Notes on MVA (Minimum Variance Analysis)

Chen Shi

October 7, 2016

Assume that we have a set of measured magnetic field vectors:

$$\vec{B}_1,\, \vec{B}_2,\, \cdots \vec{B}_M$$

We want to find a unit vector $\vec{n}$ such that the variance of the magnetic fields along $\vec{n}$

$$\sigma^2(\vec{n}) = \frac{1}{M}\sum_{m=1}^{M} \left|(\vec{B}_m - <\vec{B}>)\cdot\vec{n}\right|^2 \tag{1}$$

is minimum. As the problem has a constraint:

$$\Phi(\vec{n}) = n_x^2 + n_y^2 + n_z^2 = 1 \tag{2}$$

we can adopt the Lagrangian multiplier method, define:

$$\begin{aligned} F(n_x,\, n_y,\, n_z) &= \sigma^2(\vec{n}) - \lambda\Phi(\vec{n}) \\ &= \frac{1}{M}\sum_{m=1}^{M} \left|(\vec{B}_m - <\vec{B}>)\cdot\vec{n}\right|^2 - \lambda(n_x^2 + n_y^2 + n_z^2) \end{aligned} \tag{3}$$

and solve the linear equations:

$$\frac{\partial F}{\partial n_x} = 0 \tag{4}$$

$$\frac{\partial F}{\partial n_y} = 0 \tag{5}$$

$$\frac{\partial F}{\partial n_z} = 0 \tag{6}$$

or:

$$\overleftrightarrow{A} \cdot \vec{n} = \lambda\vec{n} \tag{7}$$

with

$$\begin{aligned} A_{ij} &= \frac{1}{M}\sum_{m=1}^{M} (\vec{B}_m - <\vec{B}>)_i(\vec{B}_m - <\vec{B}>)_j \\ &= <B_iB_j> - <\vec{B}>_i <\vec{B}>_j \end{aligned}$$