

# PID, Fuzzy and LQR Controllers for Magnetic Levitation System

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**Abstract**—Magnetic suspension systems are highly non-linear and unstable. Magnetic levitation system is modelled here and PID, FUZZY and LQR are designed to control the system. All the controllers are designed in matlab and checked in real time. Rise time, peak overshoot and settling time are compared with each controller and result is noted and compared.

**Keywords**—Magnetic levitation system; PID; Fuzzy; LQR

## I. INTRODUCTION

A Maglev is a system that uses magnetic field to levitate an object in a particular position. If an object is placed very far from the magnetic source, the magnetic field is too weak to support the weight of the object. If placed too close, the magnetic field becomes too strong and causes the object to move towards the source until it makes physical contact with the magnet. Maglev device is an example of an inherently unstable system. Magnetic levitation system can be categorized based on whether the force of attraction or repulsion is used for the levitation of the ball. Here the force of attraction of the electromagnet in the maglev is used to balance against the gravity. The main components of the magnetic levitation ball system are shown in Fig. 1.

The working of magnetic levitation system is as follows. When voltage is applied to the electromagnet, the resulting current magnetizes the coil and exerts a magnetic force on the steel ball to attract it towards the magnet. This force when becomes equal to the force of gravity, the ball is successfully suspended in the required position. Theoretically, it seems very easy because it is just a problem of finding out how much is the gravitational force on the ball at a particular point. But without the help of an appropriate controller, it cannot be done. Before designing the controller, modeling of the system is done in the next Section.

## II. MODELING

K. Ishaque and P. Suster [1,2] proposes that the magnetic levitation system can be categorized into

- 1) Electrical system
- 2) Mechanical system

The ball position in the mechanical system can be controlled by the current through the electromagnet where the current through the electromagnet in the electrical system can be controlled by applying controlled voltage across the electromagnet terminals.



Fig. 1. Magnetic levitation system

The magnitude of force  $f(x,i)$  exerted across an air gap by an electromagnet through which a current  $i$  flows can be described as

$$f(h,i) = -\frac{i^2}{2} \frac{dL(h)}{dh} \quad (1)$$

The total inductance  $L$  is a function of the distance and given by

$$L(h) = L_1 + \frac{L_0 H_0}{h} \quad (2)$$

Where,  $L_1$  is the inductance of the electromagnet coil in the absence of levitated object,  $L_0$  is the additional inductance contributed by its presence, and  $X_0$  is the equilibrium position. The parameters are determined by the geometry and construction of electromagnet and can be determined experimentally.

Substituting equation (2) in (1) yields

$$f = \frac{L_0 X_0}{2} \left[ \frac{i}{h} \right]^2 = \beta \left[ \frac{i}{h} \right]^2 \quad (3)$$

$$\beta = \frac{L_0 H_0}{2} \quad (4)$$

By using Taylors theorem, the equation of the force is linearized.

$$f = f_0 + \left( \frac{\partial f}{\partial L_0} \right) \Delta L_0 + \left( \frac{\partial f}{\partial H_0} \right) \Delta H_0 + \left( \frac{\partial f}{\partial i} \right) \Delta i + \left( \frac{\partial f}{\partial h} \right) \Delta h \quad (5)$$

Eliminating the higher order terms give

$$f = f_0 + \left( \frac{\partial f}{\partial i} \right) \Delta i + \left( \frac{\partial f}{\partial h} \right) \Delta h \quad (6)$$

Evaluating equation (6) using (4) and (5) yields

$$f = \beta \left( \frac{I_0}{H_0} \right) + \frac{2\beta I_0}{H_0^2} \Delta i - \left( \frac{2\beta I_0^2}{H_0^3} \right) \Delta h \quad (7)$$

Where  $I_0$  is the equilibrium value. At equilibrium, the weight of the object is suspended by the electromagnet force  $f_0$ . The force required to maintain equilibrium  $f_1$  is

$$f_1 = f - f_0 \quad (8)$$

Combining equations (7) and (8) gives

$$f_1 = \left( \frac{2\beta I_0}{H_0^2} \right) \Delta i - \left( \frac{2\beta I_0^2}{H_0^3} \right) \Delta h \quad (9)$$

Now we are considering the electrical system. The voltage equation of the electromagnetic coil is

$$V = iR + L(h) \frac{di}{dt} \quad (10)$$

Assuming that the suspended object remains close to its equilibrium position

$$L(h) = L_1 + L_0 \quad (11)$$

Also, assuming  $L_1$  is very large compared to  $L_0$ , equation (10) can be simplified as

$$V = iR + L_1 \frac{di}{dt} \quad (12)$$

By taking Laplace transform we get

$$\frac{I(s)}{V(s)} = \frac{1}{L_1 s + R} \quad (13)$$

Now, the main equation of the suspended object comes by applying Newton's second law of motion. For this single degree of freedom system, a force balance taken at the centre of gravity of the object yields

$$M \frac{d^2 h}{dt^2} = -f_1 \quad (14)$$

Applying Laplace transform, we get

$$\frac{H(s)}{I(s)} = \frac{-K_1 I(s)}{Ms^2 - K_2} \quad (15)$$

Where

$$K_1 = \frac{2\beta I_0}{H_0^2}$$

and

$$K_2 = \frac{2\beta I_0^2}{H_0^3}$$

There is a sensor in the model of the system. It can be modeled as

$$V_s = K_s h \quad (16)$$

The overall transfer function of the plant is

$$G(s) = \frac{V_s(s)}{V(s)} = \frac{\frac{-K_s K_1}{ML_1}}{\left(s + \frac{R}{L_1}\right) \left(s^2 - \frac{K_2}{M}\right)} \quad (17)$$

Real time system is designed in such a way that, the current produced in the electrical system is proportional to the voltage given. It is experimentally verified that the proportionality constant is 1. Also, sensor gain is not there since voltage to position and vice-versa is done in input and output sides.

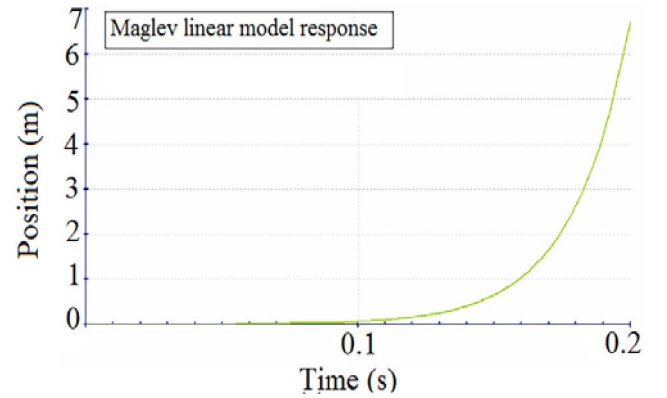


Fig. 2. Response of the linear Maglev model when 0 control voltage is applied.

### III. CONTROLLER DESIGN

Without a controller, when we give voltage to the electromagnet, the steel ball will either fall down or go and stick to the electromagnet. So, we use a PID, fuzzy logic and LQR controllers which successfully control the voltage given to the electromagnet and thereby controlling the ball in steady state after the transient time.

### A. PID Controller

The schematic diagram of PID controller is given in Fig. 3.

The transfer function of PID can be written as

$$K(s) = K_p + \frac{K_i}{s} + K_d s$$

Step response of the system in simulation with PID controller is given in Fig. 4.

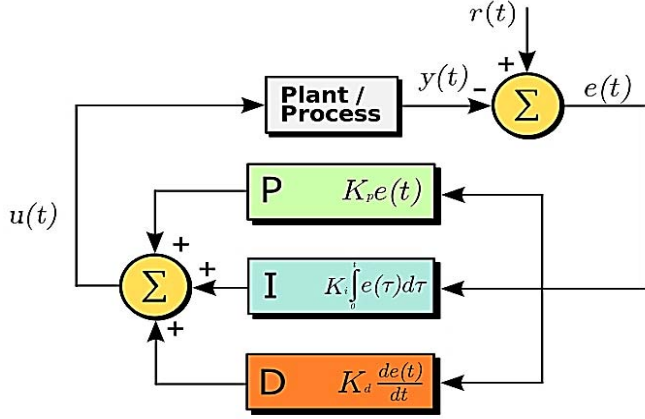


Fig. 3. Schematic diagram of PID controller

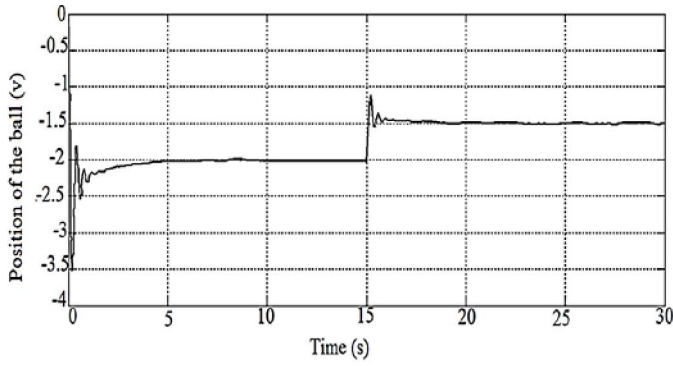


Fig. 4. Position of the ball using a PID controller

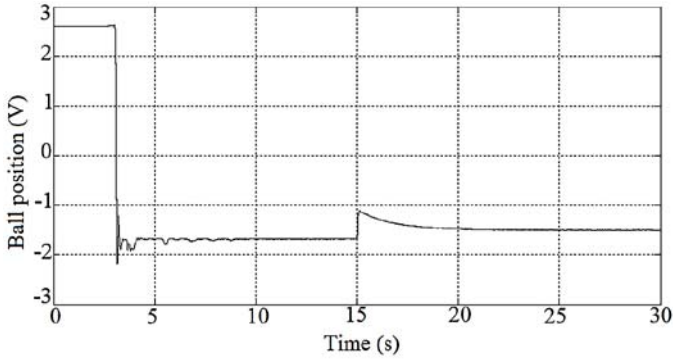


Fig. 5. The real time response of maglev with PID controller

### B. Fuzzy logic controller

AnFuzzy logic System (FLS) is a rule-based system that implements a nonlinear mapping between inputs and outputs.

Error in position and change of error in position are the two inputs taken as the input to the controller, whereas voltage is taken to be the output of the controller. The membership functions of the input and output are taken as triangular type whose range can be decided by observing the response of the system on varying the range of each membership function. Fig. 7 gives the membership plot of the input variable 'error', fig 8 gives the membership plot of input variable 'change of error' and fig 9 gives the membership plot of output variable 'output'.

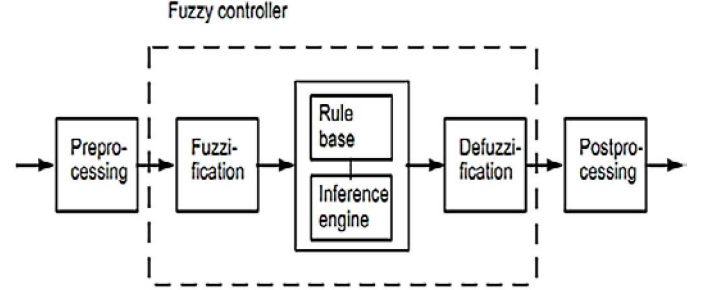


Fig. 6. Structure of a fuzzy logic system

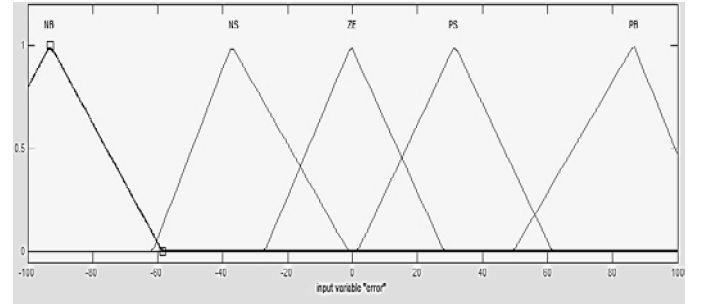


Fig. 7. Membership plot of input variable 'error'

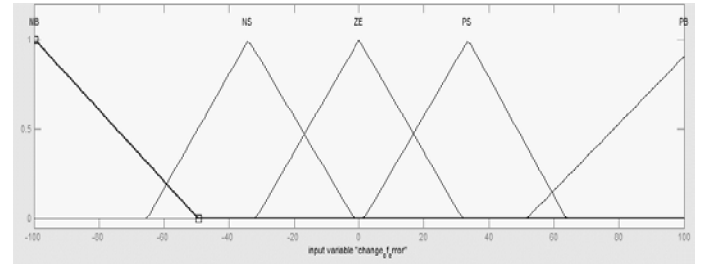


Fig. 8. Membership plot of input variable 'change of error'

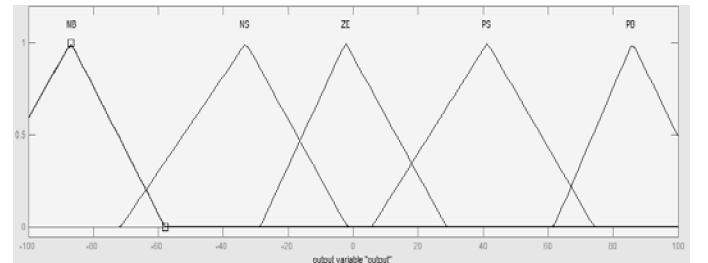


Fig. 9. Membership plot of output variable 'output'

TABLE 1- RULE BASE

$\frac{e}{de/dt}$	NB	NS	Z	PS	PB
NB	PB	PB	PB	PS	Z
NS	PB	PB	PS	Z	NS
Z	PB	PS	Z	NS	NB
PS	PS	Z	NS	NB	NB
PB	Z	NS	NB	NB	NB

The rule base is shown in Table 1.

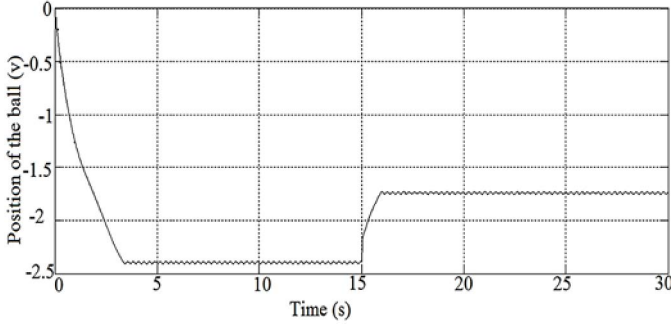


Fig .10. Position of the ball of maglev using FuzzyLogic Controller

Fig. 11. gives the real time step response of maglev using Fuzzy Logic Controller.

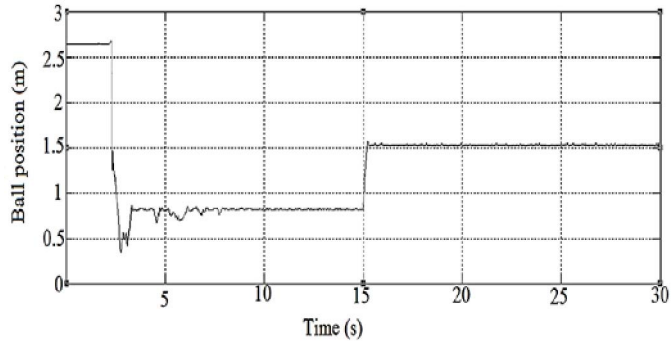


Fig. 11. The real time step response of maglev with fuzzy logic controller

### C. Linear Quadratic Regulator(LQR)

A system can be expressed in state variable form as,

$$\dot{x} = Ax + Bu \quad (18)$$

with  $x(t) \in Rn, u(t) \in Rm$ .

Assuming here that all the states are measurable and seek to find a state-variable feedback (SVFB) control,

$$u = -Kx + v \quad (19)$$

gives desirable closed-loop properties. The closed-loop system using this control becomes

$$\dot{x} = (A - BK)x + Bv = A_c x + Bv \quad (20)$$

With  $A_c$  the closed-loop plant matrix and  $v(t)$  the new command input.

To design a State Variable Feed Back(SVFB) that is optimal, performance index(PI)is defined as

$$J = \frac{1}{2} \int_0^{\infty} (x^T Qx + u^T Ru) dt \quad (21)$$

Substituting the feedback control into this yields

$$J = \frac{1}{2} \int_0^{\infty} x^T (Q + K^T RK) x dt \quad (22)$$

$v(t)$  is assumed to be zero since only internal stability properties of the closed loop system is considered.

The objective of optimal design is to select the feedback gain K that minimizes the performance index J. Depending on the design parameters of the two matrices Q ( $n \times n$  matrix) and R ( $m \times m$  matrix), the closed-loop system will exhibit a different response.Q should be always positive semi-definite and R should be positive definite. Since the plant is assumed to be linear and the performance index PI is quadratic, the problem of determining the feedback gain K to minimize J is called the Linear Quadratic Regulator (LQR).

To find the optimal feedback K, a constant matrix P is selected such that,

$$\frac{d}{dt} (x^T Px) = -x^T (Q + K^T RK) x \quad (23)$$

Substituting (23) into (22) yields,

$$J = \frac{1}{2} \int_0^{\infty} \frac{d}{dt} (x^T Px) dt = \frac{1}{2} x^T(0) Px(0) \quad (24)$$

J is made independent of K. It is a constant that depends only on the auxiliary matrix P and the initial conditions. Solving all the equations (20), (22) and (24), following equation can be obtained

$$A^T P + PA + Q + K^T RK - K^T B^T P - PBK = 0 \quad (25)$$

Where,  $K = R^{-1} B^T P$

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \quad (26)$$

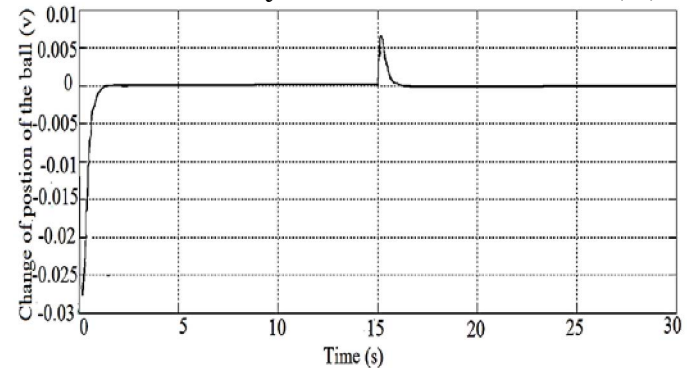


Fig .12. Simulation result of error in position ofthe ball for a step input using LQR

Equation (9) is known as the *algebraic Riccati equation (ARE)*.

The simulation result of the step response of maglev with LQR is given in Fig. 12.

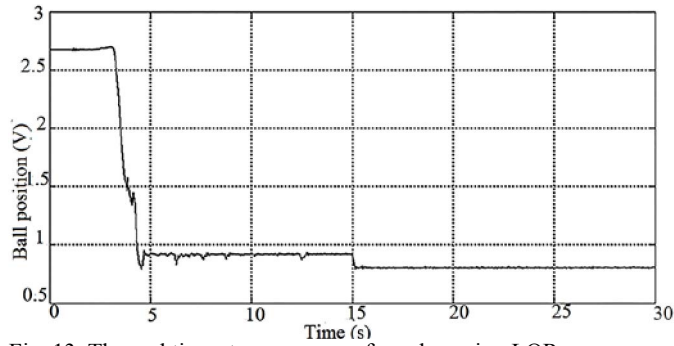


Fig. 13. The real time step response of maglev using LQR

#### IV. CONCLUSION

1) LQR and Fuzzy logic controller are applied successfully to control the ball position in the magnetic levitation system in real time.

2) Three different parameters of PID controller, Fuzzy logic controller and LQR are compared. The parameters compared are rise time, percentage peak overshoot and settling time. Comparison of the simulation of the test bed model is given in table 2 and comparison of the real time response is given as shown in Table 3

TABLE 2

Controller	Rise time(s)	Peak overshoot %Mp	Settling time(s)
PID	.068	25%	4
LQR	.09	.4%	.5
Fuzzy Logic	.52	.1%	1.7

Comparing the real bed results of all the three controllers, from Tables 2 and 3, it is seen that, for both simulation and real time results, rise time is less for PID controller, Percentage peak overshoot is less in fuzzy logic controller and settling time is less in LQR control. FLC is having an extra advantage that the exact model of the system is not required whereas LQR requires a thorough understanding of the mathematical model it is very easy to tune FLC since it deals with linguistic variables.

TABLE 3

Controller	Rise time(s)	Peak overshoot %Mp	Settling time(s)
PID	.042	34%	5.2
LQR	.166	3.1%	.166
Fuzzy Logic	.166	.005%	1.16

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