

A Novel Friction Model for Stick-Slip Driving

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Abstract - As more and more high requirements for positioning accuracy in many scientific fields, stick-slip actuator with long-range and high resolution has drawn wide attention. The performance of the stick-slip actuator is mainly affected by two reasons: the accuracy of the friction model and the accuracy of the actuator kinetic model. Considering about the opinions above, this paper presents a new friction model which is used to depict the friction of the stick-slip actuator. There are only 7 parameters in LS model, which captures some important properties of the friction phenomena such as the Stribeck effect, the hysteresis of the friction, rate independent property and nonlocal memory property. With the new friction model, a kinetic model of the stick-slip actuator is established, and its performance is obtained by simulation. The results of the simulation indicate that the kinetic model can accurately depict the actuator's performance, and the LS model with typical friction properties has a widely implementation in friction calculation.

Index Terms - Stick-slip driving, LuGre-MMS model, nonlocal memory.

I. INTRODUCTION

In modern scientific work, micro/ nanomanipulation field requires more and more high positioning accuracy, which makes an actuator based on stick-slip principle draw a great attention for its long-range and high resolution. The working principle of the stick-slip actuator is shown in Fig.1, the driven unit of the stick-slip actuator is a piezoelectric actuator, which has a rigid connection with the rod. As the driving voltage increase slowly, the slider yields a displacement ΔX along with the rod. When the driving voltage shrinks to 0 suddenly, the piezoelectric actuator and the rod return back to the origin position. However, the slider remains the position as before due to the slider's large inertia, which leads to a displacement ΔX of the slider with a respect to its origin position in a period. Thus a continuous stick-slip motion can be obtained by applying a period driving voltage mentioned above on the stick-slip actuator. The performance of the stick-slip actuator is mainly affected by two reasons [1]: the accuracy of the friction model and the accuracy of the actuator kinetic model.

According to the working principle of the stick-slip actuator, the displacement of the slider is caused by the friction between the slider and the rod, so the investigation about the friction becomes crucial to the whole system. Friction is the result caused by the unpredictable asperity interactions on the two contact surfaces [2]. The investigation about friction is generally divided into two regimes: the presliding regime and the gross sliding regime. In presliding

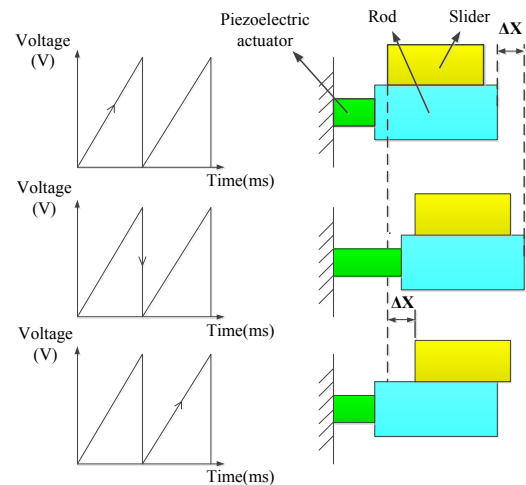


Fig.1 The working principle of the stick-slip actuator

regime, friction is mainly generated by the interactions of the asperities on the contact surfaces. As the relative micro-displacement between the two surfaces increases, an elastic-plastic deformation occurs on the surface asperity, which causes a reaction force. With the increasing of the elastic-plastic deformation, the static interactions between the asperities break away, generating a gross sliding. When all the static interactions were broken away, the friction then enters the gross sliding regime. In the gross sliding regime, the friction is mainly determined by the relative velocity between the two surfaces. During the presliding regime, the main characteristics are presliding hysteresis characteristic, nonlocal memory phenomenon and rate dependent phenomenon. In the gross sliding regime, the main characteristic is Stribeck effect, where friction varies with the changing of the relative velocity [3]. Due to the large quantities investigation about friction, there are several typical friction models proposed in recent decades. However, since these friction models could not depict the friction characteristics completely, or have difficulties for implementation [7], the applications of these models are limited in some situation. Based on the above seasons, this paper propose a new friction model—LS model, which can depict each regime of the friction behavior by large investigation about the typical friction models. Analysis each properties of the friction model by a series of mechanics simulations. As important as the friction model, the dynamic modeling about the stick-slip actuator also has a great effect on the performance of the system. The dynamic model reflects the working principle and the properties of the system which

is the key section in the investigation of the micro/nanomanipulation field.

II. A NEW FRICTION MODEL BASED ON LUGRE AND MMS

A. The New Friction Model

Among the current friction models, the Dahl model [4] and the LuGre model [5] can't depict the nondrifting phenomenon in the presliding regime. The elastoplastic model [10] can suppress the nondrifting phenomenon well, but both the elastoplastic model and the LuGre model could not depict the nonlocal memory phenomenon in presliding regime. Though the Leuven model [10] [11] and the Maxwell model [12] could depict each property of the friction process accurately, these two models have much difficulty in implementation during identify the parameters. Since there are lots of assumptions during the modeling process, the Modified Maxwell Slip model (MMS) [13] can only be applied to describe the friction behavior in the presliding regime.

According to the above analysis, the LuGre model could not depict the nonlocal memory phenomenon [7], but it can describe the other properties during the friction process well, such as the Stribeck effect. The reason for this phenomenon is that the LuGre model in the presliding regime is too simple [8]. During the presliding regime, the friction caused by the damp $\sigma_1(dz/dt) + \sigma_2(dx/dt)$ can be neglected normally, so the LuGre model only uses the initial stiffness of the system σ_0 to depict the properties in presliding regime. It is obviously not capable enough for the complex position-force relationship in presliding regime [10]. However, during the gross sliding regime, the intermediate function $g(v)$ depict the relationship between the friction and the velocity well, which reflects the dynamic property in the gross sliding process. In contrast with the LuGre model, the Modified Maxwell Slip model (MMS) can give an accurate description of friction in presliding regime, which is identical to the friction curve obtained by experimentalism, such as the nonlocal memory phenomenon. However, since there are lots of assumptions during the modeling process, the Modified Maxwell model (MMS) can only be applied to describe the friction behavior in the presliding regime [13]. If there is a need to investigate the complete process of the friction, the MMS model can be used as part of the overall model to describe the whole friction process.

According to the characteristics of the MMS model and the LuGre model, consider to combine the two models together: Since the LuGre model can describe all the properties of friction except the presliding regime; and the new Modified Maxwell Slip model (MMS) [13] which could depict the nonlocal memory phenomenon can only be used in presliding regime. So overcome the shotcoming of the two models and combine them together—utilize Modified Maxwell Slip model to describe the behavior of the presliding regime, and utilize LuGre model to describe the behavior of the gross sliding regime making friction be accurate in overall process.

Based on above analysis, the following are assumed:

(1) The friction hysteresis in presliding is rate independent. According to the experiment results, the friction-displacement hysteresis curves remain a constant shape under the excitation signals with different frequencies [14]. Since in the microcosmic situation, the inertial force can be normally neglected in presliding regime, which allows the nonlinear friction-displacement curve not change in time domain [12]. The Dahl model and the Maxwell model are both rate independent. But the LuGre model, Leuven model and the GMS model are shown to be rate dependent [15].

(2) The asperity of the contact surface yield an elastic-plastic deformation in presliding regime, and the maximum friction based on the deformation does not exceed the maximum static friction force F_s . During the transition between the presliding regime and the gross sliding regime, the asperity yields a deformation with the increasing relative displacement. The deformation causes an increase of friction until reaching the maximum static friction force F_s . Where, F_s is proportional to the normal force.

(3) The parameter K of the MMS model is identical to the parameter σ_0 in the LuGre model. Both of these two parameters represent the initial stiffness of the system. For a transition between the two models, replace K by the stiffness parameter σ_0 of the LuGre model.

According to the above assumptions, this paper proposes a new friction model—LS model:

$$F = F_1 + \sigma_1 \cdot \dot{z} + \sigma_2 \cdot v \quad (1)$$

$$\dot{F}_1 = |\Omega| \cdot \sigma_0 \cdot \exp(-\sigma_0 |x_r|) \quad (2)$$

$$\Omega = \text{sgn}(v) \cdot F_s - F_1 \quad (3)$$

$$x_r = 0 \rightarrow \text{velocity reverse} \quad (4)$$

$$x_r = \int_{t_r}^t v dt \rightarrow \text{other situation} \quad (5)$$

$$\dot{z} = v - \frac{|v| \cdot \sigma_0}{g(v)} \cdot z \quad (6)$$

$$g(v) = F_c + (F_s - F_c) \cdot e^{-(v/v_s)^2} \quad (7)$$

Where, F_1 is the friction of the MMS model in presliding regime. With the increase of the relative displacement, F_1 gradually converge to the maximum static friction force F_s . v represents the relative velocity. x_r represents the relative displacement after the velocity reverse, which is reset to zero at the moment of velocity reverse, and then begins to calculate the relative displacement. σ_0 represents the initial stiffness of the contact surface. State variables z represents the average deformation of the bristles. σ_1 is the damping coefficient in the micro-displacement. σ_2 is the high speed damping coefficient. F_c is the Coulomb friction. F_s is the maximum static friction force. v_s represent the velocity when the Stribeck effect occur. F_c and F_s are both proportional to the normal force applying between the two contact surface, $F_c = \mu_c \cdot N$, $F_s = \mu_s \cdot N$.

In presliding regime, the damping force $\sigma_1(dz/dt) + \sigma_2(dx/dt)$ can be neglect in equation

$F=F_1+\sigma_1(dz/dt)+\sigma_2(dx/dt)$ [9], only leaving F_1 (MMS) to describe the friction behavior. In gross sliding regime, the friction is mainly described by the damping force $\sigma_1(dz/dt)+\sigma_2(dx/dt)$. Where $g(v)=F_c+(F_s-F_c)\exp(-(v/v_s)^2)$ could depicts the Stribeck effect [14]. Thus a complete friction model is obtained for the whole process from the presliding regime to the gross sliding regime.

B. Modeling and analysis of the LS model

The friction process is divided into the presliding regime and the gross sliding regime. During the presliding regime, the main characteristics are presliding hysteresis characteristic, nonlocal memory phenomenon and rate independent phenomenon. In the gross sliding regime, the main characteristic is Stribeck effect, which describes the relationship between the friction force and the relative velocity [6]. So make a simulation for LS model, observing the behavior in the above 4 characteristics. In order to reflect the characteristics of the LS model, compare the LuGre model with the LS model, observing the difference between the two models.

C. Stribeck effect of LS model

Firstly, investigate the characteristic of LS model in gross sliding regime. In the gross sliding regime, the main characteristic is Stribeck effect, which describes the relationship between the friction force and the relative velocity that the friction force firstly decreases and then increases with the increasing velocity as depicted in Fig.2. Under the acceleration of $1 \times 10^{-5} \text{ mm/s}^2$, make a simulation about LS model between the friction force and the relative velocity. Fig.3 is the simulation result of the Stribeck effect about LS model. As shown in Fig.3, the LS model could describe the Stribeck effect and the transition from the presliding regime to the gross sliding regime.

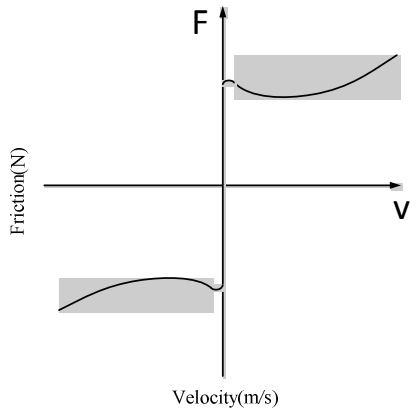


Fig.2 The Stribeck effect curve of friction.

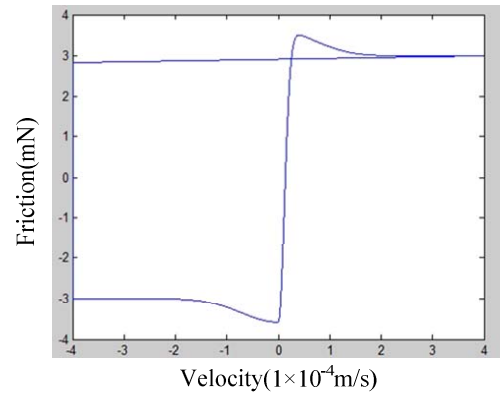


Fig.3 The Stribeck effect curve of the LS model.

D. Hysteresis characteristic of the LS model

The experiment reveals that there is a hysteresis relationship between friction and velocity [13]. In order to validate the property of the LS model, make a hysteresis simulation for the LS model with a sinusoidal velocity input signal. Fig.4 is the simulation result about the friction-displacement curve. As shown in Fig.4, with the increasing and decreasing of the velocity, there is a hysteresis deviation at the same velocity. So the LS model possesses the hysteresis characteristic.

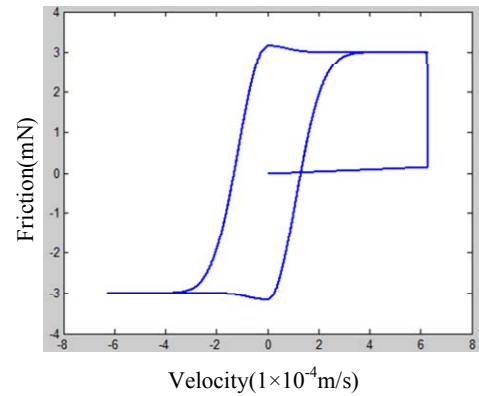


Fig.4 LS hysteresis characteristic simulation

E. Nonlocal memory phenomenon of the LS model

The LuGre model could not describe the nonlocal memory phenomenon, but the LS model is an improvement on the LuGre model. Make a simulation in presliding regime for the LuGre model and the LS model respectively, comparing the performance on the nonlocal memory phenomenon between the two models. Apply a random periodic wave excitation as the displacement input signal of the LuGre model and the LS model, shown as Fig.5, observing the calculate results generated by the two friction models. Fig.6 shows the friction-displacement curve of the LuGre model. After the velocity reverse, the model produced a drift in the presliding regime; Fig.7 shows the friction-displacement curve of the LS model. After the velocity reverse, the model almost produced no drift in the presliding regime. The analysis about the friction-displacement curves of

the two friction models shows that: the LS model can accurately describe the nonlocal memory phenomenon in presliding regime.

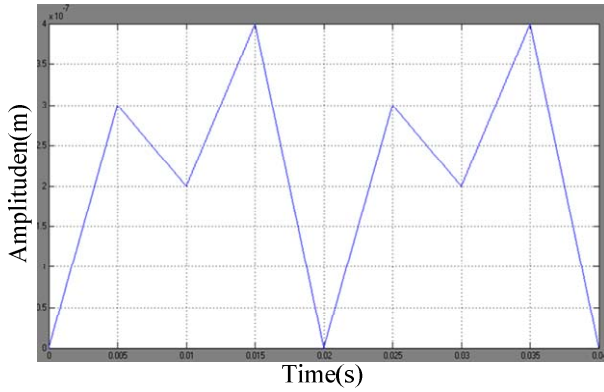


Fig.5 The random periodic wave excitation

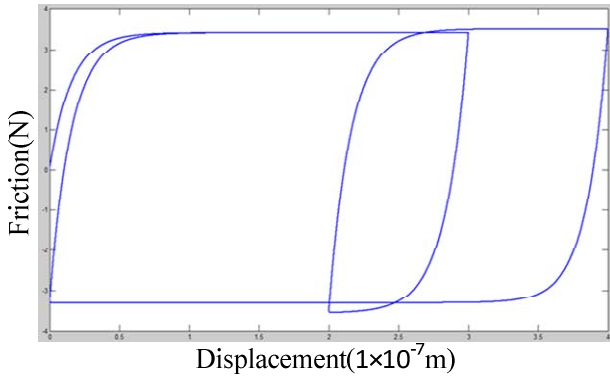


Fig.6 The friction-displacement curve of the LuGre model with no nonlocal memory.

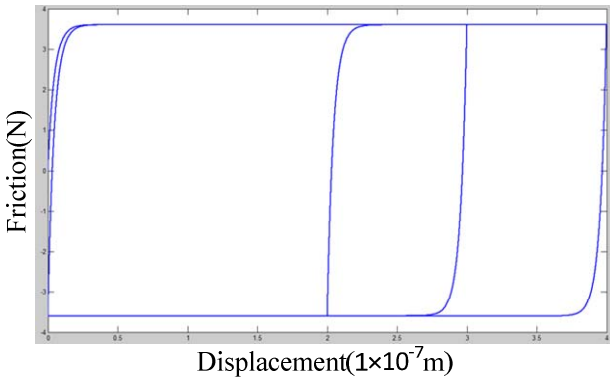


Fig.7 The friction-displacement curve of the LS model with nonlocal memory.

F. Rate independent characteristic of the LS model

According to the results of experiments [14], friction-displacement curve does not change with the frequency of the input signal. But in the traditional LuGre model, friction-displacement curve does not have a rate independent property [15]. In order to verify the LS model is rate independent, apply a series of sinusoidal excitation signals with frequency of 10Hz, 100Hz, 1000Hz for a simulation on the LuGre model and the LS model respectively, and observe the performance

of the output curves. Fig.8 shows the friction-displacement curves of the LuGre model, in which the shape of the curves change with the input frequencies obviously. Fig.9 shows the friction-displacement curves of the LS model, in which the curves appear the same pattern with the input frequencies changing. The simulation results indicate that the LS model has a property of rate independent.

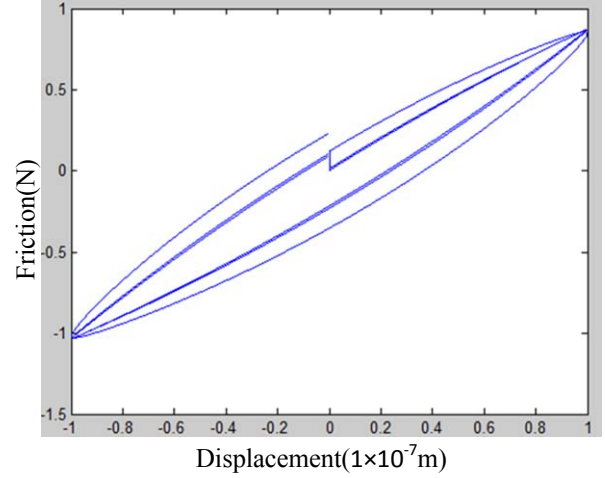


Fig.8 The friction-displacement curves of the LuGre model at a frequency of 10Hz,100Hz,1000Hz.

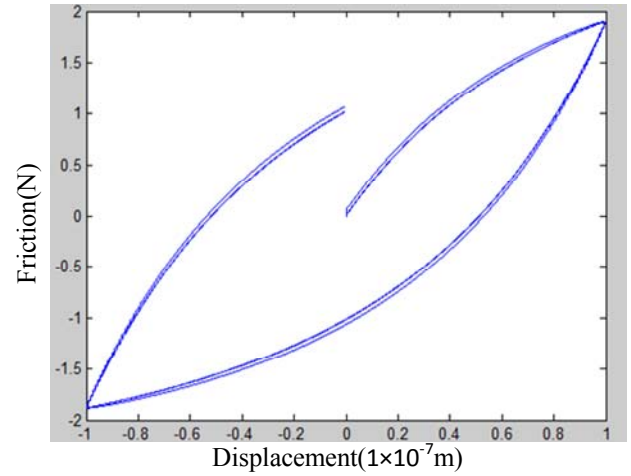


Fig.9 The friction-displacement curves of the LS model at a frequency of 10Hz,100Hz,1000Hz.

III. KINETIC SIMULATION OF THE STICK-SLIP ACTUATOR

Obtaining an accurate friction model, establish a dynamic model for the stick-slip actuator. Analyze the characteristics of the system and validate the practicability of the LS model simultaneously. Fig.10 is the stick-slip actuator. Fig.11 is the schematic diagram of the stick-slip actuator which consists of piezoelectric ceramic, sliding column and slider. Firstly, simplify the structure of the actuator, as shown in Fig.12.

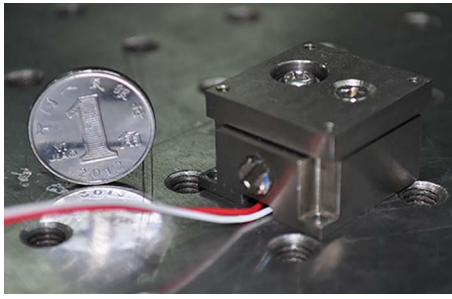


Fig.10 The stick-slip actuator.

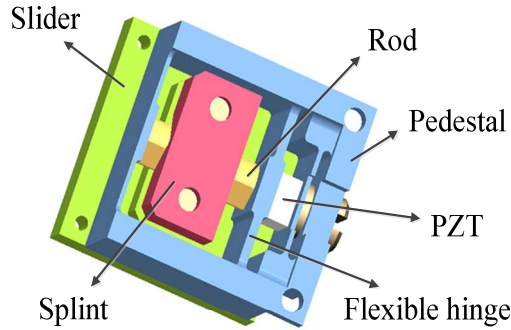


Fig.11 The schematic diagram of the stick-slip actuator

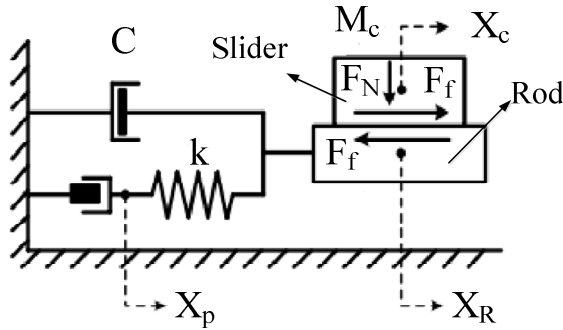


Fig.12 simplified model of the stick-slip actuator

Simplify the piezoelectric ceramic into a spring-damper system. k is the spring stiffness. c is the damping coefficient. M_R is the mass of the sliding column. M_c is the mass of the slider. F_f is the friction. F_N represents the normal force between the sliding column and the slider. x_p is the Ideal strain of the ceramics. x_R represents the sliding column displacement. x_c is the slider displacement. According to Newton's second law, establish the dynamics equation of the stick-slip actuator:

$$k \cdot (x_p - x_R) - C \cdot \dot{x}_R - F_f = M_R \cdot \ddot{x}_R \quad (7)$$

$$F_f = M_c \cdot \ddot{x}_c \quad (8)$$

The friction in the equation is replaced by the LS model. The slider achieves a horizontal displacement by stick-slip motion. The slider displacement can be obtained according to (9). The related parameters of the actuator are shown in Table I.

$$x_c = \int \int_0^t \frac{F_f}{M_c} \quad (9)$$

Table I
THE RELATED PARAMETERS OF THE ACTUATOR

parameter	Parameter Symbol	Unit	Value
Coefficient of dynamic friction	μ_d		0.3
Coefficient of static friction	μ_s		0.36
Clamping force	F_n	N	10
Piezoelectric ceramic stiffness	k	N/ m	2.08×10^8
Piezoelectric ceramic damping	C	kg/s	6.2
Mass of sliding column	m_R	kg	1.1×10^{-3}
Mass of slider	M_c	kg	1.38×10^{-2}
Stribeck effect Velocity	v_s	m/s	0.4×10^{-3}
Stiffness coefficient	0	N/ m	1×10^7
Low-speed viscosity	1	Ns/m	80
High-speed viscosity	2	Ns/m	8

According to the system of kinetic equations and the parameters, make a dynamic simulation for the stick-slip actuator. The ideal displacement of piezoelectric ceramic is a periodic input signal with amplitude $4.4 \mu\text{m}$ with the frequency of 500 Hz. As shown in Fig.13, the simulation result is identical with the waveform in [16]. On the one hand, the result illustrates the correctness of the dynamic model; on the other hand, it also shows that LS model can accurately describe the phenomenon of friction. So LS model can be applied to the related study of friction.

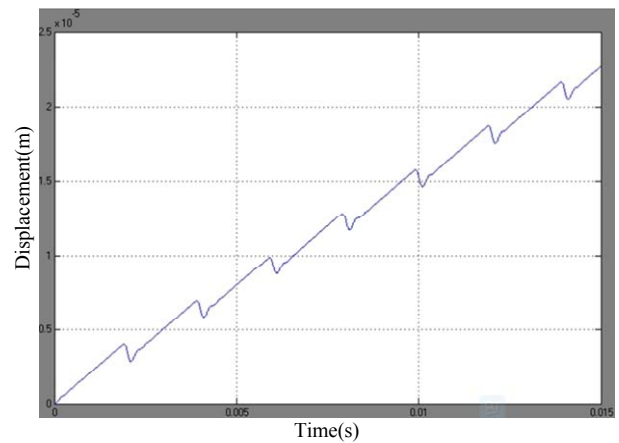


Fig.13 The displacement simulation result of the actuator under a periodic wave excitation

IV. CONCLUSION

For stick-slip actuator, this paper proposes a new friction model (LS model) based on the LuGre model and the MMS model. This model consists of two parts, illustrating the MMS model describing the presliding regime and illustrating the LuGre model describing the sliding regime. Through a series of simulations, the LS model is verified to capture the typical friction properties such as the nonlocal memory phenomenon, Stribeck effect, friction hysteresis and the rate independence. And the LS model has a simple formula, containing only seven parameters, whose high computational efficiency is very suitable for the simulations of friction. Utilize the LS model to establish the dynamic model of the stick-slip actuator. According to the simulation results, the dynamic model is proved to be capable of describing the mechanical properties of the stick-slip actuator correctly. Moreover, since the LS model can accurately describe the phenomenon of friction, the new frictional model can be applied in related studies.

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