CHAPTER 3

THE INSTANTANEOUS POWER THEORY

As discussed in the previous chapter, research on the calculation and physical interpretation of energy flow in an electric circuit dates back to the 1920s. It is amazing to find excellent early works dealing with most of the important aspects of this energy flow. However, the basic concern was related to the average power or rms value of voltage and current. Although the concept presented by Fryze [1] uses rms values for power analysis, it treats a three-phase circuit as a unit, not a superposition of three single-phase circuits. The development of power electronics devices and their associated converters has brought new boundary conditions to the energy flow problem. This is not exactly because the problem is new, but because these converters behave as nonlinear loads and represent a significant amount of power compared with other traditional linear loads. The speed response of these converters and the way they generate reactive power and harmonic components have made it clear that conventional approaches to the analysis of power are not sufficient in terms of taking average or rms values of variables. Therefore, time-domain analysis has evolved as a new manner to analyze and understand the physical nature of the energy flow in a nonlinear circuit. This chapter is dedicated to the time-domain analysis of power in a three-phase electric circuit.

The theories that deal with instantaneous power can be mainly classified into the following two groups. The first one is developed based on the transformation from the *abc* phases to three-orthogonal axes, and the other is done directly on the *abc* phases. The first one is what will be called the *p-q* Theory that is based on the *abc* to $\alpha\beta 0$ transformation. The second one has no specific name. Because it deals directly with the *abc* phases, it will be called the *abc* Theory in this book. Finally, comparisons between the two theories will be presented.

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3.1. BASIS OF THE *p-q* THEORY

The *p-q* Theory is based on a set of instantaneous powers defined in the time domain. No restrictions are imposed on the voltage or current waveforms, and it can be applied to three-phase systems with or without a neutral wire for three-phase generic voltage and current waveforms. Thus, it is valid not only in the steady state, but also in the transient state. As will be seen in the following chapters, this theory is very efficient and flexible in designing controllers for power conditioners based on power electronics devices.

Other traditional concepts of power are characterized by treating a three-phase system as three single-phase circuits. The *p-q* Theory first transforms voltages and currents from the *abc* to $\alpha\beta0$ coordinates, and then defines instantaneous power on these coordinates. Hence, this theory always considers the three-phase system as a unit, not a superposition or sum of three single-phase circuits.

3.1.1. Historical Background of the *p-q* Theory

The *p-q* Theory in its first version was published in the Japanese language in July 1982 in a local conference and later in the journal *Transactions of the IEE-Japan* [3]. With a minor time lag it was published in 1983 in an international conference [4], and, in 1984, in the *IEEE Transactions on Industry Applications,* including experimental verification [5]. The development of this theory was based on various previous works written by power electronics specialists interested in reactive-power compensation. From the end of 1960s to the beginning of 1970s, some papers related to what can be considered as a basic principle of reactive-power compensation were published [6,7,8]. The authors of [6] presented some basic ideas like ". . . compensation of distortive power are unknown to date. . . ." They also assured that "a nonlinear resistor behaves like a reactive-power generator while having no energy-storing elements," and presented the very first approach to power-factor correction. Fukao and his coauthors in [8], the authors said, ". . . by connecting a reactive-power source in parallel with the load and by controlling it in such a way as to supply reactive power to the load, the power network will only supply active power. Therefore, an ideal power transmission would be possible."

Gyugyi and Pelly in [9] presented the idea that reactive power could be compensated by a naturally commuted cycloconverter without energy storage elements. Generation of reactive power without energy storage elements was also investigated in [10]. This idea was explained from a physical point of view, but no specific mathematical proof was presented. In 1976, Harashima and his coauthors presented in [11], probably for the first time, the term "instantaneous reactive power" for a single-phase circuit. In that same year, Gyugyi and Strycula [12] used the term "active ac power filters" for the first time. In 1981, Takahashi and his coauthors in [13] and [14] gave a hint to the emergence of the *p-q* Theory. The formulation they reached is in fact a subset of the *p-q* Theory. However, no physical meaning of the variables introduced in the two papers was explained. This theory will be explained in the next sections.

The *p-q* Theory uses the $\alpha\beta 0$ transformation, also known as the Clarke transformation [15], which consists of a real matrix that transforms three-phase voltages and currents into the $\alpha\beta 0$ stationary reference frames. Therefore, the presentation of the *p-q* Theory will start with this transformation, followed by the theory itself, its physical meanings and interpretations, and comparisons with conventional power theories.

3.1.2. The Clarke Transformation

The $\alpha\beta 0$ transformation or the Clarke transformation [15] maps the three-phase instantaneous voltages in the *abc* phases, v_a, v_b, and v_c, into the instantaneous voltages on the $\alpha\beta 0$-axes v_α, v_β, and v_0. The Clarke Transformation and its inverse transformation of three-phase generic voltages are given by

$$\begin{bmatrix} v_0 \\ \hline v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \left[\begin{array}{ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \hline 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{array}\right] \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{3.1}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \left[\begin{array}{c|cc} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{array}\right] \begin{bmatrix} v_0 \\ \hline v_\alpha \\ v_\beta \end{bmatrix} \tag{3.2}$$

Similarly, three-phase generic instantaneous line currents, i_a, i_b, and i_c, can be transformed on the $\alpha\beta 0$ axes by

$$\begin{bmatrix} i_0 \\ \hline i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \left[\begin{array}{ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \hline 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{array}\right] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{3.3}$$

and its inverse transformation is

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \left[\begin{array}{c:cc} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{array}\right] \left[\begin{array}{c} i_0 \\ \hdashline i_\alpha \\ i_\beta \end{array}\right] \tag{3.4}$$

One advantage of applying the $\alpha\beta 0$ transformation is to separate zero-sequence components from the abc-phase components. The α and β axes make no contribution to zero-sequence components. No zero-sequence current exists in a three-phase, three-wire system, so that i_0 can be eliminated from the above equations, thus resulting in simplification. If the three-phase voltages are balanced in a four-wire system, no zero-sequence voltage is present, so that v_0 can be eliminated. However, when zero-sequence voltage and current components are present, the complete transformation has to be considered.

If v_0 can be eliminated from the transformation matrixes, the Clarke transformation and its inverse transformation become

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{3.5}$$

and

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \tag{3.6}$$

Similar equations hold in the line currents.

The real matrices in (3.5) and (3.6) suggest an axis transformation as shown in Fig. 3-1. They are stationary axes and should not be confused with the concepts of voltage or current phasors, like those illustrated in Chapter 2 (Fig. 2-2). Here, instantaneous values of phase voltages and line currents referred to the abc stationary axes are transformed into the $\alpha\beta$ stationary axes, or vice-versa. The a, b, and c axes are spatially shifted by $2\pi/3$ rad from each other while the α and β axes are orthogonal, and the α axis is parallel to the a axis. The direction of the β axis was chosen in such a way that if voltage or current spatial vectors on the abc coordinates rotate

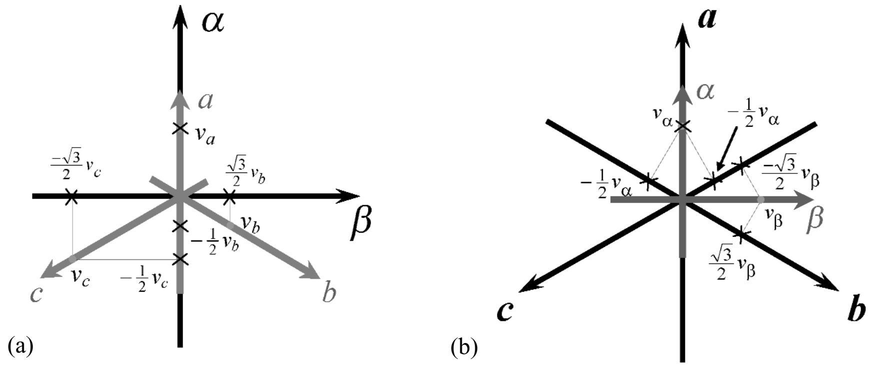


Figure 3-1. Graphical representations. (a) The *abc* to $\alpha\beta$ transformation (Clarke transformation). (b) Inverse $\alpha\beta$ to *abc* transformation (inverse Clarke transformation).

in the *abc* sequence, they would rotate in the $\alpha\beta$ sequence on the $\alpha\beta$ coordinates. This idea is well explained in the following section.

3.1.2.A. Calculation of Voltage and Current Vectors When Zero-Sequence Components are Excluded

If v_0 can be neglected, an instantaneous voltage vector is defined from the instantaneous α- and β-voltage components, that is,

$$\mathbf{e} = v_\alpha + jv_\beta \tag{3.7}$$

Similarly, if i_0 can be neglected, the instantaneous current vector is defined as

$$\mathbf{i} = i_\alpha + ji_\beta \tag{3.8}$$

The above instantaneous vectors can be represented in a complex plane, where the real axis is the α axis, and the imaginary axis is the β axis of the Clarke transformation. It should be noted that the vectors defined above are functions of time, because they consist of the Clarke components of the instantaneous phase voltages and line currents in a three-phase system. Therefore, they should not be misinterpreted as phasors. The usefulness of this vector definition is illustrated in the following example. Later, it will be used also to define a new concept of instantaneous complex apparent power.

Consider the following sinusoidal balanced phase voltages and line currents of a three-phase linear circuit.

$$\begin{cases} v_a(t) = \sqrt{2}V\cos(\omega t + \phi_V) \\ v_b(t) = \sqrt{2}V\cos\left(\omega t + \phi_V - \dfrac{2\pi}{3}\right) \\ v_c(t) = \sqrt{2}V\cos\left(\omega t + \phi_V + \dfrac{2\pi}{3}\right) \end{cases} \qquad \begin{cases} i_a(t) = \sqrt{2}I\cos(\omega t + \phi_I) \\ i_b(t) = \sqrt{2}I\cos\left(\omega t + \phi_I - \dfrac{2\pi}{3}\right) \\ i_c(t) = \sqrt{2}I\cos\left(\omega t + \phi_I + \dfrac{2\pi}{3}\right) \end{cases} \tag{3.9}$$

The angles ϕ_V and ϕ_I are the voltage and current phases, respectively, with respect to a given reference.

The above voltages and currents consist of a single symmetrical component in the fundamental positive sequence (see Fig. 2-7). Thus, they are sinusoidal and balanced. These voltages and currents can be transformed to the $\alpha\beta$ reference frames by using (3.5) and its similar current equations. The above three-phase voltages and currents transformed into the α–β reference frames are given by:

$$\begin{cases} v_\alpha = \sqrt{3}V\cos(\omega t + \phi_V) \\ v_\beta = \sqrt{3}V\sin(\omega t + \phi_V) \end{cases} \quad \text{and} \quad \begin{cases} i_\alpha = \sqrt{3}I\cos(\omega t + \phi_I) \\ i_\beta = \sqrt{3}I\sin(\omega t + \phi_I) \end{cases}$$

Now, a voltage vector **e** and current vector **i** are derived as follows:

$$\mathbf{e} = v_\alpha + jv_\beta \Rightarrow \left| \begin{array}{l} \mathbf{e} = \sqrt{3}V[\cos(\omega t + \phi_V) + j\sin(\omega t + \phi_V)] \\ \mathbf{e} = \sqrt{3}V\, e^{j(\omega t + \phi_V)} \end{array} \right. \tag{3.10}$$

and

$$\mathbf{i} = i_\alpha + ji_\beta \Rightarrow \left| \begin{array}{l} \mathbf{i} = \sqrt{3}I[\cos(\omega t + \phi_I) + j\sin(\omega t + \phi_I)] \\ \mathbf{i} = \sqrt{3}I\, e^{j(\omega t + \phi_I)} \end{array} \right. \tag{3.11}$$

Therefore, in the case of a three-phase balanced sinusoidal system the voltage and current vectors have constant amplitudes and rotate in the clockwise direction, or in the $\alpha \rightarrow \beta$ sequence, at the angular frequency ω, as suggested in Fig. 3-2.

The same result is found if the instantaneous *abc* voltages and currents are taken directly to compose vectors on a complex plane. To confirm the above result obtained by using the Clarke Transformation, the relative positions of the *a*, *b*, and *c* axes and the α and β axes must be kept as suggested in Fig. 3-1. The *a* axis must be coincident with the α axis that is the real axis of the complex plane. The β axis is the imaginary axis shifted by $\pi/2$ from the real α axis. Each time function of the *abc*-phase voltages is multiplied by a proper unitary complex phase shifter to lay in the

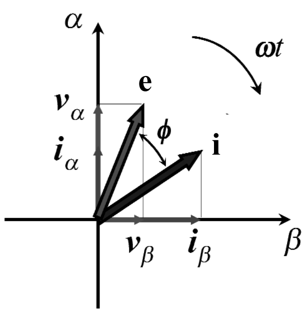


Figure 3-2. Vector representation of voltages and currents on the α-β reference frames.

corresponding axis direction. Thus, the complex voltage vector can be composed by

$$\mathbf{e}_{abc} = v_a e^{j0} + v_b e^{j2\pi/3} + v_c e^{-j2\pi/3} \tag{3.12}$$

By replacing the time functions for v_a, v_b, and v_c given in (3.9) and making some manipulation, the following equation is obtained:

$$\mathbf{e}_{abc} = \frac{3\sqrt{2}}{2} V[\cos(\omega t + \phi_V) + j\sin(\omega t + \phi_V)] = \frac{3\sqrt{2}}{2} V\, e^{j(\omega t + \phi_V)} \tag{3.13}$$

Note that the voltage vector $\mathbf{e}_{abc}$ in (3.13) has a constant amplitude, and rotates in the clockwise direction at the angular frequency ω, like $\mathbf{e}$ defined in (3.10). Therefore, if $\mathbf{e}$ turns clockwise, in the $\alpha \rightarrow \beta$ sequence, at the synchronous angular speed, $\mathbf{e}_{abc}$ does the same, passing sequentially through $a \rightarrow b \rightarrow c$ axes at the same angular speed. By comparison, the following relation can be written:

$$\mathbf{e} = \sqrt{\frac{2}{3}}\, \mathbf{e}_{abc} \tag{3.14}$$

The vector representation of three-phase instantaneous voltages and currents has been used increasingly in the field of power electronics. For instance, it has been used in vector control of ac motor drives, in space-vector pulse-width-modulation (PWM) of power converters, as well as in control of power conditioners.

3.1.3. Three-Phase Instantaneous Active Power in Terms of Clarke Components

The Clarke transformation and its inverse transformation as used in (3.1) to (3.4) have the property of being invariant in power. This feature is very suitable when the focus is put on the analysis of instantaneous power in three-phase systems.

All traditional power definitions summarized in the last chapter require as a precondition that the system be in the steady state. The three-phase instantaneous active power has a clear and universally accepted physical meaning, and is also valid during transient states.

> For a three-phase system with or without a neutral conductor in the steady state or during transients, the three-phase instantaneous active power $p_{3\phi}(t)$ describes the total instantaneous energy flow per second between two subsystems.

The three-phase instantaneous active power, $p_{3\phi}(t)$, is calculated from the instantaneous phase voltages and line currents as

$$\begin{gathered} p_{3\phi}(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) \\ \Updownarrow \\ p_{3\phi} = v_a i_a + v_b i_b + v_c i_c \end{gathered} \tag{3.15}$$

where v_a, v_b, and v_c are the instantaneous phase voltages and i_a, i_b, and i_c the instantaneous line currents, as shown in Fig. 3-3. In a system without a neutral wire, v_a, v_b, and v_c are measured from a common point of reference. Sometimes, it is called the "ground" or "fictitious star point." However, this reference point can be set arbitrarily and $p_{3\phi}$, calculated from (3.15), always results in the same value for all arbitrarily chosen reference points for voltage measurement. For instance, if the b phase is chosen as a reference point, the measured "phase voltages" and the three-phase instantaneous active power, $p_{3\phi}$, are calculated as

$$p_{3\phi} = (v_a - v_b)i_a + (v_b - v_b)i_b + (v_c - v_b)i_c = v_{ab}i_a + v_{cb}i_c \tag{3.16}$$

This explains why it is possible to use $(n - 1)$ wattmeters to measure the active power in n-wire systems.

The three-phase instantaneous active power can be calculated in terms of the $\alpha\beta 0$ components if (3.2) and (3.4) are used to replace the abc variables in (3.15).

$$p_{3\phi} = v_a i_a + v_b i_b + v_c i_c \Leftrightarrow p_{3\phi} = v_\alpha i_\alpha + v_\beta i_\beta + v_0 v_0 \tag{3.17}$$

The property of power invariance as a result of using the Clarke transformation is shown in (3.17). The p-q Theory will exploit this feature.

3.1.4. The Instantaneous Powers of the *p-q* Theory

The p-q Theory is defined in three-phase systems with or without a neutral conductor. Three instantaneous powers—the instantaneous zero-sequence power p_0, the instantaneous real power p, and the instantaneous imaginary power q—are defined from the instantaneous phase voltages and line currents on the $\alpha\beta 0$ axes as

$$\begin{bmatrix} p_0 \\ \hdashline p \\ q \end{bmatrix} = \left[\begin{array}{c:cc} v_0 & 0 & 0 \\ \hdashline 0 & v_\alpha & v_\beta \\ 0 & v_\beta & -v_\alpha \end{array}\right] \begin{bmatrix} i_0 \\ \hdashline i_\alpha \\ i_\beta \end{bmatrix} \tag{3.18}$$

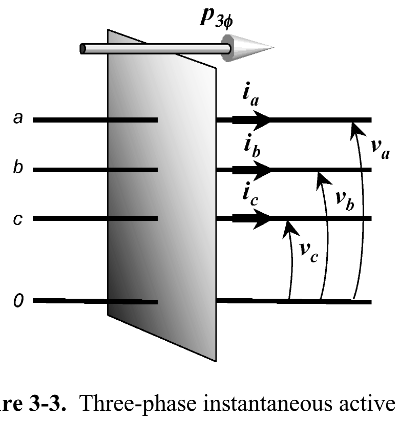


Figure 3-3. Three-phase instantaneous active power.

There are no zero-sequence current components in three-phase, three-wire systems, that is, $i_0 = 0$. In this case, only the instantaneous powers defined on the $\alpha\beta$ axes exist, because the product $v_0 i_0$ in (3.17) is always zero. Hence, in three-phase, three-wire systems, the instantaneous real power p represents the total energy flow per time unity in terms of $\alpha\beta$ components. In this case, $p_{3\phi} = p$.

The instantaneous imaginary power q has a nontraditional physical meaning that will be explained later in the next section.

3.2. THE *p-q* THEORY IN THREE-PHASE, THREE-WIRE SYSTEMS

Another way to introduce the p-q Theory for three-phase, three-wire systems is to use the instantaneous voltage and current vectors defined in (3.7) and (3.8). The conventional concept of the complex power defined in (2.12) uses a voltage phasor and the conjugate of a current phasor. Thus, it is valid only for a system in the steady state with a fixed line frequency. A new definition of *instantaneous complex power* is possible, using the instantaneous vectors of voltage and current. The instantaneous complex power **s** is defined as the product of the voltage vector **e** and the conjugate of the current vector **i***, given in the form of complex numbers:

$$\mathbf{s} = \mathbf{e} \cdot \mathbf{i}^* = (v_\alpha + jv_\beta)(i_\alpha - ji_\beta) = \underbrace{(v_\alpha i_\alpha + v_\beta i_\beta)}_{p} + j\underbrace{(v_\beta i_\alpha - v_\alpha i_\beta)}_{q} \tag{3.19}$$

The instantaneous real and imaginary powers defined in (3.18) are part of the instantaneous complex power, **s**, defined in (3.19). Since instantaneous voltages and currents are used, there are no restrictions in **s**, and it can be applied during steady states or during transients.

The original definition of p and q in [2]–[5] was based on the following equation:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{3.20}$$

Equations (3.18) and (3.20) are applicable to the analysis and design in the same manner. However, this book adopts (3.18), because a positive value of the instantaneous imaginary power q in (3.18) corresponds to the product of a positive-sequence voltage and a lagging (inductive) positive-sequence current, in agreement with the conventional concept of reactive power.

In the following explanation, the $\alpha\beta$ currents will be set as functions of voltages and the real and imaginary powers p and q. This is very suitable for better explaining the physical meaning of the powers defined in the p-q Theory. From (3.18), it is possible to write

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \tag{3.21}$$

The right-hand side of (3.21) can have its terms expanded as

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} p \\ 0 \end{bmatrix} + \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} 0 \\ q \end{bmatrix} \triangleq \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \tag{3.22}$$

The above current components can be defined as shown below.

Instantaneous active current on the α axis $i_{\alpha p}$:

$$i_{\alpha p} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} p \tag{3.23}$$

Instantaneous reactive current on the α axis $i_{\alpha q}$:

$$i_{\alpha q} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} q \tag{3.24}$$

Instantaneous active current on the β axis $i_{\beta p}$:

$$i_{\beta p} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} p \tag{3.25}$$

Instantaneous reactive current on the β axis $i_{\beta q}$:

$$i_{\beta q} = \frac{-v_\alpha}{v_\alpha^2 + v_\beta^2} q \tag{3.26}$$

The instantaneous power on the α and β coordinates are defined as p_α and p_β, respectively, and are calculated from the instantaneous voltages and currents on the $\alpha\beta$ axes as follows:

$$\begin{bmatrix} p_\alpha \\ p_\beta \end{bmatrix} = \begin{bmatrix} v_\alpha i_\alpha \\ v_\beta i_\beta \end{bmatrix} = \begin{bmatrix} v_\alpha i_{\alpha p} \\ v_\beta i_{\beta p} \end{bmatrix} + \begin{bmatrix} v_\alpha i_{\alpha q} \\ v_\beta i_{\beta q} \end{bmatrix} \tag{3.27}$$

Note that, in the three-phase, three-wire system, the three-phase instantaneous active power in terms of Clarke components in (3.17) is equal to the instantaneous real power defined in (3.18). From (3.27) and (3.18), the real power can be given by the sum of p_α and p_β. Therefore, rewriting this sum by using (3.27) yields the following equation:

$$\begin{aligned} p &= v_\alpha i_{\alpha p} + v_\beta i_{\beta p} + v_\alpha i_{\alpha q} + v_\beta i_{\beta q} \\ &= \frac{v_\alpha^2}{v_\alpha^2 + v_\beta^2} p + \frac{v_\beta^2}{v_\alpha^2 + v_\beta^2} p + \frac{v_\alpha v_\beta}{v_\alpha^2 + v_\beta^2} q + \frac{-v_\alpha v_\beta}{v_\alpha^2 + v_\beta^2} q \end{aligned} \tag{3.28}$$

In the above equation, there are two important points. One is that the instantaneous real power p is given only by

$$v_\alpha i_{\alpha p} + v_\beta i_{\beta p} = p_{\alpha p} + p_{\beta p} = p \tag{3.29}$$

The other is that the following relation exists for the terms dependent on q

$$v_\alpha i_{\alpha q} + v_\beta i_{\beta q} = p_{\alpha q} + p_{\beta q} = 0 \tag{3.30}$$

The above equations suggest the separation of the powers in the following types.

Instantaneous active power on the α axis $p_{\alpha p}$

$$p_{\alpha p} = v_\alpha \cdot i_{\alpha p} = \frac{v_\alpha^2}{v_\alpha^2 + v_\beta^2} p \tag{3.31}$$

Instantaneous reactive power on the α axis $p_{\alpha q}$

$$p_{\alpha q} = v_\alpha \cdot i_{\alpha q} = \frac{v_\alpha v_\beta}{v_\alpha^2 + v_\beta^2} q \tag{3.32}$$

Instantaneous active power on the β axis $p_{\beta p}$

$$p_{\beta p} = v_\beta \cdot i_{\beta p} = \frac{v_\beta^2}{v_\alpha^2 + v_\beta^2} p \tag{3.33}$$

Instantaneous reactive power on the β axis $p_{\beta q}$

$$p_{\beta q} = v_\beta \cdot i_{\beta q} = \frac{-v_\alpha v_\beta}{v_\alpha^2 + v_\beta^2} q \tag{3.34}$$

It should be noted that the watt [W] can be used as the unit of all the powers, $p_{\alpha p}$, $p_{\alpha q}$, $p_{\beta p}$, and $p_{\beta q}$, because each power is defined by the product of the instantaneous voltage on one axis and a part of the instantaneous current on the same axis.

The above equations lead to the following important conclusions:

- The instantaneous current i_α is divided into the instantaneous active component $i_{\alpha p}$ and the instantaneous reactive component $i_{\alpha q}$ as shown in (3.23) and (3.24). This same division is made for the currents on the β axis.
- The sum of the α axis instantaneous active power $p_{\alpha p}$, given in (3.31), and the β axis instantaneous active power $p_{\beta p}$, given in (3.33), corresponds to the instantaneous real power p.
- The sum of $p_{\alpha q}$ and $p_{\beta q}$ is always zero. Therefore, they neither make a contribution to the instantaneous nor average energy flow between the source and the load in a three-phase circuit. This is the reason that they were named instantaneous *reactive* power on the α and β axes. The instantaneous imaginary power q is a quantity that gives the magnitude of the powers $p_{\alpha q}$ and $p_{\beta q}$.
- Because the sum of $p_{\alpha q}$ and $p_{\beta q}$ is always zero, their compensation does not need any energy storage system, as will be shown later.

If the $\alpha\beta$ variables of the instantaneous imaginary power q as defined in (3.18) are replaced by their equivalent expressions referred to the abc axes using (3.5) and similarly for the currents, the following relation can be found:

$$\begin{aligned} q = v_\beta i_\alpha - v_\alpha i_\beta &= \frac{1}{\sqrt{3}}[(v_a - v_b)i_c + (v_b - v_c)i_a + (v_c - v_a)i_b] \\ &= \frac{1}{\sqrt{3}}(v_{ab}i_c + v_{bc}i_a + v_{ca}i_b) \end{aligned} \tag{3.35}$$

Note that q, on the $\alpha\beta$ reference frames, is defined as the sum of products of voltages and currents on different axes. Likewise, the imaginary power q, when calculated directly from the abc phase voltages and line currents, results from the sum of products of line voltages and line currents in different phases. This expression is similar to that implemented in some instruments for measuring the three-phase reactive power. The difference is that voltage and current phasors are used in those instruments. Here, instantaneous values of voltage and current are used instead. As was shown, the imaginary power q does not contribute to the total energy flow between the source and the load, and vice-versa. The imaginary power q is a new quantity, and needs a unit to distinguish this power from the traditional reactive power. The authors propose the use of the unit "Volt-Ampere Imaginary" and the symbol "vai," making an analogy to the symbol "var" of the traditional unit "Volt-Ampere Reactive."

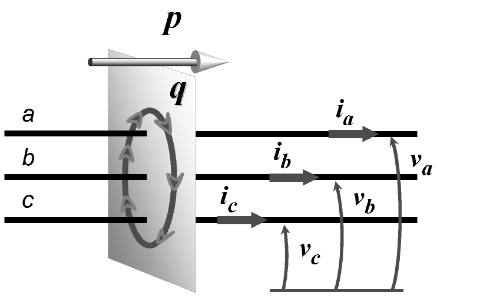
From now on, whenever no doubt can arise, the *instantaneous* zero-sequence power, the *instantaneous* imaginary power, and the *instantaneous* real power defined in the *p-q* Theory will be called *zero-sequence power, imaginary power,* and *real power,* respectively.

At this point, the following important remark can be written.

The imaginary power q is proportional to the quantity of energy that is being exchanged between the phases of the system. It does not contribute to the energy transfer* between the source and the load at any time.

Figure 3-4 summarizes the above explanations about the real and imaginary power. It is important to note that the conventional power theory defined reactive power as a component of the instantaneous (active) power, which has an *average* value equal to zero. Here, it is not so. The imaginary power means a sum of products of *instantaneous* three-phase voltage and current portions that do not contribute to the energy transfer between two subsystems at any time. In this book, the term "*instantaneous reactive power*" in a three-phase system is used as a synonym of the imaginary power. Therefore, they have the same physical meaning.

*The term "energy transfer" is used here in a general manner, meaning not only the energy delivered to the load, but also the energy oscillating between the source and the load.



p : instantaneous total energy flow per time unit;
q : energy exchanged between the phases without transferring energy.

Figure 3-4. Physical meaning of the instantaneous real and imaginary powers.

3.2.1. Comparisons with the Conventional Theory

To better understand the meaning of p and q, some examples will be presented. The first one is for a linear circuit with sinusoidal voltages and currents, and the others are for sinusoidal voltages supplying nonlinear loads.

3.2.1.A. Example #1—Sinusoidal Voltages and Currents

Suppose a three-phase ideal voltage source supplying power to a three-phase balanced impedance. The phase voltages and line currents can be expressed as

$$\begin{cases} v_a(t) = \sqrt{2}V\sin(\omega t) \\ v_b(t) = \sqrt{2}V\sin\left(\omega t - \dfrac{2\pi}{3}\right) \\ v_c(t) = \sqrt{2}V\sin\left(\omega t + \dfrac{2\pi}{3}\right) \end{cases} \quad \text{and} \quad \begin{cases} i_a(t) = \sqrt{2}I\sin(\omega t + \phi) \\ i_b(t) = \sqrt{2}I\sin\left(\omega t - \dfrac{2\pi}{3} + \phi\right) \\ i_c(t) = \sqrt{2}I\sin\left(\omega t + \dfrac{2\pi}{3} + \phi\right) \end{cases} \tag{3.36}$$

The $\alpha\beta$ transformation of the above voltages and currents are:

$$\begin{cases} v_\alpha = \sqrt{3}V\sin(\omega t) \\ v_\beta = -\sqrt{3}V\cos(\omega t) \end{cases} \quad \text{and} \quad \begin{cases} i_\alpha = \sqrt{3}I\sin(\omega t + \phi) \\ i_\beta = -\sqrt{3}I\cos(\omega t + \phi) \end{cases} \tag{3.37}$$

The above two equations make it possible to calculate the real and imaginary powers:

$$\begin{cases} p = 3VI\cos\phi \\ q = -3VI\sin\phi \end{cases} \tag{3.38}$$

Both instantaneous powers are constant in this example. The real power p is equal to the conventional definition of the three-phase active power $P_{3\phi}$, whereas the imaginary power q is equal to the conventional three-phase reactive power $Q_{3\phi}$ [compare with that in (2.42)]. This example shows the correspondences between the *p-q* Theory and the conventional theory in the case of having sinusoidal balanced voltages and linear loads.

If the load has inductive characteristics, the imaginary power q has a positive value, and if the load is capacitive, its value is negative, in concordance with the most common definition of reactive power.

3.2.1.B. Example #2—Balanced Voltages and Capacitive Loads

To better explain the concepts behind the *p-q* Theory, the following two situations are examined: (i) a three-phase balanced voltage with a three-phase balanced capacitive load (capacitance C), and (ii) an unbalanced load (just one capacitor connected between two phases).

In the first case, the load is balanced under steady-state conditions. The following real and imaginary powers are obtained:

$$\begin{cases} p = 0 \\ q = -3\dfrac{V^2}{X_C} \end{cases} \tag{3.39}$$

The term X_C represents the reactance of the capacitor. As expected, in this case there is no power flowing from the source to the load. Moreover, the imaginary power is constant and coincident with the conventional three-phase reactive power.

In the second case, a capacitor (capacitance C) is connected between phases a and b. The instantaneous real and imaginary powers are given by

$$\begin{aligned} p &= \frac{3V^2}{X_C}\sin\left(2\omega t + \frac{\pi}{3}\right) \\ q &= -\frac{3V^2}{X_C}\left[1 + \cos\left(2\omega t + \frac{\pi}{3}\right)\right] \end{aligned} \tag{3.40}$$

Each power in the above equation has no constant part, and presents an oscillating part. From the conventional power theory, it would be normal to expect only a reactive power (average imaginary power) and no real power at all. However, the results are different, and should be discussed. The reason that the real power is not zero is because the capacitor terminal voltage is varying as a sinusoidal wave and, therefore, it is being charged and discharged, justifying the energy flow given by p. In fact, if it is considered that a turbine is powering the generator, it will have to produce a positive torque when the capacitor is charging, or a negative torque when it is discharging. Of course, there will be torque ripples in its shaft. In the previous example having three balanced capacitors, one capacitor is discharging while the others are charging. In steady-state conditions, there is no total (three-phase) energy flow from the source to the capacitors.

The instantaneous imaginary power also varies with time, a nonzero average value equal to that of the previous case of the balanced three-phase capacitor bank [compare with (3.39)]. It is clear from this example that under unbalanced load conditions the *p-q* Theory presents some important insights that cannot be seen with the conventional frequency-domain theory under unbalanced load conditions.

3.2.1.C. Example #3—Sinusoidal Balanced Voltage and Nonlinear Load

Now, it will be assumed that a three-phase voltage source is balanced and purely sinusoidal, as in (3.36), and the load is a thyristor rectifier operating with a firing angle equal to 30°. The commutation angle is assumed to be null and the smoothing reactor at the dc side large enough to eliminate totally the ripple in the dc current. Figure 3-5 shows the idealized circuit.

Figure 3-6(a) shows the rectifier dc output voltage waveform v_d and Fig. 3-6(b) shows the a-phase voltage and the ideal current waveforms. It is well known that this three-phase typical current waveform contains, besides the fundamental, harmonics on the order ($6n \pm 1$; $n = 1, 2, 3 \ldots$). The ($6n - 1$)th harmonics are of the negative-sequence type, whereas the ($6n + 1$)th harmonics are of the positive-sequence type (see Fig. 2-7). The line currents of a six-pulse thyristor rectifier operating with a firing angle equal to 30° can be represented as:

$$\begin{cases} i_a(t) = \sqrt{2}I_1 \sin\left(\omega t - \dfrac{\pi}{6}\right) - \sqrt{2}I_5 \sin\left(5\omega t - \dfrac{\pi}{6}\right) + \sqrt{2}I_7 \sin\left(7\omega t - \dfrac{\pi}{6}\right) - \ldots \\ i_b(t) = \sqrt{2}I_1 \sin\left(\omega t - \dfrac{2\pi}{3} - \dfrac{\pi}{6}\right) - \sqrt{2}I_5 \sin\left(5\omega t + \dfrac{2\pi}{3} - \dfrac{\pi}{6}\right) \\ \qquad + \sqrt{2}I_7 \sin\left(7\omega t - \dfrac{2\pi}{3} - \dfrac{\pi}{6}\right) - \ldots \\ i_c(t) = \sqrt{2}I_1 \sin\left(\omega t + \dfrac{2\pi}{3} - \dfrac{\pi}{6}\right) - \sqrt{2}I_5 \sin\left(5\omega t - \dfrac{2\pi}{3} - \dfrac{\pi}{6}\right) \\ \qquad + \sqrt{2}I_7 \sin\left(7\omega t + \dfrac{2\pi}{3} - \dfrac{\pi}{6}\right) - \ldots \end{cases} \tag{3.41}$$

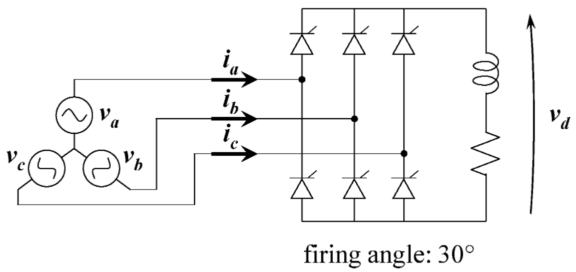


Figure 3-5. Three-phase voltage source supplying a thyristor rectifier.

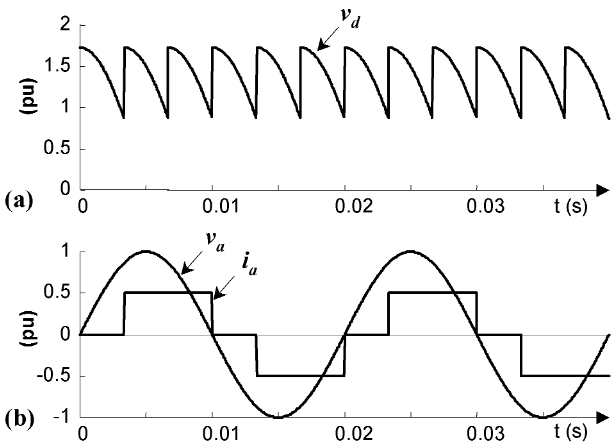


Figure 3-6. (a) Rectifier dc output voltage v_d and (b) a-phase voltage and current waveforms.

Figures 3-7(a) and (b) show the real power p and the imaginary power q, respectively. The real power was calculated using the voltages and currents at the ac side, and it is the same as if it would be calculated by the product of the dc voltage v_d and the dc current. The instantaneous input power at the ac side of the rectifier is equal to the output power at the dc side if there are no losses in the rectifier. The imaginary power is only defined for ac multiphase circuits, so it can be only calculated at the ac side.

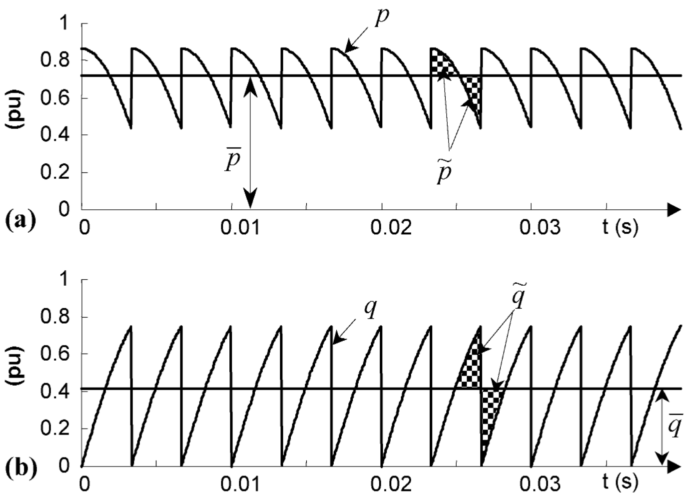


Figure 3-7. (a) Real power p and (b) imaginary power q.

These two powers have constant values and a superposition of oscillating components. Therefore, it is interesting to separate p and q into two parts:

$$\begin{array}{llcccccl} \text{Real power:} & p & = & \bar{p} & + & \tilde{p} & & \\ \text{Imaginary power:} & q & = & \bar{q} & + & \tilde{q} & & \\ & & & \text{Average powers} & & \text{Oscillating powers} & & \end{array} \tag{3.42}$$

where $\bar{p}$ and $\tilde{p}$ represent the average and oscillating parts of p, whereas $\bar{q}$ and $\tilde{q}$ represent the average and oscillating parts of q.

The real power p represents the total (three-phase) energy flow per time unity in the circuit. The average value $\bar{p}$ represents the energy flowing per time unity in one direction only. If $\bar{p}$ and $\bar{q}$ are calculated in terms of the *abc* components from (3.36) and (3.41), they result in

$$\begin{cases} \bar{p} = 3VI_1 \cos\dfrac{\pi}{6} \\ \bar{q} = 3VI_1 \sin\dfrac{\pi}{6} \end{cases} \tag{3.43}$$

Again, the average values of the real and imaginary power given by the *p-q* Theory agree with the conventional definition of three-phase active and reactive powers given in Chapter 2 (Budeanu approach). The oscillating part $\tilde{p}$ represents the oscillating energy flow per time unity, which naturally produces a zero average value, representing an amount of additional power flow in the system without effective contribution to the energy transfer from the source to the load or from the load to the source.

The imaginary power q gives the magnitude of the instantaneous reactive powers $p_{\alpha q}$ and $p_{\beta q}$, or the magnitude of the corresponding powers in the *abc* system. As explained before, although the powers $p_{\alpha q}$ and $p_{\beta q}$ exist in each axis, their sum is zero all the time. The average value of the imaginary power $\bar{q}$ corresponds to the conventional three-phase reactive power and does not contribute to energy transfer. The oscillating component of the imaginary power $\tilde{q}$ corresponds also to a power that is being exchanged among the three phases, without transferring any energy between source and load. In the present example, both oscillating real ($\tilde{p}$) and imaginary ($\tilde{q}$) powers are related to the presence of harmonics exclusively in the load currents. Later, general equations for $\bar{p}$, $\tilde{p}$, $\bar{q}$, and $\tilde{q}$ including harmonic voltage and current simultaneously will be presented.

Figure 3-8 shows the a-phase current component responsible for producing the average real power $\bar{p}$ and the average imaginary power $\bar{q}$. For reference, the voltages in this same phase are plotted together. These currents were calculated using (3.23) and (3.25), for $p = \bar{p}$, and (3.24) and (3.26), for $q = \bar{q}$, followed by the $\alpha\beta$ to *abc* transformation. The current $i_{a\bar{p}}$ is perfectly in phase with the voltage v_a, whereas $i_{a\bar{q}}$ is delayed exactly by 90°, acting like an inductive load, as is well known for a line-commutated thyristor rectifier.

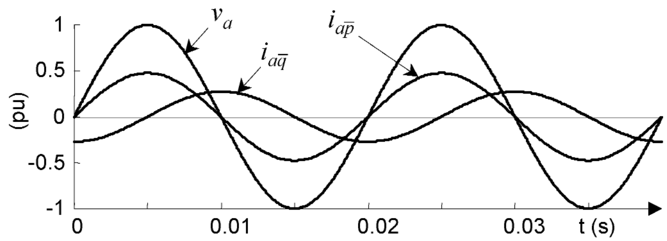


Figure 3-8. Voltage in *a* phase; currents $i_{a\bar{p}}$ and $i_{a\bar{q}}$.

Figure 3-9 (a), (b), and (c) show the *a*-phase current $i_{a\tilde{p}}$ responsible for producing the oscillating power $\tilde{p}$, the current $i_{\alpha\tilde{q}}$ that produces the oscillating power $\tilde{q}$, and the sum of $i_{a\tilde{p}}$ and $i_{a\tilde{q}}$, respectively. These currents were calculated, as in the previous case, using (3.23) and (3.25) for $p = \tilde{p}$, and (3.24) and (3.26) for $q = \tilde{q}$, followed by the $\alpha\beta$ to *abc* transformation. The sum of the four components, $i_{a\bar{p}} + i_{a\bar{q}} + i_{a\tilde{p}} + i_{a\tilde{q}}$, is equal to the original square wave current shown in Fig. 3-6 (b).

The *p-q* Theory has the prominent merit of allowing complete analysis and real-time calculation of various powers and respective currents involved in a three-phase circuit. However, this is not the main point. Knowing in real time the values of undesirable currents in a circuit allow us to eliminate them. For instance, if the oscillating powers are undesirable, by compensating the currents $i_{a\tilde{p}}$ and $i_{a\tilde{q}}$ of the load and their correspondent currents in phases *b* and *c* the compensated current drawn from the network would become sinusoidal. It can be easily shown that $i_a - (i_{a\tilde{p}} + i_{a\tilde{q}})$ produces a purely sinusoidal waveform. This is one of the basic ideas of active filtering that will be presented in detail in the next chapter.

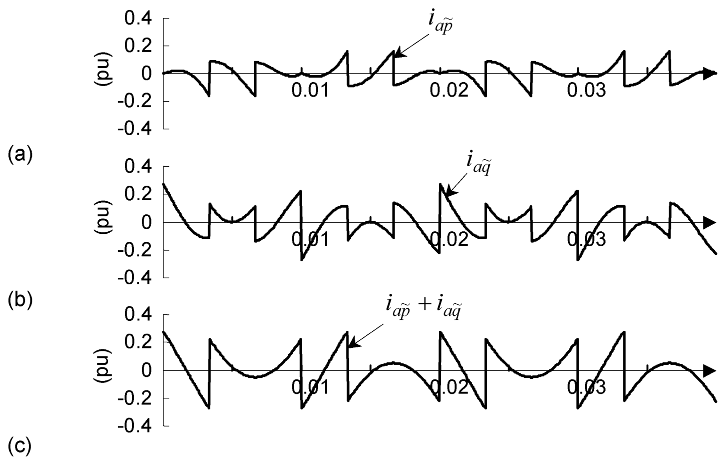


Figure 3-9. Currents $i_{a\tilde{p}}$, $i_{a\tilde{q}}$, and the sum $(i_{a\tilde{p}} + i_{a\tilde{q}})$.

3.2.2. Use of the *p-q* Theory for Shunt Current Compensation

One important application of the *p-q* Theory is the compensation of undesirable currents. Various examples of this kind of compensation will be presented in the next chapters. Figure 3-10 illustrates the basic idea of shunt current compensation. It shows a source (power generating system) supplying a nonlinear load that is being compensated by a shunt compensator. A kind of shunt compensator is the active filter that will be presented in detail in the next chapter. For the sake of simplicity, it is assumed that the shunt compensator behaves as a three-phase, controlled current source that can draw any set of arbitrarily chosen current references i^*_{Ca}, i^*_{Cb}, and i^*_{Cc}.

Figure 3-11 shows a general control method to be used in the controller of a shunt compensator. The calculated real power p of the load can be separated into its average ($\bar{p}$) and oscillating ($\tilde{p}$) parts. Likewise, the load imaginary power q can be separated into its average ($\bar{q}$) and oscillating ($\tilde{q}$) parts. Then, undesired portions of the real and imaginary powers of the load that should be compensated are selected. The powers to be compensated are represented by $-p^*_c$ and $-q^*_c$ in the controller shown in Fig. 3-11. The reason for including minus signals in the compensating powers is to emphasize that the compensator should draw a compensating current that produces exactly the inverse of the undesirable powers drawn by the nonlinear load. Note that the adopted current convention in Fig. 3-10 is such that the compensated current, that is, the source current, is the sum of the load current and the compensating current. Then, the inverse transformation from $\alpha\beta$ to abc is applied to calculate the instantaneous values of the three-phase compensating current references i^*_{Ca}, i^*_{Cb}, and i^*_{Cc}.

Figures 3-12 to 3-16 show various possible compensation results obtained from Fig. 3-10, considering the same nonlinear load as in Fig. 3-5. Each figure shows the current that should be eliminated from the load current, the compensated source

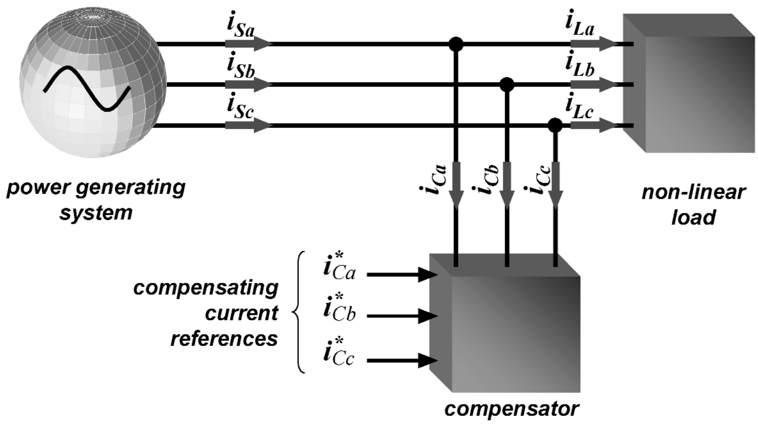


Figure 3-10. Basic principle of shunt current compensation.

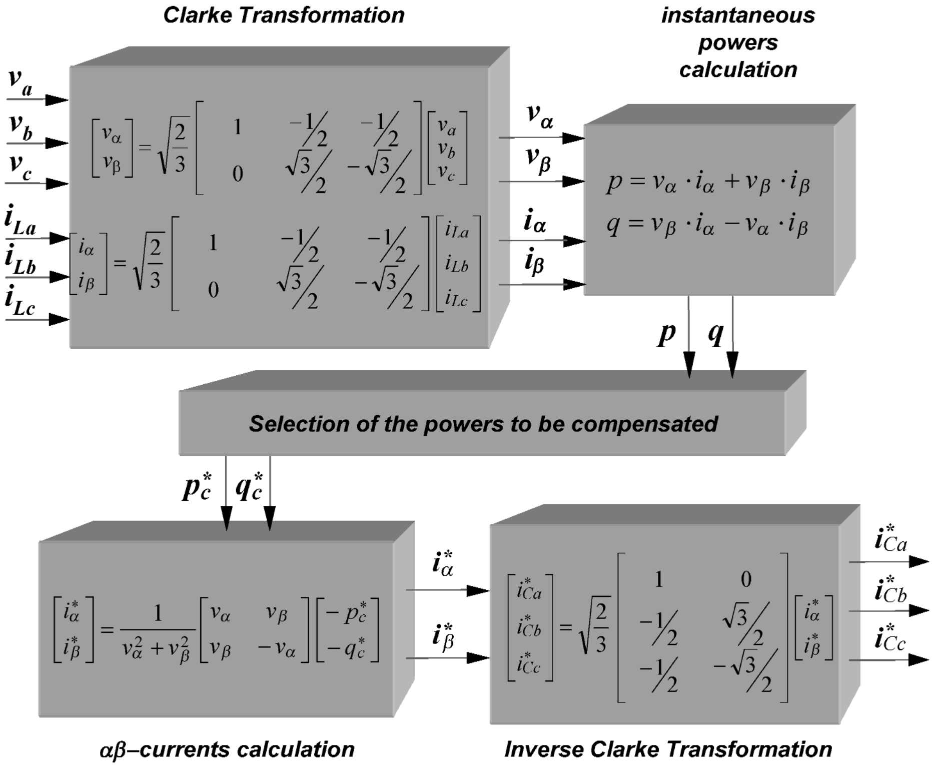


Figure 3-11. Control method for shunt current compensation based on the *p-q* Theory.

current, and p_S and q_S at the source side, along with the *a*-phase voltage. Note that the compensator must reproduce the inverse of that current to be eliminated, which is not be shown in the mentioned figures. The ideal compensated current can be calculated simply by *subtracting* the eliminated current from the load current. This result is the same if the control method shown in Fig. 3-11 is used to generate the compensating current that produces the *inverse* of the powers to be eliminated ($-p_c^*$ and $-q_c^*$). Then, this compensating current is *summed* to the load current. The powers p_S and q_S correspond to the new powers delivered from the source after compensation.

Figure 3-12 shows the compensation of the load imaginary power q. In this case, all current components that do not transfer energy, although they may produce losses in the network, are eliminated. The real power p_S, produced by the compensated current, is equal to that produced by the load current, whereas the imaginary power q_S is zero in the source. One interesting point is that the compensator acting as a controlled current sink draws a compensating current that depends only on q, so that no energy is flowing out or into this compensator. This means that, in principle, this compensator does not need any power source or energy storage system to realize

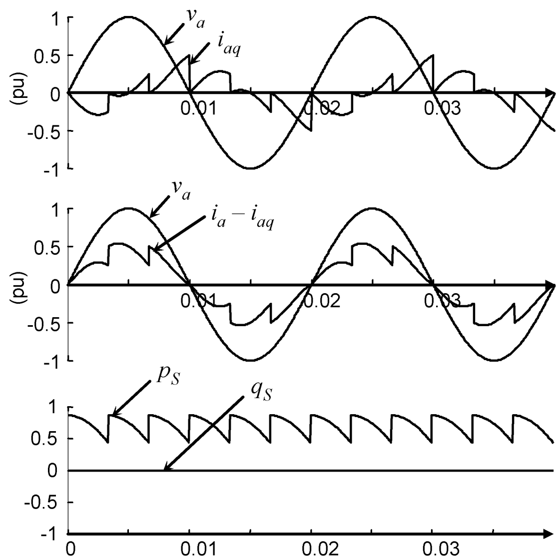


Figure 3-12. Eliminated current i_{aq}, compensated current $i_a - i_{aq}$, and the real and imaginary powers produced by the compensated current.

the compensation of q. Since the compensator is drawing currents that correspond to the load imaginary power q, including all the harmonics related to $q = \bar{q} + \tilde{q}$, this compensator must have, ideally, an infinite frequency response.

Figure 3-13 shows the same compensation as that in the previous case, except for using only the average imaginary power $\bar{q}$. In this case, the compensating current $i_{a\bar{q}}$ has no harmonic components and, therefore, the compensator draws sinusoidal currents at the line frequency. As expected, the real power p_S at the source side is equal to the real power p of the load. The imaginary power q_S has only an oscillating part as the average value ($\bar{q}$) of the load imaginary power is being compensated.

Figure 3-14 shows the case of compensation only for $\tilde{p}$. This may not be not a common situation, although it is very interesting from a theoretical point of view. Taking the current related to this power as the compensating current reference to the compensator makes constant (without ripple) the three-phase instantaneous real power that is equal to the calculated real power. The imaginary power is the same as that of the load. If the primary power source consists of a turbine generator, this kind of power compensation eliminates the torque ripple in the rotor axis, thus resulting in producing no undesirable shaft vibration. It is interesting to note that although the compensated current contains significant harmonic content, these harmonics do not influence the real power. The compensator has to supply exactly the oscillating real power $\tilde{p}$ that is being eliminated from the power source. Therefore, it must have the capability to supply and absorb energy, but with zero average value. Hence, this compensator must be coupled with an energy storage system.

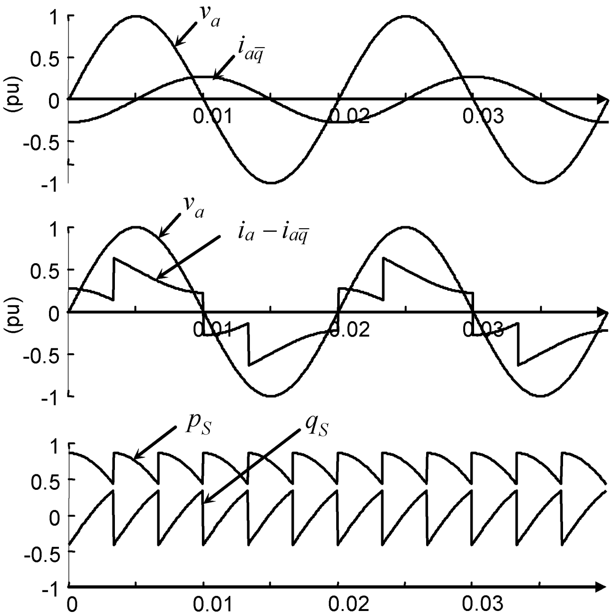


Figure 3-13. Eliminated current $i_{a\bar{q}}$, compensated current $i_a - i_{a\bar{q}}$, and the real and imaginary powers produced by the compensated current.

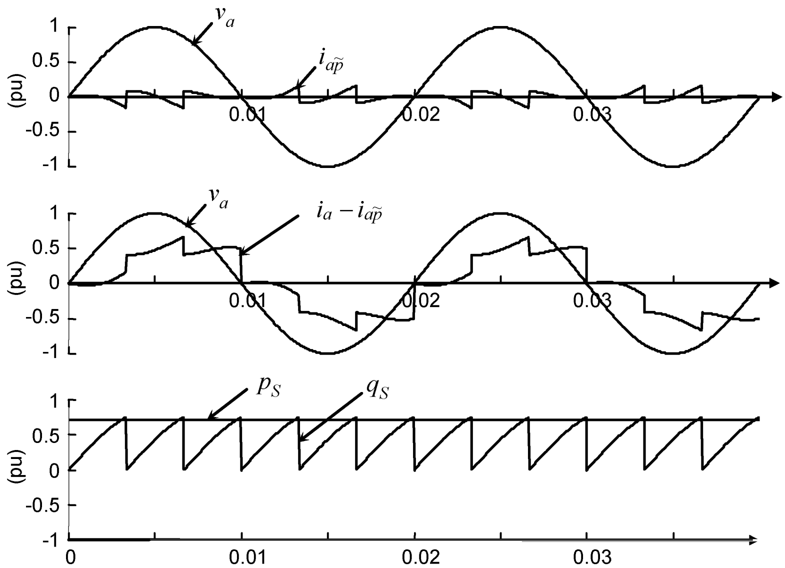


Figure 3-14. Eliminated current $i_{a\tilde{p}}$, compensated current $i_a - i_{a\tilde{p}}$, and the real and imaginary powers produced by the compensated current.

Figure 3-15 shows the case of compensation for $\tilde{p}$ and $\tilde{q}$. This means that the currents shown in Fig. 3-9 are eliminated. Now, the source current becomes purely sinusoidal. This kind of compensation is applicable when harmonic elimination is the most important issue. Although the resulting real and imaginary powers have no ripple, the current related to the average imaginary power $\bar{q}$ is still flowing out of the network, making the power factor lower than unity. In terms of conventional concepts of powers, the reactive current is not being compensated.

Figure 3-16 shows the case of compensation for $\tilde{p}$ and q. This means that all the undesirable current components of the load are being eliminated. The compensated current is sinusoidal, produces a constant real power, and does not generate any imaginary power. The nonlinear load and the compensator form an ideal, linear, purely resistive load. The source current has a minimum rms value that transfers the same energy as the original load current that produce the average real power $\bar{p}$. This is the best compensation that can be made from the power-flow point of view, because it smoothes the power drawn from the generator system. Besides, it eliminates all the harmonic currents. However, it should be pointed out that this is a particular situation in which no unbalances or distortions are present in the system voltages. Cases involving three-phase unbalanced and distorted voltages will be studied later in this chapter. It will be clear that, under nonsinusoidal system voltages, it is impossible to guarantee *simultaneously* constant real power and sinusoidal currents drawn from the network.

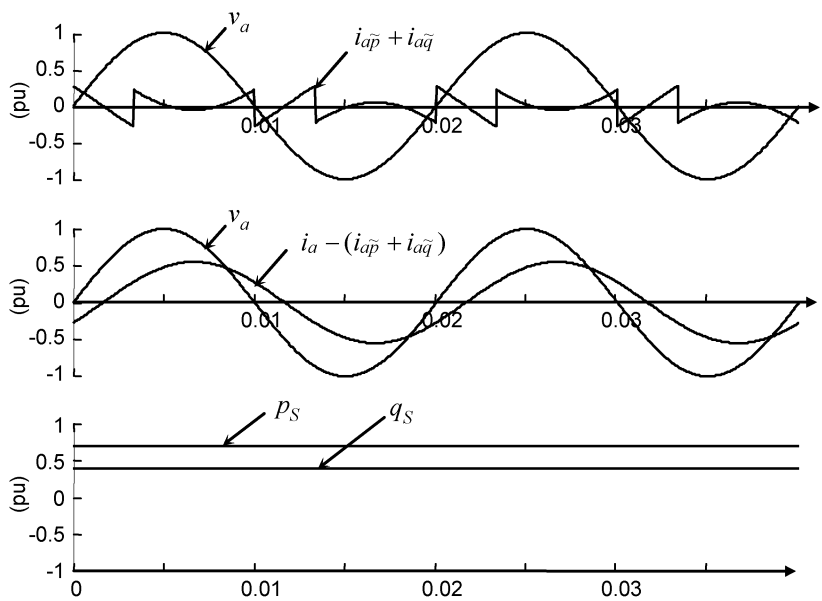


Figure 3-15. Eliminated current $i_{a\tilde{p}} + i_{a\tilde{q}}$, compensated current $i_a - (i_{a\tilde{p}} + i_{a\tilde{q}})$, and the real and imaginary powers produced by the compensated current.

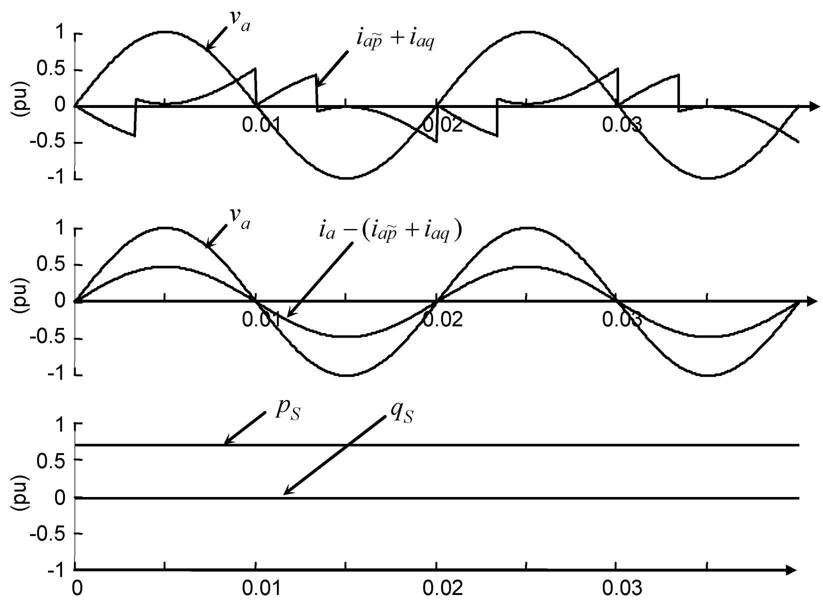


Figure 3-16. Eliminated current $i_{a\tilde{p}} + i_{aq}$, compensated current $i_a - (i_{a\tilde{p}} + i_{aq})$, and the real and imaginary powers produced by the compensated current.

3.2.2.A. Examples of Appearance of Hidden Currents

As will be shown in detail in the next chapter, there are many works showing that the *p-q* Theory is very precise for the calculation of compensating currents for shunt active filters. The compensation may be perfect if the power converter used to synthesize the compensating currents is able to generate them with high fidelity. However, one interesting phenomenon occurs when the percentage of oscillating real power $\tilde{p}$ being compensated is different from that of the oscillating imaginary power $\tilde{q}$. This phenomenon is maximized in the case when only $\tilde{p}$ or only $\tilde{q}$ is compensated. Although it was not highlighted, this situation was already verified in Fig. 3-12 and in Fig. 3-14, where the harmonic spectra of the compensated current (source current) are different from that of the original load current. In fact, when an unequal percentage of compensation of oscillating powers is used, the filtering algorithm may introduce some harmonics that are not present in the original load currents. Although some authors criticize the *p-q* Theory for this point, this phenomenon is not really a problem, and depends on what one wants to have as a result of the compensation. To simplify the analysis, a circuit with only the fifth harmonic current of the negative-sequence component will be analyzed. Then, a seventh harmonic component of a positive-sequence type is analyzed.

3.2.2.A.1. Presence of the Fifth Harmonic in Load Current. Here, it will be assumed that the three-phase balanced voltage consists only of the fundamental positive-sequence voltage, as given in (3.36). Besides its fundamental component, the nonlinear load current generally contains a large harmonic spectrum. However, for this analysis, only the fifth-order harmonic component will be considered. The phase angle displacements are chosen such that this current has only a negative-sequence component given by

$$\begin{cases} i_{a5}(t) = \sqrt{2}I_{-5}\sin(5\omega t + \delta_{-5}) \\ i_{b5}(t) = \sqrt{2}I_{-5}\sin\left(5\omega t + \delta_{-5} + \dfrac{2\pi}{3}\right) \\ i_{c5}(t) = \sqrt{2}I_{-5}\sin\left(5\omega t + \delta_{-5} - \dfrac{2\pi}{3}\right) \end{cases} \tag{3.44}$$

Subscript "–" indicates a negative-sequence component and subscript "5" indicates a fifth-order harmonic frequency. The Clarke transformation of the currents in (3.44) results in

$$\begin{cases} i_{\alpha 5} = \sqrt{3}I_{-5}\sin(5\omega t + \delta_{-5}) \\ i_{\beta 5} = \sqrt{3}I_{-5}\cos(5\omega t + \delta_{-5}) \end{cases} \tag{3.45}$$

The $\alpha\beta$ transformation of the voltage results in (3.37). From (3.37) and (3.45), the real and imaginary powers are calculated as the following oscillating components:

$$\begin{aligned} \tilde{p} &= -3V_{+1}I_{-5}\cos(6\omega t + \delta_{-5}) \\ \tilde{q} &= -3V_{+1}I_{-5}\sin(6\omega t + \delta_{-5}) \end{aligned} \tag{3.46}$$

Note that the real and imaginary powers have only an oscillating component at six times the line frequency. Now, from these $\tilde{p}$ and $\tilde{q}$ components and using (3.23) to (3.26), the instantaneous active and reactive currents on the $\alpha\beta$ axes are calculated as follows:

$$i_{\alpha p5} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2}\tilde{p} = \frac{\sqrt{3}}{2}I_{-5}\sin(5\omega t + \delta_{-5}) - \frac{\sqrt{3}}{2}I_{-5}\sin(7\omega t + \delta_{-5}) \tag{3.47}$$

$$i_{\alpha q5} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2}\tilde{q} = \frac{\sqrt{3}}{2}I_{-5}\sin(5\omega t + \delta_{-5}) + \frac{\sqrt{3}}{2}I_{-5}\sin(7\omega t + \delta_{-5}) \tag{3.48}$$

$$i_{\beta p5} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2}\tilde{p} = \frac{\sqrt{3}}{2}I_{-5}\cos(5\omega t + \delta_{-5}) + \frac{\sqrt{3}}{2}I_{-5}\cos(7\omega t + \delta_{-5}) \tag{3.49}$$

$$i_{\beta q5} = \frac{-v_\alpha}{v_\alpha^2 + v_\beta^2}\tilde{q} = \frac{\sqrt{3}}{2}I_{-5}\cos(5\omega t + \delta_{-5}) - \frac{\sqrt{3}}{2}I_{-5}\cos(7\omega t + \delta_{-5}) \tag{3.50}$$

where

$$\begin{cases} i_{\alpha 5} = i_{\alpha p5} + i_{\alpha q5} \\ i_{\beta 5} = i_{\beta p5} + i_{\beta q5} \end{cases} \tag{3.51}$$

Observing the above equations makes it possible to note two interesting facts:

1. $i_{\alpha p5}$ and $i_{\alpha q5}$, as well as $i_{\beta p5}$ and $i_{\beta q5}$ contain seventh-harmonic components that were not present in the original currents.
2. In the α axis, the seventh harmonic in $i_{\alpha p5}$ is equal to the seventh harmonic in $i_{\alpha q5}$, but with the opposite signal. Therefore, they normally sum zero and do not appear in the circuit. The same is valid for the β-axis current components $i_{\beta p5}$ and $i_{\beta q5}$.

If *p-q* Theory is used to compensate for the currents that are dependent on $\tilde{p}$ and $\tilde{q}$, it is possible to define compensating currents using gains k_p and k_q for the above current components given in (3.51). Thus, the compensating currents would be given by

$$\begin{cases} i_{C\alpha 5} = k_p \cdot i_{\alpha p5} + k_q \cdot i_{\alpha q5} \\ i_{C\beta 5} = k_p \cdot i_{\beta p5} + k_q \cdot i_{\beta q5} \end{cases} \tag{3.52}$$

In this case, the source current would be

$$\begin{cases} i_{S\alpha 5} = i_{\alpha 5} - i_{C\alpha 5} \\ i_{S\beta 5} = i_{\beta 5} - i_{C\beta 5} \end{cases} \tag{3.53}$$

If $k_p = k_q$, the seventh-harmonic component is totally eliminated in the source current. However, if $k_p \neq k_q$, the seventh-harmonic component in $i_{\alpha p5}$ does not cancel the seventh-harmonic component in $i_{\alpha q5}$. Therefore, a harmonic component that was not present in the original currents is introduced in the source currents. For this reason, this seventh-harmonic component this type of current component will be called “hidden currents.”

Part of the fifth-harmonic current component produces oscillating real power $\tilde{p}$ and is responsible for the oscillating energy flowing in a three-phase circuit. The other part of the fifth-harmonic current component does not transport energy at all, because it produces an oscillating imaginary power $\tilde{q}$.

In the filtering process, the worst situation occurs when the *p-q* Theory is used to compensate only for oscillating imaginary power $\tilde{q}$, or only for the oscillating real power $\tilde{p}$. In these cases, the seventh-harmonic component (the hidden current) in $i_{\alpha p5}$ and $i_{\beta p5}$ will not be cancelled by the seventh-harmonic component in $i_{\alpha q5}$ and $i_{\beta q5}$, respectively. Therefore, the hidden current will appear with maximum magnitude. It is important to note that a fifth-harmonic negative-sequence component is considered and, in this case, the hidden-current component is at a higher (seventh harmonic) frequency.

When only $\tilde{q}$ is used to filter the current, the source current after compensation will contain the hidden-current component. In principle, it is not possible to say that this is a bad or good thing. What can be said truly is that the imaginary power in the source is zero. In other words, the source currents after compensation consist only of components that contribute to the energy flow between the source and the load.

When only $\tilde{p}$ is compensated, the source current will also contain hidden currents, as in the previous case. However, all oscillating real power in the source is eliminated. Therefore, if the objective is to eliminate the oscillating energy flow in the circuit, this is the solution. This compensation technique may be interesting when dealing with motor drives or specific generation systems. In fact, this procedure is important when torque ripple in a motor or generator has to be eliminated.

On the other hand, if the objective of the filter is to eliminate partially or all of the fifth harmonic without introducing any hidden current, the oscillating real power and the oscillating imaginary power must be compensated with $k_p = k_q$. This is the case using a passive filter that has no capability to filter only the active or reactive portion of a harmonic current.

It is clear from the above analysis that the p-q Theory intended for current compensation brings more flexibility to the filter design.

3.2.2.A.2. Presence of the Seventh Harmonic in Load Current. Next, a seventh-harmonic positive-sequence current component is analyzed. These currents are given by

$$\begin{cases} i_{a7}(t) = \sqrt{2}I_{+7}\sin(7\omega t + \delta_{+7}) \\ i_{b7}(t) = \sqrt{2}I_{+7}\sin\left(7\omega t + \delta_{+7} - \dfrac{2\pi}{3}\right) \\ i_{c7}(t) = \sqrt{2}I_{+7}\sin\left(7\omega t + \delta_{+7} + \dfrac{2\pi}{3}\right) \end{cases} \tag{3.54}$$

The Clarke transformation of the currents in (3.54) is given by

$$\begin{cases} i_{\alpha 7} = \sqrt{3}I_{+7}\sin(7\omega t + \delta_{+7}) \\ i_{\beta 7} = -\sqrt{3}I_{+7}\cos(7\omega t + \delta_{+7}) \end{cases} \tag{3.55}$$

Again, the $\alpha\beta$ transformation of the voltage is given in (3.37), and the real and imaginary powers are calculated as

$$\begin{aligned} \tilde{p} &= 3V_{+1}I_{+7}\cos(6\omega t + \delta_{+7}) \\ \tilde{q} &= -3V_{+1}I_{+7}\sin(6\omega t + \delta_{+7}) \end{aligned} \tag{3.56}$$

Note that the real and imaginary powers have only oscillating components at six times the system frequency. From these $\tilde{p}$ and $\tilde{q}$ components and using (3.23) to (3.26), the instantaneous active and reactive currents on the $\alpha\beta$ axes can be calculated:

$$i_{\alpha p7} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2}\,\tilde{p} = -\frac{\sqrt{3}}{2}\,I_{+7}\sin(5\omega t + \delta_{+7}) + \frac{\sqrt{3}}{2}\,I_{+7}\sin(7\omega t + \delta_{+7}) \tag{3.57}$$

$$i_{\alpha q7} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2}\,\tilde{q} = \frac{\sqrt{3}}{2}\,I_{+7}\sin(5\omega t + \delta_{+7}) + \frac{\sqrt{3}}{2}\,I_{+7}\sin(7\omega t + \delta_{+7}) \tag{3.58}$$

$$i_{\beta p7} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2}\,\tilde{p} = -\frac{\sqrt{3}}{2}\,I_{+7}\cos(5\omega t + \delta_{+7}) - \frac{\sqrt{3}}{2}\,I_{+7}\cos(7\omega t + \delta_{+7}) \tag{3.59}$$

$$i_{\beta q7} = \frac{-v_\alpha}{v_\alpha^2 + v_\beta^2}\,\tilde{q} = \frac{\sqrt{3}}{2}\,I_{+7}\cos(5\omega t + \delta_{+7}) - \frac{\sqrt{3}}{2}\,I_{+7}\cos(7\omega t + \delta_{+7}) \tag{3.60}$$

where

$$\begin{cases} i_{\alpha 7} = i_{\alpha p7} + i_{\alpha q7} \\ i_{\beta 7} = i_{\beta p7} + i_{\beta q7} \end{cases} \tag{3.61}$$

The above equations show that there are also hidden currents associated with positive-sequence harmonic currents. They have similar properties to those in the case of the previous fifth-order harmonic negative-sequence currents. The difference is that the frequency of the hidden currents is lower than the frequency of the original, positive-sequence harmonic current. All conclusions made for the case of the fifth-order harmonic, shown in (3.44) to (3.53), are valid for the seventh-order harmonic, shown in (3.54) to (3.61).

3.2.3. The Dual *p-q* Theory

The original *p-q* Theory was defined with the most common case of three-phase systems comprising only voltage sources in mind. However, it may be interesting to also present its dual theory that would be suitable for the cases in which three-phase current sources are present, or in which it is desirable to perform *series voltage* compensation instead of *shunt current* compensation.

In the previous section, current components were calculated as function of the $\alpha\beta$ voltages, the real power, and the imaginary power. Those equations are suitable for applications to the control method of shunt current compensation. In the *dual p-q Theory,* it is assumed that the currents, and the real and imaginary powers are known, and the voltage components should be calculated or compensated. One possible application of this dual *p-q* Theory is to the case of series voltage compensation that is the dual of shunt current compensation.

For simplicity, only the case of a three-phase, three-wire system will be analyzed. Therefore, no zero-sequence voltage and current components are present. For this condition, the following equation can be derived from (3.19):

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} i_\alpha & i_\beta \\ -i_\beta & i_\alpha \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \tag{3.62}$$

Considering that the real and imaginary powers, as well as the currents, are known, the voltages can be calculated as function of these variables. Multiplying both sides of (3.62) by the inverse matrix of currents, the voltages are determined as functions of currents and powers by

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{1}{i_\alpha^2 + i_\beta^2} \begin{bmatrix} i_\alpha & -i_\beta \\ i_\beta & i_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \tag{3.63}$$

The right-hand side of (3.63) can be decomposed as

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{1}{i_\alpha^2 + i_\beta^2} \begin{bmatrix} i_\alpha & -i_\beta \\ i_\beta & i_\alpha \end{bmatrix} \begin{bmatrix} p \\ 0 \end{bmatrix} + \frac{1}{i_\alpha^2 + i_\beta^2} \begin{bmatrix} i_\alpha & -i_\beta \\ i_\beta & i_\alpha \end{bmatrix} \begin{bmatrix} 0 \\ q \end{bmatrix} \tag{3.64}$$

From (3.64), the following voltage components can be defined:

- Instantaneous active voltage on the α axis $v_{\alpha p}$

$$v_{\alpha p} = \frac{i_\alpha}{i_\alpha^2 + i_\beta^2} p \tag{3.65}$$

- Instantaneous reactive voltage on the α axis $v_{\alpha q}$

$$v_{\alpha q} = \frac{-i_\beta}{i_\alpha^2 + i_\beta^2} q \tag{3.66}$$

- Instantaneous active voltage on the β axis $v_{\beta p}$

$$v_{\beta p} = \frac{i_\beta}{i_\alpha^2 + i_\beta^2} p \tag{3.67}$$

- Instantaneous reactive voltage on the β axis $v_{\beta q}$

$$v_{\beta q} = \frac{i_\alpha}{i_\alpha^2 + i_\beta^2} q \tag{3.68}$$

The following equation is valid:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} v_{\alpha p} \\ v_{\beta p} \end{bmatrix} + \begin{bmatrix} v_{\alpha q} \\ v_{\beta q} \end{bmatrix} \tag{3.69}$$

The above equations can be applied when the load or the source can be modeled as a current source.

The voltage components on the $\alpha\beta$ reference frames are derived from (3.63), whereas the current components are derived from (3.21). The voltage components defined in (3.65) to (3.68) correspond to the dual of those current components defined in (3.23) to (3.26), in the *p-q* Theory. Hence, all physical meanings associated with those current components are valid here for their dual voltage components. Furthermore, all examples of separation of load *current* components are applicable here, but now to load *voltage* components.

In Section 3.2.2, the basic principle of shunt current compensation was introduced and illustrated in Fig. 3-10. Now, a complementary principle of *series voltage* compensation is derived. Figure 3-17 illustrates this dual principle of compensation.

The shunt compensator draws a current to compensate for undesirable power components produced by the load current. The ideal series compensator of Fig. 3-17 behaves as a controlled voltage source to compensate for undesirable power components produced by the load voltage. This dual principle can determine the compensating voltage v_C^* directly from the current and the power portions to be compensated.

A general control method for calculating the compensating voltage v_C^* is illustrated in Fig. 3-18. It is the dual of that compensation method shown in Fig. 3-11 for shunt current compensation. The phase voltages at the load terminal and the line currents are measured and transformed into the $\alpha\beta$ reference frames. Then, the real and imaginary powers of the load are calculated, and the undesirable power portions are selected. From these power portions of the load powers and the line currents, the compensating voltages are calculated and inserted "instantaneously" in the power system by the series compensator. Hence, the compensated voltages v_{Sa},

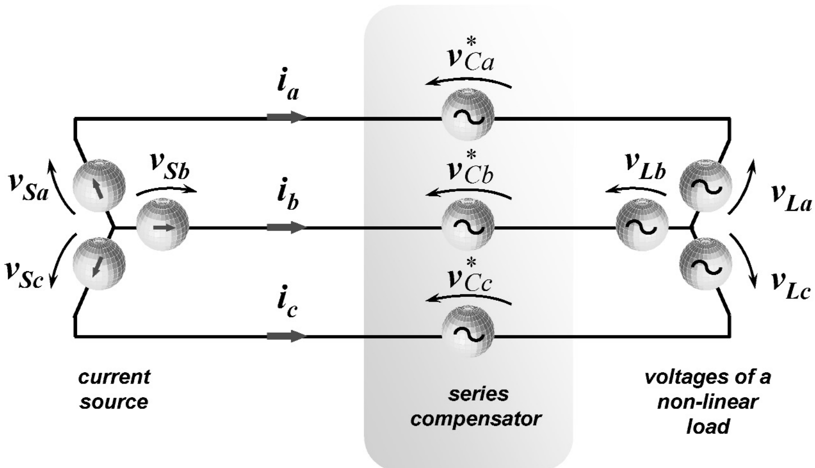


Figure 3-17. Basic principle of series voltage compensation.

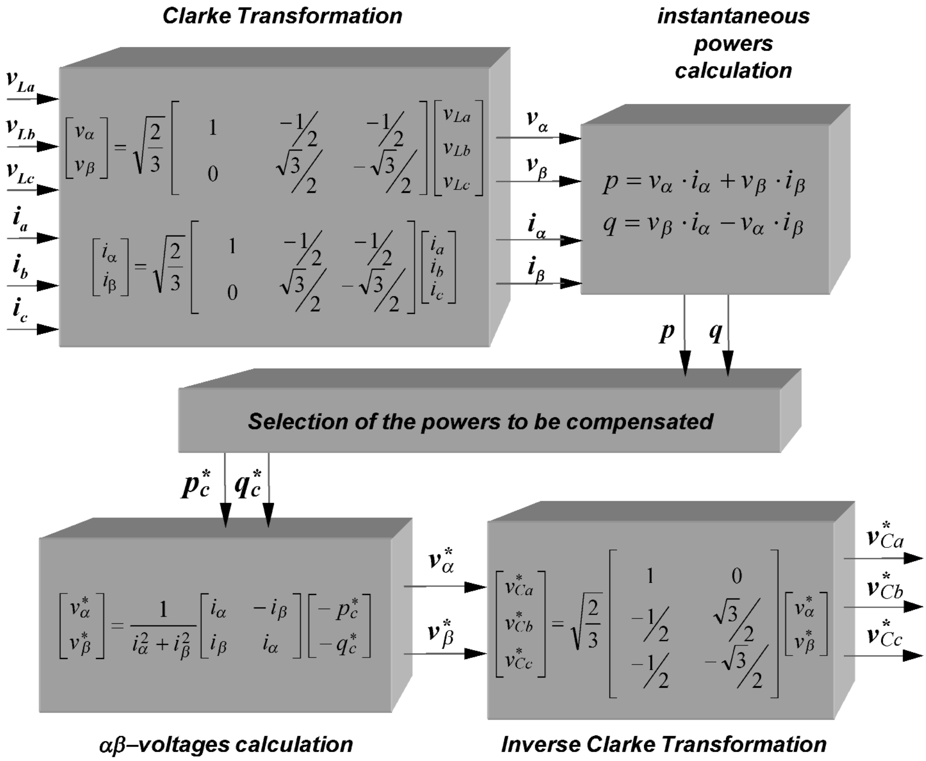


Figure 3-18. Control method for series voltage compensation based on the *p-q* Theory.

v_{Sb}, and v_{Sc} do not produce any undesirable power portions with the line currents at the load terminal.

One could prefer to design a compensator not for directly compensating portions of power but rather to guarantee, for instance, sinusoidal compensated voltage. This point and other compensation features will be explored in the next chapters. In practical cases, it is difficult to implement a control algorithm using the compensation method shown in Fig. 3-18. If this control algorithm is implemented in a controller for the series compensator, the $\alpha\beta$ voltage calculation block may realize a division by zero under noload conditions. However, this is the complement of a short-circuit situation in the case of a shunt active filter (Fig. 3-11), where the $\alpha\beta$-current calculation block would realize a division by zero.

3.3. THE *p-q* THEORY IN THREE-PHASE, FOUR-WIRE SYSTEMS

The previous section presented the *p-q* Theory for three-phase, three-wire systems. The physical meaning of the instantaneous real and imaginary power was discussed and clarified with examples. However, the presence of a fourth conductor, namely the neutral conductor, is very common in low-voltage distribution systems, in addition to the cases of grounded transmission systems. These systems are classified as three-phase, four-wire systems. The simplified transformation and equations used in the previous section are not applicable to these cases. This section presents cases involving the three instantaneous powers p, q, and p_0 of the p-q Theory.

The three-phase, four-wire systems can include both zero-sequence voltage and current as a generic case. These systems allow all the three line currents i_a, i_b, and i_c to be independent, whereas two of the three line currents are independent in three-phase, three-wire systems. Therefore, to represent the system correctly, the instantaneous zero-sequence power p_0 defined on the $\alpha\beta 0$ reference frames, p_0 has to be introduced as the third instantaneous power in addition to the instantaneous real power p and the instantaneous imaginary power q.

Mathematically, they are defined as:

$$\begin{bmatrix} p_0 \\ \hline p \\ q \end{bmatrix} = \left[\begin{array}{c|cc} v_0 & 0 & 0 \\ \hline 0 & v_\alpha & v_\beta \\ 0 & v_\beta & -v_\alpha \end{array}\right] \begin{bmatrix} i_0 \\ \hline i_\alpha \\ i_\beta \end{bmatrix} \tag{3.70}$$

The real and imaginary powers have the same physical meaning as before. The difference is the additional definition of the zero-sequence power. Before explaining it, the three-phase instantaneous active power shown in (3.17) should be re-written in terms of the $\alpha\beta 0$ components:

$$p_{3\phi} = v_a i_a + v_b i_b + v_c i_c = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 = p + p_0 \tag{3.71}$$

This equation shows that the three-phase instantaneous active power $p_{3\phi}$ is equal to the sum of the real power p and the zero-sequence power p_0. In the case of a three-phase, three-wire circuit, the power p_0 does not exist, and so $p_{3\phi}$ is equal to p.

3.3.1. The Zero-Sequence Power in a Three-Phase Sinusoidal Voltage Source

To understand the nature of the zero-sequence power, it is considered that a three-phase sinusoidal voltage source consists of positive- and zero-sequence voltages at the angular frequency ω. The symmetrical components of this voltage source were calculated based on their phasors (see Fig. 2-2). Although this analysis is valid only for the steady-state condition, it is very elucidating. These symmetrical components were then transformed back to the time domain and rewritten as time functions (this procedure is better explained in the next section) as

$$\begin{aligned} v_a &= \sqrt{2}V_+ \sin(\omega t + \phi_{v+}) + \sqrt{2}V_0 \sin(\omega t + \phi_{v0}) \\ v_b &= \sqrt{2}V_+ \sin(\omega t - 2\pi/3 + \phi_{v+}) + \sqrt{2}V_0 \sin(\omega t + \phi_{v0}) \\ v_c &= \sqrt{2}V_+ \sin(\omega t + 2\pi/3 + \phi_{v+}) + \sqrt{2}V_0 \sin(\omega t + \phi_{v0}) \end{aligned} \tag{3.72}$$

It is assumed that the current also has positive- and zero-sequence components, that is,

$$\begin{aligned} i_a &= \sqrt{2}I_+ \sin(\omega t + \phi_{i+}) + \sqrt{2}I_0 \sin(\omega t + \phi_{i0}) \\ i_b &= \sqrt{2}I_+ \sin(\omega t - 2\pi/3 + \phi_{i+}) + \sqrt{2}I_0 \sin(\omega t + \phi_{i0}) \\ i_c &= \sqrt{2}I_+ \sin(\omega t + 2\pi/3 + \phi_{i+}) + \sqrt{2}I_0 \sin(\omega t + \phi_{i0}) \end{aligned} \tag{3.73}$$

The subscripts + and 0 are used to define the positive- and zero-sequence components. Applying the Clarke transformation, the following voltages and currents on the $\alpha\beta 0$ reference frames are obtained:

$$\begin{aligned} v_\alpha &= \sqrt{3}V_+ \sin(\omega t + \phi_{v+}) \\ v_\beta &= -\sqrt{3}V_+ \cos(\omega t + \phi_{v+}) \\ v_0 &= \sqrt{6}V_0 \sin(\omega t + \phi_{v0}) \end{aligned} \tag{3.74}$$

and

$$\begin{aligned} i_\alpha &= \sqrt{3}I_+ \sin(\omega t + \phi_{i+}) \\ i_\beta &= -\sqrt{3}I_+ \cos(\omega t + \phi_{i+}) \\ i_0 &= \sqrt{6}I_0 \sin(\omega t + \phi_{i0}) \end{aligned} \tag{3.75}$$

Since the real and imaginary powers defined by (3.70) depend only on the positive-sequence voltage and current, these powers are similar in content and in meaning to those analyzed in the previous section. Therefore, the analysis here will be focused on the zero-sequence power p_0 that is given by

$$p_0 = 3V_0I_0 \cos(\phi_{v0} - \phi_{i0}) - 3V_0I_0 \cos(2\omega t + \phi_{v0} + \phi_{i0}) = \bar{p}_0 + \tilde{p}_0 \tag{3.76}$$

This power has the same characteristics as the instantaneous power in a single-phase circuit. It has an average value and an oscillating component at twice the line frequency. The average value $\bar{p}_0$ represents a unidirectional energy flow. It has the same characteristics as the conventional (average) active power. The oscillating component $\tilde{p}_0$ also transfers energy instantaneously. However, it has an average value equal to zero, because it is oscillating. The analysis shows that, in principle, the average value of the zero-sequence power helps to increase the total energy transfer, and in this sense, it can be considered as a positive point. However, even for the simplest case of a zero-sequence component in the voltage and current, the zero-sequence power p_0 cannot produce constant power $\bar{p}_0$ alone. In other words, p_0 always consists of $\tilde{p}_0$ plus $\bar{p}_0$, if $\cos(\phi_{v0} - \phi_{i0}) \neq 0$. The elimination of $\tilde{p}_0$ is accompanied by the elimination of $\bar{p}_0$ together. This is one interesting characteristic of this power, and this is one of the reasons why it is not welcome in most circuits. In summary, the zero-sequence power p_0 exists only if there are zero-sequence voltage and current. It is an instantaneous *active* power contributing to energy flow, just like in a single-phase circuit. The average zero-sequence power $\bar{p}_0$ is always associated to the oscillating component $\tilde{p}_0$. Therefore, there is no way to eliminate the oscillating component and keep the average part alone.

3.3.2. Presence of Negative-Sequence Components

For a three-phase, balanced, positive-sequence voltage, the presence of the negative-sequence components may be a serious problem. This section will analyze the case in which the voltages are sinusoidal waveforms with a frequency of ω, consisting of positive-, negative-, and zero-sequence components as given below:

$$\begin{aligned} v_a &= \sqrt{2}V_+ \sin(\omega t + \phi_{v+}) + \sqrt{2}V_- \sin(\omega t + \phi_{v-}) + \sqrt{2}V_0 \sin(\omega t + \phi_{v0}) \\ v_b &= \sqrt{2}V_+ \sin(\omega t - 2\pi/3 + \phi_{v+}) + \sqrt{2}V_- \sin(\omega t + 2\pi/3 + \phi_{v-}) \\ &\quad + \sqrt{2}V_0 \sin(\omega t + \phi_{v0}) \\ v_c &= \sqrt{2}V_+ \sin(\omega t + 2\pi/3 + \phi_{v+}) + \sqrt{2}V_- \sin(\omega t - 2\pi/3 + \phi_{v-}) \\ &\quad + \sqrt{2}V_0 \sin(\omega t + \phi_{v0}) \end{aligned} \tag{3.77}$$

and the currents consist of

$$\begin{aligned} i_a &= \sqrt{2}I_+ \sin(\omega t + \phi_{i+}) + \sqrt{2}I_- \sin(\omega t + \phi_{i-}) + \sqrt{2}I_0 \sin(\omega t + \phi_{i0}) \\ i_b &= \sqrt{2}I_+ \sin(\omega t - 2\pi/3 + \phi_{i+}) + \sqrt{2}I_- \sin(\omega t + 2\pi/3 + \phi_{i-}) \\ &\quad + \sqrt{2}I_0 \sin(\omega t + \phi_{i0}) \\ i_c &= \sqrt{2}I_+ \sin(\omega t + 2\pi/3 + \phi_{i+}) + \sqrt{2}I_- \sin(\omega t - 2\pi/3 + \phi_{i-}) \\ &\quad + \sqrt{2}I_0 \sin(\omega t + \phi_{i0}) \end{aligned} \tag{3.78}$$

Applying the Clarke transformation yields the following voltages and currents:

$$\begin{aligned} v_\alpha &= \sqrt{3}V_+ \sin(\omega t + \phi_{v+}) + \sqrt{3}V_- \sin(\omega t + \phi_{v-}) \\ v_\beta &= -\sqrt{3}V_+ \cos(\omega t + \phi_{v+}) + \sqrt{3}V_- \cos(\omega t + \phi_{v-}) \\ v_0 &= \sqrt{6}V_0 \sin(\omega t + \phi_{v0}) \end{aligned} \tag{3.79}$$

and

$$\begin{aligned} i_\alpha &= \sqrt{3}I_+ \sin(\omega t + \phi_{i+}) + \sqrt{3}I_- \sin(\omega t + \phi_{i-}) \\ i_\beta &= -\sqrt{3}I_+ \cos(\omega t + \phi_{i+}) + \sqrt{3}I_- \cos(\omega t + \phi_{i-}) \\ i_0 &= \sqrt{6}I_0 \sin(\omega t + \phi_{i0}) \end{aligned} \tag{3.80}$$

The zero-sequence voltage and current are the same as those in the previous case; the zero-sequence power is equal to that in the previous case. Nothing changed in it due to the presence of negative-sequence components, as expected. On the other hand, the real and imaginary powers changed considerably. The powers given below are separated in their average and oscillating components:

$$\begin{cases} \bar{p} = 3V_+I_+\cos(\phi_{v+} - \phi_{i+}) + 3V_-I_-\cos(\phi_{v-} - \phi_{i-}) \\ \bar{q} = 3V_+I_+\sin(\phi_{v+} - \phi_{i+}) - 3V_-I_-\sin(\phi_{v-} - \phi_{i-}) \\ \tilde{p} = -3V_+I_-\cos(2\omega t + \phi_{v+} + \phi_{i-}) - 3V_-I_+\cos(2\omega t + \phi_{v-} + \phi_{i+}) \\ \tilde{q} = -3V_+I_-\sin(2\omega t + \phi_{v+} + \phi_{i-}) + 3V_-I_+\sin(2\omega t + \phi_{v-} + \phi_{i+}) \end{cases} \tag{3.81}$$

The following conclusions can be written from the above equations for the real and imaginary powers:

1. The positive- and negative-sequence components in voltages and currents may contribute to the average real and imaginary powers.
2. The instantaneous real and imaginary powers contain oscillating components due to the cross product of the positive-sequence voltage and the negative-sequence current, and the negative-sequence voltage and the positive-sequence current. Hence, even circuits without harmonic components may have oscillating real or imaginary powers.

3.3.3. General Case Including Distortions and Imbalances in the Voltages and in the Currents

The three-phase, four-wire system with sinusoidal positive-, negative-, and zero-sequence voltage and current components at the fundamental frequency was analyzed in the previous section. This section will discuss the generalized three-phase, four-wire system including not only those components at the fundamental frequency, but also harmonics components.

General equations relating the instantaneous powers in the *p-q* Theory and the theory of symmetrical components (also called *the Fortescue components* [36]) are valid in the steady state. The theory of symmetrical components based on phasors is valid only in the steady state. However, these general equations are fundamental in elucidating some important characteristics of the *p-q* Theory that are valid even during transients. Moreover, the general equations will be useful in understanding control methods for shunt and series active filters and other active power-line conditioners introduced in the following chapters.

For the present analysis, a three-phase, four-wire system considered in the steady state has generic, but periodic, voltages and currents. They may include the fundamental component as well as harmonic components. Further, each three-phase group of phasors in a given frequency may be unbalanced, which means that it may consist of positive-, negative-, and zero-sequence components, according to the symmetrical component theory. Generic periodic voltages and currents can be decomposed into *Fourier series* as

$$v_k(t) = \sum_{n=1}^{\infty} \sqrt{2} V_{kn} \sin(\omega_n t + \phi_{kn}) \qquad k = (a, b, c) \tag{3.82}$$

$$i_k(t) = \sum_{n=1}^{\infty} \sqrt{2} I_{kn} \sin(\omega_n t + \phi_{kn}) \qquad k = (a, b, c) \tag{3.83}$$

where n indicates the harmonic order. Equations (3.82) and (3.83) can be written in terms of phasors, including the fundamental (n = 1) and harmonic phasors, as follows:

$$\dot{V}_k = \sum_{n=1}^{\infty} V_{kn} \angle \phi_{kn} = \sum_{n=1}^{\infty} \dot{V}_{kn} \qquad k = (a, b, c) \tag{3.84}$$

$$\dot{I}_k = \sum_{n=1}^{\infty} I_{kn} \angle \delta_{kn} = \sum_{n=1}^{\infty} \dot{I}_{kn} \qquad k = (a, b, c) \tag{3.85}$$

Then, the symmetrical-components transformation [36] is applied to each *a-b-c*-harmonic group of phasors of voltages or currents to determine their positive-, negative-, and zero-sequence components, that is,

$$\begin{bmatrix} \dot{V}_{0n} \\ \dot{V}_{+n} \\ \dot{V}_{-n} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} \dot{V}_{an} \\ \dot{V}_{bn} \\ \dot{V}_{cn} \end{bmatrix} \tag{3.86}$$

The subscripts "0," "+," and "–" correspond to the zero-, positive-, and negative-sequence components, respectively. The complex number α in the transformation matrix is the 120° phase-shift operator:

$$\alpha = 1 \angle 120° = e^{j(2\pi/3)} \tag{3.87}$$

The inverse transformation of (3.86) is given by

$$\begin{bmatrix} \dot{V}_{an} \\ \dot{V}_{bn} \\ \dot{V}_{cn} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} \dot{V}_{0n} \\ \dot{V}_{+n} \\ \dot{V}_{-n} \end{bmatrix} \tag{3.88}$$

Equivalent functions of time can be derived from the phasors given by (3.88). Hence, rewriting the harmonic voltages in terms of symmetrical components in the time domain, yield the following expressions for the nth *a-b-c* group of harmonic voltages:

$$\begin{cases} v_{an}(t) = \sqrt{2}V_{0n}\sin(\omega_n t + \phi_{0n}) + \sqrt{2}V_{+n}\sin(\omega_n t + \phi_{+n}) \\ \qquad + \sqrt{2}V_{-n}\sin(\omega_n t + \phi_{-n}) \\ v_{bn}(t) = \sqrt{2}V_{0n}\sin(\omega_n t + \phi_{0n}) + \sqrt{2}V_{+n}\sin[\omega_n t + \phi_{+n} - (2\pi/3)] \\ \qquad + \sqrt{2}V_{-n}\sin[\omega_n t + \phi_{-n} + (2\pi/3)] \\ v_{cn}(t) = \sqrt{2}V_{0n}\sin(\omega_n t + \phi_{0n}) + \sqrt{2}V_{+n}\sin[\omega_n t + \phi_{+n} + (2\pi/3)] \\ \qquad + \sqrt{2}V_{-n}\sin[\omega_n t + \phi_{-n} - (2\pi/3)] \end{cases} \tag{3.89}$$

Similarly, the instantaneous line currents are found to be

$$\begin{cases} i_{an}(t) = \sqrt{2}I_{0n}\sin(\omega_n t + \delta_{0n}) + \sqrt{2}I_{+n}\sin(\omega_n t + \delta_{+n}) \\ \qquad + \sqrt{2}I_{-n}\sin(\omega_n t + \delta_{-n}) \\ i_{bn}(t) = \sqrt{2}I_{0n}\sin(\omega_n t + \delta_{0n}) + \sqrt{2}I_{+n}\sin[\omega_n t + \delta_{+n} - (2\pi/3)] \\ \qquad + \sqrt{2}I_{-n}\sin[\omega_n t + \delta_{-n} + (2\pi/3)] \\ i_{cn}(t) = \sqrt{2}I_{0n}\sin(\omega_n t + \delta_{0n}) + \sqrt{2}I_{+n}\sin[\omega_n t + \delta_{+n} + (2\pi/3)] \\ \qquad + \sqrt{2}I_{-n}\sin[\omega_n t + \delta_{-n} - (2\pi/3)] \end{cases} \tag{3.90}$$

The above decomposition into symmetrical components allows the analysis of an unbalanced three-phase system as a sum of two balanced three-phase systems plus the zero-sequence component.

The harmonic voltages and currents in terms of symmetrical components, as given in (3.89) and (3.90), can replace the terms in the series given in (3.82) and (3.83), respectively. If the $\alpha\beta 0$ transformation, as defined in (3.1) and (3.3), is applied, the following expressions for the generic voltage and current transformed in the $\alpha\beta 0$-reference frames can be obtained:

$$\begin{cases} v_\alpha = \sum_{n=1}^{\infty} \sqrt{3}V_{+n}\sin(\omega_n t + \phi_{+n}) + \sum_{n=1}^{\infty} \sqrt{3}V_{-n}\sin(\omega_n t + \phi_{-n}) \\ v_\beta = \sum_{n=1}^{\infty} -\sqrt{3}V_{+n}\cos(\omega_n t + \phi_{+n}) + \sum_{n=1}^{\infty} \sqrt{3}V_{-n}\cos(\omega_n t + \phi_{-n}) \\ v_0 = \sum_{n=1}^{\infty} \sqrt{6}V_{0n}\sin(\omega_n t + \phi_{0n}) \end{cases} \tag{3.91}$$

$$\begin{cases} i_\alpha = \sum_{n=1}^{\infty} \sqrt{3}I_{+n}\sin(\omega_n t + \delta_{+n}) + \sum_{n=1}^{\infty} \sqrt{3}I_{-n}\sin(\omega_n t + \delta_{-n}) \\ i_\beta = \sum_{n=1}^{\infty} -\sqrt{3}I_{+n}\cos(\omega_n t + \delta_{+n}) + \sum_{n=1}^{\infty} \sqrt{3}I_{-n}\cos(\omega_n t + \delta_{-n}) \\ i_0 = \sum_{n=1}^{\infty} \sqrt{6}I_{0n}\sin(\omega_n t + \delta_{0n}) \end{cases} \tag{3.92}$$

It is possible to see that the positive- and negative-sequence components contribute to the α- and β-axis voltages and currents, whereas the 0-axis voltage and current comprise only zero-sequence components.

Further, the real power p, the imaginary power q, and the zero-sequence power p_0, as defined in (3.70), can be calculated by using the generic voltages and currents in terms of symmetrical components given by (3.91) and (3.92).

The relation between the conventional concepts of powers and the new powers defined in the p-q Theory is better visualized if the powers p, q, and p_0 are separated in their average values $\bar{p}$, $\bar{q}$, $\bar{p}_0$, and their oscillating parts $\tilde{p}$, $\tilde{q}$, $\tilde{p}_0$.

$$
\begin{array}{lccccc}
\text{Real power:} & p & = & \bar{p} & + & \tilde{p} \\
\text{Imaginary power:} & q & = & \bar{q} & + & \tilde{q} \\
\text{Zero-sequence power:} & p_0 & = & \bar{p}_0 & + & \tilde{p}_0 \\
 & & & \text{Average powers} & & \text{Oscillating powers}
\end{array}
\tag{3.93}
$$

The resulting power expressions are as follows:

$$\bar{p}_0 = \sum_{n=1}^{\infty} 3V_{0n}I_{0n}\cos(\phi_{0n} - \delta_{0n}) \tag{3.94}$$

$$\bar{p} = \sum_{n=1}^{\infty} 3V_{+n}I_{+n}\cos(\phi_{+n} - \delta_{+n}) + \sum_{n=1}^{\infty} 3V_{-n}I_{-n}\cos(\phi_{-n} - \delta_{-n}) \tag{3.95}$$

$$\bar{q} = \sum_{n=1}^{\infty} 3V_{+n}I_{+n}\sin(\phi_{+n} - \delta_{+n}) + \sum_{n=1}^{\infty} -3V_{-n}I_{-n}\sin(\phi_{-n} - \delta_{-n}) \tag{3.96}$$

$$
\begin{aligned}
\tilde{p}_0 = \Bigg\{ & \sum_{\substack{m=1 \\ m\neq n}}^{\infty} \left[\sum_{n=1}^{\infty} 3V_{0m}I_{0n}\cos\big((\omega_m - \omega_n)t + \phi_{0m} - \delta_{0m}\big) \right] + \\
& + \sum_{m=1}^{\infty} \left[\sum_{n=1}^{\infty} -3V_{0m}I_{0n}\cos\big((\omega_m + \omega_n)t + \phi_{0m} + \delta_{0m}\big) \right] \Bigg\}
\end{aligned}
\tag{3.97}
$$

$$
\begin{aligned}
\tilde{p} = \Bigg\{ & \sum_{\substack{m=1 \\ m\neq n}}^{\infty} \left[\sum_{n=1}^{\infty} 3V_{+m}I_{+n}\cos\big((\omega_m - \omega_n)t + \phi_{+m} - \delta_{+n}\big) \right] + \\
& + \sum_{\substack{m=1 \\ m\neq n}}^{\infty} \left[\sum_{n=1}^{\infty} 3V_{-m}I_{-n}\cos\big((\omega_m - \omega_n)t + \phi_{-m} - \delta_{-n}\big) \right] + \\
& + \sum_{m=1}^{\infty} \left[\sum_{n=1}^{\infty} -3V_{+m}I_{-n}\cos\big((\omega_m + \omega_n)t + \phi_{+m} + \delta_{-n}\big) \right] + \\
& + \sum_{m=1}^{\infty} \left[\sum_{n=1}^{\infty} -3V_{-m}I_{+n}\cos\big((\omega_m + \omega_n)t + \phi_{-m} + \delta_{+n}\big) \right] \Bigg\}
\end{aligned}
\tag{3.98}
$$

$$
\begin{aligned}
\tilde{q} = \Bigg\{ & \sum_{\substack{m=1 \\ m \neq n}}^{\infty} \left[\sum_{n=1}^{\infty} 3V_{+m}I_{+n} \sin\big((\omega_m - \omega_n)t + \phi_{+m} - \delta_{+n}\big) \right] + \\
& + \sum_{\substack{m=1 \\ m \neq n}}^{\infty} \left[\sum_{n=1}^{\infty} -3V_{-m}I_{-n} \sin\big((\omega_m - \omega_n)t + \phi_{-m} - \delta_{-n}\big) \right] + \\
& + \sum_{m=1}^{\infty} \left[\sum_{n=1}^{\infty} -3V_{+m}I_{-n} \sin\big((\omega_m + \omega_n)t + \phi_{+m} + \delta_{-n}\big) \right] + \\
& + \sum_{m=1}^{\infty} \left[\sum_{n=1}^{\infty} 3V_{-m}I_{+n} \sin\big((\omega_m + \omega_n)t + \phi_{-m} + \delta_{+n}\big) \right] \Bigg\}
\end{aligned}
\tag{3.99}
$$

These generic power expressions elucidate the relations between the conventional and the instantaneous concepts of active and reactive power. For instance, it is possible to see that the well-known three-phase fundamental active power ($P = 3VI \cos \varphi$) is one term of the average real power $\bar{p}$, whereas the three-phase reactive power ($Q = 3VI \sin \varphi$) is included in the average imaginary power $\bar{q}$. All harmonics in voltage and current can contribute to the average powers $\bar{p}$ and $\bar{q}$ if they have the same frequency and have the same sequence component (positive or negative), as shown in (3.95) and (3.96). The presence of more than one harmonic frequency and/or sequence components also produce $\tilde{p}$ and $\tilde{q}$, according to (3.98) and (3.99). On the other hand, the zero-sequence power $p_0 = \bar{p}_0 + \tilde{p}_0$, that is, the sum of (3.94) and (3.97), always has the average part associated with an oscillating part $\tilde{p}_0$. Therefore, if the oscillating part is eliminated by a compensator, this compensator should be able to also deal with the average part $\bar{p}_0$ that may be present. Contrarily, Section 3.2.2 showed briefly that it is always possible to compensate for only $\tilde{p}$ or $\tilde{q}$ and leave $\bar{p}$ or $\bar{q}$ to be supplied by the source. These points will be discussed in detail in the following chapter.

3.3.4. Physical Meanings of the Instantaneous Real, Imaginary, and Zero-Sequence Powers

Before the use of the *p-q* Theory to develop control circuits for active power-line conditioners for current or voltage compensation, the physical meaning of all the instantaneous powers must be clearly explained. Fig. 3-19 summarizes the concepts involved in these powers.

The following conclusions are similar to all the conclusions obtained so far. However, this time they were obtained for generic voltage and current waveforms.

The instantaneous powers that the *p-q* Theory defines in the time domain are independent of the rms values of voltages and currents. This theory includes the conventional frequency-domain concepts of active and reactive power defined for three-phase sinusoidal balanced systems as a particular case. Therefore, the *p-q* Theory in the time domain is not contradictory but complementary to the conventional theories in the frequency domain

The general equations for the real, imaginary, and zero-sequence powers given in (3.94) to (3.99) are the basis for understanding the energy transfer in a three-

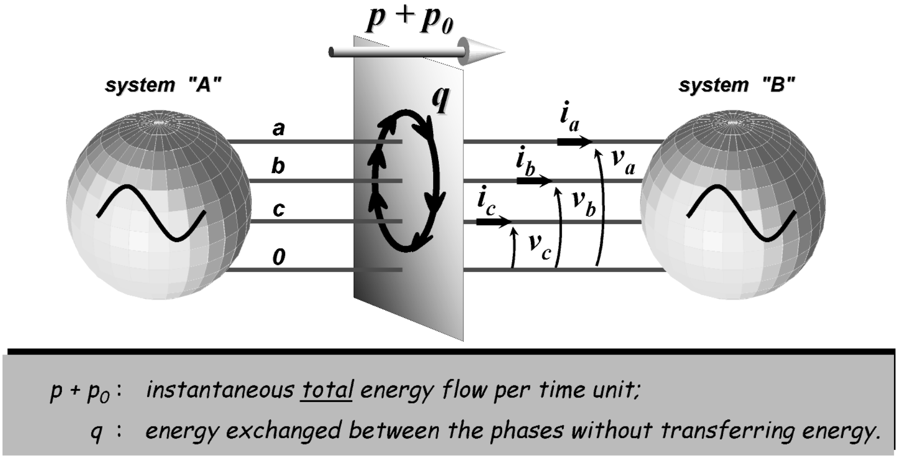


Figure 3-19. Physical meaning of the instantaneous powers defined in the α-β-0 reference frame.

- Zero-sequence components in the fundamental voltage and current and/or in the harmonics do not contribute to the real power p or to the imaginary power q.
- The total instantaneous energy flow per time unit, that is, the three-phase instantaneous active power, even in a distorted and unbalanced system, is always equal to the sum of the real power and the zero-sequence power ($p_{3\phi} = p + p_0$), and may contain average and oscillating parts.
- The imaginary power q, independent of the presence of harmonic or unbalances, represents the energy quantity that is being exchanged between the phases of the system. This means that the imaginary power does not contribute to energy transfer* between the source and the load at any time.

phase system. It is possible to understand how all the voltage and current components achieve energy transfer or induce energy exchange (imaginary power) in a three-phase circuit. Note that these voltages and currents may be at the same or different frequencies and at the same or different sequences, including the fundamental frequency.

3.3.5. Avoiding the Clarke Transformation in the *p-q* Theory

The *p-q* Theory in a three-phase, four-wire system defines the real power, the imaginary power, and the zero-sequence power as functions of voltages and currents in the $\alpha\beta0$-reference frames. Some expressions relating these powers in terms of phase mode, that is, the *abc* variables, were presented in the previous sections, and the two most useful ones are rewritten below:

*The term "energy transfer" is used here in a general manner, referring not only to the energy delivered to the load, but also to the energy oscillation between source and load as well.

$$p_{3\phi} = v_a i_a + v_b i_b + v_c i_c = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 = p + p_0 \tag{3.100}$$

$$q = v_\beta i_\alpha - v_\alpha i_\beta = \frac{1}{\sqrt{3}}[(v_a - v_b)i_c + (v_b - v_c)i_a + (v_c - v_a)i_b] \tag{3.101}$$

$$q = \frac{1}{\sqrt{3}}(v_{ab}i_c + v_{bc}i_a + v_{ca}i_b)$$

Moreover, it might be interesting to determine active (real) and reactive (imaginary) current components directly from the instantaneous *abc* voltages and currents. For convenience, the active (real) and reactive (imaginary) current decomposition defined in (3.22) is repeated here:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \underbrace{\frac{1}{v_\alpha^2 + v_\beta^2}\begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix}\begin{bmatrix} p \\ 0 \end{bmatrix}}_{\text{real currents}} + \underbrace{\frac{1}{v_\alpha^2 + v_\beta^2}\begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix}\begin{bmatrix} 0 \\ q \end{bmatrix}}_{\text{imaginary currents}} \tag{3.102}$$

Applying the appropriate inverse Clarke transformation in (3.102) and taking only the first term in right side of the expression, determine the the *abc* real currents as

$$\begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \frac{v_\alpha i_\alpha + v_\beta i_\beta}{v_\alpha^2 + v_\beta^2}\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \tag{3.103}$$

The *abc* real currents in (3.103) can be calculated directly from the *abc* voltages and currents by using (3.5) and its corresponding matrix transformation for the currents. Thus, the $\alpha\beta$ variables can be set in terms of *abc* variables. After some manipulations, the real currents are found to be

$$\begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = \frac{(v_{ab} - v_{ca}) \cdot i_a + (v_{bc} - v_{ab}) \cdot i_b + (v_{ca} - v_{bc}) \cdot i_c}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2}\begin{bmatrix} (v_{ab} - v_{ca})/3 \\ (v_{bc} - v_{ab})/3 \\ (v_{ca} - v_{bc})/3 \end{bmatrix} \tag{3.104}$$

Note that line voltages, that is, $v_{ab} = v_a - v_b$, $v_{bc} = v_b - v_c$, $v_{ca} = v_c - v_a$, are used in (3.104). This expression confirms a fundamental concept introduced in the *p-q* Theory, which establishes that zero-sequence components do not contribute to the real power. Thus, the *abc* real currents i_{ap}, i_{bp}, and i_{cp} that correspond to the components in the original currents i_a, i_b, and i_c. Moreover, the real currents are not influenced by zero-sequence voltage or current components, neither in voltages nor in currents. The reason is that line voltages never contain zero-sequence components, because $v_{ab} + v_{bc} + v_{ca} = 0$. Further, even if zero-sequence currents are considered in (3.104), it follows that

$$(v_{ab} - v_{ca}) \cdot i_0 + (v_{bc} - v_{ab}) \cdot i_0 + (v_{ca} - v_{bc}) \cdot i_0 = 0 \tag{3.105}$$

which results in zero real current.

If the second term in (3.102) is considered, the *abc* imaginary currents can be determined by using the same procedure as the above. After some manipulations, they result in

$$\begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} = \frac{v_{bc} \cdot i_a + v_{ca} \cdot i_b + v_{ab} \cdot i_c}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2} \begin{bmatrix} v_{bc} \\ v_{ca} \\ v_{ab} \end{bmatrix} \tag{3.106}$$

As expected, (3.106) leads to the following conclusion: the *abc* imaginary currents i_{aq}, i_{bq}, and i_{cq} correspond to the components in the original currents i_a, i_b, i_c, which generate only imaginary power. They are not influenced by zero-sequence voltage or current components.

The inverse Clark transformation as given in (3.4) can be decomposed into the sum of two terms, as follows:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} i_0 \\ i_0 \\ i_0 \end{bmatrix} + \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{3.107}$$

On the other hand, (3.102) separates i_α and i_β into two components, the real and the imaginary currents. One is dependent only on the real power, and the other is dependent only on the imaginary power. These current components can be calculated directly from the instantaneous *abc* voltages and currents by using (3.104) and (3.106). These equations together with (3.102) and (3.107) allow us to write

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} i_0 \\ i_0 \\ i_0 \end{bmatrix} + \begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} + \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} \tag{3.108}$$

3.3.6. Modified *p-q* Theory

This chapter describes the basic principles of shunt current compensation and series voltage compensation based on the *p-q* Theory, which are applicable to all controllers for active filters and active power-line conditioners that will be presented in the following chapters. The reader should notice that the authors use the $\alpha\beta 0$ transformation to deal properly with zero-sequence components that are separated from the $\alpha\beta$ components. In this way, zero-sequence voltage and current components are separated from the $\alpha\beta$ components, and treated as "single-phase variables." The positive- and negative-sequence components are naturally kept in the $\alpha\beta$ axes.

As will be shown in the next chapters, the approach adopted in the above sections is perfectly sufficient to deal with three-phase circuits for both three-wire and four-wire systems. Nevertheless, in 1994 and 1995, references [48] and [49] introduced a *modified p-q theory* in 1994 and 1995. In fact, they expanded the concept of the imaginary power q defined in (3.20).

A three-dimensional, instantaneous voltage vector $\mathbf{e}_{\alpha\beta 0}$ and a current vector $\mathbf{i}_{\alpha\beta 0}$ can be defined from the instantaneous voltages and currents transformed into the $\alpha\beta 0$ reference frames. These instantaneous vectors are defined as

$$\mathbf{e}_{\alpha\beta 0} = [v_\alpha, v_\beta, v_0]^T; \qquad \mathbf{i}_{\alpha\beta 0} = [i_\alpha, i_\beta, i_0]^T \tag{3.109}$$

The original imaginary power q defined in (3.20) can be understood as the cross product of instantaneous vectors $\mathbf{e}$ and $\mathbf{i}$ defined in (3.10) and (3.11), respectively. On the other hand, the real power of the *p-q* Theory can be interpreted as the scalar product (dot product) of those vectors. Similarly, the modified *p-q* theory defines three instantaneous imaginary powers as derived from the cross product of the vectors defined in (3.109), whereas the three-phase instantaneous active power $p_{3\phi}$ defined in (3.71) represents the scalar product of these same vectors. Here, the three-phase instantaneous active power $p_{3\phi}$ is represented simply by the symbol p in the modified *p-q* theory. However, it should not be confused with the real power p defined in the original *p-q* Theory that excludes the zero-sequence power p_0 from the real power p. Thus, the modified *p-q* theory defines only a single instantaneous active power that is the sum of the real and zero-sequence powers in the original *p-q* Theory, that is,

$$p = \mathbf{e}_{\alpha\beta 0} \cdot \mathbf{i}_{\alpha\beta 0} = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 \tag{3.110}$$

On the other hand, the modified *p-q* theory defines an instantaneous imaginary-power vector composed of three elements, q_0, q_α, and q_β, as follows:

$$\mathbf{q} = \mathbf{e}_{\alpha\beta 0} \times \mathbf{i}_{\alpha\beta 0} = \begin{bmatrix} q_\alpha \\ q_\beta \\ q_0 \end{bmatrix} = \begin{bmatrix} \begin{vmatrix} v_\beta & v_0 \\ i_\beta & i_0 \end{vmatrix} \\ \begin{vmatrix} v_0 & v_\alpha \\ i_0 & i_\alpha \end{vmatrix} \\ \begin{vmatrix} v_\alpha & v_\beta \\ i_\alpha & i_\beta \end{vmatrix} \end{bmatrix} \tag{3.111}$$

The instantaneous powers defined above are combined in a matrix expression as follows:

$$\begin{bmatrix} p \\ q_\alpha \\ q_\beta \\ q_0 \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta & v_0 \\ 0 & -v_0 & v_\beta \\ v_0 & 0 & -v_\alpha \\ -v_\beta & v_\alpha & 0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \tag{3.112}$$

Note that the new imaginary power q_0 is the same as the original imaginary power q defined in (3.20). The other two imaginary powers q_α and q_β relate α and β components with a zero-sequence voltage and current, which are not considered in the original *p-q* Theory. The norm of the instantaneous imaginary-power vector expresses the "total" instantaneous imaginary power q as follows:

$$q = |\mathbf{q}| = \sqrt{q_\alpha^2 + q_\beta^2 + q_0^2} \tag{3.113}$$

The inverse transformation of (3.112) is performed as follows:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \frac{1}{v_{\alpha\beta 0}^2} \begin{bmatrix} v_\alpha & 0 & v_0 & -v_\beta \\ v_\beta & -v_0 & 0 & v_\alpha \\ v_0 & v_\beta & -v_\alpha & 0 \end{bmatrix} \begin{bmatrix} p \\ q_\alpha \\ q_\beta \\ q_0 \end{bmatrix} \tag{3.114}$$

where

$$v_{\alpha\beta 0}^2 = v_\alpha^2 + v_\beta^2 + v_0^2 \tag{3.115}$$

From (3.114), active and reactive current components can be derived. For instance, the zero-sequence current i_0 is divided in its active part i_{0p} and reactive part i_{0q}, where $i_0 = i_{0p} + i_{0q}$. They are calculated as follows:

- Instantaneous zero-sequence active current i_{0p}

$$i_{0p} = \frac{v_0}{v_{\alpha\beta 0}^2}\, p \tag{3.116}$$

- Instantaneous zero-sequence reactive current i_{0q}

$$i_{0q} = \frac{v_\beta}{v_{\alpha\beta 0}^2}\, q_\alpha - \frac{v_\alpha}{v_{\alpha\beta 0}^2}\, q_\beta \tag{3.117}$$

Similarly, instantaneous active and reactive currents on the α and β axes can be redefined. Unlike the active and reactive current components defined in (3.23) to (3.26) in the original *p-q* Theory, they also depend on zero-sequence components [50,51]:

- Instantaneous active current on the α axis $i_{\alpha p}$

$$i_{\alpha p} = \frac{v_\alpha}{v_{\alpha\beta 0}^2}\, p \tag{3.118}$$

- Instantaneous reactive current on the α axis $i_{\alpha q}$

$$i_{\alpha q} = \frac{v_0}{v_{\alpha\beta 0}^2}\, q_\beta - \frac{v_\beta}{v_{\alpha\beta 0}^2}\, q_0 \tag{3.119}$$

- Instantaneous active current on the β axis $i_{\beta p}$

$$i_{\beta p} = \frac{v_\beta}{v_{\alpha\beta 0}^2}\, p \tag{3.120}$$

- Instantaneous reactive current on the β axis $i_{\beta q}$

$$i_{\beta q} = \frac{v_\alpha}{v_{\alpha\beta 0}^2}\, q_0 - \frac{v_0}{v_{\alpha\beta 0}^2}\, q_\alpha \tag{3.121}$$

The modified *p-q* theory presented above provides the basic equations to develop a new control method for shunt current compensation. The method based on the *p-q* Theory, presented in Figure 3-11, can be modified by using (3.112) and (3.114). Fig. 3-20 shows the control method for shunt current compensation based on the modified *p-q* theory.

The control method based on the modified *p-q* theory has the same flexibility as that based on the *p-q* Theory. Powers or even portions of powers, like the average or oscillating component of the above-defined active power p, and the elements of the imaginary-power vector **q** can be selected and compensated independently from each other. However, it should be kept in mind that the zero-sequence voltage and current are now manipulated together with the positive-sequence and negative-sequence components. In some cases, this can produce undesirable compensation effects, which will be clarified in the next section.

If zero-sequence components—generically said to be homopolar mode—do not represent a problem when manipulated together with positive-sequence and negative-sequence components—generically said to be nonhomopolar modes—the modified *p-q* theory can be simplified. The use of the Clarke transformation can be avoided. Peng and his coauthors [52,53] established a set of power definitions directly in the *abc*-phase mode and correlated it with the modified *p-q* theory presented above. The basic idea consists in defining the three-phase instantaneous active power, as well as the instantaneous imaginary-power vector, directly from instantaneous voltage and current vectors formed by the instantaneous phase voltages v_a, v_b, and v_c, and the instantaneous line currents i_a, i_b, and i_c, instead of their corresponding $\alpha\beta 0$ variables. Both approaches in the modified theory differ from the approach preferred in this book in the sense that they consider the homopolar mode as containing the same properties as the nonhomopolar modes. It was shown that the Clarke transformation is useful to separate the nonhomopolar modes that form the $\alpha\beta$ variables, from the zero-sequence components.

In 1999, a deep discussion on these different definitions was presented in [51]. It shows that the definitions of the imaginary power, given in (3.20), as well as the

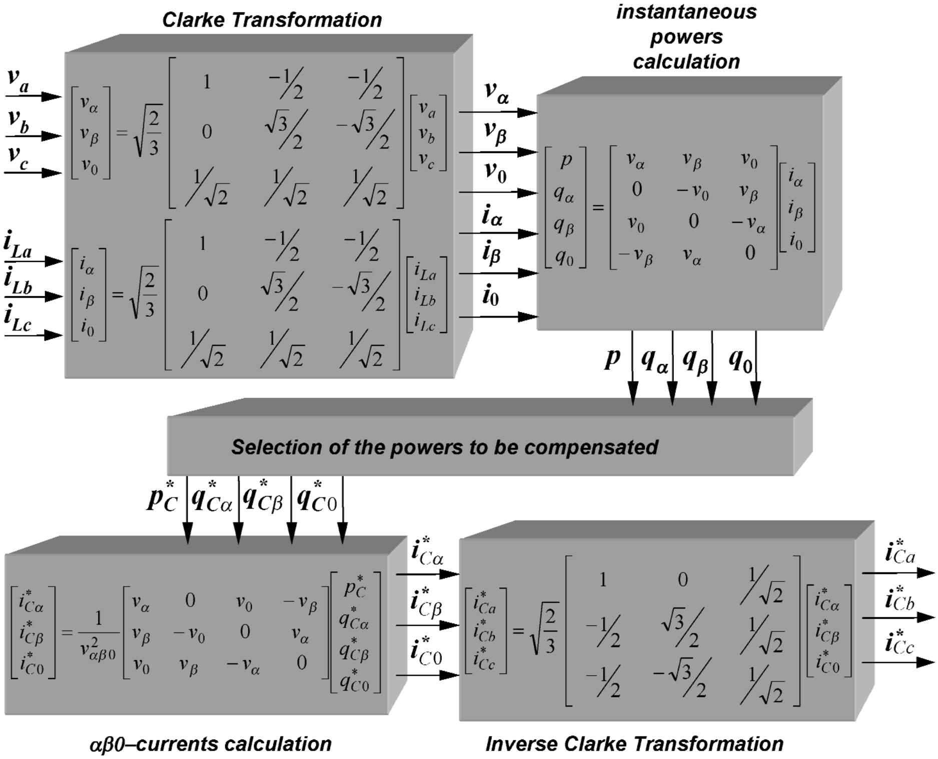


Figure 3-20. Control method for shunt current compensation based on the modified *p-q* theory.

imaginary-power vector, given in (3.111), or that one given in [52], are correct. However, they lead to some different interpretations. For instance, the *p-q* Theory suggests that zero-sequence currents should be completely eliminated and never treated as containing "active" and "reactive" portions that could be separated from each other. Contrarily, the modified theories allow the compensation of parts of the zero-sequence current, depending on the presence or absence of zero-sequence voltage. Moreover, the presence of zero-sequence components in voltage and current affect the $\alpha\beta$ variables—the nonhomopolar quantities—in the modified theories.

When $v_0 = 0$, the zero-sequence current is simply an "instantaneous current" that does not transfer any energy from the source to the load, since $p_0 = 0$. Contrarily, when $v_0 \neq 0$, the zero-sequence current becomes an "instantaneous active current," because p_0 is not zero, and energy is transferred from the source to the load in the same way as in a single-phase circuit [38]. The next chapter confirms the advantage of the *p-q* Theory when applied to controllers of three-phase, four-wire shunt active filters.

The imaginary-power vector given in (3.111) treats the zero-sequence current as an "instantaneous reactive current" when $v_0 = 0$. A compensator without an energy storage element can be designed to completely compensate for i_0, under the condition of $v_0 = 0$, by using either the *p-q* Theory or the modified *p-q* theory. However, the authors believe that working with zero-sequence components separated from the $\alpha\beta$ components makes it possible to better understand the physical meaning of the problem resulting from the presence of v_0 and i_0, and to take measures to eliminate them. For instance, as will be shown in the next chapters, the current i_0 can be compensated easily by a shunt active filter with or without the presence of v_0. In the case of series active filter, the zero-sequence voltage v_0 can be compensated in a dual way. References [48], [49], [50], [52] and [53] proposed expanded imaginary powers. However, they do not consider the elimination of v_0.

In 2004, Dai and coauthors [54] published a paper giving a more generic view of the definition of the "instantaneous reactive quantities for multiphase power systems." Instead of using a vector as result of the cross product, which is valid only for three-dimensional spaces, they define an imaginary-power tensor, which has the advantage of being applicable to generic multiphase systems. This is also an interesting approach for those who will work with power systems with more than three phases. However, this problem will not be addressed in this book.

3.4. INSTANTANEOUS *abc* THEORY

As explained in the introduction to this chapter, the instantaneous power theory can be separated into two groups: one defining the powers on the $\alpha\beta0$-reference frame and the other defining them directly in the *abc* phases, that is, the use of instantaneous phase voltages and instantaneous line currents. A summary of the theories working directly in the *abc* axes are presented here. To maintain a clear contrast with the *p-q* Theory, these sets of power definitions and current decompositions in the *abc* axes will be called the *abc* Theory.

Until the end of the 1960s, the concept of reactive power was related to a time-independent variable. However, with an increase in the problems caused by power electronics devices, the term "*instantaneous reactive power*" appeared in the literature as an extension of the old concept [10,11]. The work presented by Erlicki and Emanuel-Eigeles, in 1968, introduced the idea of compensation of "instantaneous" reactive power [6]. However, the analysis was done in the frequency domain and the compensation in the time domain. This means that the concept of instantaneous reactive power was not well defined at that time. One of the first works to relate the possibility of reactive power generation by means of power electronic converters without direct association with the energy storage capacity of dc reactors of a thyristor converter was presented by Depenbrock, in 1962 [10]. Recently, all these issues have been well addressed by means of the *p-q* Theory.

The *p-q* Theory gave the well-defined concept of imaginary power a clear physical meaning. However, it requires the use of the $\alpha\beta0$ transformation (Clarke transformation) in this theory. For some analysis, or even for some cases of harmonic elimination, this transformation may be seen as an extra calculation effort that should be avoided. In fact, it is possible to avoid the $\alpha\beta0$ transformation in the *p-q* Theory, as shown in (3.104), (3.106), and (3.108).

A summary of some original research has been made, and will be presented below as an alternative approach to calculate the *active* and *nonactive* current portions of a generic load current. Instead of using the Clarke transformation and the real and imaginary power calculation, the following approach calculates the active and nonactive current components of a generic load current directly from the *abc*-phase voltages and line currents. This approach can be generally understood as being obtained from the application of a minimization method to the load current, as will be clarified in the following sections.

Although Fryze did not mention the concept of "*instantaneous*" reactive power in his original work, he used the idea of decomposition into active current component ("*Wirkstrom,*" in German) and reactive current component ("*Blindstrom,*" in German) [1]. An extension to active and reactive voltage components was also addressed. Based on these components, the active power, P_w, and the reactive power, P_q, were consistently defined, as summarized in (2.26) to (2.32).

The following *abc* Theory consists in determining instantaneously the active portion of a generic load current. In other words, a minimized, instantaneous, active current component is determined with the constraint that it should transfer the same amount of energy as the uncompensated (original) load current. The difference between the instantaneous original load current and the calculated instantaneous active current (minimized current) is the instantaneous nonactive current that is part of the original load current. To determine the instantaneous active current, a minimization method (the *Lagrange Multiplier Method*) formulation can be used.

A concept of *three-phase instantaneous reactive power* can be derived from the above-defined concept of active and nonactive currents as follows.

> The three-phase instantaneous reactive power comprises products of voltage and current components that do not contribute to the three-phase instantaneous active power.

Fryze used similar words to define its reactive power, P_q [1]. The difference is that here instantaneous values of voltages and currents are considered.

It should be remarked that here the traditional idea that reactive power is related to an oscillating energy flow, as introduced in Chapter 2 (Fig. 2-1), is abandoned. If an oscillating energy flow exists between two subsystems, irrespective of whether it is in a three-phase three-wire or four-wire system, it is treated here as a three-phase instantaneous *active* power portion that has a zero average value.

Since instantaneous values of voltages and current are used, the above concepts of instantaneous decomposition into active and nonactive currents, along with the instantaneous active and reactive powers, are valid during transient periods or in steady-state conditions. Further, no restrictions are imposed on their waveforms, and they can be used under nonsinusoidal or unbalanced conditions.

At this point, the reader could be asking the following questions. What is the difference between the *p-q* Theory and the *abc* Theory? What is the difference between the current decomposition made in (3.23) to (3.26) and the concept of active and nonactive current? What is the relation between the imaginary power and the above-defined concept of three-phase instantaneous reactive power?

The physical meaning related to the imaginary power in the *p-q* Theory cannot be directly extended to the above-defined three-phase instantaneous reactive power. The results are the same if no zero-sequence components are present, whereas they are different in the presence of zero-sequence components in current and/or voltage. During the presentation of the *abc* Theory, these differences from the *p-q* Theory will be clarified through hypothetical examples of current compensation.

3.4.1. Active and Nonactive Current Calculation by Means of a Minimization Method

The instantaneous nonactive current in a three-phase system is the component of the load current that does not produce three-phase instantaneous active power, although it increases the current amplitude and consequently also increases the losses in the network. The instantaneous nonactive current can be determined by applying a minimization method. For the formulation of the problem, a three-phase hypothetical load current i_k, $k = (a, b, c)$, is assumed to consist of an *active* portion i_{wk} and a *nonactive* portion i_{qk}, that is,

$$i_k = i_{wk} + i_{qk}; \qquad k = (a, b, c) \tag{3.122}$$

The method involves minimizing load currents under the constraint that the *nonactive* current components i_{qa}, i_{qb}, and i_{qc} do not generate three-phase instantaneous active power. Thus, the task consists of finding the minimum of

$$L(i_{qa}, i_{qb}, i_{qc}) = (i_a - i_{qa})^2 + (i_b - i_{qb})^2 + (i_c - i_{qc})^2$$

constrained by

$$g(i_{qa}, i_{qb}, i_{qc}) = v_a i_{qa} + v_b i_{qb} + v_c i_{qc} = 0 \tag{3.123}$$

The problem can be solved by applying the Lagrange Multiplier Method, which leads to the following system of equations:

$$\begin{bmatrix} 2 & 0 & 0 & v_a \\ 0 & 2 & 0 & v_b \\ 0 & 0 & 2 & v_c \\ v_a & v_b & v_c & 0 \end{bmatrix} \begin{bmatrix} i_{qa} \\ i_{qb} \\ i_{qc} \\ \lambda \end{bmatrix} = \begin{bmatrix} 2i_a \\ 2i_b \\ 2i_c \\ 0 \end{bmatrix} \tag{3.124}$$

Solving (3.124) for λ gives

$$\lambda = \frac{2(v_a i_a + v_b i_b + v_c i_c)}{v_a^2 + v_b^2 + v_c^2} = \frac{2p_{3\phi}}{v_a^2 + v_b^2 + v_c^2} \tag{3.125}$$

By replacing (3.125) in (3.124), the instantaneous nonactive currents are found to be:

$$\begin{bmatrix} i_{qa} \\ i_{qb} \\ i_{qc} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \frac{p_{3\phi}}{v_a^2 + v_b^2 + v_c^2} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{3.126}$$

From (3.122) and (3.126), the instantaneous active currents are

$$\begin{bmatrix} i_{wa} \\ i_{wb} \\ i_{wc} \end{bmatrix} = \frac{p_{3\phi}}{v_a^2 + v_b^2 + v_c^2} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{3.127}$$

Therefore, the restriction imposed in the minimization method forces the active currents calculated in (3.127) and the original load currents i_a, i_b, and i_c to produce the same three-phase instantaneous active power ($p_{3\phi}$) when multiplied by their respective phase voltage v_a, v_b, v_c, that is,

$$p_{3\phi} = v_a i_a + v_b i_b + v_c i_c = v_a i_{wa} + v_b i_{wb} + v_c i_{wc} \tag{3.128}$$

Hence, they are equivalent from an energy transfer point of view. The difference is that the active currents i_{wa}, i_{wb}, and i_{wc} do not generate any three-phase instantaneous reactive power and have smaller rms values. The active currents determined in (3.127), contrary to the *abc* real currents given in (3.104), are influenced by zero-sequence components in the voltages and currents.

If i_{qa}, i_{qb}, and i_{qc} are compensated close to the load terminals, then the power generating system supplies only i_{wa}, i_{wb}, and i_{wc}, reducing losses in the network. The basic principle of shunt current compensation is illustrated in Fig. 3-10. This principle can be used for the compensation of nonactive currents as calculated in (3.126). In this case, the original load currents $i_a = i_{wa} + i_{qa}$, $i_b = i_{wb} + i_{qb}$ and $i_c = i_{wc} + i_{qc}$, are compensated by making $i^*_{Ca} = -i_{qa}$, $i^*_{Cb} = -i_{qb}$ and $i^*_{Cc} = -i_{qc}$, forcing the source currents to become $i_{Sa} = i_{wa}$, $i_{Sb} = i_{wb}$, $i_{Sc} = i_{wc}$. Since i_{qa}, i_{qb}, and i_{qc} do not produce active power, the compensator does not supply any energy to the system all the time. Hence, the compensator does not need any energy storage capability.

The *abc* Theory described above is based on the principle of load-current decomposition into active and nonactive current portions. The concept of three-phase instantaneous reactive power given above is derived from this current decomposition. In fact, there is no equation for directly calculating the three-phase instantaneous reactive power, in contrast to the well-defined imaginary power in the *p-q* Theory.

Much research on power theories can be found in the literature. Although the above-described *abc* Theory does not go in its particularities, it tries to summarize the results of the research. Some recent power formulations that resulted in formulations similar to those given in (3.126) and (3.127) were presented by Furuhashi et al. [41], Tenti et al. [42,43,44], van Wyk et al. [45,46,47], Depenbrock et al. [10,25,40] and Czarnecki [26,27].

If the three-phase system is balanced and sinusoidal, $p_{3\phi}$ is constant. In this case, the quadratic sum ($v_a^2 + v_b^2 + v_c^2$) is also constant. Thus, the minimized instantaneous currents i_{wa}, i_{wb}, and i_{wc} are *proportional* to the phase voltages v_a, v_b, and v_c, respectively. Hence, the line current in a phase is sinusoidal and in phase with the corresponding phase (line-to-neutral) voltage. This gives rise to the idea that "*the best kind of load is a purely resistive one.*"

The main objectives of the compensation strategy based on the *abc* Theory are: (i) to obtain the compensated current proportional to the voltage and (ii) to obtain the compensated current with a minimum rms value, capable of delivering the same active power as that of the original current of the load.

However, in most cases under nonsinusoidal conditions, $p_{3\phi}$ and $(v_a^2 + v_b^2 + v_c^2)$ vary, so that the linearity between voltages and currents no longer exists. An example is given in Fig. 3-21. When the hypothetical phase voltages v_a, v_b and v_c, and the generic load currents i_a, i_b and i_c are used in (3.126) and (3.127), neither the waveforms of the active currents, nor those of the nonactive currents have any similarity to those of the voltages. Although the waveforms of the generic load currents and the minimized active currents i_{wa}, i_{wb}, and i_{wc} differ significantly, (3.128) is always valid and they produce the same three-phase instantaneous active power ($p_{3\phi}$), as shown in Fig. 3-21.

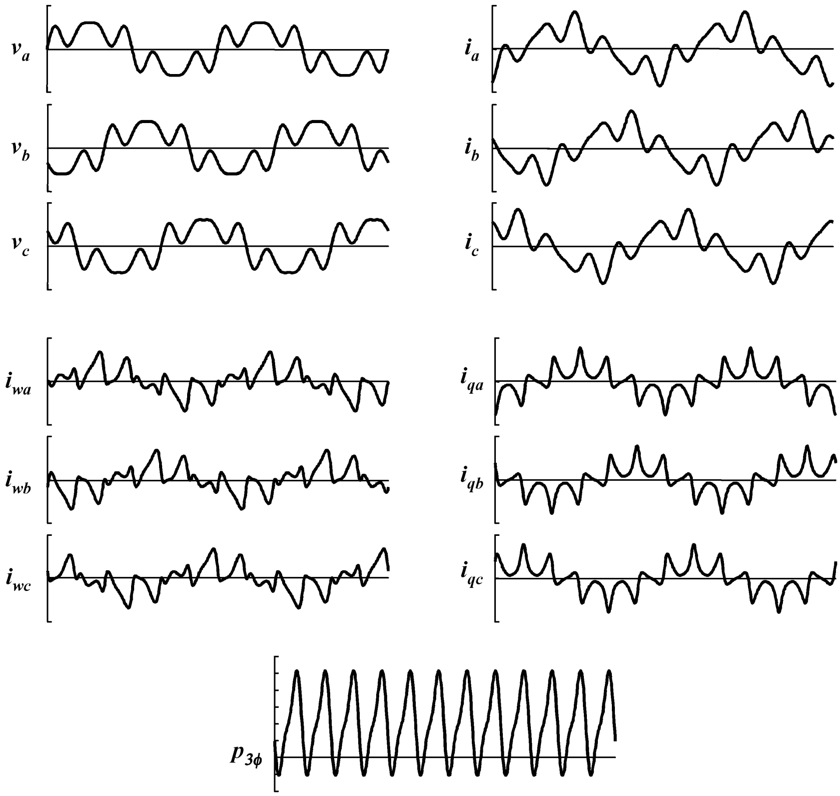


Figure 3-21. An example of minimized currents i_{wa}, i_{wb}, and i_{wc}.

Another point that should be reinforced is that the linearity between voltages and currents cannot guarantee *constant* three-phase instantaneous active power. An example is given in Fig. 3-22. This important feature must be kept in mind when designing controllers for active power-line conditioners. Although the linearity does not guarantee constant instantaneous active power, there are some special cases. In these cases, even under nonsinusoidal voltages, proportional currents can produce constant instantaneous active power, as well as zero three-phase instantaneous reactive power. One interesting case is given in Fig. 3-23. This figure suggests that, from an energy-flow point of view, rectangular supply voltages might be adequate to the voltage source for three-phase diode bridges. A problem might be encountered in that the rectangular voltages could not be transmitted for long distances, keeping the same waveshape along the transmission line.

In practice, most cases do not produce constant three-phase active power by making the currents proportional to the voltages. Anyway, the minimized active currents from (3.127) never generate three-phase instantaneous reactive power. This will be shown later by using the *p-q* Theory, because the *abc* Theory does not have a well-accepted expression for reactive-power calculation.

It was seen that the linearity between voltage and current waveforms can or cannot draw constant three-phase active power from a generic voltage source. To finalize this discussion, the case shown in Fig. 3-24 is an example of voltage and current waveforms that are different even though they draw constant three-phase active power from the source.

The three-phase phase voltages of the previous examples shown in Fig. 3-22 and Fig. 3-24 include a fundamental component and fifth- and seventh-harmonic components. Both harmonic components have equal amplitude and are negative- and positive-sequence components, respectively. Moreover, they have the same phase angle, that is, they are in phase with each other. This is a very particular case. These voltages produce a constant three-phase active power if multiplied by sinusoidal and balanced line currents, as can be seen in Fig. 3-24. Normally, nonsinusoidal voltages do not produce constant three-phase active power if multiplied by sinu-

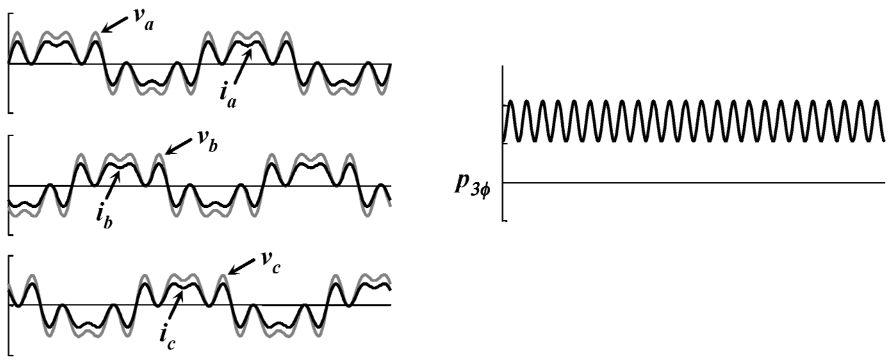


Figure 3-22. Currents proportional to the phase voltages.

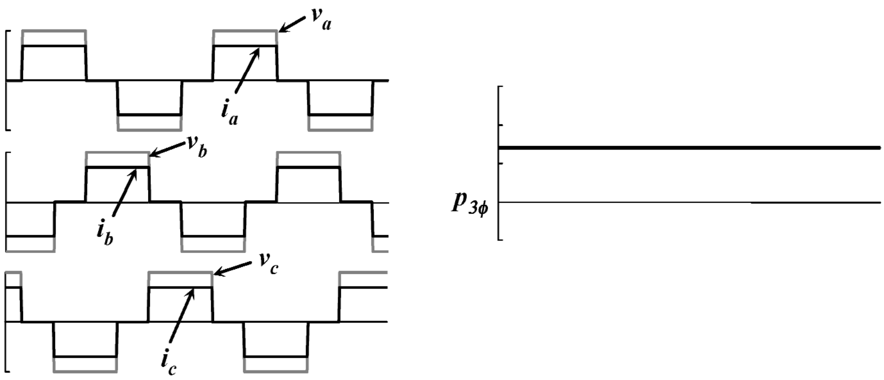


Figure 3-23. Special case of linearity between voltages and currents that produces constant three-phase active power.

soidal currents, or vice-versa. However, this is a very special case that can be better understood by using the general power equations in the *p-q* Theory in terms of symmetrical components. The currents in Fig. 3-24 consist only of a fundamental positive-sequence component, $\dot{I}_{+1} = I_{+1} \angle 0°$, whereas the voltages have three components: $\dot{V}_{+1} = V_{+1} \angle 0°$, $\dot{V}_{-5} = V \angle 0°$, and $\dot{V}_{+7} = V \angle 0°$. Thus, from (3.96) it is possible to see that the unique term that could produce constant imaginary power $\overline{q}$ is related to the phasors $\dot{I}_{+1}$ and $\dot{V}_{+1}$. However, they have the same phase angle and, therefore, $\overline{q}$ is zero. Contrarily, the constant real power $\overline{p}$ reaches the maximum due to the product of parallel phasors, as can be seen in (3.95).

The cross products of voltage and current phasors at different frequencies and/or from different sequence components appear in the oscillating real ($\tilde{p}$) and imaginary ($\tilde{q}$) powers. Surprisingly, as shown in Fig. 3-24 and proven in (3.98), the oscillating real power $\tilde{p}$ is zero, because $\dot{V}_{-5} = V \angle 0°$ and $\dot{V}_{+7} = V \angle 0°$ have the same amplitude and phase angle. Both products of $\dot{I}_{+1}$ with $\dot{V}_{-5}$, and with $\dot{V}_{+7}$ generate oscillating

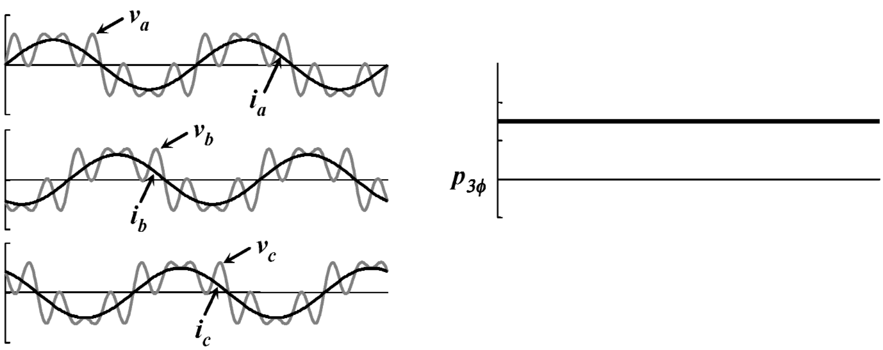


Figure 3-24. Special case under nonsinusoidal conditions.

powers at 6ω, where ω is the fundamental angular frequency. These power terms are summed up in (3.99) for determining the oscillating imaginary power $\tilde{q}$, whereas they are subtracted in (3.98) for calculation of $\tilde{p}$.

With certain restrictions, the calculation of the imaginary power in the *p-q* Theory is being used to analyze power behaviors derived from the decomposition of currents into active and nonactive parts. A significant difference would exist between the active currents i_{wa}, i_{wb}, and i_{wc} given in (3.127) and the real currents i_{ap}, i_{bp}, and i_{cp} determined from (3.104) if zero-sequence components were present. The same situation occurs with the nonactive currents i_{qa}, i_{qb}, and i_{qc} given in (3.126) and the imaginary currents i_{aq}, i_{bq}, and i_{cq} determined from (3.106). Hence, the imaginary power as calculated in the *p-q* Theory should be applied with limitations to the analysis of the concept of three-phase instantaneous reactive power as described above for the *abc* Theory.

3.4.2. Generalized Fryze Currents Minimization Method

If linearity between voltage and current is desired when performing power compensation, an extension of the minimization method shown above may be used to guarantee the linearity, even under distorted and/or unbalanced source voltages. This alternative method of compensation is known as *the generalized Fryze currents minimization method* [40,44].

The basic idea consists of determining the minimized currents from the *average value* of the three-phase instantaneous active power ($\bar{p}_{3\phi}$), instead of using the three-phase instantaneous active power ($p_{3\phi}$) as in (3.126) and (3.127). Moreover, in order to obtain an average value for the equivalent conductivity ($i = G_e v$), the instantaneous sum of squared phase voltages have also to be replaced by a sum of the squared rms values of voltages, as shown below. The new minimized currents—*the generalized Fryze currents*—are represented by the symbols $i_{\overline{w}a}$, $i_{\overline{w}b}$, and $i_{\overline{w}c}$, and are defined as:

$$i_{\overline{w}k} = G_e v_k; \qquad k = (a, b, c) \tag{3.129}$$

If the admittance G_e is represented by an average value instead of a varying instantaneous value, the linearity between voltage and current is ensured. No restriction should be given to the three-phase voltage, allowing it to be distorted and/or unbalanced. The average admittance G_e is obtained from an old concept of aggregate voltage,* as follows:

$$G_e = \frac{\bar{p}_{3\phi}}{V_\Sigma^2} \tag{3.130}$$

*Buchholz [39] established the concept of aggregate current and voltage in 1919 to define apparent power for ac systems under nonsinusoidal conditions. Recently, Emanuel [22], Depenbrock [40], and Czarnecki [28] have also used this concept.

where

$$\bar{p}_{3\phi} = \frac{1}{T}\int_0^T p_{3\phi}(t)dt = \frac{1}{T}\int_0^T (v_a i_a + v_b i_b + v_c i_c)dt \tag{3.131}$$

and

$$V_\Sigma = \sqrt{V_a^2 + V_b^2 + V_c^2} = \sqrt{\frac{1}{T}\left(\int_0^T v_a^2(t)dt + \int_0^T v_b^2(t)dt + \int_0^T v_c^2(t)dt\right)} \tag{3.132}$$

Alternatively, in order to reduce computation efforts in a real implementation of a controller for an active power-line conditioner, the average admittance G_e can be obtained by means of a low-pass filter or a moving-average filter that determines

$$G_e = \frac{1}{T}\int_0^T g_e(t)dt = \frac{1}{T}\int_0^T \left(\frac{v_a i_a + v_b i_b + v_c i_c}{v_a^2 + v_b^2 + v_c^2}\right)dt \tag{3.133}$$

The currents in Fig. 3-22 and Fig. 3-23 could be considered to be the result of the above minimization method (generalized Fryze currents). Unfortunately, this extended method imposes some dynamics in a real implementation, because some time is needed to measure and calculate the rms values of the phase voltages that are used in the calculation of the aggregate voltage V_Σ. Some dynamics are also associated with the extraction of the average value ($\bar{p}_{3\phi}$) from the three-phase instantaneous active power. Alternatively, a low-pass filter can be used to determine the average admittance G_e, defined in (3.133). This is not the case if the method given by (3.127) is used, since it deals with instantaneous voltage and current values. Therefore, (3.127) can be considered as an "instantaneous" control algorithm for minimizing load currents, while (3.129) cannot.

In most cases, the generalized Fryze currents given in (3.129) and the minimization method given in (3.127) produce different compensated (minimized) currents under nonsinusoidal conditions. This point is evidenced in Fig. 3-25, where they are applied to compensate for the load currents under nonsinusoidal source voltages. Although the minimized currents produce different instantaneous powers, both currents produce exactly the same three-phase *average* active power ($\bar{p}_{3\phi}$) required by the load. Thus, the generic load currents i_a, i_b, and i_c, the active currents i_{wa}, i_{wb}, and i_{wc}, as well as the generalized Fryze currents $i_{\bar{w}a}$, $i_{\bar{w}b}$, and $i_{\bar{w}c}$ are well equivalent from the point of view of average energy transfer.

The generalized Fryze currents given in (3.129) and the active currents given in (3.127) do not produce any reactive power, whereas the generalized Fryze currents method offers the best result when reduction in losses is the main goal. Fig. 3-25 shows that this method produces the lowest rms currents values to transmit the same three-phase average active power. Further, it is possible to demonstrate that the aggregate values of the currents $i_{\bar{w}a}$, $i_{\bar{w}b}$, and $i_{\bar{w}c}$ are smaller than i_{wa}, i_{wb},

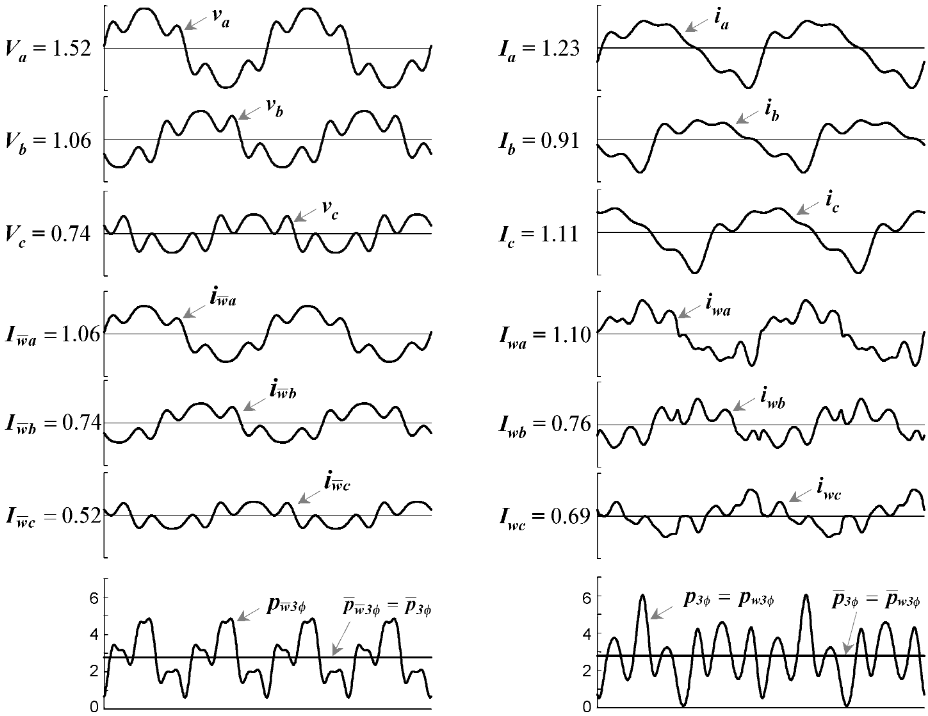


Figure 3-25. Comparison between the generalized Fryze currents and the minimization method.

and i_{wc}. For the sinusoidal case, (3.127) and (3.129) produce the same result. Thus,

$$I_{\overline{w}\Sigma} = \sqrt{I_{\overline{w}a}^2 + I_{\overline{w}b}^2 + I_{\overline{w}c}^2} \le I_{w\Sigma} = \sqrt{I_{wa}^2 + I_{wb}^2 + I_{wc}^2} \tag{3.134}$$

The example given above proves that both methods of minimization may not be used to compensate for the load currents if a *constant* three-phase instantaneous active power drawn from the source is required under nonsinusoidal conditions. Moreover, it has been proved that the linearity between voltage and current is not sufficient to guarantee an optimal (constant) power flow to the network.

Three-phase, four-wire systems may contain zero-sequence components. If these components are simultaneously present in voltage and current, they produce zero-sequence power. The zero-sequence power is a kind of active power and never contains any reactive portion according to the definition given above. Zero-sequence components should be avoided in three-phase systems, because they cannot produce three-phase constant power, as explained in the general equations of the *p-q* Theory in terms of symmetrical components. The minimization methods given in the *abc* Theory are inappropriate to deal with zero-sequence components. Some extra efforts have to be made with the above-described minimization methods in order to deal properly with zero-sequence components.

Figure 3-26 shows an example that evidences some drawbacks of the minimization methods defined in the *abc* Theory when applied in the presence of zero-sequence voltage components. This figure shows three-phase, arbitrarily chosen voltages with fundamental negative- and zero-sequence components, and distorted with second- and third-harmonic components. The load currents are also shown in Fig. 3-26. These load currents are similar to those in a three-phase thyristor rectifier. Thus, the load currents contain no zero-sequence component, although a three-phase four-wire compensator and network should be considered to agree with the results shown in Fig. 3-26. The phase angle of the currents corresponds to a firing angle of 45° in the thyristor rectifier. Therefore, the load currents produce a large amount of reactive power. For comparison, both minimization methods were applied to compensate for the reactive power of the load, since the fundamental component of compensated currents i_{wa}, i_{wb}, and i_{wc}, and $i_{\bar{w}a}$, $i_{\bar{w}b}$, and $i_{\bar{w}c}$ are in phase with the fundamental phase voltages. However, the sum $i_{wa} + i_{wb} + i_{wc} = i_{So}$, as well

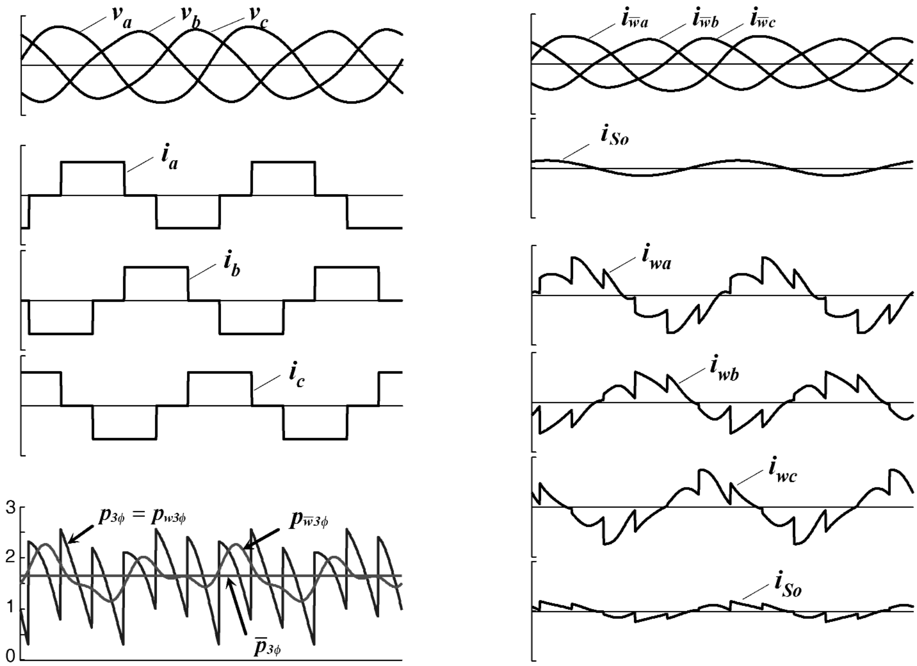


Figure 3-26. Application example of the generalized Fryze currents method and the active current minimization method in the presence of zero-sequence components only in the voltages.

as the sum $i_{\overline{w}a} + i_{\overline{w}b} + i_{\overline{w}c} = i_{So}$, are not equal to zero. In other words, the minimization methods force the compensator to draw undesirable zero-sequence currents (neutral currents) from the network, which are not present in the original load currents.

3.5. COMPARISONS BETWEEN THE *p-q* THEORY AND THE *abc* THEORY

The *p-q* Theory can identify voltage and current components on the $\alpha\beta$ axes, which are dependent only on the real power p or on the imaginary power q. In other words, it is possible to separate the real and the imaginary voltages, as well as the real and imaginary currents, which contribute only to the real or to the imaginary powers. Similarly, the *abc* Theory determines the active currents i_{wa}, i_{wb}, and i_{wc} that contribute only to the three-phase instantaneous active power, and the nonactive currents i_{qa}, i_{qb}, and i_{qc} that contribute only to the three-phase instantaneous reactive power. The current decomposition given in (3.22) is rewritten here as

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \qquad (3.135)$$

A compensation algorithm to determine the instantaneous imaginary currents can be derived directly from (3.135) by just calculating the inverse transformation of $i_{\alpha q}$ and $i_{\beta q}$, that is,

$$\begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \qquad (3.136)$$

Therefore, in analogy to compensation of the nonactive current given by (3.126) in the *abc* Theory, a compensation algorithm based on the *p-q* Theory can be realized to compensate only for the imaginary power. If a shunt compensator, as shown in Fig. 3-10, draws the imaginary current $i_{Ca} = -i_{aq}$, $i_{Cb} = -i_{bq}$, and $i_{Cc} = -i_{cq}$, the network supplies only real current (i_{ap}, i_{bp}, and i_{cp}) of a generic load current (i_a, i_b, and i_c). This case disregards zero-sequence components. Figure 3-27 illustrates the whole algorithm that determines the instantaneous values of compensating currents that should be drawn by the shunt compensator. Although this control algorithm demands more calculation efforts than those for the nonactive current compensation based on the *abc* Theory, it involves only algebraic operations, that is, it contain no dynamic blocks. This means that the instantaneous imaginary currents, as well as the nonactive currents, can be determined instantaneously. If no zero-sequence components are present, the following relation for the compensation method based on the *p-q* Theory is valid:

$$\begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} \tag{3.137}$$

Otherwise, (3.108) should be considered.

In contrast to the instantaneous nonactive currents given in (3.126), the instantaneous imaginary currents obtained in Fig. 3-27 are not influenced by zero-sequence components. The *abc*-phase voltages, as well as the three-phase instantaneous active power $p_{3\phi}$ used in (3.126), can contain zero-sequence components, whereas the $\alpha\beta$ voltages and the imaginary power q in Fig. 3-27 are not affected by these components. This difference is evidenced in Fig. 3-28, where the compensated current (source current) derived from both methods are plotted together for the same case of phase voltages and load currents considered in Fig. 3-26. No neutral current is drawn by the shunt compensator if its controller is based on the *p-q* Theory, whereas an undesirable neutral current appears in the case of the *abc* Theory.

Although the load currents i_a, i_b, and i_c in Fig. 3-26 and in Fig. 3-28 are strongly distorted to compensate, they do not contain any zero-sequence current. That is, i_a+

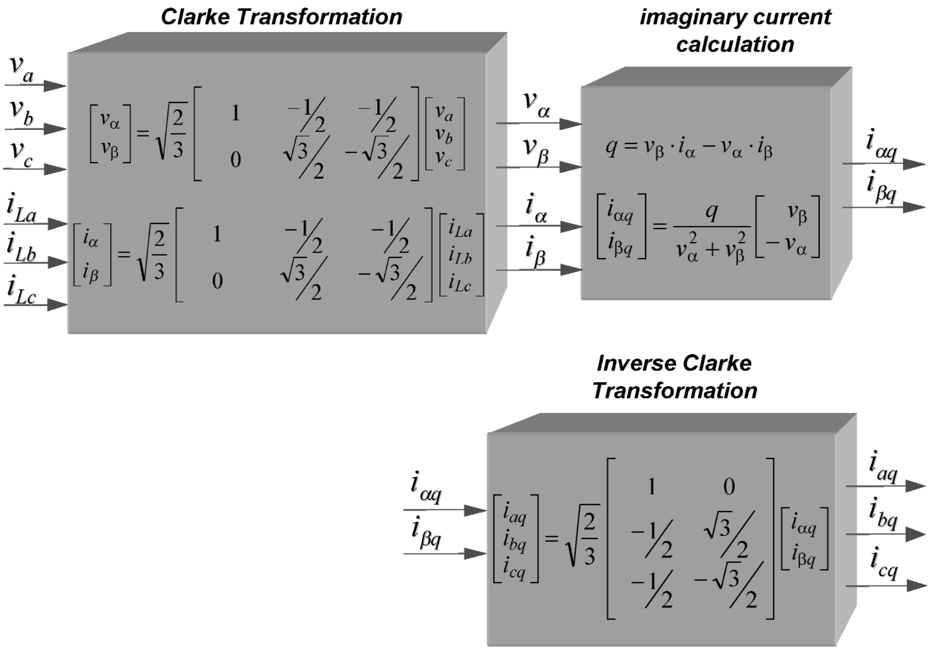


Figure 3-27. Control algorithm for imaginary current compensation based on the *p-q* Theory.

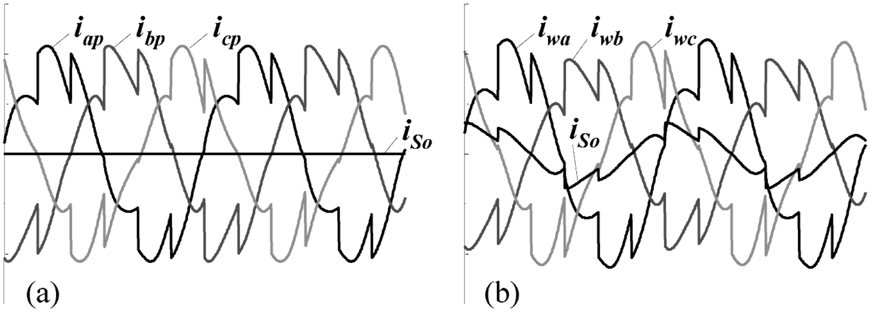


Figure 3-28. Compensated currents. (a) Real currents $i_{kp} = i_k - i_{kq}$, $k = a, b, c$ (*p-q* Theory). (b) Active currents $i_{wk} = i_k - i_{qk}$, $k = a, b, c$ (minimization method).

$i_b + i_c = 0$. Unfortunately, an undesirable neutral current would be generated by a shunt compensator controlled by the minimization method given in (3.126). This neutral current flows through the source, since $i_{So} \neq 0$, as shown in Fig. 3-28(b). Contrarily, the control algorithm based on the *p-q* Theory can compensate properly for the load currents in the presence of zero-sequence voltages, without generating zero-sequence currents to the source. That is, $i_{So} = i_{ap} + i_{bp} + i_{cp} = 0$, as can be seen in Fig. 3-28(a).

If no zero-sequence components are present in both phase voltages and line currents, the imaginary current determined from (3.136) is identical to the nonactive current from (3.126). Consequently, the active current $i_{wk} = i_k - i_{qk}$, $k = (a, b, c)$, and the real current $i_{kp} = i_k - i_{kq}$, $k{=}(a, b, c)$ are also the same in this case. However, in the presence of zero-sequence components, the compensation methods result in two different solutions, although neither i_{wk} nor i_{kp} produce imaginary power q, and neither i_{qk} nor i_{kq} produce active power, that is,

$$\begin{cases} q = \dfrac{1}{\sqrt{3}}(v_{ab}i_{wc} + v_{bc}i_{wa} + v_{ca}i_{wb}) = 0 \\[2ex] q = \dfrac{1}{\sqrt{3}}(v_{ab}i_{cp} + v_{bc}i_{ap} + v_{ca}i_{bp}) = 0 \end{cases} \tag{3.138}$$

$$\begin{cases} p_{3\phi} = v_a i_{qa} + v_b i_{qb} + v_c i_{qc} = 0 \\ p_{3\phi} = v_a i_{aq} + v_b i_{bq} + v_c i_{cq} = 0 \end{cases} \tag{3.139}$$

Equation (3.17) shows that $p_{3\phi} = p + p_0$. On the other hand (3.139) shows that the shunt compensator does not supply any energy to the load all the time. Therefore, the three-phase instantaneous active power ($p_{3\phi}$) of the source is identical to that of the load for both compensation methods used in Fig. 3-28. Since the minimized currents i_{wa}, i_{wb}, and i_{wc}, as well as the supply voltages, contain zero-sequence components, the source will supply a zero-sequence power through its own zero-sequence voltage components. Contrarily, the source does not supply a zero-sequence power if the shunt compensator is controlled by the algorithm given in Fig. 3-27. Therefore, the currents i_{wa}, i_{wb}, and i_{wc}, and i_{ap}, i_{bp}, and i_{cp} produce different real and zero-sequence powers. However, their sum, $p + p_0$, must be equal to $p_{3\phi}$ all the time. Note that the considered load currents i_a, i_b, and i_c do not contain zero-sequence components. Thus, the real power produced by i_a, i_b, and i_c, and i_{ap}, i_{bp}, and i_{cp} must be identical.

The example given above evidences that the *p-q* Theory is more efficient to deal with nonsinusoidal currents, particularly when zero-sequence components are present. One advantage of using the power definitions given in the *p-q* Theory is the possibility of compensating separately the powers p, q, and p_0. This is the reason why the *p-q* Theory can compensate load currents to provide constant instantaneous power to the source, even under distorted and/or unbalanced voltages. The controllers of the active power-line conditioners presented in the next chapters exploit widely this feature.

The next chapter deals with shunt active filters. It shows that the *p-q* Theory can deal properly with zero-sequence components. Moreover, it can compensate for load currents so as to produce sinusoidal currents to the source, even if the voltages are distorted and/or unbalanced. If desired, the *p-q* Theory can also be used to force the compensated currents to draw constant three-phase instantaneous active power from the source, even in the presence of distorted and/or unbalanced voltages.

3.5.1. Selection of Power Components to be Compensated

One significant advantage of using the *p-q* Theory in designing controllers for active power-line conditioners is the possibility of independently selecting the portions of real, imaginary, and zero-sequence powers to be compensated. Some times, it is convenient to separate these powers into their average and oscillating parts, that is,

$$\begin{array}{lccccc}
\text{Real power:} & p & = & \bar{p} & + & \tilde{p} \\
\text{Imaginary power:} & q & = & \bar{q} & + & \tilde{q} \\
\text{Zero-sequence power:} & p_0 & = & \bar{p}_0 & + & \tilde{p}_0 \\
 & & & \text{Average powers} & & \text{Oscillating powers}
\end{array} \tag{3.140}$$

The idea is to compensate all undesirable power components generated by nonlinear loads that can damage or make the power system overloaded or stressed by harmonic pollution. In this way, it would be desirable for a three-phase balanced power-generating system to supply only the average real power $\bar{p}$ of the load. Thus, all other power components required by the nonlinear load, that is, $\tilde{p}$, $\bar{q}$, $\tilde{q}$, $\bar{p}_0$, and $\tilde{p}_0$, should be compensated by a shunt compensator connected as close as possible to this load.

In Fig. 3-25 and Fig. 3-26, examples are shown to prove that the current minimization methods based on the *abc* Theory cannot guarantee constant instantaneous power drawn from the source, principally in the presence of zero-sequence voltage components. Even more difficult is to find a solution based on the current minimization methods to compensate zero-sequence currents in the presence of zero-sequence voltages, such that constant real (active) power is drawn from the source.

If there is no zero-sequence current to be compensated, as in the example given in Fig. 3-26, the zero-sequence power p_0 is always zero. In this case, a shunt compensator as shown in Fig. 3-10 should be controlled to compensate the oscillating real power $\tilde{p}$ and the whole imaginary power $q = \overline{q} + \tilde{q}$. This guarantees constant instantaneous power ($\overline{p}$) drawn from the source with reduced losses in the transmission system, since the imaginary power of the load is also being compensated. This is done by selecting the powers $\tilde{p}$ and q of the load to be compensated in the control algorithm shown in Fig. 3-11, thus making the following compensating current calculation:

$$\begin{bmatrix} i_{c\alpha} \\ i_{c\beta} \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} \tag{3.141}$$

$$\begin{bmatrix} i_{Ca}^* \\ i_{Cb}^* \\ i_{Cc}^* \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{c\alpha} \\ i_{c\beta} \end{bmatrix} \tag{3.142}$$

Now, the new compensated currents i_{Sa}, i_{Sb}, and i_{Sc} ($i_{Sk} = i_k - i_{Ck}$, $k = a, b, c$) that flow in the source are those shown in Fig. 3-29 if the same load currents and source voltages of the example given in Fig. 3-26 are considered.

In contrast to the currents compensated by the generalized Fryze current method, the compensation algorithm based on the *p-q* Theory forces the power source to provide currents that are not proportional to its phase voltages. Hence, a group of nonlinear loads compensated by a shunt compensator with a controller based on (3.141) and (3.142) are not "seen" as linear impedances or pure resistive loads by the power source. Nevertheless, this controller guarantees that the compensated currents draw *constant* instantaneous real power p from the source. Although the generalized Fryze current method provides the smallest rms values of currents to draw the same average active power, it cannot guarantee *constant* instantaneous real power p drawn from the source. Since the load currents do not have zero-sequence components, the three-phase instantaneous active power $p_{S3\phi}$ of the source is equal to the average real power $\overline{p}$ of the load.

3.6. SUMMARY

In this chapter, several important of power definitions have been covered, emphasizing their use in the control of active power-line conditioners. Some important conclusions are summarized below.

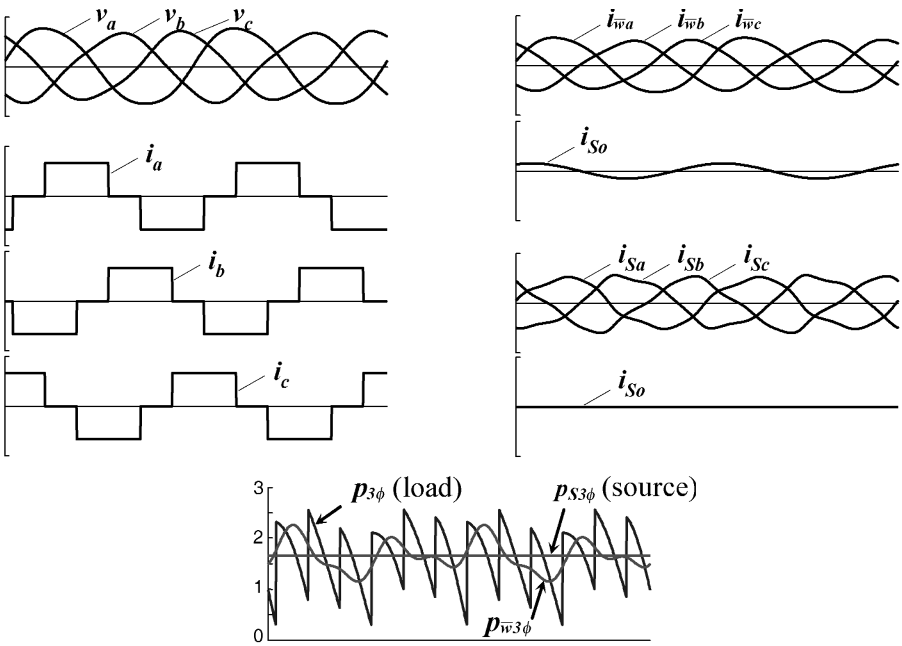


Figure 3-29. Compensated currents: (a) by the Generalized Fryze Currents Method ($i_{\overline{w}a}$, $i_{\overline{w}b}$, and $i_{\overline{w}c}$, and source power $p_{\overline{w}3\phi}$), and (b) by the *p-q* Theory (i_{Sa}, i_{Sb}, i_{Sc}, and source power $p_{S3\phi}$).

1. The instantaneous real and imaginary powers defined in the time domain form a consistent basis for efficient algorithms to be applied to the control of active power-line conditioners.
2. Clear physical meanings are assigned to the real power, the imaginary power, and the zero-sequence power in the *p-q* Theory.
3. The compensation algorithms established through minimization methods are relatively simple to implement. However, they are inapplicable to three-phase, four-wire systems, and cannot guarantee constant active power to the source.
4. The generalized Fryze current method results in line currents proportional to the phase voltages and gives the smallest rms value for the compensated currents. However, it is not an "instantaneous" algorithm (it is not an algebraic set of equations).
5. The compensation algorithm based on the *p-q* Theory is very flexible. The undesirable powers to be compensated can be conveniently selected. The instantaneous imaginary power is calculated without time delay ("instantaneously"). The compensation algorithm using the $\alpha\beta0$ transformation can allow three-phase loads to provide constant instantaneous active power to the source, even if the supply voltages are unbalanced and/or contain harmonics.

However, it is more complex than the other methods mentioned in (3) and (4).

REFERENCES

[1] S. Fryze, "Wirk-, Blind- und Scheinleistung in elektrischen Stromkreisen mit nicht-sinusförmigem Verlauf von Strom und Spannung," *ETZ-Arch. Elektrotech.,* vol. 53, 1932, pp. 596–599, 625–627, 700–702.

[2] H. Akagi, Y. Kanazawa, and A. Nabae, "Principles and Compensation Effectiveness of a Instantaneous Reactive Power Compensator Devices," in *Meeting of the Power Semiconductor Converters Researchers—IEE-Japan,* SPC-82-16, 1982 (in Japanese).

[3] H. Akagi, Y. Kanazawa, and A. Nabae, "Generalized Theory of Instantaneous Reactive Power and Its Applications," *Transactions of the IEE-Japan, Part B,* vol. 103, no.7, 1983, pp. 483–490 (in Japanese).

[4] H. Akagi, Y. Kanazawa, and A. Nabae, "Generalized Theory of the Instantaneous Reactive Power in Three-Phase Circuits," in *IPEC'83—International Power Electronics Conference,* Tokyo, Japan, 1983, pp. 1375–1386.

[5] H. Akagi, Y. Kanazawa, and A. Nabae, "Instantaneous Reactive Power Compensator Comprising Switching Devices Without Energy Storage Components," *IEEE Trans. Ind. Appl.,* vol. IA-20, no. 3, 1984, pp. 625–630.

[6] M. S. Erlicki and A. Emanuel-Eigeles, "New Aspects of Power Factor Improvements Part I—Theoretical Basis," *IEEE Trans. on Industry and General Applications,* vol. IGA-4, 1968, July/August, pp. 441–446.

[7] H. Sasaki and T. Machida, "A New Method to Eliminate AC Harmonic by Magnetic Compensation—Consideration on Basic Design," *IEEE Trans. on Power Apparatus and Syst.,* vol. 90, no. 5, 1970, pp. 2009–2019.

[8] T. Fukao, H. Iida, and S. Miyairi, "Improvements of the Power Factor of Distorted Waveforms by Thyristor Based Switching Filter," *Transactions of the IEE-Japan, Part B,* vol. 92, no.6, 1972, pp. 342–349 (in Japanese).

[9] L. Gyugyi and B. R. Pelly, *Static Power Frequency Changers: Theory, Performance and Application,* Wiley, 1976.

[10] M. Depenbrock, *Untersuchungen über die Spannungs- und Leistungsverhältnisse bei Umrichtern ohne Energiespeicher,* Ph.D. Thesis, TH Hannover, Germany, 1962.

[11] F. Harashima, H Inaba, and K. Tsuboi, "A Closed-loop Control System for the Reduction of Reactive Power Required by Electronic Converters," IEEE Trans. IECI, vol. 23, no. 2, May 1976, pp. 162–166.

[12] L. Gyugyi and E. C. Strycula, "Active ac Power Filters," in *Proceedings IEEE Industrial Applications Annual Meeting,* vol. 19-C, 1976, pp. 529–535.

[13] I. Takahashi and A. Nabae, "Universal Reactive Power Compensator," in *IEEE—Industry Application Society Annual Meeting Conference Record,* 1980, pp. 858–863.

[14] I. Takahashi, K. Fujiwara, and A. Nabae, "Distorted Current Compensation System Using Thyristor Based Line Commutated Converters," *Transactions of the IEE-Japan, Part B,* vol. 101, no. 3, 1981, pp. 121–128 (in Japanese).

[15] E. Clarke, *Circuit Analysis of A-C Power Systems, Vol. I—Symmetrical and Related Components,* Wiley, 1943.

[16] J. L. Willems, “Instantaneous Sinusoidal and Harmonic Active and Deactive Currents in Three-phase Power Systems,” *ETEP—Eur. Trans. Elect. Power Eng.,* vol. 4, no. 5, Sep./Oct. 1994, pp. 335–346.

[17] W. V. Lyon, “Reactive Power and Unbalanced Circuits,” *Electrical World,* vol. 75, no. 25, 1920, pp. 1417–1420.

[18] F. Buchholz, “Die Drehstrom-Scheinleistung bei ungleichmäßiger Belastung der drei Zweige,” *Licht und Kraft, Zeitschrift für elekt. Energie-Nutzung,* no. 2, Jan. 1922, pp. 9–11.

[19] C. I. Budeanu, “Puissances reactives et fictives,” *Instytut Romain de l’Energie,* pub. no. 2, Bucharest, 1927.

[20] C. I. Budeanu, “The Different Options and Conceptions Regarding Active Power in Non-sinusoidal Systems”, *Instytut Romain de l’Energie,* pub. no. 4, Bucharest, 1927.

[21] R. M. Kerchner, G. F. Corcoran, *Alternating-Current Circuits,* Wiley, 1938.

[22] A. E. Emanuel, “Apparent and Reactive Powers in Three-Phase Systems: In Search of a Physical Meaning and a Better Resolution,” *ETEP—Eur. Trans. Elect. Power Eng.,* vol. 3, no. 1, Jan./Feb. 1993, pp. 7–14.

[23] L. S. Czarnecki, “What Is Wrong with the Budeanu Concept of Reactive and Distortion Power and Why It Should Be Abandoned,” *IEEE Trans. Instr. Meas.,* vol. IM-36, no. 3, 1987, pp. 834–837.

[24] P. Filipski and R. Arseneau, “Definition and Measurement of Apparent Power under Distorted Waveform Conditions,” *IEEE Tutorial Course on Non-sinusoidal Situations,* 90EH0327-7, 1990, pp. 37–42.

[25] M. Depenbrock, D. A. Marshall, and J. D. van Wyk, “Formulating Requirements for a Universally Applicable Power Theory as Control Algorithm in Power Compensators,” *ETEP—Eur. Trans. Elect. Power Eng.,* vol. 4, no. 6, Nov./Dec. 1994, pp. 445–455.

[26] L. S. Czarnecki, “Comparison of Power Definitions for Circuits with Nonsinusoidal Waveforms,” *IEEE Tutorial Course on Non-sinusoidal Situations,* 90EH0327-7, 1990, pp. 43–50.

[27] L. S. Czarnecki, “Misinterpretations of Some Power Properties of Electric Circuits,” *IEEE Trans. Power Delivery,* vol. 9, no. 4, Oct. 1994, pp. 1760–1769.

[28] L. S. Czarnecki, “Power Related Phenomena in Three-Phase Unbalanced Systems,” *IEEE Trans. Power Delivery,* vol. 10, no. 3, July 1995, pp. 1168–1176.

[29] *IEEE Standard Dictionary of Electrical and Electronics Terms,* 3rd ed., IEEE/Wiley-Interscience, 1984.

[30] H. Schering, “Die Definition der Schein- und Blindleistung sowie des Leistungsfaktors bei Mehrphasenstrom,” *Elektrotechnische Zeitschrift,* vol. 27., July 1924, pp. 710–712.

[31] V. N. Nedelcu, “Die einheitlische Leistungstheorie der unsymmetrischen und mehrwelligen Mehrphasensysteme,” *ETZ-Arch. Elektrotech.* vol. 84, no. 5, 1963, pp. 153–157.

[32] Z. Nowomiejski, “Generalized Theory of Electric Power,” *Arch. für Elektrotech.,* vol. 63, 1981, pp. 177–182.

[33] A. E. Emanuel, “Energetical Factors in Power Systems with Nonlinear Loads,” *Arch. für Elektrotech.,* vol. 59, 1977, pp. 183–189.

[34] R. Arseneau and P. Filipski, “Application of a Three-Phase Nonsinusoidal Calibration System for Testing Energy and Demand Meters under Simulated Field Conditions,” *IEEE Trans. Power Delivery,* vol. 3, July 1988, pp. 874–879.

[35] E. W. Kimbark, *Direct Current Transmission,* vol. 1, Wiley, 1971.

[36] C. L. Fortescue, "Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks," *A.I.E.E. Trans.,* vol. 37, June 1918, pp. 1027–1140.

[37] E. H. Watanabe, R. M. Stephan, and M. Aredes, "New Concepts of Instantaneous Active and Reactive Powers in Electrical Systems with Generic Loads," *IEEE Trans. Power Delivery,* vol. 8, no. 2, Apr. 1993, pp. 697–703.

[38] M. Aredes and E. H. Watanabe, "New Control Algorithms for Series and Shunt Three-Phase Four-Wire Active Power Filters," *IEEE Trans. Power Delivery,* vol. 10, no. 3, July 1995, pp. 1649–1656.

[39] F. Buchholz, "Die Darstellung der Begriffe Scheinleistung und Scheinarbeit bei Mehrphasenstrom," *Elektro-J. 9,* 1929, pp. 15–21.

[40] M. Depenbrock, "The FBD-Method, a Generally Applicable Tool for Analysing Power Relations," *IEEE Trans. Power Systems,* vol. 8, no. 2, May 1993, pp. 381–387.

[41] T. Furuhashi, S. Okuma, and Y. Uchikawa, "A Study on the Theory of Instantaneous Reactive Power," *IEEE Trans. Ind. Elect.,* vol. 37, no. 1, Feb. 1990, pp. 86–90.

[42] L. Malesani, L. Rossetto, and P. Tenti, "Active Filter for Reactive Power and Harmonics Compensation," in *IEEE-PESC'86—Power Electronics Special Conference,* 86CH2310-1, pp. 321–330.

[43] L. Rossetto and P. Tenti, "Evaluation of Instantaneous Power Terms in Multi-Phase Systems: Techniques and Application to Power-Conditioning Equipments," *ETEP—Eur. Trans. Elect. Power Eng.,* vol. 4, no. 6, Nov./Dec. 1994, pp. 469–475.

[44] P. Tenti, "The Relation Between Global Performance Indexes: Power Factor and FBD Factor," *ETEP—European Transactions on Electrical Power Engineering,* vol. 4, no. 6, Nov./Dec. 1994, pp. 505–507.

[45] J. H. R. Enslin and J. D. van Wyk, "Measurement and Compensation of Fictitious Power under Nonsinusoidal Voltage and Current Conditions," *IEEE Trans. Instr. Meas.,* vol. IM-37, no. 3, 1988, pp. 403–408.

[46] M. A. Slonim and J. D. van Wyk, "Power Components in Systems with Sinusoidal and Nonsinusoidal Voltage and/or Current Conditions," *IEE Proc.,* vol. 135, Pt. B, no. 2, Mar. 1988, pp. 76–84.

[47] D. A. Marshall and J. D. van Wyk, "An Evaluation of the Real-Time Compensation of Fictitious Power in Electric Energy Networks," *IEEE Trans. Power Delivery,* vol. 6, no. 4, Oct. 1991, pp. 1774–1780.

[48] S. Togasawa, T. Murase, H. Nakano, and A. Nabae, "Reactive Power Compensation Based on a Novel Cross-Vector Theory," *Trans. on Ind. App. of the IEE—Japan (IEEJ),* vol. 114, no. 3, Mar. 1994, pp. 340–341 (in Japanese).

[49] A. Nabae, L. Cao, and T. Tanaka, "An instantaneous distortion current compensator without any coordinate transformation," in *Proceedings of the International Power Electronics Conference (IPEC-Yokohama),* 1995, pp. 1651–1655.

[50] A. Nabae and T. Tanaka, "New definition of instantaneous active-reactive current and power based on instantaneous space vectors on polar coordinates in three-phase circuits," *IEEE Trans. Power Delivery,* vol. 11, no. 3, pp. 1238–1243, July 1996.

[51] H. Akagi, S. Ogasawara, and H. Kim, "The Theory of Instantaneous Power in Three-Phase Four-Wire Systems: A Comprehensive Approach," in *IEEE IAS Annual Meeting,* 1999, pp. 431–439.

[52] F. Z. Peng and J. S. Lai, "Generalized Instantaneous Reactive Power Theory for Three-

Phase Power System," *IEEE Trans. on Instr. and Meas.,* vol. 45, no. 1, Feb. 1996, pp. 293–297.

[53] F. Z. Peng, G. W. Ott Jr., and D. J. Adams, "Harmonic and Reactive Power Compensation Based on the Generalized Instantaneous Reactive Power Theory for Three-Phase Four-Wire Systems," *IEEE Trans. on Power Electronics,* vol. 13, no. 6, Nov. 1998, pp. 1174–1181.

[54] X. Dai, G. Liu, and R. Gretsh, "Generalized Theory of Instantaneous Reactive Quantity for Multiphase Power System," *IEEE Trans. on Power Delivery,* vol. 19, no. 3, July 2004, pp. 965–972.