Longest Path NP Complete Analysis

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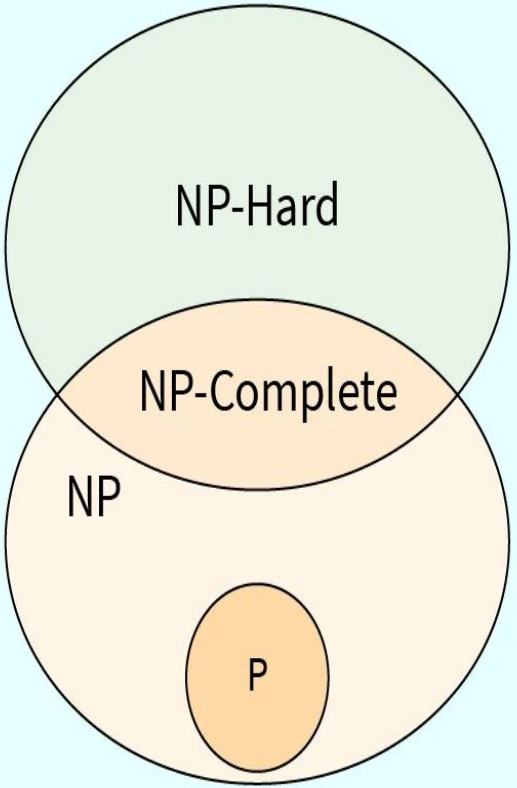
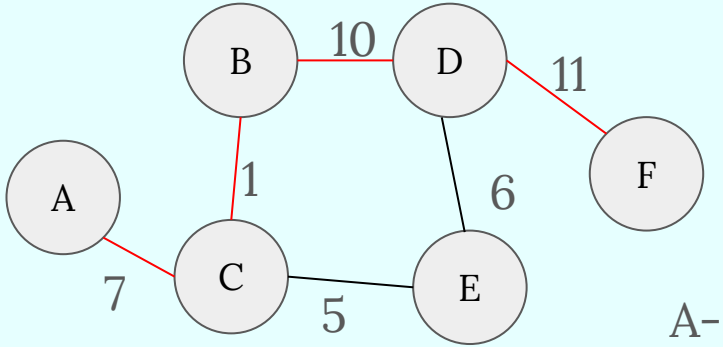


Exact Solution

Goal: Find the simple path (no repeated vertices) between any two vertices with maximum total weight.

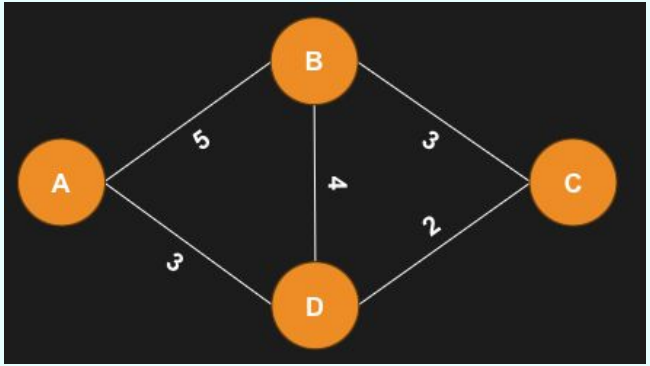
- Decision: YES/NO
- Optimization: What is the longest path

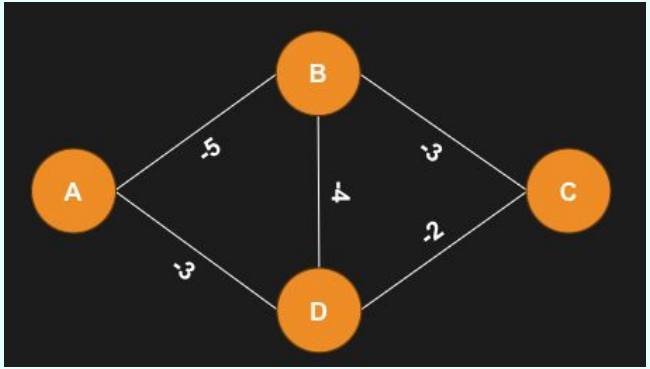


A->C->B->D->F: 29

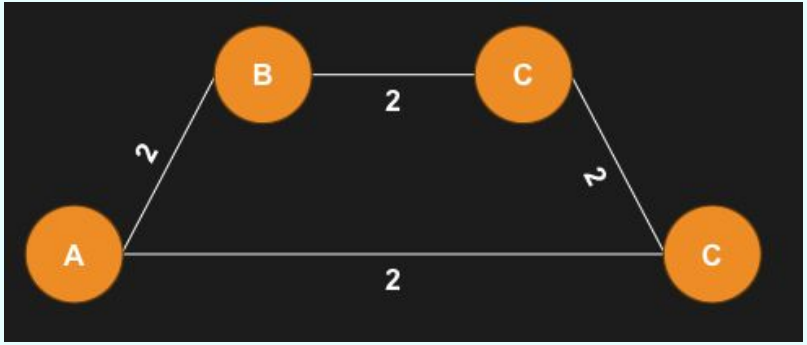
Intuitive Solutions That Don't Work

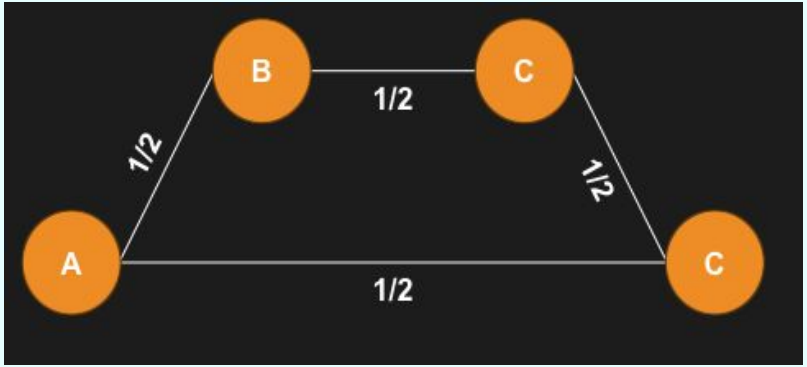
Inverted Edges





Reciprocal Edges





Certifier is Polynomial

Certifier(G, s, t, k, path):

1. Check that the path starts at s and ends at t.
   If not, reject the certificate.

2. Check that no vertex appears more than once in the path.
   (A simple path cannot repeat vertices.)
   If any vertex repeats, reject the certificate.

3. For each pair of consecutive vertices in the path:
   Check that an edge actually exists between them in the graph.
   If any required edge is missing, reject the certificate.

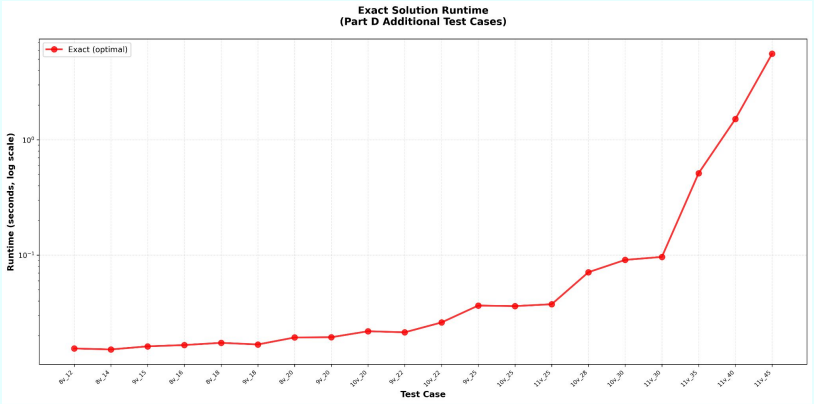
4. Count how many edges the path contains.
   If the length of this path is at least k, accept the certificate.
   Otherwise, reject it.

Time Complexity:
O(n)

Big-O Runtime Analysis

```
def find_longest_path(vertices, graph):
    ...

    for start in vertices: # multiplies total cost by n
        ...

        def dfs(current_vertex, path_length, path):
            ...

        # EXPONENTIAL BRANCHING
        for neighbor, weight in graph[current_vertex].items():
                if neighbor not in visited:
                    …

                    dfs(neighbor, …) # EXPONENTIAL RECURSION


                    …

        dfs(start, 0, [start])
       ...
```

Time Complexity:
O(n! * n)



Hamiltonian Path => Longest Path

1. Copy all vertices in G (O(n))
2. Copy every edge with weight 1 (O(m))
3. Calculate k (O(n))