

# Robust Adaptive Stick-Slip Friction Compensation

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**Abstract**—In this paper, a robust adaptive tracking control scheme is proposed for compensation of the stick-slip friction in a mechanical servo system. The control scheme has a sliding control input to compensate friction forces. The gain of the sliding control input is adjusted adaptively to estimate the linear bound of the stick-slip friction. By introducing the sliding control input, the global stability and the tracking error asymptotic convergence to the predetermined boundary are established via Lyapunov's stability theorem. The proposed scheme is shown to be robust to variations of the system and/or friction characteristics, and a bounded external disturbance. Computer simulations and experiments on an X-Y table verify the effectiveness of the proposed scheme.

## I. INTRODUCTION

**F**RICTION is a natural resistance to relative motion between two contacting bodies. The friction model has been widely studied by numerous researchers [1], [2]. It is commonly modeled as a linear combination of Coulomb friction, stiction, viscous friction, and Stribeck effect.

In servo systems, when the controller is designed without consideration of the friction, the closed-loop systems show steady-state tracking errors and/or oscillations. In addition, the friction characteristics may be changed easily due to the environment's changes, for instance, the variations of the load, temperature, and humidity. So the friction compensation is not an easy problem. To compensate the friction, adaptive schemes were developed in optical tracking telescopes [3], robot manipulators in low velocity [4], and X-Y tables [5]. In [6], a comparative work of several adaptive friction compensation techniques was presented. A nonlinear compensation technique which has a nonlinear proportional feedback control force for the regulation of the one degree of freedom ("1-DOF") mechanical system was proposed in [7].

In this paper, we propose a robust adaptive compensation technique for tracking of the 1-DOF mechanical system with unknown stick-slip friction. In the proposed scheme, the linear bound of the friction is identified adaptively. Using the estimated bound, a sliding control input is calculated such that the tracking error is forced to remain in the preassigned boundary. The boundedness of all signals of the closed-loop system is guaranteed in the sense of Lyapunov. The proposed scheme is also robust to the variation of the friction, even in the presence of a bounded external disturbance. We demonstrate the performance of our scheme via computer simulations and experiments performed on a 1-DOF mechanical plant called the X-Y table.

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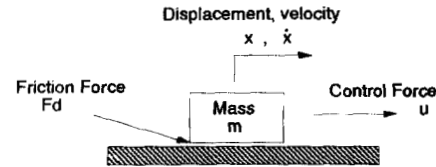


Fig. 1. The 1-DOF mass system.

The remainder of this paper is organized as follows: In Section II, a mechanical system with friction and the investigation of the model of stick-slip friction are described. In Section III, an estimation algorithm and a control law for friction compensation in details are proposed. The stability of the closed-loop system via Lyapunov's direct stability theorem is also proved. In Section IV, the simulation and experimental results are provided to illustrate the performances of the developed scheme. Finally, conclusions are remarked in Section V.

## II. SYSTEM MODELS

The 1-DOF mechanical system under investigation is a mass contained to move in one dimension with stick-slip friction present between the mass and the supporting surface as shown in Fig. 1.

The equation for this model is described as follows:

$$m\ddot{x}(t) + F_f(\cdot) + \nu(t) = u(t) \quad (1)$$

where  $m$  is a mass,  $x(t)$  is a relative displacement,  $F_f(\cdot)$  is a friction force,  $u(t)$  is a control force, and  $\nu(t)$  is a bounded external disturbance which represents the equivalent total disturbance due to the measurement noise and noises in the power sources, etc. The external disturbance is assumed to be bounded within the unknown upper bound  $\epsilon_d$  as follows:

$$|\nu(t)| < \epsilon_d, \quad \text{for all } t > 0. \quad (2)$$

The stick-slip friction force  $F_f(\cdot)$  is assumed to be modeled as follows [7]:

$$F_f(\cdot) = F_{\text{slip}}(\dot{x})[\lambda(\dot{x})] + F_{\text{stick}}(u)[1 - \lambda(\dot{x})] \quad (3)$$

where

$$\lambda(\dot{x}) = \begin{cases} 1 & |\dot{x}| > \alpha, \quad \alpha > 0 \\ 0 & |\dot{x}| \leq \alpha. \end{cases}$$

The above model allows us to evaluate the friction force during sticking and slipping motions. The conceptual stick-slip friction model can be obtained from (3) by limiting  $\alpha \rightarrow 0$ . The artificial parameter  $\alpha$  is taken to be identically zero throughout the analytic design of the stick-slip friction compensator. Nonzero values of  $\alpha$  are only used in the

computer simulation to insure that the numerical integration algorithms remain stable.

The sticking function,  $F_{\text{stick}}(u)$ , provides the value of the friction force at zero velocity. This term is used to determine whether the mass will stick or break free from the static friction forces. The positive and negative limits on the static friction forces are given by  $F_s^+$  and  $F_s^-$ , respectively. Generally they are not equal in magnitude, therefore the model should consider these asymmetries. It is modeled as follows:

$$F_{\text{stick}}(u) = \begin{cases} F_s^+, & u(t) \geq F_s^+ > 0 \\ u(t), & F_s^- < u(t) < F_s^+ \\ F_s^-, & u(t) \leq F_s^- < 0. \end{cases} \quad (4)$$

The mass cannot move until the applied force is greater in magnitude than the respective static friction force.

The slipping function,  $F_{\text{slip}}(\dot{x})$ , provides values of the friction force at nonzero velocity, and is represented by

$$\begin{aligned} F_{\text{slip}}(\dot{x}) &= F_d^+(\dot{x})\mu(\dot{x}) + F_d^-(\dot{x})\mu(-\dot{x}) \\ \mu(\dot{x}) &= \begin{cases} 1 & \dot{x} > 0 \\ 0 & \dot{x} \leq 0 \end{cases} \\ F_d^+(\dot{x}) &= F_s^+ - \Delta F^+ [1 - e^{-(\dot{x}/\dot{x}_{cr}^+)}] + b^+ \dot{x} \\ F_d^-(\dot{x}) &= F_s^- - \Delta F^- [1 - e^{-(\dot{x}/\dot{x}_{cr}^-)}] + b^- \dot{x} \end{aligned} \quad (5)$$

where  $\Delta F^+$  and  $\Delta F^-$  are the respective drops from the static to the kinetic force level,  $\dot{x}_{cr}^+$  and  $\dot{x}_{cr}^-$  are the critical Stribeck velocities, and  $b^+$  and  $b^-$  are the viscous friction coefficients. The friction force is modeled as a summation of the Coulomb friction, viscous friction, and the Stribeck effect. In this case, the Coulomb friction level is presented as  $F_C^{+(-)} = F_s^{+(-)} - \Delta F^{+(-)}$ . As shown in the circled part of Fig. 2, the Stribeck effect is that the friction force is in many cases decreasing with increased relative velocity [8]. It is caused by increased fluid lubrication. An example of the slipping friction force plot with  $\alpha = 0$  is shown in Fig. 2.

The slipping force is assumed to dissipate energy at all the nonzero velocities, and therefore is bounded within the first and third quadrants. It is assumed that there exist the following constants:

$$\begin{aligned} b_1 &> \max(b^+, b^-), \\ F_0 &\geq \max(|F_s^+|, |F_s^-|), \quad F_0 > 0 \end{aligned} \quad (6)$$

which define the following piecewise linear bounds for the slipping functions as shown in Fig. 2:

$$\begin{aligned} F_d^+(\dot{x}) &\leq F_0 + b_1 \dot{x}(t), \quad \forall \dot{x}(t) > 0 \\ -F_0 + b_1 \dot{x}(t) &\leq F_d^-(\dot{x}), \quad \forall \dot{x}(t) < 0. \end{aligned}$$

Therefore, we can say that the stick-slip friction is bounded within the linear bound as follows:

$$|F_f(\cdot)| \leq F_0 + b_1 |\dot{x}(t)|, \quad \forall \dot{x}(t). \quad (7)$$

The following section describes an estimation algorithm for the linear bound described as above and the control scheme using the estimated bound to guarantee the stability of the overall system.

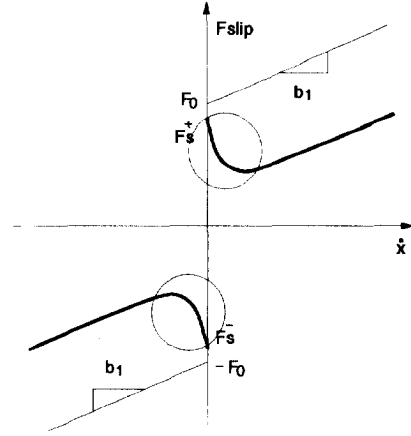


Fig. 2. The graph of a slipping friction force.

### III. ROBUST ADAPTIVE CONTROL LAW

The control objective is to force the plant state vector,  $\mathbf{x}(t) \triangleq [x(t), \dot{x}(t)]^T$ , to follow a desired trajectory,  $\mathbf{x}_d(t) \triangleq [x_d(t), \dot{x}_d(t)]^T$ . Let us define the tracking errors and the error metric,  $s(t)$ , as follows:

$$\begin{aligned} e(t) &= x_d(t) - x(t) \\ \dot{e}(t) &= \dot{x}_d(t) - \dot{x}(t) \\ s(t) &= \dot{e}(t) + \lambda e(t), \quad \lambda > 0. \end{aligned} \quad (8)$$

The equation  $s(t) = 0$  defines a time-varying hyper plane in  $\mathbb{R}^2$  on which the tracking error vector decays exponentially to zero. If the magnitude of  $s(t)$  can be shown to be bounded by a constant  $\delta$ , then the actual tracking errors are asymptotically bounded as described in [9]

$$\begin{aligned} |e(t)| &\leq \lambda^{-1} \delta \\ |\dot{e}(t)| &\leq 2\delta. \end{aligned} \quad (9)$$

Here, we define another error metric,  $s_\Delta(t)$ , as in [10]

$$s_\Delta = s(t) - \delta \text{sat}(s(t)/\delta) \quad (10)$$

where  $\text{sat}(\cdot)$  is a saturation function defined as

$$\text{sat}(x) = \begin{cases} x & \text{if } |x| < 1 \\ \text{sign}(x) & \text{otherwise.} \end{cases}$$

The function  $s_\Delta$  has the following useful properties:

- (i) if  $|s| < \delta$ , then  $\dot{s}_\Delta = s_\Delta = 0$ ,
- (ii) if  $|s| > \delta$ , then  $\dot{s}_\Delta = \dot{s}$  and  $|s_\Delta| = |s| - \delta$ ,
- (iii)  $s_\Delta \text{sat}(s/\delta) = |s_\Delta|$ .

Thus the problem is to design a control law  $u(t)$  which ensures that the tracking error metric  $s(t)$  lies in the predetermined boundary  $\delta$  for all time  $t > 0$ .

As shown in Fig. 3, the control input  $u(t)$  is designed as

$$\begin{aligned} u(t) &= \hat{m}(t)\ddot{x}_r(t) + k_d s_\Delta(t) \\ &\quad + \hat{k}(\dot{x}(t), t) \text{sat}(s(t)/\delta) \end{aligned} \quad (12)$$

where  $\hat{m}(t)$  is the estimate of the mass  $m$ ,  $\ddot{x}_r(t) \triangleq \ddot{x}_d(t) + \lambda \dot{e}(t)$ ,  $k_d$  is a positive constant feedback gain, and  $\hat{k}(\dot{x}(t), t)$  is

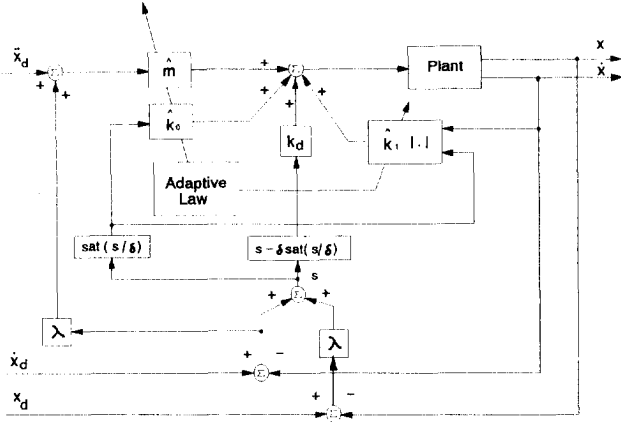


Fig. 3. The block diagram of the control scheme.

a gain of the sliding control input which is the estimated linear bound of the stick-slip friction as discussed in the previous section. The  $\hat{k}(\dot{x}(t), t)$  is defined as

$$\hat{k}(\dot{x}(t), t) \triangleq \hat{k}_0(t) + \hat{k}_1(t)|\dot{x}(t)| \quad (13)$$

where  $\hat{k}_0(t)$  represents the estimate of  $k_0$  which is defined as the summation of the upper bound of the static friction level  $F_0$  and the bounded external disturbance  $\epsilon_d$

$$k_0 \triangleq F_0 + \epsilon_d \quad (14)$$

and  $\hat{k}_1(t)$  is the estimate of the linear bound's slope  $b_1$ . The sliding control input having time-varying gain is introduced to guarantee that the tracking error is within the predetermined boundary  $\delta$ .

Using the control law, the time derivative of the error metric  $s(t)$  can be obtained as

$$\begin{aligned} m\dot{s}(t) = & \tilde{m}(t)\ddot{x}_r(t) + F_f(\cdot) + \nu(t) \\ & - k_d s_{\Delta}(t) - \hat{k}(\dot{x}(t), t) \text{sat}(s(t)/\delta) \end{aligned} \quad (15)$$

where  $\tilde{m}(t) \triangleq m - \hat{m}(t)$  is the estimation error of the mass  $m$ .

The following estimation algorithms are used to estimate the mass and the parameters of the sliding control gain:

$$\begin{aligned} \dot{\hat{m}}(t) &= \eta_1 s_{\Delta}(t) \ddot{x}_r(t) \\ \dot{\hat{k}}_0(t) &= \eta_2 |s_{\Delta}(t)| \\ \dot{\hat{k}}_1(t) &= \eta_3 |s_{\Delta}(t)| |\dot{x}(t)| \end{aligned} \quad (16)$$

where  $\eta_i$ ,  $i = 1, 2, 3$  are positive adaptive gains. Then we can get the following theorem.

**Theorem 1:** Consider the closed-loop adaptive control system consisting of the plant (1), the control law (12), and the estimation algorithms (16). Then all the states in the closed-loop system remain bounded and the tracking error metric  $s(t)$  asymptotically converge to the predetermined boundary  $\delta$ .

*Proof:* Let us define a Lyapunov function candidate as

$$V(t) = \frac{1}{2} \left( m s_{\Delta}^2(t) + \frac{1}{\eta_1} \tilde{m}^2(t) + \frac{1}{\eta_2} \tilde{k}_0^2(t) + \frac{1}{\eta_3} \tilde{k}_1^2(t) \right) \quad (17)$$

where  $\tilde{k}_0(t) \triangleq k_0 - \hat{k}_0(t)$  and  $\tilde{k}_1(t) \triangleq b_1 - \hat{k}_1(t)$  are the estimation errors of the linear bound of the stick-slip friction.

$(\frac{d}{dt})s_{\Delta}^2$  is well defined and continuous everywhere except at  $|s| = \delta$ . It can be written  $(\frac{d}{dt})s_{\Delta}^2 = 2s_{\Delta}\dot{s}$  from (11).

Therefore, using (16) and (11),  $\dot{V}(t) = 0$  when  $|s| < \delta$ . When  $|s| > \delta$ , the time derivative using (12), (15), and (16) can be easily derived as

$$\begin{aligned} \dot{V} = & -k_d s_{\Delta}^2 + \tilde{m} s_{\Delta} \ddot{x}_r + F_f(\cdot) s_{\Delta} \\ & + \nu s_{\Delta} - \hat{k}_0 |s_{\Delta}| - \hat{k}_1 |s_{\Delta}| |\dot{x}| \\ & - \frac{1}{\eta_1} \tilde{m} \dot{\tilde{m}} - \frac{1}{\eta_2} \tilde{k}_0 \dot{\tilde{k}}_0 - \frac{1}{\eta_3} \tilde{k}_1 \dot{\tilde{k}}_1 \\ \leq & -k_d s_{\Delta}^2(t) + (F_0 + \epsilon_d) |s_{\Delta}| + b_1 |\dot{x}| |s_{\Delta}| \\ & - k_0 |s_{\Delta}| - \hat{k}_1 |s_{\Delta}| |\dot{x}| - \frac{1}{\eta_2} \tilde{k}_0 \dot{\tilde{k}}_0 - \frac{1}{\eta_3} \tilde{k}_1 \dot{\tilde{k}}_1 \\ = & -k_d s_{\Delta}^2 < 0, \quad \text{for all } t > 0. \end{aligned} \quad (18)$$

If the initial values of  $s_{\Delta}$ ,  $\tilde{m}$ ,  $\tilde{k}_0$ , and  $\tilde{k}_1$  are bounded, they remain bounded for all  $t > 0$ . Since  $s_{\Delta}(t)$  is uniformly bounded, if  $e(0)$  and  $\dot{e}(0)$  are bounded, then  $e(t)$  and  $\dot{e}(t)$  are also bounded for all  $t > 0$ . Thus, as  $x_d(t)$  is bounded by design,  $x(t)$  is bounded as well.

Since  $V(t)$  is a positive monotonically decreasing function, the limit  $V(\infty)$  is well defined and

$$-\frac{1}{k_d} \int_0^{\infty} \dot{V} dt = \int_0^{\infty} s_{\Delta}^2 dt < \infty$$

that is,  $s_{\Delta} \in L_2$ .

By definition,  $\dot{s}_{\Delta}(t)$  is either 0 or  $\dot{s}(t)$ , where  $\dot{s}(t)$  is given in (15). From boundedness of  $s_{\Delta}$ ,  $\dot{x}(t)$ , the estimation errors  $\tilde{m}$ ,  $\tilde{k}_0$ ,  $\tilde{k}_1$  and  $\ddot{x}_d(t)$ ,  $\dot{s}(t)$  is bounded. By Babalat's lemma,  $s_{\Delta}(t) \rightarrow 0$  as  $t \rightarrow \infty$ . This means that the inequality  $|s(t)| \leq \delta$  is obtained asymptotically and the asymptotic bounded tracking errors follow from (9).  $\square$

**Remark:** One should choose small  $\delta$  to ensure the tracking errors in small boundary as shown in Theorem 1. A small  $\delta$  may cause that the sliding control input is likely to act as a discontinuous function. The discontinuous control input may cause the degradation of system reliability. Therefore, there should be a "trade-off" between the desired tracking error tolerance and the discontinuity of the control input.

#### IV. SIMULATION AND EXPERIMENT

In this section, we illustrate the effectiveness of the proposed control scheme by computer simulation and an experiment on an X-Y table. For simplicity, the proposed scheme was applied to only an X-axis positioning (1-DOF) system. Performance of the proposed scheme will be compared with that of a PD controller. The PD controller is used instead of a complete PID controller, because it is more convenient to compare PD due to the similarity of its structure to the proposed controller. The integral term in a PID controller is mainly used for eliminating the steady-state error in point-to-point control tasks and it is not much useful in tracking tasks.

The desired position and velocity trajectories are shown in Fig. 4. They are computed by integrating the following acceleration profile:

$$\ddot{x}_d(t) = \begin{cases} A \sin(4\pi t/T), & \text{for } 0 < t < T/2, \\ -A \sin(4\pi t/T), & \text{for } T/2 < t < T. \end{cases}$$

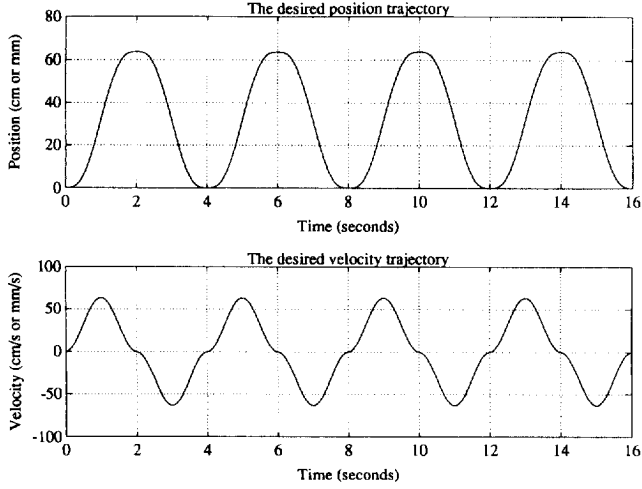
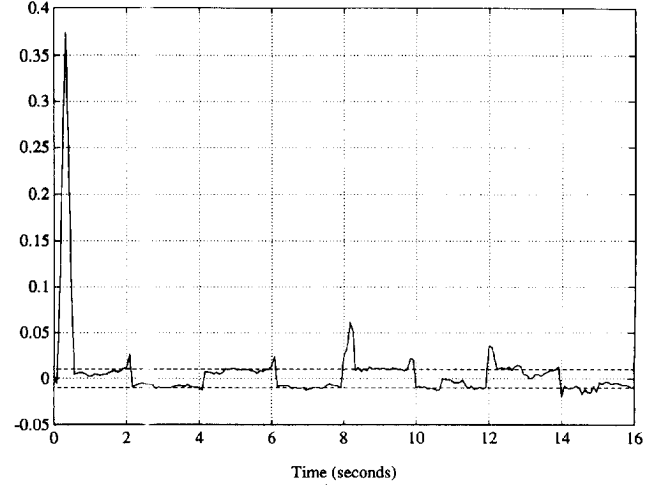
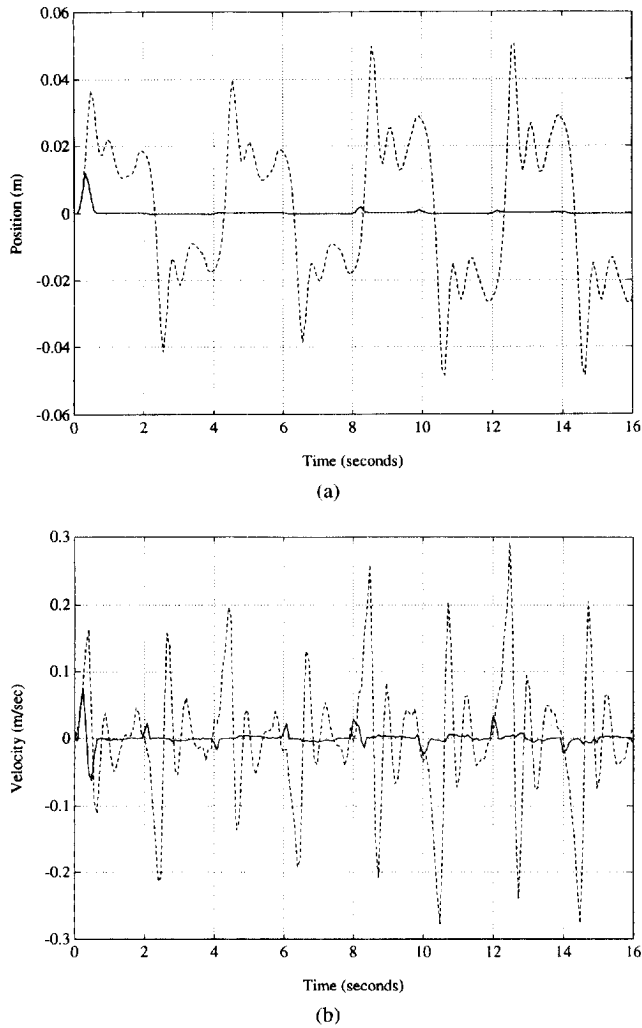


Fig. 4. The desired position and velocity trajectories.

Fig. 6. The error metric  $s(t)$  trajectory from computer simulation.Fig. 5. The simulation results of the proposed and the PD controllers: (a) position tracking errors  $e(t)$ , (b) velocity tracking errors  $\dot{e}(t)$ , (solid line: proposed controller, dashed line: PD controller).

where  $A$  is a set to 1 ( $\text{m/s}^2$ ) and 0.1 ( $\text{m/s}^2$ ) for the simulations and the experiments, respectively.  $T$  represents a period, which is set to 4 (s).

TABLE I  
SIMULATION PARAMETERS FOR STICK-SLIP FRICTION MODEL

Time (seconds)		$0 \leq t \leq 8$	$t > 8$
mass	$m$	1.0 kg	1.2 kg
Stick-Slip	$F^+$	4.2 N	5.4 N
	$\Delta F^+$	1.8 N	2.5 N
	$\dot{x}_{cr}^+$	0.1 m/s	0.2 m/s
	$b^+$	0.5 Ns/m	0.4 Ns/m
Friction	$F^-$	-4.0 N	-5.0 N
	$\Delta F^-$	-1.7 N	-2.0 N
	$\dot{x}_{cr}^-$	0.1 m/s	0.2 m/s
	$b^-$	0.5 Ns/m	0.4 Ns/m
$\alpha$		0.001 m/s	0.001 m/s

#### A. Computer Simulation

Let us consider the parameters of the 1-DOF mechanical system (see Fig. 1) and the stick-slip friction model presented in Table I. We changed the parameters of the system at time  $t = 8$  s to show the adaptability and the robustness of the proposed scheme. The external disturbance  $\nu(t)$  is zero mean Gaussian noise whose standard deviation is set to 0.1.

The designer's parameters of the controller are selected as

$$k_d = 5, \quad \lambda = 30, \quad \eta_1 = 10, \quad \eta_2 = 50, \quad \eta_3 = 50$$

and the boundary of the error metric  $\delta$  is set to 0.01. For fairness of comparison, the gains of the PD controller are chosen to the identical gains of the PD part ( $k_d s(t)$  term) in the proposed scheme, i.e.,  $K_P = k_d \lambda = 150$ ,  $K_D = k_d = 5$ . The initial values of the estimates are chosen to be zero, i.e.

$$\hat{m}(0) = \hat{k}_0(0) = \hat{k}_1(0) = 0.$$

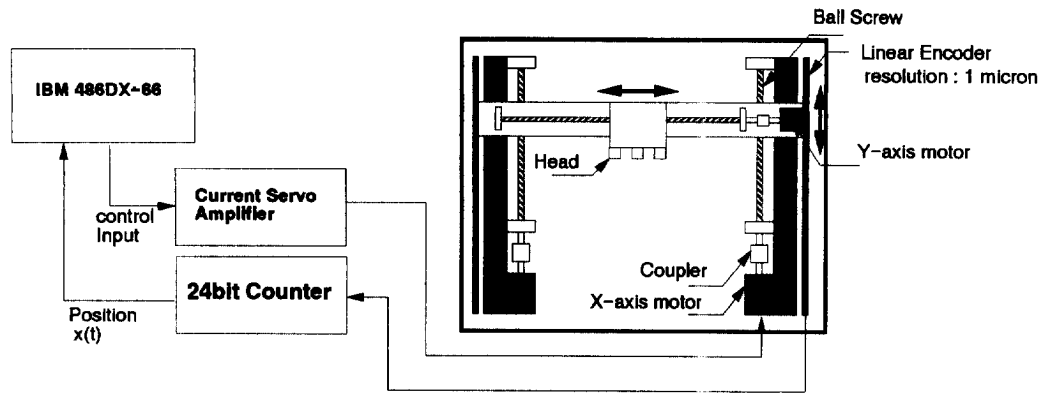


Fig. 7. Experiment setup.

The Euler's approximation method is used for integration with a sampling period of 4 (ms).

Fig. 5 shows the tracking errors of the proposed and the PD controllers. The solid and the dashed lines represent the output responses under the proposed controller and the PD controller, respectively. From Fig. 5, it can be seen that large tracking errors whenever the velocity becomes zero. Since the control input becomes smaller than the friction force as the velocity decreases, the mass becomes stuck. However, such a stiction can be avoided in the proposed controller. It demonstrates the superiority and the adaptability of the proposed scheme. The tracking errors of the proposed scheme are reduced remarkably. It can be seen that the tracking errors of the PD controller are increased after the system parameters are changed in the PD controller. However, the proposed scheme is robust to the variations of the system and/or stick-slip friction characteristics and the bounded external disturbance. We proved also that the tracking errors converge asymptotically to the preassigned boundary in the previous section. Fig. 6 shows the tracking error metric  $s(t)$  which does indeed demonstrate the asymptotic convergence as we expected.

### B. Experiment

The experimental setup is shown in Fig. 7. The system consists of the positioning mechanism, a position sensor system, a servo amplifier and an IBM PC equipped with a custom board. The X-Y table has two linear motion mechanisms which are composed of a dc motor, a screw, and a linear encoder. The linear encoder which has one micron ( $1 \mu\text{m}$ ) resolution is equipped in X-axis mechanism. In order to obtain velocity measurement, numerical differentiation of position measurement was used. The obtained velocity information was then filtered by a second-order digital low-pass filter to alleviate the effects of noise. The main control algorithm is implemented with a 250 Hz sampling rate via an IBM PC with an Intel i486DX-66 microprocessor. The PC is interfaced to the current servo amplifier and the sensor through a custom card containing a 24-b counter and one-channel DA conversion circuits for one-channel analog output. The proposed algorithm is written in language C.

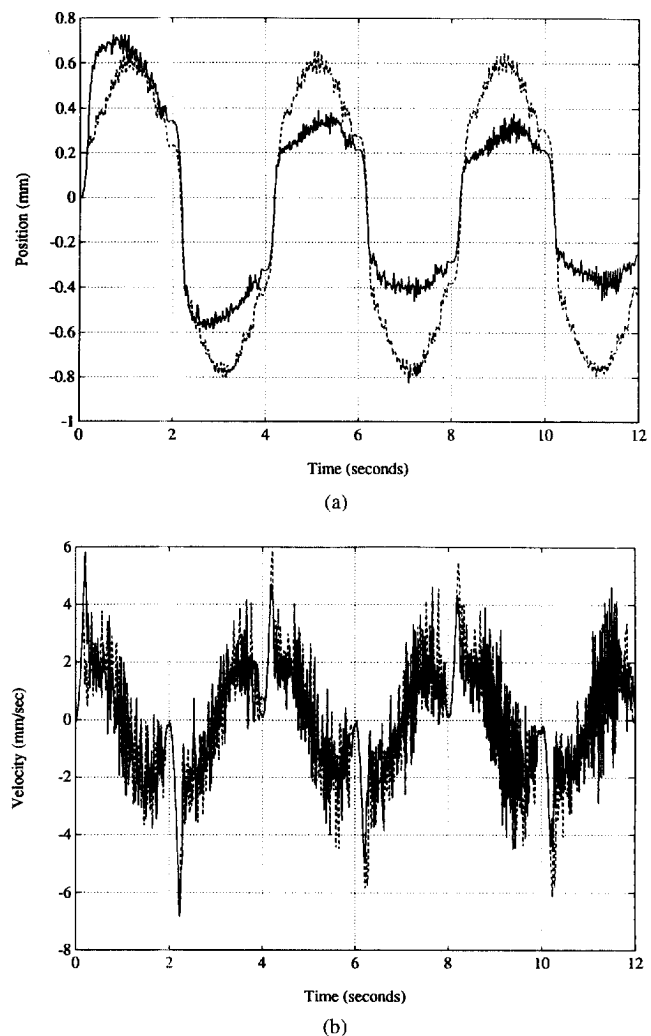


Fig. 8. The experiment results of the proposed and the PD controllers: (a) position tracking errors  $e(t)$ ; (b) velocity tracking errors  $\dot{e}(t)$ , (solid line: proposed controller, dashed line: PD controller).

The parameters of the controller are selected as

$$k_d = 0.1, \quad \lambda = 30, \quad \eta_1 = 5 \times 10^{-8}, \\ \eta_2 = 0.061, \quad \eta_3 = 10^{-6}$$

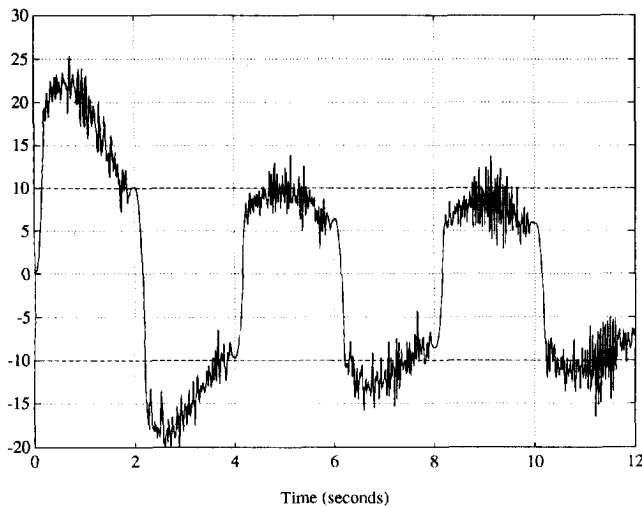


Fig. 9. The error metric  $s(t)$  trajectory from experiment.

and the desired error tolerance  $\delta$  is set to 10, which is experimentally determined so that the control input  $u(t)$  is smooth as discussed in the remark in Section III. The identical gains of the PD part in the proposed controller are used by the PD controller, i.e.,  $K_P = 3$ ,  $K_D = 0.1$ . The initial values of the estimates are chosen to be zero (no prior knowledge) to highlight the effect of the adaptation. Since the mechanical structure and other components in the system have inherent unmodeled high-frequency dynamics which should not be excited, small adaptation gain  $\eta_1$  should be used for trajectories with high acceleration  $\ddot{x}_r(t)$  in order to keep the bandwidth of the closed-loop system, and thus the excitation of the unmodeled dynamics, at the same level [11].

Fig. 8 shows the position and the velocity tracking errors of the well-tuned PD controller and the proposed controller, respectively. From Fig. 8(a), it can be seen that a maximum position error of the PD controller is about 0.7 mm. On the other hand, the proposed controller allows us to obtain a maximum position error of less than 0.3 mm after 4 s, representing around two times reduction in the tracking error. Although it is hard to distinguish the tracking performance in velocity from the Fig. 8(b), the proposed controller guarantees slightly reduced velocity error, compared to the PD controller. As was confirmed in computer simulation, Fig. 9 shows the asymptotic convergence of the error metric  $s(t)$  to the desired error tolerance  $\delta = 10$ . The result demonstrates the usefulness of the proposed controller such that we can handle the tracking performance within the prespecified error tolerance.

## V. CONCLUSION

In this paper, the development and implementation of a robust adaptive tracking control scheme to compensate the unknown stick-slip friction was presented. A linear bound of the stick-slip friction is estimated adaptively using an estimation algorithm based on Lyapunov's stability theorem. The proposed control scheme shows the tracking error is bounded in the predetermined tolerance in the presence of the external disturbance. The global stability of the overall system

is established in the Lyapunov's sense. Finally, simulation and experimental results have verified the effectiveness of the proposed control scheme and have shown that the compensation technique guarantees superior tracking performance compared to the conventional PD controller.

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