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Closed-loop stable model predictive control of integrating systems with dead time

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ABSTRACT

Model predictive control (MPC) applications in the process industry usually deal with process systems that show time delays (dead times) between the system inputs and outputs. Also, in many industrial applications of MPC, integrating outputs resulting from liquid level control or recycle streams need to be considered as controlled outputs. Conventional MPC packages can be applied to time-delay systems but stability of the closed loop system will depend on the tuning parameters of the controller and cannot be guaranteed even in the nominal case. In this work, a state space model based on the analytical step response model is extended to the case of integrating time systems with time delays. This model is applied to the development of two versions of a nominally stable MPC, which is designed to the practical scenario in which one has targets for some of the inputs and/or outputs that may be unreachable and zone control (or interval tracking) for the remaining outputs. The controller is tested through simulation of a multivariable industrial reactor system.

1. Introduction

In the chemical industry, one can frequently find integrating processes related to liquid level in process tanks, gas pressure in vessels and component concentration in systems containing recycle streams. Time delays between the process inputs and outputs are frequently present in process systems and are mainly associated with fluid transport delays that, in the control loop, may be reflected as input delays, and measurement delays that may be translated into output delays. The presence of time delays in the process model does not constitute a major obstacle to the implementation of the conventional MPC approaches mainly for open loop stable systems and when the controller is based on the step response model [1]. However, in the conventional MPC, stability of the closed-loop system will depend on the selected tuning parameters. So, it is interesting to develop MPC strategies where nominal stability is guaranteed. In the classical approach, the stability of the resulting control loop is forced through indirect methods such as those based on the existence of a Lyapunov function related to the closed loop system. Following this approach, Keerthi and Gilbert [2] showed that the stability of the closed loop system with MPC is obtained when the terminal state is constrained to the origin. If the control problem with the terminal constraint remains feasible, it is easy to show that the optimal cost behaves as a Lyapunov function and stability is obtained. The limitation of this approach is that the terminal constraint can turn the control problem infeasible because of the presence of input constraints and the control horizon being limited. An extension of the terminal state technique to produce a stable MPC was proposed by Michalska and Mayne [3] that converted the terminal state constraint to a terminal set constraint, where the state at the end of the prediction horizon is forced to lie in a set that contains the desired equilibrium state. The terminal set must be a control invariant set in the sense that, once the state enters the terminal set, there will be a linear controller that will keep the state inside the terminal set. This controller is also known as the dual MPC because one has two control laws, one outside the terminal set and other inside the terminal set. Rawlings and Muske [4] proposed a predictive regulator with infinite output horizon (IHMPC) and input and output constraints that for stable systems can be reduced to a finite horizon regulator with a terminal weight. They showed that this controller has recursive feasibility, which means that if, at time step k, the control problem is feasible, then it will remain feasible at any subsequent time step $k+1$, $k+2, \ldots$. They also showed that in these conditions, the control cost function is strictly decreasing and can be interpreted as a Lyapunov function for the closed loop system. The same authors [5] extended the infinite horizon regulator to the output tracking case and to the case where there are unmeasured disturbances. The approach consists in including an additional layer in the controller where the system steady state is updated based on the desired set point and estimated disturbances. At each sampling step the calculated steady state is fed to the infinite horizon regulator that drives the

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system to the computed set point without offset. The main question about their approach is that, since the two layers are executed at the same sampling period, it is not clear if the interaction between the two layers can disrupt the stability of the infinite horizon regulator. Carrapiço and Odloak [6] extended the work of Rawlings and Muske [4] and Muske and Rawlings [5] to the case of output tracking for systems with unknown or unreachable steady-states. They proposed to separate the IHMPC problem of integrating processes in two sub-problems. In the first sub-problem the objective is to zero the integrating states of the system and in the second sub-problem the system outputs are led to the desired values. The work of Carrapiço and Odloak [6] was extended in Alvarez et al. [7] to integrating systems with zone control of the outputs and systems with optimizing input targets.

The controller proposed here extends the MPC presented in Alvarez et al. [7] for time-delayed systems with stable and integrating modes. This case encompasses the large majority of the process systems found in industry as open loop unstable systems are not often found in process industry. The controller is offset free as the adopted state space model is incremental in the input and there is no need to include an intermediate layer in the control structure to compute a feasible steady state as proposed in Muske and Rawlings [5] and Kassmann et al. [8]. For the outputs with targets, the integration with RTO is obtained by fixing the output set point at the optimum value. Stability of the proposed MPC results from the consideration of an infinite output horizon adapted to the output tracking case. To accommodate the practical application, it is included the zone control strategy as in González and Odloak [9].

In the second section of this work, the model considered in the proposed controller is detailed and a nominally stable MPC for integrating systems with dead time is proposed. Two variations of the proposed optimization problem that defines the controller are considered. In Section 3, the stability of the proposed controller is discussed and in Section 4, the two versions of the controller are compared through the simulation of an industrial reactor for the production of ethylene oxide. Finally, in Section 5, the paper is concluded.

2. IHMPC for integrating systems with time delay

To circumvent the need to compute the system steady state that is required to apply the infinite horizon regulator, Lee et al. [10] proposed to formulate the process model in the incremental form as follows

$$x(k+1) = A\,x(k) + B\,\Delta u(k) \quad y(k) = C\,x(k) \tag{1}$$

With the model defined in (1), any steady state corresponds to a point where $\Delta u(k) = 0$ and there is no need to know the explicit value of u at the steady state corresponding to a particular output set-point. In this section a state space model of the form defined in (1) is developed for integrating systems with time delays. As the step response model has been widely used in the MPC technology, this model can be interpreted as a generalization of the step response model that is represented in the analytical form. This model can be applied to time delayed stable, unstable and integrating systems.

2.1. The analytical step response model for systems with dead-time

Initially, the state space model proposed in Rodrigues and Odloak [11] is extended to the case of systems with time delay. For this purpose, assume that the multivariable system has nu inputs and ny outputs from which ni are integrating. Now, consider that for each pair (y_i, u_j), one has a transfer function of the form

$$G_{i,j}(s) = \frac{b_{i,j,0} + b_{i,j,1}s + \cdots + b_{i,j,nb}s^{nb}}{s(s-r_{i,j,1})(s-r_{i,j,2})\cdots(s-r_{i,j,na})}e^{-\theta_{i,j}s}$$

where it is assumed that the poles of $G_{i,j}$ are non-repeated. The step response of the above transfer function can be represented as follows

$$S_{i,j}(s) = \frac{G_{i,j}(s)}{s} = \frac{d^0_{i,j}}{s}e^{-\theta_{i,j}s} + \frac{d^d_{i,j,1}}{s-r_{i,j,1}}e^{-\theta_{i,j}s} + \cdots + \frac{d^d_{i,j,na}}{s-r_{i,j,na}}e^{-\theta_{i,j}s} + \frac{d^i_{i,j}}{s^2}e^{-\theta_{i,j}s} \tag{2}$$

Assuming that Δt is the sampling time (2) is equivalent to

$$S_{i,j}(k\Delta t) = 0, \quad \text{if} \quad k\Delta t \le \theta_{i,j}$$

and

$$S_{i,j}(k\Delta t) = d^0_{i,j} + d^d_{i,j,1}e^{r_{i,j,1}k\Delta t - \theta_{i,j}} + \cdots + d^d_{i,j,na}e^{r_{i,j,na}k\Delta t - \theta_{i,j}} + d^i_{i,j}(k\Delta t - \theta_{i,j}) \tag{3}$$

if $k\Delta t > \theta_{i,j}$

Following the approach of Rodrigues and Odloak [11], for the multivariable system, the step response defined in (2) can be translated into the model:

$$\begin{bmatrix} x^s(k+1) \\ x^d(k+1) \\ x^i(k+1) \end{bmatrix} = \begin{bmatrix} I_{ny} & 0 & \Delta t\, I^*_{ny} \\ 0 & F & 0 \\ 0 & 0 & I^*_{ny} \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \\ x^i(k) \end{bmatrix} + \begin{bmatrix} B^s_0 \\ B^d_0 \\ B^i_0 \end{bmatrix} \Delta u(k) + \begin{bmatrix} B^s_1 \\ B^d_1 \\ B^i_1 \end{bmatrix} \Delta u(k-1) + \cdots + \begin{bmatrix} B^s_{\theta_{\max}} \\ B^d_{\theta_{\max}} \\ B^i_{\theta_{\max}} \end{bmatrix} \Delta u(k-\theta_{\max}) \tag{4}$$

$$y(k) = \begin{bmatrix} I_{ny} & \Psi & 0_{ny\times ny} \end{bmatrix} x(k)$$

where $x^s \in \Re^{ny}$; $x^d \in C^{nd}$, $nd = ny\,nu\,na$; $x^i \in \Re^{ny}$; $y \in \Re^{ny}$ and $\theta_{\max}$ is the largest time delay between any input and any output.

The state vector component $x^s(k)$ corresponds to the integrating states introduced into the model through the adopted incremental form of the input. The state components $x^d(k)$ and $x^i(k)$ correspond respectively to the stable and integrating states of the original system. I^*_{ny} is a diagonal matrix with ones in the entries corresponding to the integrating outputs and zeros in the remaining positions. If the stable poles of the system are non-repeated, matrix F can be represented as follows

$$F = \mathrm{diag}(e^{r_{1,1,1}\Delta t} \ldots e^{r_{1,1,na}\Delta t} \ldots e^{r_{1,nu,1}\Delta t} \ldots e^{r_{1,nu,na}\Delta t} \ldots e^{r_{ny,1,1}\Delta t} \ldots e^{r_{ny,1,na}\Delta t} \ldots e^{r_{ny,nu,1}\Delta t} \ldots e^{r_{ny,nu,na}\Delta t}) \quad F \in C^{nd\times nd}$$

Matrices B^s_l and B^i_l, with $l = 1, \ldots, \theta_{\max}$ can be computed as follows:

$$\text{If } l \ne \theta_{i,j}, \quad \text{then} \quad B^s_l = 0; \quad B^i_l = 0$$

$$\text{If } l = \theta_{i,j}, \quad \text{then} \quad [B^s_l]_{i,j} = d^0_{i,j} + \Delta t d^i_{i,j}; \quad [B^i_l]_{i,j} = d^i_{i,j}.$$

Construction of matrices B^d_l is a little more subtle. If there were no dead times ($l = 0$) then $B^d_0 = D^d F N$, where matrices D^d and N are computed as follows:

$$D^d = \operatorname{diag}(d^d_{1,1,1}\cdots d^d_{1,1,na}\cdots d^d_{1,nu,1}\cdots d^d_{1,nu,na}\cdots d^d_{ny,1,1}\cdots d^d_{ny,1,na}\cdots d^d_{ny,nu,1}\cdots d^d_{ny,nu,na}) \quad D^d \in C^{nd\times nd}$$

$$N = \left.\begin{bmatrix} J \\ J \\ \vdots \\ J \end{bmatrix}\right\} ny, \quad N \in \Re^{nd\times nu}; \quad J = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \quad J \in \Re^{nu\,na\times nu}$$

Alternatively, if $l \neq 0$, then each matrix B^d_l would have the same dimension as $D^d\,FN$ where those elements corresponding to transfer functions with dead time different from l are replaced with zeros.

Finally, matrix Ψ that appears in the output matrix C is given by

$$\Psi = \begin{bmatrix} \Phi & & 0 \\ & \ddots & \\ 0 & & \Phi \end{bmatrix}, \quad \Psi \in \Re^{ny\times nd}, \quad \Phi = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}, \quad \Phi \in \Re^{nu\,na}$$

The model defined in (4) can be represented in the conventional state space representation defined in (1) when the following state is defined:

$$x(k) = \begin{bmatrix} x^s(k)^T & x^d(k)^T & x^i(k)^T & z_1(k)^T & z_2(k)^T & \cdots & z_{\theta_{\max}}(k)^T \end{bmatrix}^T \tag{5}$$

and the following matrices state are considered:

$$A = \begin{bmatrix} I_{ny} & 0 & \Delta t I^* & B^s_1 & B^s_2 & \cdots & B^s_{\theta_{\max}-1} & B^s_{\theta_{\max}} \\ 0 & F & 0 & B^d_1 & B^d_2 & \cdots & B^d_{\theta_{\max}-1} & B^d_{\theta_{\max}} \\ 0 & 0 & I^*_{ny} & B^i_1 & B^i_2 & \cdots & B^i_{\theta_{\max}-1} & B^i_{\theta_{\max}} \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & I_{nu} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & I_{nu} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} B^s_0 \\ B^d_0 \\ B^i_0 \\ I_{nu} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{6}$$

$$C = \begin{bmatrix} I_{ny} & \Psi & 0_{ny\times(ny+nu\theta_{\max})} \end{bmatrix}$$

where $x \in C^{nx}$, $nx = 2ny + nd + nu\,\theta_{\max}$, $z_1, \ldots, z_{\theta_{\max}} \in \Re^{nu}$.

It should be observed that, in comparison to the model defined in (4), the additional components of the state defined in (5) are $z_1, \ldots, z_{\theta_{\max}}$, which have a clear physical interpretation as these components correspond to the past input moves, or $z_j(k) = \Delta u(k-j)$.

Remark 1. Any state space model to be used in the MPC control strategy needs to be detectable. A system is detectable if it is observable, or, all the non-observable states converge to zero [12]. A previous extension of the state space model equivalent to the step response model proposed in Rodrigues and Odloak [11] was proposed in [13]. But, it can be shown that, if their approach is followed to build a model for integrating systems with time delays, then the resulting model may not be detectable when the time delays are not the same for all the inputs. In the Appendix section, it is shown through an example that the model resulting from the approach presented in [13] can be undetectable while the approach proposed here always results in a detectable model

Remark 2. With the model structure defined in (5) and (6), one can show that the equilibrium state corresponding to output y_{ss} is represented as follows

$$x_{ss} = \begin{bmatrix} y^T_{ss} & 0^T & \cdots & 0^T \end{bmatrix}^T$$

This means that, with the proposed model structure, state component x^s represents the output at steady state.

2.2. The proposed MPC for integrating systems with time-delay

Advanced control structures are usually divided into layers [8,13,14] and the process optimizer lies at the top of this structure. The process optimizer defines the optimal operating conditions of the process system. Here, it is assumed that these optimal conditions are passed to a single layer MPC that computes at each sampling step the control inputs that will drive the process to the optimal condition. The optimal operating conditions are usually translated into targets to the manipulated inputs and/or controlled outputs and the controller must be capable of dealing with these targets while preserving stability and performance. Usually, the process system has a number of controlled outputs that is larger than the number of manipulated inputs. Thus the process outputs have to be controlled following a zone control strategy [9]. Methods to design feasible output zones can be found in Lima and Georkakis [15] and Shead et al. [16]. The implementation of unreachable output tracking approaches in model predictive controller has been studied by several authors [17–19] and the approach followed here was presented in Alvarez et al. [7].

The control cost of the MPC proposed here considers an infinite output horizon, zone control of the outputs and possible input and/or output targets for systems with stable and integrating poles. It can be written as follows:

$$\begin{aligned} &\min_{\Delta u_k, y_{sp,k}, \delta_{y,k}, \delta_{i,k}, \delta_{u,k}} V_{1,k} \\ &= \sum_{j=0}^{\infty} ||y(k+j|k) - y_{sp,k} - \delta_{y,k} - (j-\theta_{\max}-m)\Delta t \delta_{i,k}||^2_{Q_y} \\ &+ \sum_{j=0}^{\infty} ||u(k+j|k) - u_{des} - \delta_{u,k}||^2_{Q_u} \\ &+ \sum_{j=0}^{m-1} \left\| \Delta u(k+j|k) \right\|^2_R + ||\delta_{y,k}||^2_{S_y} + ||\delta_{u,k}||^2_{S_u} + ||\delta_{i,k}||^2_{S_i} \end{aligned} \tag{7}$$

Observe that the control objective defined in (7) includes slack variables $\delta_{y,k}$, $\delta_{u,k}$ and $\delta_{i,k}$, which extends the feasibility region of the proposed controller as will be shown later. The variable $y(k+j|k)$ corresponds to the output prediction at time step $k+j$ calculated at time k. The control sequence is defined as follows $\Delta u_k = \begin{bmatrix} \Delta u(k|k)^T & \cdots & \Delta u(k+m-1|k)^T \end{bmatrix}^T$ and u_{des} is the input target. The weighting matrices $Q_y, Q_u, R, S_y, S_u, S_i$ are additional tuning parameters of the controller. Note that in (7), with the adopted zone control strategy $y_{sp,k}$ becomes a constrained decision variable of the control problem instead of a fixed set point.

From the model defined in (6), it is easy to show that

$$\begin{aligned} y(k+\theta_{\max}+m+j|k) &= x^s(k+\theta_{\max}+m|k) \\ &+ j\Delta t D^i x^i(k+\theta_{\max}+m|k) \\ &+ \Psi F^j x^d(k+\theta_{\max}+m|k) \end{aligned} \tag{8}$$

Substituting (8) into the first term on the right hand side of (6), it is easy to see that the infinite sum will be bounded only if the following constraints are included into the control problem:

$$x^s(k+m+\theta_{\max}|k) - y_{sp,k} - \delta_{y,k} = 0 \tag{9}$$

$$x^i(k+m+\theta_{\max}|k) - \delta_{i,k} = 0 \tag{10}$$

If the constraints defined in (9) and (10) are satisfied, the infinite output error sum in (7) can be written as follows

$$\sum_{j=0}^{\infty} ||y(k+j|k) - y_{sp,k} - \delta_{y,k} - (j-\theta_{\max}-m)\Delta t \delta_{i,k}||^2_{Q_y}$$
$$= \sum_{j=0}^{\theta_{\max}+m} ||y(k+j|k) - y_{sp,k} - \delta_{y,k} - (j-\theta_{\max}-m)\Delta t \delta_{i,k}||^2_{Q_y}$$
$$+ \sum_{j=1}^{\infty} ||\Psi F^j x^d(k+\theta_{\max}+m|k)||^2_{Q_y} \tag{11}$$

Analogously, the second term on the right hand side of (7) will be bounded only if the following constraint is satisfied:

$$u(k+m-1|k) - u_{des} - \delta_{u,k} = 0 \tag{12}$$

If (12) is satisfied, then the second infinite sum on the right hand side of (7) can be written as follows

$$\sum_{j=0}^{\infty} ||u(k+j|k) - u_{des} - \delta_{u,k}||^2_{Q_u} = \sum_{j=0}^{m-1} ||u(k+j|k) - u_{des} - \delta_{u,k}||^2_{Q_u} \tag{13}$$

Other constraints that need to be satisfied by the decision variables of the control problem are related to the input bounds and the output zone control. These constraints are the following:

$$-\Delta u_{\max} \le \Delta u(k+j|k) \le \Delta u_{\max}, \quad j = 0, 1, \ldots, m-1 \tag{14}$$

$$u_{\min} \le u(k+j|k) \le u_{\max}, \quad j = 0, 1, \ldots, m-1 \tag{15}$$

$$y_{\min} \le y_{sp,k} \le y_{\max} \tag{16}$$

$$y_{sp,k,i} = y_{des,i} \quad i \in \Omega \tag{17}$$

Observe that constraint (17) is written only for those outputs that have explicit optimizing targets and Ω is the set of outputs that have targets. It is clear that the number of inputs and outputs that receive optimizing targets should not be larger than the number of manipulated inputs.

Substituting (11) and (13) into the control cost defined in (7), $V_{1,k}$ is a quadratic function and the constraints defined in (9), (10), (12), (14)–(17) are all linear in the decision variables of the control problem. Thus one can formulate the optimization problem that defines the proposed controller as follows:

Problem P1.

$$\min_{\Delta u_k, y_{sp,k}, \delta_{y,k}, \delta_{i,k}, \delta_{u,k}} V_{1,k}$$

Subject to (9), (10), (12), (14)–(17)

Remark 3. Problem P1 is always feasible because of the slack variables that are included in the control cost and automatically transferred to the equality constraints. However, there is no guarantee that the control law resulting from the solution to Problem P1 will force the controlled outputs to converge to their control zones/targets and the inputs to converge to their targets. This convergence will be proved for the case where the slack vector $\delta_{i,k}$, which is related the integrating modes of the system, is reduced to zero at the end of the control horizon. This approach gives rise to the two-step strategy that is presented next.

2.3. The two-step MPC for integrating systems with time delays

In the two-step MPC proposed in Carrapiço and Odloak [6], the first step minimizes a norm of slack $\delta_{i,k}$ that is related to the integrating states, which in the case considered here is equivalent to minimizing $x^i(k+m+\theta_{\max}|k)$ because of constraint (10). Next, in the second step, the control cost defined in (7) is minimized but without disrupting the convergence of the problem solved in the first step. Analogously, the two-step controller proposed here can be based on the sequential solution of the following problems:

Problem P2a.

$$\min_{\Delta u_a(k+j|k),\, j=0,\ldots,m-1} V_{2a,k} = \left\| x^i_a(k+m+\theta_{\max}|k) \right\|^2_{Q_i}$$

subject to

$$-\Delta u_{\max} \le \Delta u_a(k+j|k) \le \Delta u_{\max}, \; j = 0, 1, \ldots, m-1$$
$$u_{\min} \le u_a(k+j|k) \le u_{\max}, \; j = 0, 1, \ldots, m-1$$

Problem P2b.

$$\min_{\Delta u_{b,k}, y_{sp,k}, \delta_{y,k}, \delta_{u,k}} V_{2b,k} = \sum_{j=0}^{\infty} ||y(k+j|k) - y_{sp,k} - \delta_{y,k}||^2_{Q_y}$$
$$+ \sum_{j=0}^{\infty} ||u_b(k+j|k) - u_{des} - \delta_{u,k}||^2_{Q_u}$$
$$+ \sum_{j=0}^{m-1} ||\Delta u_b(k+j|k)||^2_R + ||\delta_{y,k}||^2_{S_y} + ||\delta_{u,k}||^2_{S_u}$$

subject to

$$-\Delta u_{\max} \le \Delta u_b(k+j|k) \le \Delta u_{\max}, j = 0, 1, \ldots, m-1$$
$$u_{\min} \le u_b(k+j|k) \le u_{\max}, j = 0, 1, \ldots, m-1$$
$$u_b(k+m-1|k) - u_{des} - \delta_{u,k} = 0 \tag{18}$$

$$y_{\min} \le y_{sp,k} \le y_{\max}$$
$$y_{sp,k,i} = y_{des,i} \quad i \in \Omega$$
$$x^s(k+m+\theta_{\max}|k) - y_{sp,k} - \delta_{y,k} = 0 \tag{19}$$

$$x^i_b(k+m+\theta_{\max}|k) = x^i_a(k+m+\theta_{\max}|k) \tag{20}$$

where $\Delta u_{b,k} = \left[\Delta u_b(k|k)^T \quad \cdots \quad \Delta u_b(k+m-1|k)^T \right]^T$.

At each time step k, Problem P2a is solved first and, assuming that the computation time necessary to solve this problem is negligible, the optimal solution to P2a is passed to Problem P2b that is solved at the same time step and produces the control sequence $\Delta u_{b,k}$. The first control move $\Delta u_b(k|k)$ is implemented in the real plant. The two problems are linked through constraint (20) that guarantees that, at time step $k+1$, it is possible to find a feasible solution to Problem P2a, corresponding to a value of the objective function V_{2a} that is not larger than the optimal value of this function at time k. Observe that both Problems P2a and P2b are QPs that can be easily solved with the available QP solvers.

3. Convergence and stability of the two-step MPC

In the two-step controller, Problem P2a is always feasible as there is at least the trivial solution $\Delta u_a = 0$ and, Problem P2b is feasible because of the slack vectors $\delta_{u,k}$ and $\delta_{y,k}$ and it can be shown that constraint (20) is also feasible. Consequently, the MPC controller defined through Problems P2a and P2b has recursive feasibility. The standard procedure to prove the stability of MPC consists in showing that the optimal cost function can be interpreted as a Lyapunov function of the closed-loop system. Here, this procedure is adapted to the controller resulting from the solution to Problems P2a and P2b. One starts by showing the convergence of the prediction of the integrating state $x^i(k)$ at the end of the delayed control horizon to zero when Problems P2a and P2b are solved sequentially.

Theorem 1. *The sequential solution of* Problems P2a and P2b *produces the convergence of the predicted integrating state* $x^i(k+m+\theta_{max}|k)$ *to zero after a finite number of time steps.*

Proof. Suppose that at time step k, Problem P2a is solved and the optimal solution is represented as $\Delta u^*_{a,k} = [\,\Delta u^*_a(k|k)^T \;\; \cdots \;\; \Delta u^*_a(k+m-1|k)^T\,]^T$ and the corresponding integrating state $x_a^{i*}(k+m+\theta_{\max}|k)$ is passed to Problem P2b, which is solved at the same time step. The solution to Problem P2b results in the control sequence $\Delta u^*_{b,k} = [\,\Delta u^*_b(k|k)^T \;\; \cdots \;\; \Delta u^*_b(k+m-1|k)^T\,]^T$ from which the first control action $\Delta u^*_b(k|k)$ is injected in the real process. At time step $k+1$, Problem P2a is solved again and the control sequence $\Delta\tilde{u}_{a,k+1} = [\,\Delta u^*_b(k+1|k)^T \;\; \cdots \;\; \Delta u^*_b(k+m-1|k)^T \;\; 0\,]^T$ is a feasible solution to this problem as it satisfies the inequality constraints related to the manipulated inputs. Let the objective function of Problem P2a corresponding to this control sequence be designated $\tilde{V}_{2a,k+1}$. Also, the prediction of the integrating state at the end of the delayed control horizon corresponding to this control sequence is given by

$$\tilde{x}^i(k+1+m+\theta_{\max}|k+1) = x^i(k+1|k+1) + D^i\Delta u^*_b(k+1|k) + \cdots + D^i\Delta u^*_b(k+m-1|k) + D^i \times 0$$

If the state is measured and the system is undisturbed, one has

$$x^i(k+1|k+1) = x^i(k+1|k)$$

Now, using the space model defined in (6), one obtains

$$x^i(k+1|k+1) = x^i(k|k) + D^i\Delta u^*_b(k|k)$$

Then,

$$\begin{aligned}\tilde{x}^i&(k+1+m+\theta_{\max}|k+1)\\ &= x^i(k|k) + D^i\Delta u^*_b(k|k) + D^i\Delta u^*_b(k+1|k) + \cdots\\ &\quad + D^i\Delta u^*_b(k+m-1|k)\end{aligned}$$

Which means that

$$\tilde{V}_{2a,k+1} = V^*_{2a,k}$$

Consequently, the objective function of Problem P2a is non-increasing and converges to zero when the system is stabilizable. In fact, it is easy to show that this convergence will happen in a finite number of time steps.□

For the undisturbed system, once the prediction of the integrating state at the end of the delayed control horizon is made equal to zero, and the control horizon is large enough, this state can be kept at zero while the control cost of Problem P2b is decreasing and, if the optimizing targets are reachable, the control cost of Problem P2b will also converge to zero as stated in the theorem below.

Theorem 2. *If in* Problem P2b, *constraint* (20) *is replaced with the constraint* $x_b{}^i\,(k+m+\theta_{max}|k)=0$ *and at time step k the resulting optimization problem is feasible, then, for the undisturbed system, it will remain feasible at subsequent time steps and the control cost* $V_{2b,k}$ *will converge to zero. If the input/output targets are reachable, then the inputs/outputs will converge to these targets. If the targets are not reachable, the system will converge to a steady state that lies at the minimum distance from the desired targets.*

Proof. Following the standard procedure to prove the convergence of MPC algorithms, suppose that at time step k, Problem P2b is solved with the modified constraint (20) and the optimum solution is defined as $\Delta u_{b,k}{}^*$, $y_{sp,k}{}^*$, $\delta_{u,k}{}^*$ and $\delta_{y,k}{}^*$. Then, the first control input $\Delta u_b{}^*(k|k)$ is injected in the true system and, at $k+1$, Problem P2b is solved again. At this time, consider the following set of decision variables inherited from the solution of P2b at time k: $\Delta\tilde{u}^b_{k+1}$, $\tilde{y}_{sp,k+1}$, $\tilde{\delta}_{u,k+1}$ and $\tilde{\delta}_{y,k+1}$

where

$$\begin{aligned}\Delta\tilde{u}_{b,k+1} &= \left[\,\Delta u^*_b(k+1|k)^T \;\; \cdots \;\; \Delta u^*_b(k+m-1|k)^T \;\; 0\,\right]^T,\\ \tilde{y}_{sp,k+1} &= y^*_{sp,k}, \quad \tilde{\delta}_{y,k+1} = \delta^*_{y,k}, \quad \tilde{\delta}_{u,k+1} = \delta^*_{u,k}\end{aligned} \tag{21}$$

One can prove that the solution proposed above is feasible. It trivially satisfies all the inequality constraints, and, for instance the constraint defined in (18) is satisfied as for the solution proposed in (21) one has

$$\tilde{u}_b(k+m|k+1) - u_{des} - \tilde{\delta}_{u,k+1} = u^*_b(k+m-1|k) - u_{des} - \delta^*_{u,k} = 0$$

Analogously, constraint (19) can be written as follows

$$\begin{aligned}x^s&(k+m+1+\theta_{\max}|k+1) - y^*_{sp,k} - \delta^*_{y,k}\\ &= x^s(k+m+\theta_{\max}|k) + \Delta t D^i x^i(k+m+\theta_{\max}|k) - y^*_{sp,k} - \delta^*_{y,k}\\ &= x^s(k+m+\theta_{\max}|k) - y^*_{sp,k} - \delta^*_{y,k} = 0\end{aligned}$$

Also, since the last control move in the control sequence defined in (21) is equal to zero, one can write

$$x^i(k+m+1+\theta_{\max}|k+1) = x^i(k+m+\theta_{\max}|k) = 0$$

Thus, since all the equality constraints of Problem P2b are also satisfied, the solution defined in (21) is feasible. Now, comparing the value of the optimum cost function of Problem P2b at time k and the value of the same cost corresponding to the proposed solution at time $k+1$, one has

$$V^*_{2b,k} - \tilde{V}_{2b,k+1} = ||y(k|k) - y^*_{sp,k} - \delta^*_{y,k}||^2_{Q_y} + ||u_b(k|k) - u_{des} - \delta^*_{u,k}||^2_{Q_u} + ||\Delta u^*(k|k)||^2_R$$

Since the weight matrices Q_y, Q_u and R are assumed positive semidefinite, one has $\tilde{V}_{2b,k+1} \le V^*_{2b,k}$, and consequently $V^*_{2b,k+1} \le V^*_{2b,k}$. This means that the cost function of Problem P2b can be interpreted as a Lyapunov function for the closed loop system with the proposed controller. Then, the cost will converge to a minimum that is equal to

$$V_{2b,\infty} = ||\delta_{y,\infty}||^2_{S_y} + ||\delta_{u,\infty}||^2_{S_u}$$

Observe that $V_{2b,\infty}$ will be equal to zero if the desired steady state is reachable, otherwise it will represent the minimum weighted distance between the desired steady state and a reachable steady state.□

The main disadvantage of the two-step controller described above is that if the integrating state x^i is large the performance of the controller can be affected while the controller is trying to reduce this state to zero. In this case, the control sequence resulting from the solution to Problem P2a is mainly directed to minimizing x^i leaving little space for the attainment of the real control objectives. Based on the observation that the convergence and stability of the IHMPC is only obtained after the zeroing of the prediction of the integrating state at the end of the control horizon extended with the time delay, one can propose an alternative formulation of the IHMPC by explicitly including in the control problem the contraction of the slack variable associated with the integrating state. This formulation of the proposed controller is based on the following optimization problem:

$$\min_{\Delta u_k, y_{sp,k}, \delta_{y,k}, \delta_{i,k}, \delta_{u,k}} V_{3,k}$$

$$= \sum_{j=0}^{\infty} \|y(k+j|k) - y_{sp,k} - \delta_{y,k} - (j-\theta_{\max}-m)\Delta t \delta_{i,k}\|^2_{Q_y}$$

$$+ \sum_{j=0}^{\infty} \|u(k+j|k) - u_{des} - \delta_{u,k}\|^2_{Q_u}$$

$$+ \sum_{j=0}^{m-1} \|\Delta u(k+j|k)\|^2_R + \|\delta_{y,k}\|^2_{S_y} + \|\delta_{u,k}\|^2_{S_u} + S_i|\delta_{i,k}| \tag{22}$$

subject to (9), (10), (12), (14)–(17) and

$$|\delta_{i,k}| \le |\tilde{\delta}_{i,k}| \tag{23}$$

where $\tilde{\delta}_{i,k}$ is computed through the following equation

$$x^i(k+m+\theta_{\max}|k-1) - D^i\Delta u(k-1) - \tilde{\delta}_{i,k} = 0$$

Observe that the inequality constraint (23) can be converted into a linear constraint by defining

$$\delta_{i,k} = \delta^+_{i,k} - \delta^-_{i,k} \quad \text{where} \quad \delta^+_{i,k} \ge 0 \quad \text{and} \quad \delta^-_{i,k} \ge 0$$

and replacing (23) with

$$\delta^+_{i,k} - \delta^-_{i,k} \le |\tilde{\delta}_{i,k}|$$

Also, in the control objective defined in (22) the term that penalizes $\delta_{i,k}$ is expressed as follows

$$S_i|\delta_{i,k}| = S_i(\delta^+_{i,k} + \delta^-_{i,k})$$

Now, the new controller is obtained from the solution to the following problem:

Problem P3.

$$\min_{\Delta u_k, y_{sp,k}, \delta_{y,k}, \delta^+_{i,k}, \delta^-_{i,k}, \delta_{u,k}} V_{3,k}$$

$$= \sum_{j=0}^{\infty} \|y(k+j|k) - y_{sp,k} - \delta_{y,k} - (j-m-\theta_{\max})\Delta t \delta_{i,k}\|^2_{Q_y}$$

$$+ \sum_{j=0}^{\infty} \|u(k+j|k) - u_{des} - \delta_{u,k}\|^2_{Q_u}$$

$$+ \sum_{j=0}^{m-1} \|\Delta u(k+j|k)\|^2_R + \|\delta_{y,k}\|^2_{S_y} + \|\delta_{u,k}\|^2_{S_u} + S_i(\delta^+_{i,k} + \delta^-_{i,k})$$

subject to (9), (10), (12), (14)–(17) and

$$\sum_{j=1}^{ny} (\delta^+_{i,k}(j) - \delta^-_{i,k}(j)) \le \sum_{j=1}^{ny} |\tilde{\delta}_{i,k}| \quad \delta^+_{i,k} \ge 0, \quad \delta^-_{i,k} \ge 0$$

Observe that Problem P3 approaches the sequential solution of Problems P2a and P2b when S_i assumes large values. If this weight is properly selected a compromise between guarantee of stability and performance can be obtained. For adequate values of S_i a moderate pace of decrease in $\delta_{i,k}$ can be obtained, which produces the stability of the closed loop system, while the control performance is not significantly affected.

4. Simulation example

To evaluate the performance of the proposed controller, one considers an industrial ethylene oxide production system represented by the following transfer function model, which is part of the system studied in Carrapiço [20]:

$$G(s) = \begin{bmatrix} \dfrac{-10^{-4}(-95s+1)e^{-s}}{32.16s^2+4.65s+1} & \dfrac{-2.3\times10^{-3}}{s} & \dfrac{-3.2\times10^{-3}(-s+1)e^{-2s}}{64.55s^2+8.83s+1} & \dfrac{-7.5\times10^{-6}}{s} \\ \dfrac{-1.69\times10^{-4}e^{-3s}}{s} & \dfrac{2.1\times10^{-4}e^{-8s}}{s} & \dfrac{-1.9\times10^{-3}(1.47s+1)}{9.67s^2+13.55s+1} & \dfrac{-1.07\cdot10^{-4}}{s} \\ \dfrac{8.1\times10^{-3}(-0.02s+1)e^{-4s}}{52.45s^2+11.92s+1} & \dfrac{-5.5\times10^{-5}e^{-15s}}{s} & \dfrac{9.6\times10^{-3}(s+1)e^{-2s}}{54.42s^2+6.58s+1} & \dfrac{-2.53\times10^{-3}e^{-10s}}{s} \\ \dfrac{-3.9\times10^{-5}e^{-4s}}{s} & \dfrac{5.7\times10^{-5}e^{-8s}}{s} & \dfrac{-1.4\times10^{-3}(s+1)}{8.67s^2+14.48s+1} & \dfrac{7.6\times10^{-5}e^{-6s}}{s} \end{bmatrix}$$

In the simulations presented here, the system outputs have to be controlled in the zones defined by the following bounds:

$$y_{\min} = \begin{bmatrix} 5.8 & 16 & 275 & 18.5 \end{bmatrix}^T, \quad y_{\max} = \begin{bmatrix} 6 & 19.7 & 280 & 19.1 \end{bmatrix}^T$$

At the beginning of the simulation, it is assumed that there are no optimizing targets for the outputs, so, the controller has solely to maintain the system outputs inside the zones defined above. Depending of the operating objective, input u_3 may have an optimizing target.

The constraints related to the manipulated inputs are the following:

$$u_{\min} = \begin{bmatrix} 5700 & 4500 & 0 & 25 \end{bmatrix}^T,$$

$$u_{\max} = \begin{bmatrix} 6900 & 5700 & 100 & 95 \end{bmatrix}^T,$$

$$\Delta u_{\max} = \begin{bmatrix} 25 & 25 & 2 & 10 \end{bmatrix}^T$$

The sampling time is $\Delta t = 1$ min and the control horizon is $m = 8$. The remaining tuning parameters are the following:

$$Q_y = \mathrm{diag}([\,8\times10^3 \quad 300 \quad 10 \quad 50\,]),$$

$$Q_i = \mathrm{diag}([9\times10^8 \quad 6\times10^4 \quad 1\times10^3 \quad 1\times10^3\,])$$

$$R = \mathrm{diag}([\,0.5 \quad 10^{-3} \quad 20 \quad 125\,]),$$

$$S_y = \mathrm{diag}([\,5\times10^9 \quad 3\times10^6 \quad 6\times10^7 \quad 7\times10^7\,])$$

$$S_i = \mathrm{diag}([9\times10^8 \quad 6\times10^4 \quad 1\times10^3 \quad 1\times10^3\,]),$$

$$Q_u = \mathrm{diag}([\,0 \quad 0 \quad 1 \quad 0\,])$$

$$S_u = \mathrm{diag}([\,0 \quad 0 \quad 100 \quad 0\,])$$

The simulation responses for the regulator operation are shown in Figs. 1–4. In these figures, one compares the results of two approaches to the MPC for the integrating system with time delays.

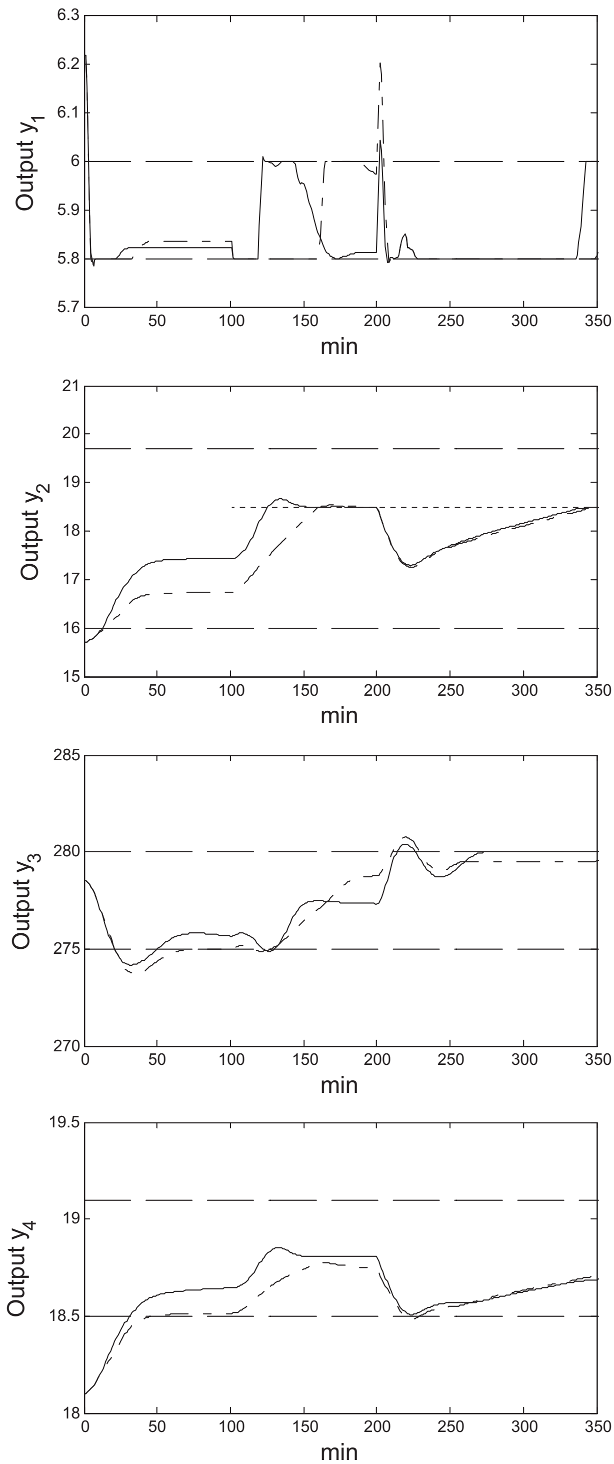

Fig. 1. System outputs for the target tracking case. Controller I (–·–·–), Controller II (—), target (· · · ·), bounds (– – –).

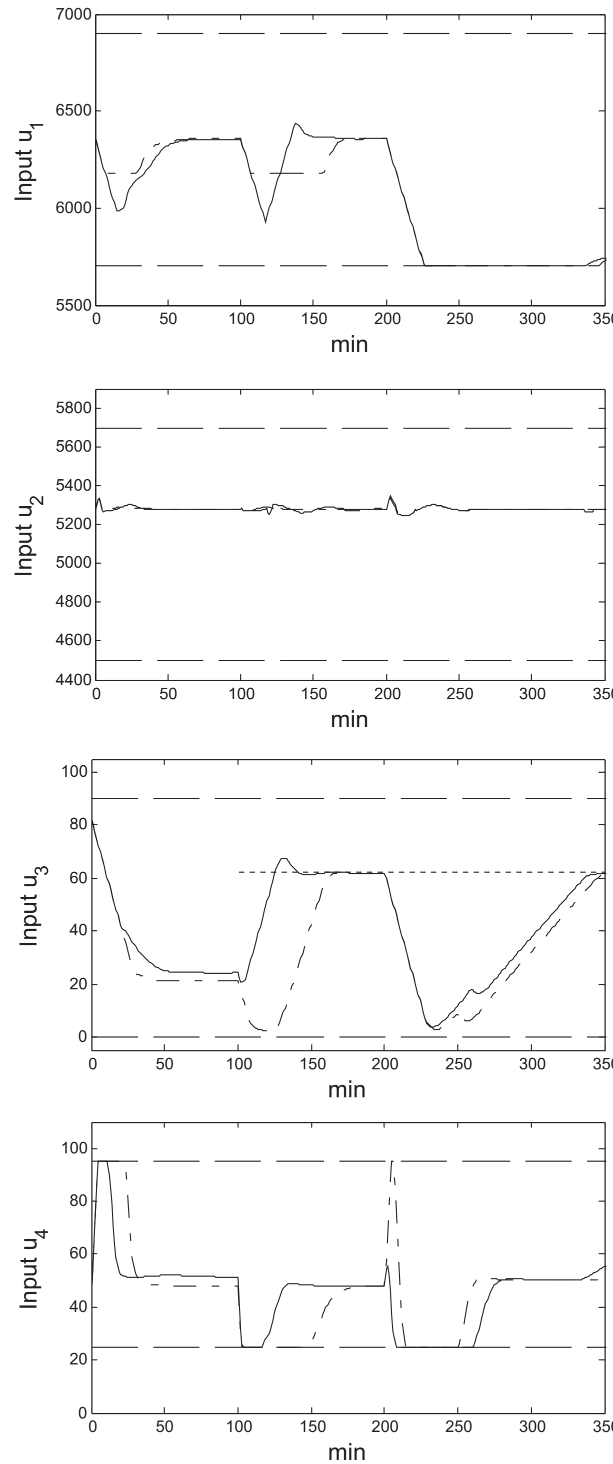

Fig. 2. System inputs for the target tracking case. Controller I (–·–·–), Controller II (—), target (· · · ·), bounds (– – –).

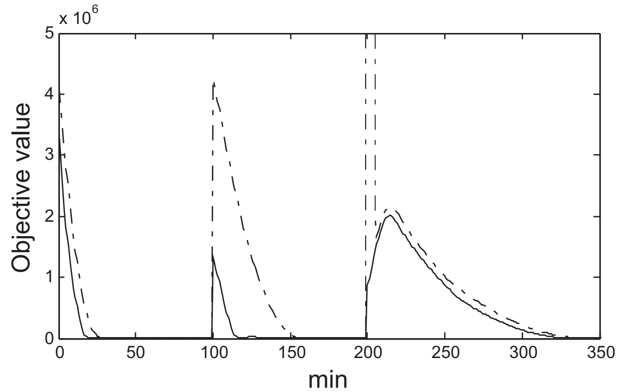

Fig. 3. Control objective for the target tracking case, Controller I (–·–·–), Controller II (—).

These approaches correspond to the solution to Problems P2a/b (Controller I) and Problem P3 (Controller II).

The system starts from the following initial conditions: $u_0 = [\,6357 \quad 5280 \quad 82 \quad 48\,]^T$, $y_0 = [\,6.22 \quad 15.7 \quad 278.5 \quad 18.1\,]^T$. Observe that at this starting point outputs y_1, y_2 and y_4 are outside their control zones. After 100 sample periods, when the system has already reached a new steady state, the optimizing targets $y_{2,\mathrm{des}} = 18.5$ and $u_{3,\mathrm{des}} = 62$ are inserted into the controller. From the beginning of the simulation until time $t = 100$ min, one can notice that the two versions of the proposed controller are efficient to bring the outputs back to inside their control zones and stabilizes the system. However, when output y_3 is considered, Controller II has a better performance than Controller I that allows y_3 to remain outside the control zone for a larger period of time, which is not desirable. The control costs of Problems P2b and P3 are compared in Fig. 3. It is rather surprising that the cost of P3 is smaller than the cost of P2b as the tuning parameters are the same and P3 has an extra term that weighs the slack $\delta_{i,k}$. From Fig. 4, one can observe that Controller II keeps this slack different from zero for quite a long time, which gives an extra degree of freedom to this controller when compared to Controller I where the main priority is to zero δ_i. At time $t = 100$ min, optimizing targets for u_3 and y_2 are introduced in the control system. After this time, the performance of Controller II is even better than the performance of Controller I if inputs u_3 and u_4 are considered. Controller I tends to move u_3 to the wrong direction until saturation is reached. The result is that it takes a lot more time to drive u_3 to the target and u_4 is kept unnecessarily saturated for a long period of time. With respect to the outputs, the two controllers perform similarly except for y_3 that is allowed to leave the control zone by Controller I, which also takes more time to drive y_2 to the target. Fig. 3 shows that the costs in both cases achieve zero because the target specification for u_3 and y_2 are consistent with all output zones. If, however, additional targets are imposed to the system, then there may be not enough degrees of freedom to meet all the requirements. In that case, the final steady state would correspond to a cost larger than zero.

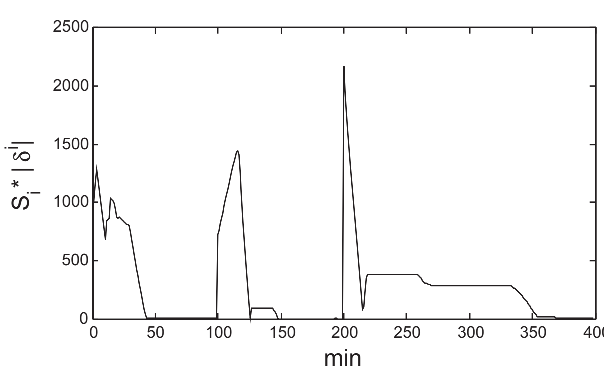

Fig. 4. Cost contribution of slacks corresponding to integrating states. Controller II.

To test the robustness of the proposed approaches to disturbances, at time step 200 min, an unmeasured disturbance equivalent to a step change of +600 kg/h in u_1 is introduced in the system. This is a particularly sever disturbance but the proposed controllers perform satisfactorily and manage to keep the controlled outputs inside their zones, Variables y_2 and u_3 are driven back to their targets and the speed of recovery is only limited by the constraints on the input moves that are defined by the process operator.

At this point, it would be interesting to compare the performance of the proposed controllers with other existing stable MPCs. This is not a simple task as the existing approaches do not consider the zone control of the outputs or input targets. However, one can show that the MPC proposed in Muske and Rawlings [5] would behave similarly to the controller resulting from the solution to Problem P1 if the slacks $\delta_{y,k}$, $\delta_{i,k}$ and $\delta_{u,k}$ are forced to remain equal to zero at any time step. However, for the control horizons that are used in practice ($m < 10$), the controller defined in [5] tends to become infeasible quite often. For instance, in the simulation presented here, the controller would become infeasible at time step 200 min where the system is disturbed. This means that the controller is not robust to disturbances. To recover feasibility, the control horizon should be increased up to $m = 30$, which is usually unacceptable in practice. Another usual "ad hoc" approach for recovering the feasibility of the control problem is by removing one or more constraints from the control problem. In this case, even if the system is stabilizable, the resulting control law does not guarantee the stability of the closed loop system.

5. Conclusion

In this work, a stable model predictive controller was presented that extends existing controllers to process systems with multiple time delays between the inputs and outputs. The case where the system has stable and integrating modes was considered. Two versions of the controller were presented focusing the general practical case of zone control of the outputs and the existence of optimizing targets for the inputs and/or outputs. One of the controller formulations is based on a two-step algorithm that has proved stability. The other version of the controller is based on a single step algorithm and has better performance than the two-step approach. The one step approach can inherit the stability property of the two-step approach with an adequate tuning of the controller. The controllers were tested through simulation of an industrial ethylene oxide reactor system. The resulting performance seems quite acceptable for the industrial implementation.

Appendix A. Appendix

Following the procedure presented in González and Odloak [13], the step response model of integrating systems with time delay as defined in Eq. (2) can be converted into an equivalent state space form if the following state and matrices are defined:

$$[x] = \begin{bmatrix} y_{k+1}^T & y_{k+2}^T & \cdots & y_{k+np}^T & x^{s^T} & x^{d^T} & x^{i^T} \end{bmatrix}^T$$

$$A = \begin{bmatrix} 0 & I_{ny} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{ny} & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I_{ny} & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & I_{ny} & \Psi(np) & \Omega(np) \\ 0 & 0 & 0 & \cdots & 0 & I_{ny} & 0 & \hat{I}\Delta t \\ 0 & 0 & 0 & \cdots & 0 & 0 & F & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & I_{ny.nu} \end{bmatrix}, \ A \in C^{ne\times ne}$$

$$B = \begin{bmatrix} S_1^T & S_2^T & \cdots & S_{np}^T & [D^0 + \Delta t \ \ D^i]^T & [D^d F \ \ N]^T & [\bar{D}^i \bar{N}]^T \end{bmatrix}^T \in C^{ne\times nu}$$

$$C = \begin{bmatrix} I_{ny} & 0 & \cdots & 0 \end{bmatrix} \in \Re^{ny\times ne}$$

$$D^0 = \begin{bmatrix} d_{1,1}^0 & \cdots & d_{1,nu}^0 \\ \vdots & \ddots & \vdots \\ d_{ny,1}^0 & \cdots & d_{ny,nu}^0 \end{bmatrix} \in \Re^{ny\times nu},$$

$$D^i = \begin{bmatrix} d_{1,1}^i & \cdots & d_{1,nu}^i \\ \vdots & \ddots & \vdots \\ d_{ny,1}^i & \cdots & d_{ny,nu}^i \end{bmatrix} \in \Re^{ny\times nu}$$

$$\bar{D}^i = \mathrm{diag}(d_{1,1}^i \dots d_{1,nu}^i \dots d_{2,1}^i \dots d_{ny,1}^i \dots d_{ny,nu}^i) \in \Re^{ny.nu\times ny.nu}$$

$$\bar{N} = \begin{bmatrix} I_{nu} \\ I_{nu} \\ \vdots \\ I_{nu} \end{bmatrix} \in \Re^{ny.nu\times nu}$$

where

$x \in \Re^{ne}$, with $ne = ny(1 + np + nu.na + nu)$ and $np > \max_{i,j} \theta_{i,j}$

$x^s \in \Re^{ny}$ and $x^d \in \Re^{nd}$ have the same interpretation as in the model defined in Section 2.1. However, when there are different time delays between the integrating output and the manipulated inputs, the dimension of x^i that is also related with the integrating modes is not the same as in the model defined in Section 2.1. In this model, we have to extend the state to include one component for each pair (input and output), and consequently the dimension of the state component associated with the integrating modes is *nynu* or $x^i \in \Re^{(ny\,nu)\times 1}$. Observe that in the model defined in (4) and (5), we have $x^i \in \Re^{ny}$.

$$\Psi(np) = \begin{bmatrix} f_{1,1,1}(np) & \cdots & f_{1,nu,na}(np) & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & f_{2,1,1}(np) & \cdots & f_{2,nu,na}(np) & \cdots & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & f_{ny,1,1}(np) & \cdots & f_{ny,nu,na}(np) \end{bmatrix}$$

$\Psi(np) \in C^{ny\times nd}$

$$\Omega(np) = \begin{bmatrix} (np-\theta_{11})\Delta t & \cdots & (np-\theta_{1nu})\Delta t & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & (np-\theta_{21})\Delta t & \cdots & (np-\theta_{2nu})\Delta t & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \ddots & \ddots & \ddots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & (np-\theta_{ny1})\Delta t & \cdots & (np-\theta_{ny.nu})\Delta t \end{bmatrix}$$

$\Omega(np) \in \Re^{ny\times(ny\,nu)}$

$$\hat{I} = \begin{bmatrix} \alpha_{1,1} & \cdots & \alpha_{1,1} & \cdots & 0 & \cdots & 0 \\ & \vdots & & \ddots & & \vdots & \\ 0 & \cdots & 0 & \cdots & \alpha_{ny,1} & \cdots & \alpha_{nu,nu} \end{bmatrix},$$ where $\alpha_{i,j} = 1$, if y_i is integrating with respect to u_j and $\alpha_{i,j} = 0$, if y_i is stable with respect to u_j.

$S_{j=1,\dots,np}$ are the step response coefficients that can be computed as follows:

$$S_j = D^0 + \Psi(j)D^d N + \Omega(j)\bar{D}^i\bar{N}, \quad S_j \in \Re^{ny\times nu}$$

D^0 and N are the same as in Section 2.1.

To illustrate the detectability problem of the model defined above, consider the following system:

$$y(s) = \underbrace{\frac{-0.19e^{-s}}{s(10s+1)}}_{G_1(s)} u_1(s) + \underbrace{\frac{0.235}{s(15+1)}}_{G_2(s)} u_2(s)$$

For this system, the step response corresponding to $G_1(s)$ is

$$S_{1,1}(t) = d_{1,1}^0 + d_{1,1,1}^d e^{r_{1,1,1}(t-\theta_{1,1})} + d_{1,1}^i(t-\theta_{1,1})$$

$$S_{1,1}(t) = 1.9 - 1.9e^{-0.1(t-1)} - 0.19(t-1)$$

Analogously, the response corresponding to $G_2(s)$ is

$$S_{1,2}(t) = d_{1,2}^0 + d_{1,2,1}^d e^{r_{1,2,1}(t-\theta_{1,2})} + d_{1,2}^i(t-\theta_{1,2})$$

$$S_{1,2}(t) = -3.525 + 3.525e^{-0.0667t} + 0.235t$$

If one considers a sampling period $\Delta t = 1$ then $np = 2$ and it is easy to show that $\Psi(np) = [\,0.818 \quad 0.8752\,]$, $\Omega(np) = [\,1 \quad 2\,]$, $\hat{I} = [\,1 \quad 1\,]$, $F = \mathrm{diag}[\,0.9048 \quad 0.9355\,]$. Consequently, the state matrix

A and the output matrix C are as follows

$$A=\begin{bmatrix}0&1&0&0&0&0&0\\0&0&1&0.8187&0.8752&1&2\\0&0&1&0&0&1&1\\0&0&0&0.9048&0&0&0\\0&0&0&0&0.9355&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{bmatrix},$$

$$C=\begin{bmatrix}1&0&0&0&0&0&0\end{bmatrix}$$

Then, applying the Popov–Belevitch–Hautus criterium for the model detectability, one obtains rank $\begin{bmatrix}A-\lambda I\\ C\end{bmatrix}=6$. As the model has 7 states, one concludes that this model is not detectable.

However, if one computes matrices A and C corresponding to the model presented in Section 2.1, one has:

$$A=\begin{bmatrix}1&0&0&1&1.71&0\\0&0.9048&0&0&-1.719&0\\0&0&0.9355&0&0&0\\0&0&0&1&-0.19&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix},$$

$$C=\begin{bmatrix}1&1&1&0&0&0\end{bmatrix}$$

Using these matrices, one concludes that rank $\begin{bmatrix}A-\lambda I\\ C\end{bmatrix}=6$, which is equal to the number of states of the model presented in Section 2.1. Then, the model is detectable and can be used in the development of the proposed MPC.

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