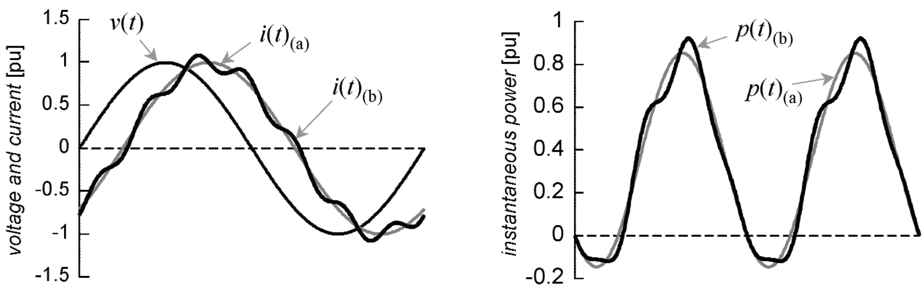


**Figure 2-4.** Instantaneous voltage, current, and power of Example #1.

waveforms of the voltage and currents considered above, and the produced instantaneous powers. The seventh harmonic current added in case (b) causes an increase of 8% in the peak value of the instantaneous power. The peak value is 0.8535 pu in case (a), whereas it is 0.9219 pu in case (b). In summary, this harmonic current induces an increase of 0.5% in the apparent power, 8% in the peak power (16 times), and produces no effect on the active or reactive powers defined in (2.17) and (2.18).

### 2.4.1.A. Power Tetrahedron and Distortion Factor

Due to the presence of the distortion power $D$ under nonsinusoidal conditions, graphical power representation is given on a three-dimensional reference frame, instead of the power triangle described in Fig. 2-3. Figure 2-5 shows the new graphical power representation that is well known as a power tetrahedron.

Under nonsinusoidal conditions, the apparent power defined in (2.7) and (2.15) differs from the complex power defined in (2.12). As a result, the equivalence between them is no longer valid, as shown in (2.13). A new complex power $\mathbf{S_{PQ}}$ can be defined from (2.17) and (2.18) that corresponds to the new active and reactive power defined by Budeanu. This new complex power $\mathbf{S_{PQ}}$ is defined as

$$\mathbf{S_{PQ}} = P + jQ = \sum_{n=1}^{\infty} P_n + j\sum_{n=1}^{\infty} Q_n = \sum_{n=1}^{\infty} V_n I_n \cos\varphi_n + j\sum_{n=1}^{\infty} V_n I_n \sin\varphi_n \qquad (2.20)$$

The relation between the apparent power $S$ and the complex power $\mathbf{S_{PQ}}$ is given by

$$S = VI = \sqrt{P^2 + Q^2 + D^2} = \sqrt{|\mathbf{S_{PQ}}|^2 + D^2} \qquad (2.21)$$

The power factor $\lambda$ is defined in (2.14) as the ratio of the active power with respect to the apparent power. Now, $\lambda$ is equal to $\cos\theta$ in the power tetrahedron of Fig. 2-5.

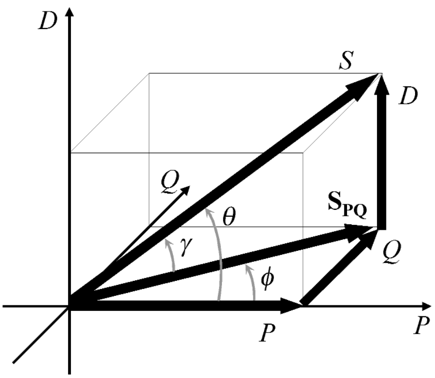


**Figure 2-5.** The power tetrahedron.

As can be seen in (2.21), the absolute value of the complex power ($|\mathbf{S_{PQ}}| = [P^2 + Q^2]^{1/2}$) is different from the apparent power $S$ under nonsinusoidal conditions. The ratio between $P$ and $|\mathbf{S_{PQ}}|$ is defined as the *displacement factor* ($\cos\phi$), although it is the power factor under sinusoidal conditions. Another term, the *distortion factor* ($\cos\gamma$), is defined as the ratio between the length of $\mathbf{S_{PQ}}$ and the apparent power $S$. These factors are listed below.

**Power factor** $\lambda$:

$$\lambda = \cos\theta = \frac{P}{S} \tag{2.22}$$

**Displacement factor** $\cos\phi$:

$$\cos\phi = \frac{P}{|\mathbf{S_{PQ}}|} \tag{2.23}$$

**Distortion factor** $\cos\gamma$:

$$\cos\gamma = \frac{|\mathbf{S_{PQ}}|}{S} \tag{2.24}$$

The following relation is valid:

$$\lambda = \cos\theta = \frac{P}{S} = \cos\phi \cdot \cos\gamma \tag{2.25}$$

***Remarks***

The reactive power $Q$ in (2.18) and the harmonic power $D$ in (2.19) are mathematical formulations that may lead to false interpretations, particularly when these concepts are extended to the analysis of three-phase circuits. The above equations treat electric circuits under nonsinusoidal conditions as a sum of several *independent* circuits excited at different frequencies. The calculated powers do not provide any consistent basis for designing passive filters or for controlling active power line conditioners.

The apparent power $S$ has at least four different definitions as it appears in a widely known dictionary [14]. One of them considers it as a number that gives the basic rating of electrical equipment. However, it is not a unique definition and, therefore, the definition given in (2.15) is just one of them, named "the rms volt-ampere" [9].

It is more difficult to find reasons for applicability of the apparent power if a three-phase, four-wire, nonsinusoidal power source is connected to generic loads [16]. These problems are reported in several works, and many researchers have tried to solve them [17,18,19,20,21]. More recently, an IEEE Trial-Use Standard [15] was published for definitions used for measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions. It lists the mathematical expressions that were used in the past, as well as new expressions, and explains the features of the new definitions.

### 2.4.2. Power Definitions by Fryze

In the early 1930s, Fryze proposed a set of power definitions based on rms values of voltage and current [3]. The basic equations according to the Fryze's approach are given below.

**Active power** $P_w$:

$$P_w = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T v(t)i(t)\,dt = V_w I = VI_w \tag{2.26}$$

where $V$ and $I$ are the voltage and current rms values and $V_w$ and $I_w$ are the active voltage and active current defined below. The rms values of voltage and current are calculated as given in (2.16). Together with the active power $P_w$, these rms values form the basis of the Fryze's approach. From them, all other parameters can be defined and calculated as follows.

**Apparent power** $P_S$:

$$P_S = VI \tag{2.27}$$

**Active power factor** $\lambda$:

$$\lambda = \frac{P_w}{P_s} = \frac{P_w}{VI} \tag{2.28}$$

**Reactive power** $P_q$:

$$P_q = \sqrt{P_s^2 - P_w^2} = V_q I = VI_q \tag{2.29}$$

where $V_q$ and $I_q$ are the reactive voltage and current as defined below.

**Reactive power factor** $\lambda_q$:

$$\lambda_q = \sqrt{1 - \lambda^2} \tag{2.30}$$

**Active voltage** $V_w$ and **active current** $I_w$:

$$V_w = \lambda \cdot V \qquad I_w = \lambda \cdot I \tag{2.31}$$

**Reactive voltage** $V_q$ and **reactive current** $I_q$:

$$V_q = \lambda_q \cdot V \qquad I_q = \lambda_q \cdot I \tag{2.32}$$

Fryze defined reactive power as comprising all the portions of voltage and current, which does not contribute to the active power $P_w$. Note that the active power $P_w$ is defined as the average value of the instantaneous active power. This concept of active and reactive power is well accepted nowadays. For instance, Czarnecki has improved this approach, going into detail by dividing reactive power $P_q$ into four subparts according to their respective origins in electric circuits [11,12,13].

It is possible to demonstrate that there is no difference between the *active* power and the *apparent* power defined by Fryze in time domain and Budeanu in frequency domain. It is easy to confirm that the active power calculated from (2.17) is always the same as from (2.26). Both apparent powers from (2.15) and from (2.27) are also the same. However, the reactive power given in (2.18) by Budeanu is different from that in (2.29) by Fryze.

Fryze verified that the active power factor $\lambda$ reaches its maximum ($\lambda = 1$) if and only if the instantaneous current is proportional to the instantaneous voltage, otherwise $\lambda < 1$ [3]. However, under nonsinusoidal conditions, the fact of having the current proportional to the voltage does not ensure an optimal power flow from the electromechanical energy conversion point of view, as will be shown later. If the concepts defined above are applied to the analysis of three-phase systems, they may lead to cases in which the three-phase instantaneous active power contains an oscillating component even if the three-phase voltage and current are proportional (unity power factor $\lambda = 1$). All these remarks will be clarified in the next chapter, which will present the instantaneous active and reactive power theory.

The above set of power definitions does not need any decomposition of generic voltage or current waveform in Fourier series, although it still requires the calculation of rms values of voltage and current. Hence, it is not valid during transient phenomena.

## 2.5. ELECTRIC POWER IN THREE-PHASE SYSTEMS

It is common to find three-phase circuits being analyzed as a sum of three *separate* single-phase circuits. This is a crude simplification, especially in cases involving power electronic devices or nonlinear loads. The total active, reactive, and apparent powers in three-phase circuits have been calculated just as three times the powers in a single-phase circuit, or the sum of the powers in the three single-phase, separated circuits. It is also common to find in the literature works that assign the same physical meaning or mathematical interpretation for the active, reactive, and apparent power in both single-phase and three-phase systems. The authors do not agree with these ideas because three-phase systems present some properties that are not observed in single-phase systems. These properties will be presented in this section.

### 2.5.1. Classifications of Three-Phase Systems

Three-phase systems can be grounded or not. If a three-phase system is grounded at more than one point under normal operation (no-fault or short-circuited operation), the ground can provide an additional path for current circulation. A three-phase system can also have a fourth conductor or the so-called "neutral wire or conductor." In both cases, the system is classified as a three-phase, four-wire system. If no ground is present or there is only one grounded node in the whole subnetwork, the system is classified as a three-phase, three-wire system or simply as a three-phase system.

So far, only single-phase systems have been considered and classified as systems "under sinusoidal conditions" or "under nonsinusoidal conditions." Hereafter, the words "distorted" system will be also used to refer to "a system under nonsinusoidal conditions." Beside these considerations, three-phase systems under sinusoidal conditions have a particular characteristic regarding the amplitude and phase angle of each phase voltage (or line current). If the amplitudes are equal and the displacement angles between the phases are equal to $2\pi/3$, the three-phase system is said to be "balanced" or "symmetrical." Otherwise, the three-phase system is "unbalanced" or "unsymmetrical." The terms balanced and unbalanced will be used in this book. The above classification is illustrated in Fig. 2-6, where three examples of three-phase voltages are given. This means that a three-phase system consisting only of a positive-sequence component or only of a negative-sequence component is a balanced system. However, only the positive-sequence component will be considered in this book as the main component.

Figure 2-6(a) and Fig. 2-6(b) show examples of three-phase balanced and unbalanced voltages, respectively. Figure 2-6(c) shows three-phase distorted and unbalanced voltages that are obtained by superposing harmonic components on the unbalanced voltages given in Fig. 2-6(b). Thus, beside the harmonics components, the voltages have also unbalanced fundamental components.

There are two kinds of imbalances, which can be better understood when the Symmetrical Components Theory [22] is applied to the three-phase phasors of voltage or current.

At first, the three-phase unbalanced phasors ($\dot{V}_a, \dot{V}_b, \dot{V}_c$) are transformed into three other phasors: the positive-sequence phasor $\dot{V}_+$, the negative-sequence phasor $\dot{V}_-$, and the zero-sequence phasor $\dot{V}_0$. They are calculated as

$$\begin{bmatrix} \dot{V}_0 \\ \dot{V}_+ \\ \dot{V}_- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} \dot{V}_a \\ \dot{V}_b \\ \dot{V}_c \end{bmatrix} \tag{2.33}$$

The constant $\alpha$ is a complex number that acts as a 120°-phase shift operator, that is,

$$\alpha = 1 \angle 120° = e^{j(2\pi/3)} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{2.34}$$

The inverse transformation of (2.33) is given by

$$\begin{bmatrix} \dot{V}_a \\ \dot{V}_b \\ \dot{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}\begin{bmatrix} \dot{V}_0 \\ \dot{V}_+ \\ \dot{V}_- \end{bmatrix} \tag{2.35}$$

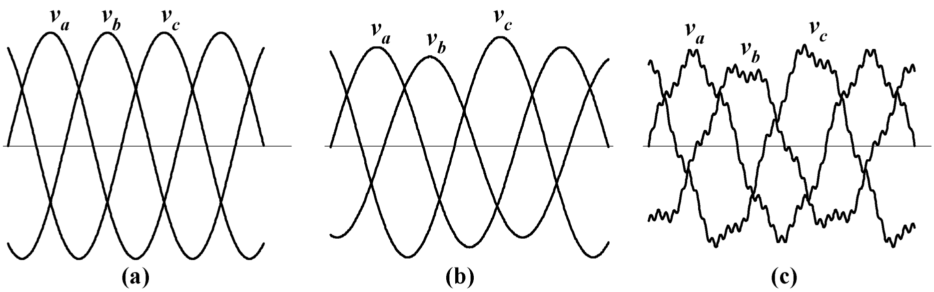


**Figure 2-6.** (a) Balanced voltages, (b) unbalanced voltages, and (c) distorted and unbalanced voltages.

Note that the zero-sequence phasor contributes to the phase voltages without 120° phase shifting, whereas the positive-sequence phasor contributes to the voltage in the form of an *abc* sequence and the negative-sequence phasor in the form of an *acb* sequence. The above idea of decomposition of unsymmetrical phasors in the positive-sequence phasor $V_+$, the negative-sequence phasor $V_-$, and the zero-sequence phasor $V_0$ can be better visualized by referring to Fig. 2-7.

Imbalances at the fundamental frequency can be caused by negative-sequence or zero-sequence components. However, it is important to note that only an imbalance from a negative-sequence component can appear in a three-phase grounded or ungrounded system. Imbalance from zero-sequence component only appears in a three-phase, four-wire (grounded) system, which induces the current flowing through the neutral wire. Contrarily, the sum of the three instantaneous phase voltages, as well as the sum of the three instantaneous line currents, is always equal to zero if they consist only of positive-sequence components and/or negative-sequence components. Hence, positive-sequence and negative-sequence components can be present in three-phase circuits with or without the neutral conductor.

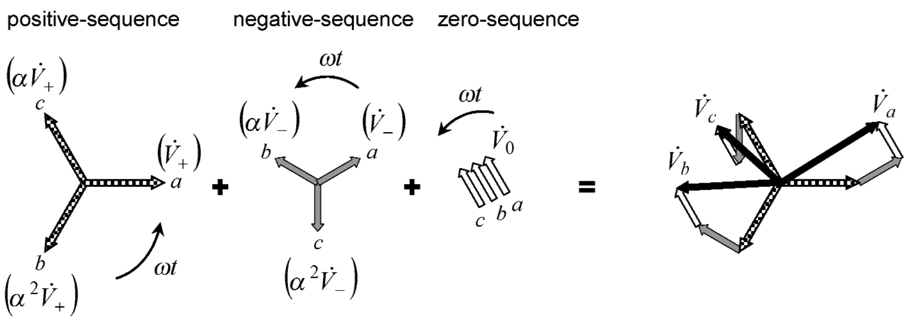


**Figure 2-7.** Positive-sequence, negative-sequence, and zero-sequence components of an unbalanced three-phase system.

Figure 2-7 shows phasors in the same angular frequency $\omega$, corresponding to the decomposition of an unbalanced three-phase system into symmetrical components. The above analysis is valid for an arbitrarily chosen value of $\omega$. For instance, three-phase, generic, periodic voltages and currents can be decomposed in Fourier series. From these series, the third ($3\omega$), fifth ($5\omega$), seventh ($7\omega$), and subsequent harmonics in a three-phase generic voltage or current can be separated in groups of *abc* harmonics that are at a given frequency. For each *abc* harmonic group, the above decomposition of phasors into symmetrical components can be applied.

### 2.5.2. Power in Balanced Three-Phase Systems

In three-phase balanced systems, the total (three-phase) active, reactive, and apparent powers have been calculated as three times the single-phase powers defined above. Although many authors assign the same physical meaning or mathematical interpretation for the active, reactive, and apparent power in both single-phase and three-phase systems, at least one remark should be made concerning the reactive power $Q$. The reactive power does not describe the same phenomenon in three-phase and in single-phase circuits [23,24]. This is confirmed below.

Three-phase voltage and line current that contain only the positive-sequence fundamental component (sinusoidal and balanced system) are given by

$$\begin{cases} v_a(t) = \sqrt{2}V_+ \sin(\omega t + \phi_{V+}) \\ v_b(t) = \sqrt{2}V_+ \sin\left(\omega t + \phi_{V+} - \dfrac{2\pi}{3}\right) \\ v_c(t) = \sqrt{2}V_+ \sin\left(\omega t + \phi_{V+} + \dfrac{2\pi}{3}\right) \end{cases} \tag{2.36}$$

$$\begin{cases} i_a(t) = \sqrt{2}I_+ \sin(\omega t + \phi_{I+}) \\ i_b(t) = \sqrt{2}I_+ \sin\left(\omega t + \phi_{I+} - \dfrac{2\pi}{3}\right) \\ i_c(t) = \sqrt{2}I_+ \sin\left(\omega t + \phi_{I+} + \dfrac{2\pi}{3}\right) \end{cases} \tag{2.37}$$

The symbols $V_+$, $I_+$, $\phi_{V+}$, and $\phi_{I+}$ with subscript "+" are used to emphasize the nature of the positive-sequence component.

For a three-phase system with or without a neutral conductor, the three-phase instantaneous active power $p_{3\phi}(t)$ describes the total instantaneous energy flow per time unit being transferred between two subsystems, and it is given by

$$p_{3\phi}(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) = p_a(t) + p_b(t) + p_c(t) \tag{2.38}$$

Substituting (2.36) and (2.37) in (2.38) results in

$$\begin{aligned} p_{3\phi}(t) = V_+ I_+ \Big[ &\cos(\phi_{V+} - \phi_{I+}) - \cos(2\omega t + \phi_{V+} + \phi_{I+}) + \\ &+ \cos(\phi_{V+} - \phi_{I+}) - \cos\left(2\omega t + \phi_{V+} + \phi_{I+} + \frac{2\pi}{3}\right) + \\ &+ \cos(\phi_{V+} - \phi_{I+}) - \cos\left(2\omega t + \phi_{V+} + \phi_{I+} - \frac{2\pi}{3}\right) \Big] \end{aligned} \tag{2.39}$$

$$p_{3\phi}(t) = 3V_+ I_+ \cos(\phi_{V+} - \phi_{I+}) = 3P$$

The sum of the three time-dependent terms in (2.39) is always equal to zero. Hence, the instantaneous active three-phase power $p_{3\phi}(t)$ is constant, that is, it is time-independent. In contrast, the single-phase power defined in (2.2) contains a time dependent term. This term was decomposed into two terms in (2.3), from which the active power $P$ and the reactive power $Q$ were defined. Here, the three-phase instantaneous active power is constant and equal to $3P$ (three times the single-phase active power). For consistency with the definitions in single-phase systems, the three-phase active (average) power $P_{3\phi}$, is defined as

$$P_{3\phi} = 3P = 3V_+ I_+ \cos(\phi_{V+} - \phi_{I+}) \tag{2.40}$$

For three-phase *balanced* systems, the three-phase apparent power can be defined from the voltage and current phasors, similar to the definition of a single-phase system under sinusoidal conditions. Because the three-phase circuit is balanced, the idea that a "three-phase circuit can be considered as three single-phase circuits" may be adopted. In this case, the idea, "what happens in a given phase also will happen in the next phase, 1/3 of the fundamental period later, and so on" is also used. Hence, the three-phase apparent power $S_{3\phi}$ is defined as

$$S_{3\phi} = 3S = 3V_+ I_+ \tag{2.41}$$

Furthermore, a three-phase complex power $\mathbf{S_{3\phi}}$ is defined as three times the single-phase complex power defined in (2.12). A definition of three-phase reactive power $Q_{3\phi}$ can be derived from the imaginary part of the definition of this three-phase complex power as follows.

**Three-phase complex power $\mathbf{S_{3\phi}}$:**

$$\begin{aligned} \mathbf{S_{3\phi}} &= 3\dot{V}_+ \dot{I}_+^* = 3V_+ \angle\, \phi_{V+}\, I_+ \angle -\phi_{I+} \\ &= \underbrace{3V_+ I_+ \cos(\phi_{V+} - \phi_{I+})}_{P_{3\phi}} + j\underbrace{3V_+ I_+ \sin(\phi_{V+} - \phi_{I+})}_{Q_{3\phi}} \end{aligned} \tag{2.42}$$

**Three-phase reactive power $Q_{3\phi}$:**

$$Q_{3\phi} = 3Q = 3V_+I_+ \sin(\phi_{V+} - \phi_{I+}) \tag{2.43}$$

The reactive power $Q_{3\phi}$ in (2.42) or in (2.43) is just a mathematical definition without any precise physical meaning. In other words, the three-phase circuit includes no oscillating power components in the three-phase instantaneous active power $p_{3\phi}(t)$ given in (2.39), in contrast to the single-phase instantaneous active power given in (2.3). Thus, there is no portion of power that can be related to a unidirectional oscillating power from which the active (average) power is defined by $P = VI \cos \phi$, or an "oscillating power component that does not realize work," defined by $Q = VI \sin \phi$.

For example, a three-phase ideal generator supplying a balanced, ideal capacitor bank produces no mechanical torque if losses are neglected. In this case, each phase voltage phasor is orthogonal to the corresponding line current phasor, and $P_{3\phi}$ is equal to zero. A generator supplying no active power means, ideally, that there is no mechanical torque. On the other hand, a single-phase ideal generator supplying an ideal capacitor has an oscillating mechanical torque due to the presence of the oscillating portion of active power, as described in (2.3). Therefore, it is not correct to think that the three-phase reactive power represents an oscillating energy between the source and the load if all the three phases of the system are considered together, not as three single-phase circuits, but as a three-phase circuit. This is the reason why a three-phase system should not be considered as a sum of three separate single-phase systems. Nevertheless, the above single-phase approach to power definitions is useful in terms of simplifying the analysis of power systems that can be approximated to balanced and sinusoidal models.

### 2.5.3. Power in Three-Phase Unbalanced Systems

The traditional concepts of apparent power and reactive power are in contradiction if applied to unbalanced and/or distorted three-phase systems. Both approaches of Budeanu in Section 2.4.1 and of Fryze in Section 2.4.2 are not adequate in unbalanced/distorted three-phase systems.

Based on rms values of voltage and current, two definitions of three-phase apparent power are commonly used by some authors [7,9,13,25,26]:

i) "Per phase" calculation:

$$S_{3\phi} = \sum_k S_k = \sum_k V_k I_k, \qquad k = (a, b, c) \tag{2.44}$$

ii) "aggregate rms value" calculation:

$$S_\Sigma = \sqrt{\sum_k V_k^2}\sqrt{\sum_k I_k^2}, \qquad k = (a, b, c) \tag{2.45}$$

Here, $V_a$, $V_b$, $V_c$ and $I_a$, $I_b$, $I_c$ are the rms values of the phase voltages (line-to-neutral voltages) and line currents, as calculated in (2.16).

It is possible to demonstrate that for a balanced and sinusoidal case, the apparent powers from (2.44) and (2.45) are equivalent. However, under nonsinusoidal or unbalanced conditions, the result always holds that $S_\Sigma \le S_{3\phi}$. The powers $S_\Sigma$ and $S_{3\phi}$ are mathematical definitions without any clear physical meaning [11,16]. However, some authors use one of them to have the sense of "maximum reachable active power at unity power factor" [7].

In (2.45), the concept of *aggregate voltage and current* is used. Schering [16] pointed out that Buchholz [2][25] introduced the concept of *aggregate* voltage and current ("Kollektivstrom" and "Kollektivspannung" in the original work written in German) in 1919 to define an apparent power for generic loads. The (rms) aggregate voltage and current are given by

$$V_\Sigma = \sqrt{V_a^2 + V_b^2 + V_c^2} \qquad \text{and} \qquad I_\Sigma = \sqrt{I_a^2 + I_b^2 + I_c^2} \tag{2.46}$$

The aggregate voltage in (2.46), which is calculated from the rms values of the phase voltages $V_a$, $V_b$, and $V_c$, gives the rms value of the line voltage if the system is sinusoidal and balanced. The same situation happens with the aggregate current that is calculated from the rms values of line currents $I_a$, $I_b$, and $I_c$. Moreover, the concept of the aggregate voltage can also be useful if instantaneous values of phase voltages are used. An example follows:

$$\left.\begin{aligned} v_a(t) &= \sqrt{2}V_+ \sin(\omega t + \phi_{V+}) \\ v_b(t) &= \sqrt{2}V_+ \sin\left(\omega t + \phi_{V+} - \frac{2\pi}{3}\right) \\ v_c(t) &= \sqrt{2}V_+ \sin\left(\omega t + \phi_{V+} + \frac{2\pi}{3}\right) \end{aligned}\right\rangle \quad v_\Sigma = \sqrt{v_a^2(t) + v_b^2(t) + v_c^2(t)} = \sqrt{3}V_+ \tag{2.47}$$

Under three-phase unbalanced/distorted systems, the instantaneous aggregate voltage has an oscillating portion that superposes the dc value given in (2.47).

It is not possible to establish a consistent set of power definitions (active power, reactive power, power factor, etc.) from the apparent power given in (2.44) or (2.45). This would lead to a subsequent definition of reactive or harmonic powers that would seem to be mathematical definitions without any clear physical meaning. Therefore, the *instantaneous active power*—a universal concept—will be chosen in the next chapter as the fundamental equation for power definitions in three-phase systems.

## 2.6. SUMMARY

This chapter presented an overview of some power theories, starting from the classical references dating from the 1920s. It was interesting to show that long ago some researchers like Fryze and Budeanu had studied the problem. Most of the concepts presented here are not only important for knowing the past work but also to better understand the new concepts that will be presented in the next chapter.

Although this chapter starts with the power definitions under sinusoidal conditions that are quite common situations and well known to electrical engineers, they are necessary to create a solid base for the rest of the theory. Voltage and current phasors, as well as the resulting complex impedance, were presented just as a review of these concepts, necessary to define the complex power and power factor. Concepts of power under nonsinusoidal conditions for conventional approaches were analyzed, starting with the concepts presented by Budeanu in the frequency domain. The power tetrahedron and the distortion factor were introduced to better fix the concepts of power under nonsinusoidal conditions. Then, power definitions by Fryze in the time domain and the resulting electric power in three-phase systems were presented, after classifications of three-phase systems with imbalances and/or distortions. Power in three-phase balanced systems and power in three-phase unbalanced and distorted systems were presented, along with detailed discussions on the concept of apparent power in both systems.

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