Approximate Solution

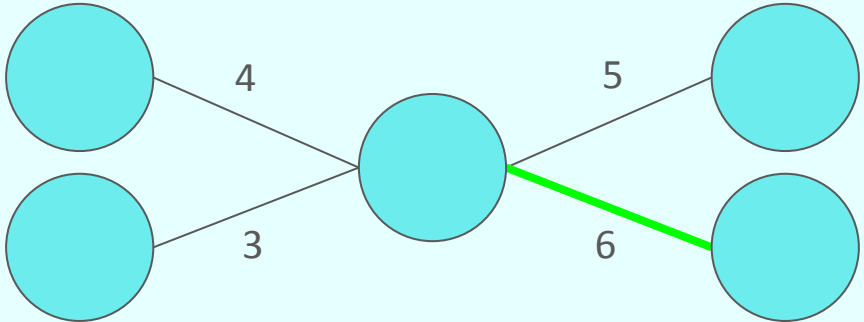
Algorithm (Approximate Greedy Longest Path):

For each vertex in the graph: → runs **n** times
  start a new path at that vertex
  mark it as visited

  While the current vertex has any unvisited neighbors: → ≤ **n** iterations (each vertex visited once)
    choose the unvisited neighbor connected by the heaviest edge → ≤ **n** (scans all neighbors)
    add that neighbor to the path
    mark it visited
    move to that neighbor

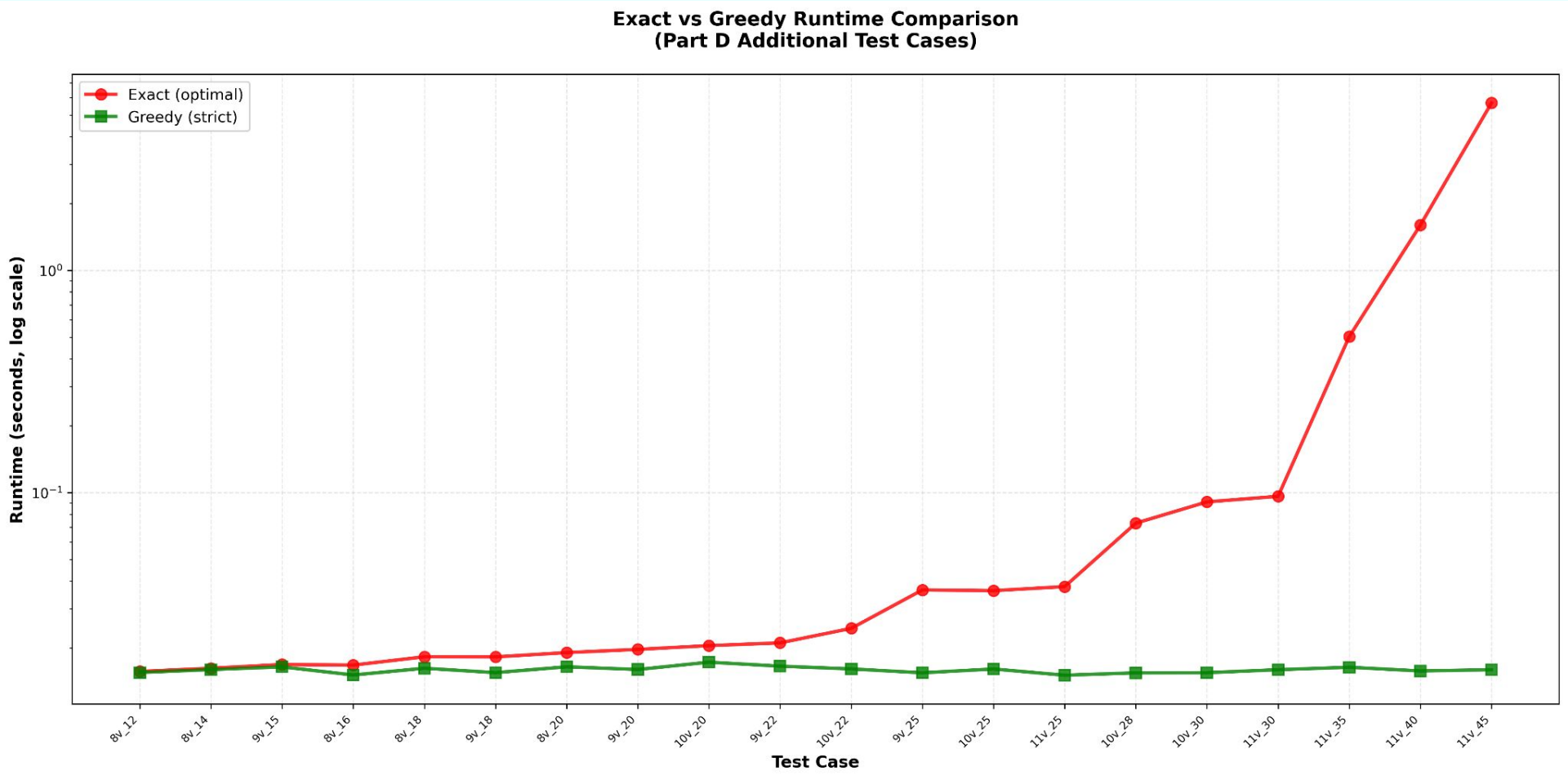
  keep track of the longest path found across all starting vertices

return the longest path discovered

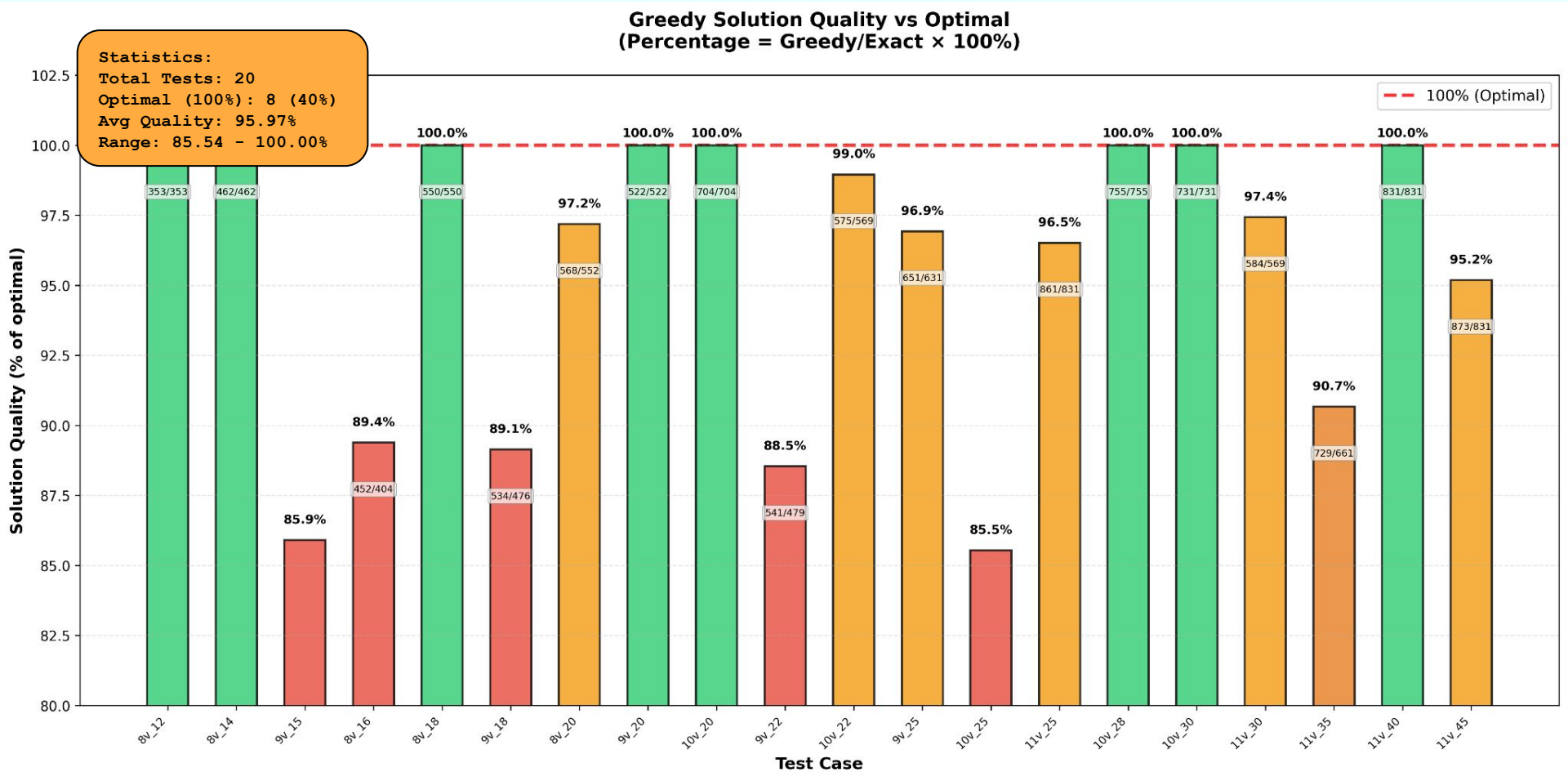


Runtime Analysis:

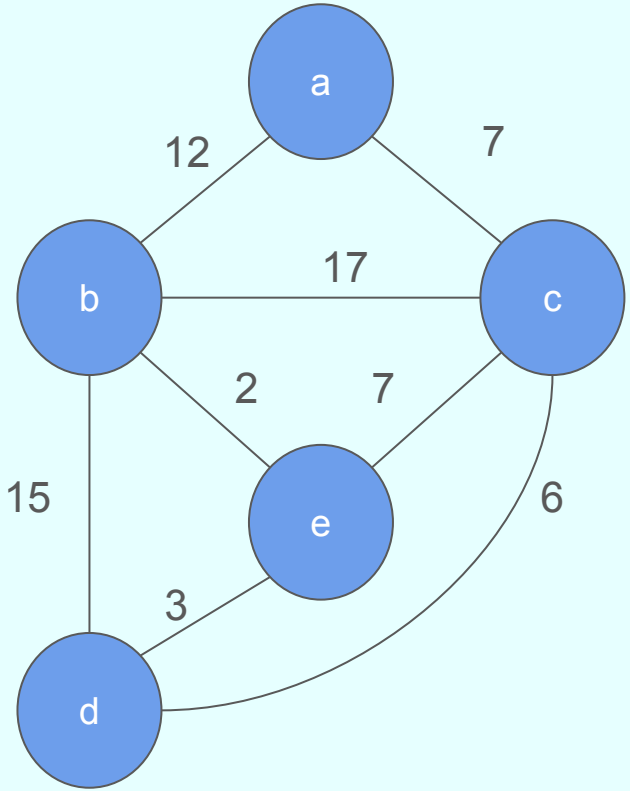
$O(n^3)$



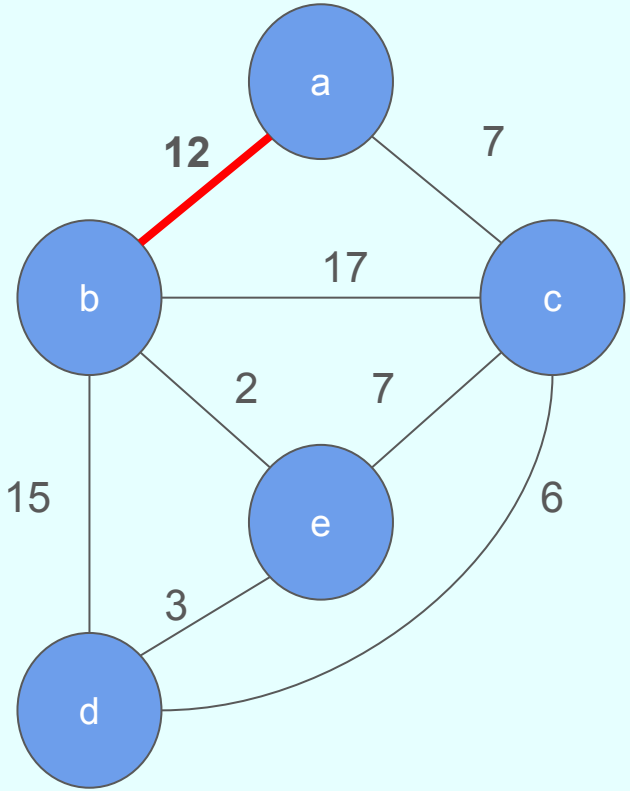
8v_16 → 8 vertices, 16 edges


Greedy Solution Quality vs Optimal
(Percentage = Greedy/Exact × 100%)
Statistics:
Total Tests: 20
Optimal (100%): 8 (40%)
Avg Quality: 95.97%
Range: 85.54 - 100.00%
100% (Optimal)
Solution Quality (% of optimal)
Test Case
8v_12
8v_14
9v_15
8v_16
8v_18
9v_18
8v_20
9v_20
10v_20
9v_22
10v_22
9v_25
10v_25
11v_25
10v_28
10v_30
11v_30
11v_35
11v_40
11v_45
100.0%
100.0%
100.0%
85.9%
89.4%
100.0%
89.1%
97.2%
100.0%
100.0%
88.5%
99.0%
96.9%
85.5%
96.5%
100.0%
100.0%
97.4%
90.7%
100.0%
95.2%
353/353
462/462
452/404
550/550
534/476
568/552
522/522
704/704
541/479
575/569
651/631
861/831
755/755
731/731
584/569
729/661
831/831
873/831

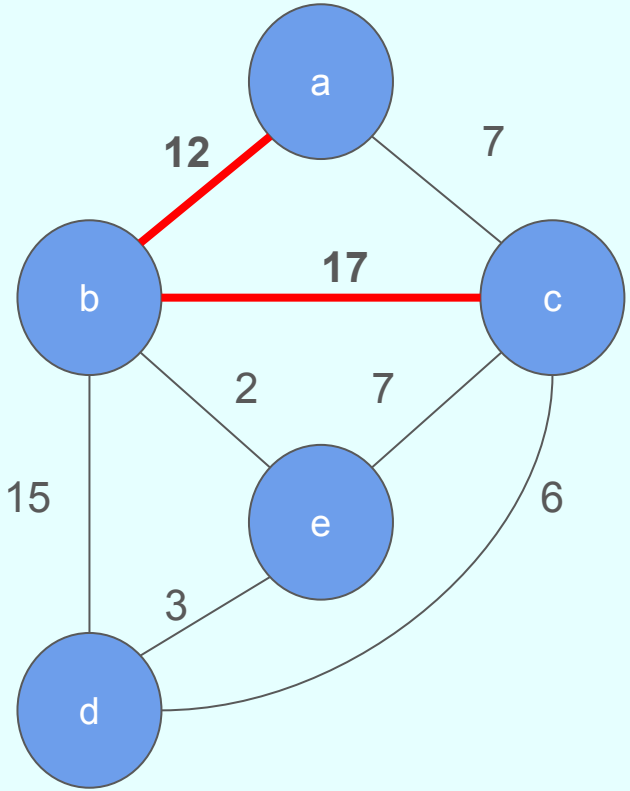
Why Greedy Can Fail



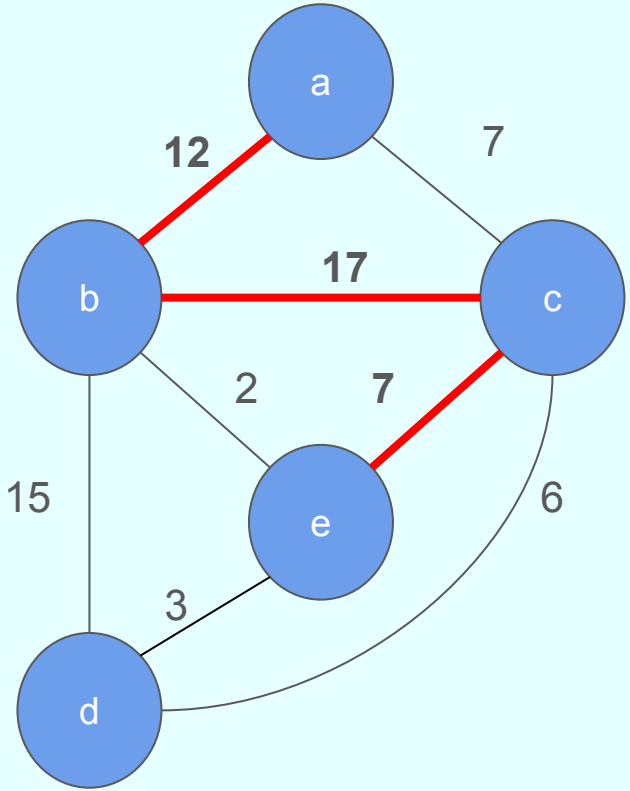
Greedy starting at a



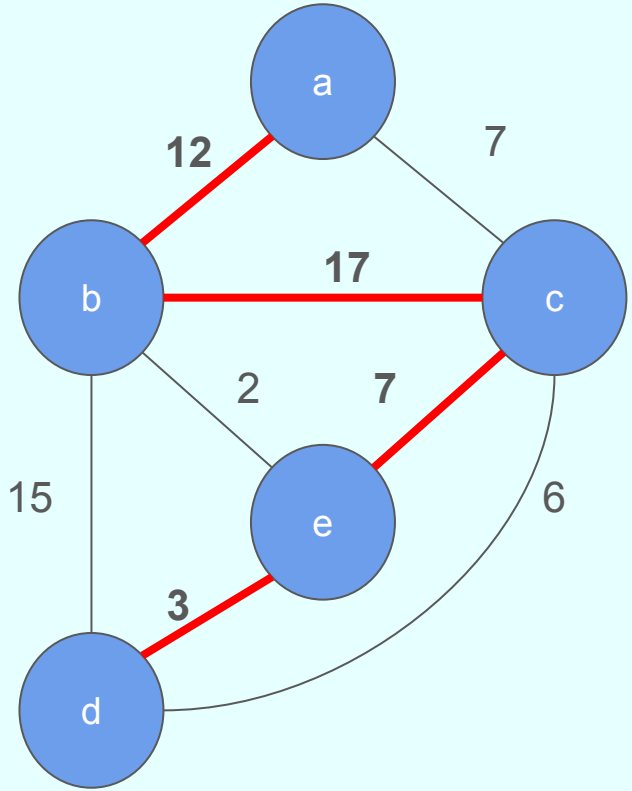
Greedy starting at a



Greedy starting at a

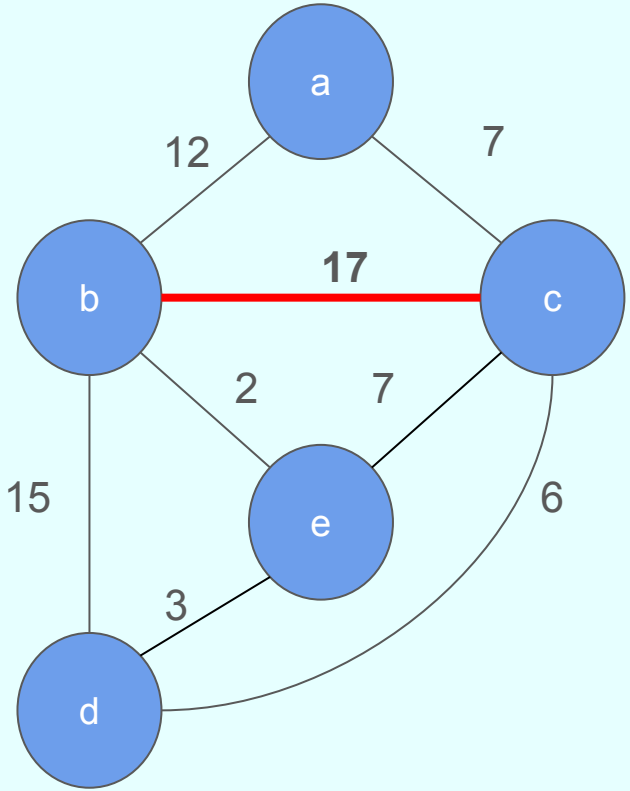


Greedy starting at a

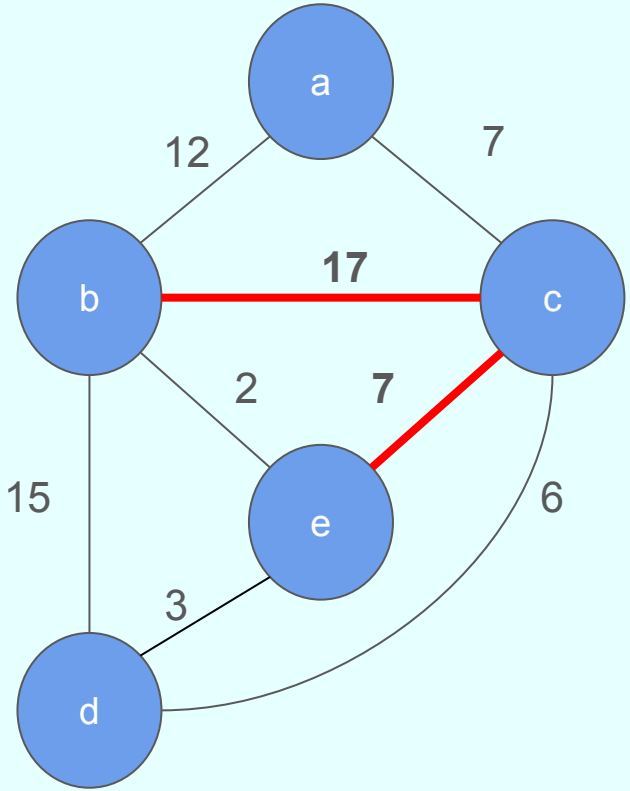


39 total

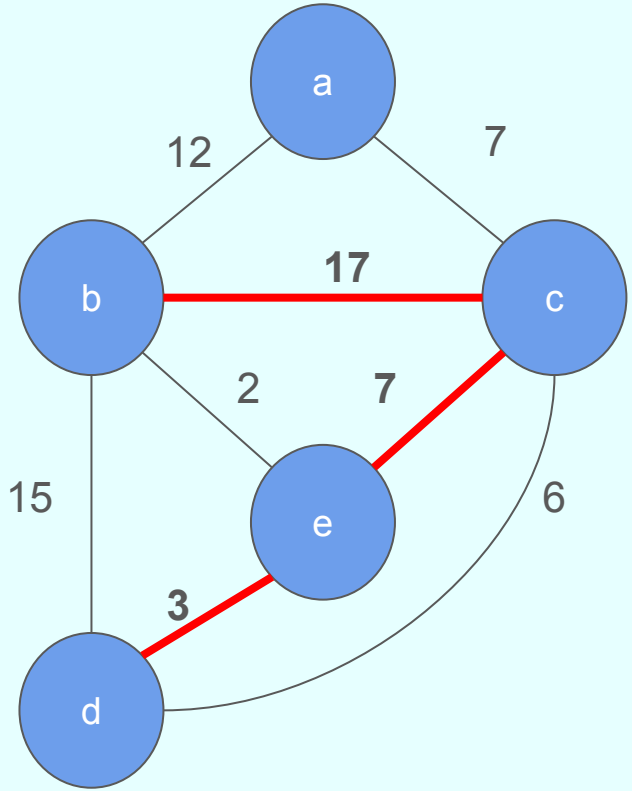
Greedy starting at b



Greedy starting at b

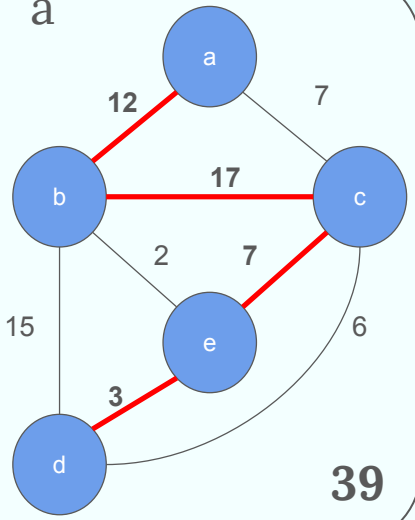


Greedy starting at b



27 total

a



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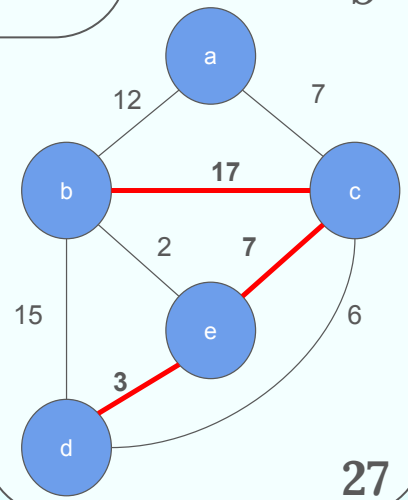
b



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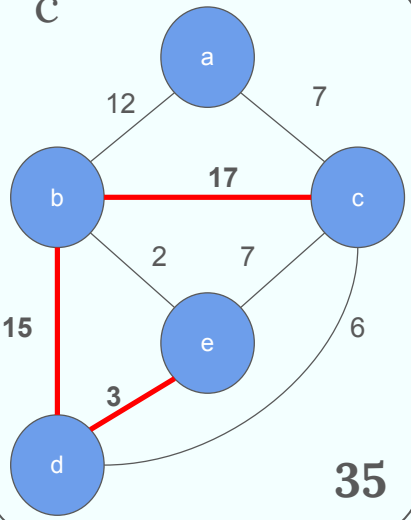
c



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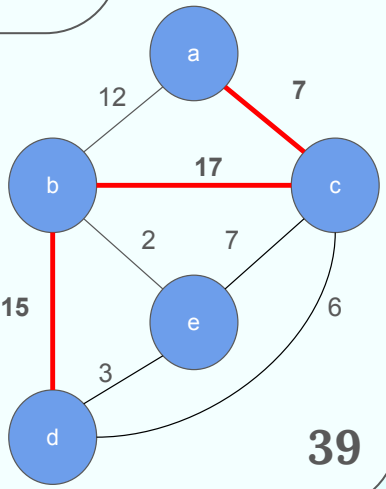
d



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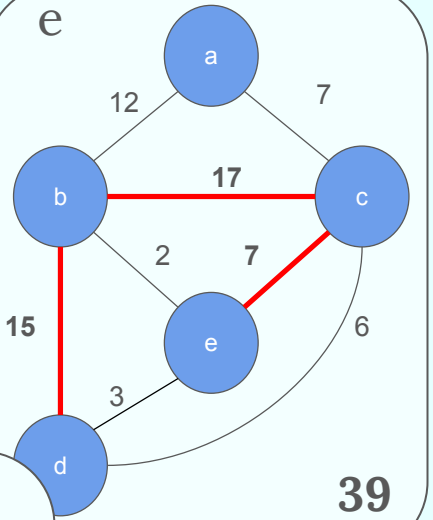
e



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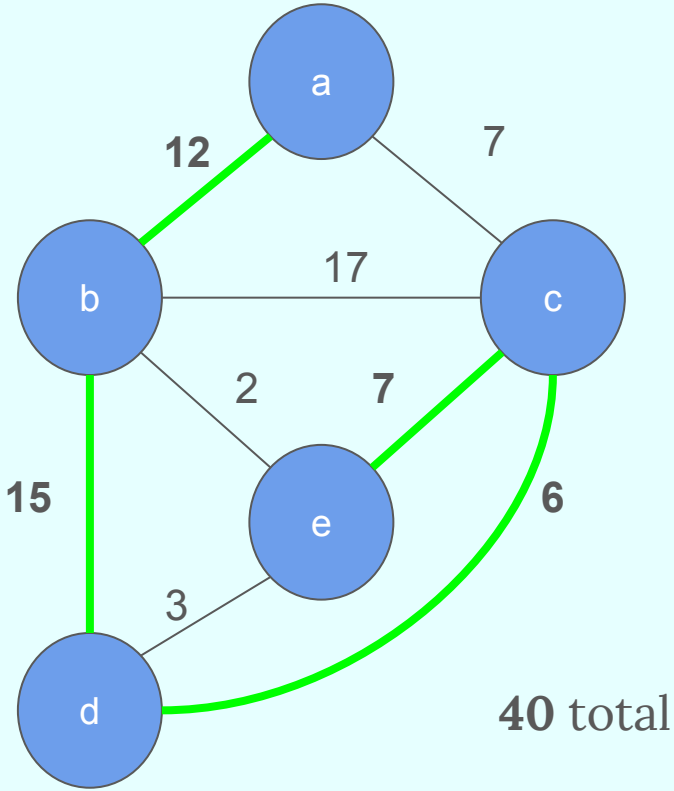
Optimal solution



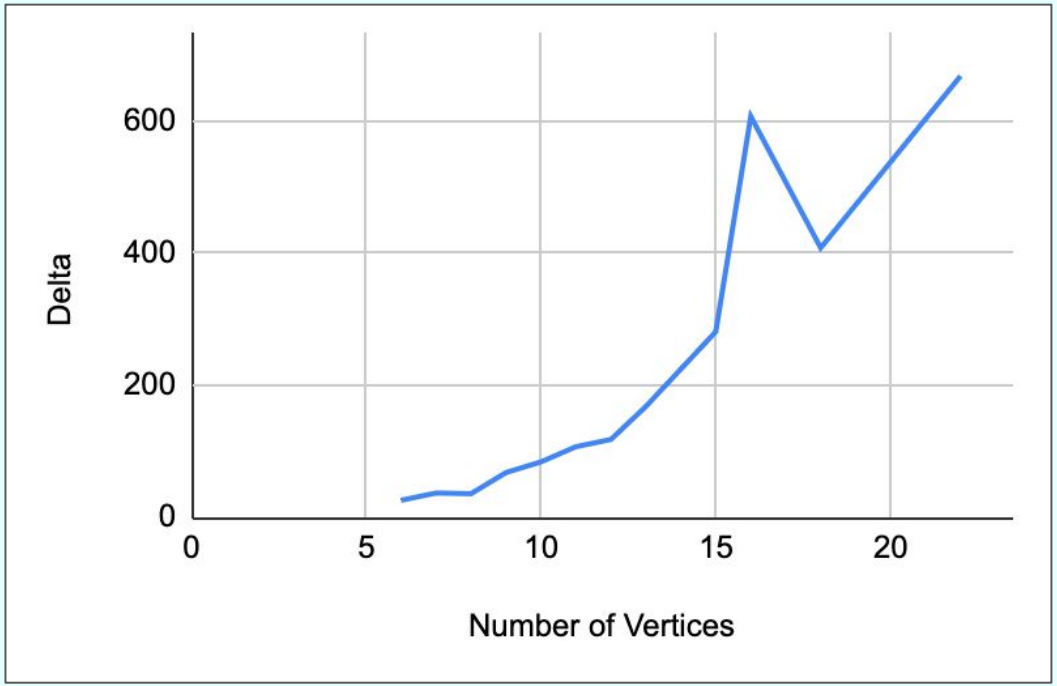
40 total

Lower Bound and Δ

- Δ = Greedy Path Weight − Lower Bound
- Maximum edge weight is valid lower bound
- Δ grows dramatically on large graphs → greedy scales well.

Case	Vertices	LB	Greedy	Δ
test_input_4	6	9	35	26
test_input_medium_1	7	12	49	37
test_input_5	8	10	46	36
test_input_medium_2	9	15	83	68
test_input_6	10	20	104	84
test_input_large_1	11	20	127	107
test_input_7	12	18	136	118
test_input_large_2	13	25	193	168
test_input_8	15	30	311	281
test_input_dense_large	16	60	667	607
test_input_large	18	34	442	408
test_input_extreme	22	45	713	668

Δ vs Number of Vertices



Δ tells how much better this solution is than the lower bound

Δ increases with vertices because greedy paths grow with the graph while lower bound remains small