Smooth Sliding Control Applied to Power Optimization via Extremum Seeking in Variable Speed Wind Turbines

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Abstract

Different methods have been examined regarding power optimization and control for wind energy conversion systems (WECS) due to the economical interest and the sustainable need for energy growth. Here, we apply extremum seeking control (ESC) in an outer loop to perform the maximum power point tracking (MPPT) and we also design a nonlinear robust controller for the inner loop. To maximize power capture, the turbine speed is tuned by the outer ESC loop for all speeds within sub-rated power operating conditions. The robust part of the controller, which maintains fast transient response, is based on sliding mode control that features a smooth control signal, free of *chattering*, previously designed for linear plants. In this sense, this work presents the first generalization of this controller for the class of nonlinear plants representing the turbine. The complete stability analysis is provided and the effectiveness of the proposed scheme is supported by analysis and simulation results.

I. INTRODUCTION

A method to estimate ground reaction forces (GRFs) in a robot/prosthesis system is presented in [**?**] using kalman filters instead of bulky load cells. The system includes a robot that emulates human hip and thigh motion, along with a powered (active) prosthesis left for transfemoral amputees, and includes four degrees of freedom (DOF): vertical hip displacement, thigh angle, knee angle and ankle angle.

Bulky load cells and sensors are often employed in robots and prosthetic legs to capture gait data, external forces (GRFs) and moments during walking **Referenciar 9 do Paper**, which will be used as feedback measurements to control the robot and prosthesis. The control parameters depend on the gait mode, which is determined on the basis of the external forces. However there are several drawbacks to the use of load cells [**?**]. Another approach would be to estimate the external forces acting on the prosthetic foot.

In [**?**], the use of Extended Kalman Filter (EKF) is proposed to estimate the GRF. However it is acknowledged two important potential drawbacks. First, the derivation of the Jacobian matrix for the linearization of the system can be complex and can cause numerical implementation difficulties. Second, linearization can lead to cumulative errors which may affect the accuracy of the estimation and consequently the stability of the estimation-based control loop.

In this paper, we propose the implementation of a High Order Observer to estimate the states of a 3DOF Robot (vertical hip displacement, thigh angle and knee angle) developed for prostesis parameters estimation and compare the results with [**?**].

II. PRELIMINARIES

The following notations and terminology are used:

- The 2-norm (Euclidean) of a vector x and the corresponding induced norm of a matrix A are denoted by $|x|$ and $|A|$, respectively. The symbol $\lambda[A]$ denotes the spectrum of A and $\lambda_m[A] = -\max_i\{Re\{\lambda[A]\}\}$.
- The $\mathscr{L}_{\infty e}$ norm of a signal $x(t) \in \mathbb{R}^n$ is defined as $\|x_t\| := \sup_{0\leq\tau\leq t} |x(\tau)|$.
- Classes of $\mathscr{K}, \mathscr{K}_\infty$ functions are defined according to [**?**, p. 144]. ISS, OSS and IOSS mean Input-State-Stable (or Stability), Output-State-Stable (or Stability) and Input-Output-State-Stable, respectively [**?**].
- (i) α denotes class-$\mathscr{K}$ functions; (ii) β denotes class-$\mathscr{K}_\infty$ functions; (iii) π denotes class-$\mathscr{K}\mathscr{L}$ functions; (iv) Ψ denotes *known* class-$\mathscr{K}$ functions; (v) $\varphi, \bar{\varphi}$ denotes *known* non-negative functions.

A. Notation and Terminology

The following notation and basic concepts are employed: **(1)** ISS means Input-to-State-Stable and classes $\mathscr{K}$, $\mathscr{K}_\infty$ functions are defined as in [**?**]. **(2)** The Euclidean norm of a vector x and the corresponding induced norm of a matrix A are denoted by $|x|$ and $|A|$, respectively. **(3)** The symbol "s" represents either the Laplace variable or the differential operator "d/dt", according to the context. **(4)** As in [**?**], [**?**] the output y of a linear time invariant (LTI) system with transfer function $H(s)$ and input u is given by $y = H(s)u$. Convolution operations $h(t) * u(t)$, with $h(t)$ being the impulse response from $H(s)$, will be eventually written, for simplicity, as $H(s) * u$. **(5)** As usual in SMC, Filippov's definition for solution of discontinuous differential equations is adopted [**?**]. **(6)** We denote by $\pi(t)$ any exponentially decreasing signal, i.e., a signal satisfying $|\pi(t)| \leq \Pi(t)$, where $\Pi(t) := Re^{-\lambda t}$, $\forall t$, for some scalars $R, \lambda > 0$.

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III. System Model

The dynamics of the machine/prosthesis system composed by a 3-link rigid body robot with prismatic-revolute-revolute (PRR) configuration, following the notation in [**?**], is given by:

$$D(q)\ddot{q}+C(q,\dot{q})\dot{q}+B(q,\dot{q})+P(\dot{q})+J_e^T F_e+g(q)=F_a\,, \tag{1}$$

where $q^T=\begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$ is the vector of joint displacements (q_1 is the vertical displacement, q_2 is the thigh angle and q_3 is the knee angle), $D(q)$ is the inertia matrix, $C(q,\dot{q})$ is the matrix of Coriolis and centrifugal forces, $B(q,\dot{q})$ is the knee damper nonlinear matrix, J_e is the kinematic Jacobian relative to the point of application of external forces F_e, $g(q)$ is the term of gravitational forces and F_a is the torque/force produced by the actuators. Here, in contrast to [**?**], we have included the term $P(\dot{q})$ in order to take explicitly into account the Coulomb friction as in [**?**]. Note that, inertial and frictional effects in the actuators can be included in this model.

To establish a basis for dynamic model derivations and to verify the leg geometry during simulations, the set of reference frames used for forward kinematics problems are the same as the ones assigned in [**?**]. Matrices $D(q)\ddot{q}$, $C(q,\dot{q})$ and $g(q)$ are obtained using the standard Newton-Euler approach and are given in Appendix , where the plant parameters were extracted from [**?**].

A. A Simplified Model

In order to illustrate the observer design proposed in this note, consider a simplified version of the machine/prosthesis system (2) where no external forces are considered ($F_e\equiv 0$), the specific leg prosthesis damping matrix is disregarded ($B(q,\dot{q})\equiv 0$) and the Coulomb friction is neglected ($P(\dot{q})\equiv 0$). In this case, the machine/prosthesis system is described by:

$$D(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=F_a\,. \tag{2}$$

The system matrices $D(q),C(q,\dot{q})$ and $g(q)$ are supposed to be uncertain, but the corresponding nominal matrices $D_n(q),C_n(q,\dot{q})$ and $g_n(q)$ are assumed known. In particular, the inertia matrix $D(q)$ which is invertible, since $D(q)=D^T(q)$ is strictly positive definite.

Introducing the variables $\xi_1:=q$ and $\xi_2:=\dot{q}$, the model (2) can be rewritten in the state-space form as:

$$\begin{aligned}\dot{\xi}_1 &= \xi_2\,, &(3)\\ \dot{\xi}_2 &= k_p(\xi,t)\,[u+d(\xi,t)]\,,\quad u:=F_a\,, &(4)\\ y &= \xi_1\,, &(5)\end{aligned}$$

or, equivalently,

$$\begin{aligned}\dot{\xi} &= A_\rho\xi+B_\rho k_p(\xi,t)[u+d(\xi,t)]\,, &(6)\\ y &= C_\rho\xi\,, &(7)\end{aligned}$$

where $\xi^T=\begin{bmatrix} \xi_1 & \xi_2 \end{bmatrix}$ is the state vector, $k_p(\xi,t)=D(\xi_1)^{-1}$, $d(\xi,t):=-C(\xi_1,\xi_2)\xi_2-g(\xi_1)$, $C_\rho=\begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}$ and the pair (A_ρ,B_ρ) is in Brunovsky?s canonical controllable form. For each solution of (6) there exists a maximal time interval of definition given by $[0,t_M)$, where t_M may be finite or infinite. Thus, finite-time escape is not precluded, *a priori*.

Remark. (**Nominal Values**) Nominal terms can be used in the HGO implementation in order to reduce conservatism in the HGO design. The plant could be rewritten as:

$$\begin{aligned}\dot{x}_1 &= x_2\,, &(8)\\ \dot{x}_2 &= f(x_1,x_2,u,t)+\delta_f(x_1,x_2,u,t)\,,\quad u:=F_a\,, &(9)\\ y &= x_1\,, &(10)\end{aligned}$$

where the nominal part of the system dynamics is represented by

$$f(x_1,x_2,u,t):=D_n^{-1}(x_1)u-D_n^{-1}(x_1)\left[C_n(x_1,x_2)x_2+g_n(x_1)\right]\,, \tag{11}$$

while the uncertainties are concentrated in the term

$$\delta_f(x_1,x_2,u,t):=\left[D^{-1}(x_1)-D_n^{-1}(x_1)\right]u+\left[D_n^{-1}(x_1)C_n(x_1,x_2)-D^{-1}(x_1)C(x_1,x_2)\right]x_2+D_n^{-1}(x_1)g_n(x_1)-D^{-1}(x_1)g(x_1)\,. \tag{12}$$

However, to simplify this presentation while keeping the main HGO design methodology, consider $C_n\equiv 0$, $g_n\equiv 0$ and, since D is assumed known, we also have $D_n=D$.

IV. HIGH GAIN OBSERVER WITH VARIABLE GAIN

The HGO is given by

$$\dot{\hat{\xi}} = A_\rho \hat{\xi} + B_\rho u + H_\mu L_o (y - C_\rho \hat{\xi}) , \tag{13}$$

where $C_\rho := [1\ 0\ \ldots\ 0]$ and L_o and H_μ are given by

$$L_o := [\ l_1 \quad \ldots \quad l_\rho\ ]^T \text{ and } H_\mu := \text{diag}(\mu^{-1}, \ldots, \mu^{-\rho}) . \tag{14}$$

The observer gain L_o is such that $s^\rho + l_1 s^{\rho-1} + \ldots + l_\rho$ is Hurwitz. In this paper, instead of using a constant μ, we introduce a *variable* parameter $\mu = \mu(t) \neq 0, \forall t \in [0, t_M)$, of the form

$$\mu(\omega, t) := \frac{\bar{\mu}}{1 + \psi_\mu(\omega, t)} , \tag{15}$$

where ψ_μ, named **domination function**, is a non-negative function (to be designed later on) continuous in its arguments, ω is an available signal obtained from a norm observer for (6) and $\bar{\mu} > 0$ is a design constant. For each system trajectory, μ is absolutely continuous and $\mu \leq \bar{\mu}$. Note that μ is bounded for t in any finite sub-interval of $[0, t_M)$. Therefore,

$$\mu(\omega, t) \in [\underline{\mu}, \bar{\mu}] , \quad \forall t \in [t_*, t_M) , \tag{16}$$

for some $t_* \in [0, t_M)$ and $\underline{\mu} \in (0, \bar{\mu})$.

A. High Gain Observer Error Dynamics

The transformation [?] [?]

$$\zeta := T_\mu \tilde{\xi} , \quad T_\mu := [\mu^\rho H_\mu]^{-1} , \quad \tilde{\xi} := \xi - \hat{\xi} , \tag{17}$$

is fundamental to represent the $\tilde{\xi}$-dynamics in convenient coordinates to allow us show that $\tilde{\xi}$ is arbitrarily small, *modulo* exponentially decaying term. First, note that:

$$(i)\ T_\mu (A_\rho - H_\mu L_o C_\rho) T_\mu^{-1} = \frac{1}{\mu} A_o , \quad (ii)\ T_\mu B_\rho = B_\rho , \quad \text{and} \quad (iii)\ \dot{T}_\mu T_\mu^{-1} = \frac{\dot{\mu}}{\mu} \Delta ,$$

where $A_o := A_\rho - L_o C_\rho$ and $\Delta := \text{diag}(1-\rho, 2-\rho, \ldots, 0)$. Then, subtracting (13) from (6) and applying the above relationships (i), (ii) and (iii), the dynamics of $\tilde{\xi}$ in the new coordinates ζ (17) is given by:

$$\mu \dot{\zeta} = [A_o + \dot{\mu}(t) \Delta] \zeta + B_\rho [\mu \nu] , \tag{18}$$

where

$$\nu := (k_p - 1) u + k_p d . \tag{19}$$

The HGO gain is inversely proportional to the small parameter μ, allowed to be time-varying in order to guarantee global tracking.Our task is to establish properties for the domination function $\psi_\mu(\omega, t)$ in (15) so that $\mu|\nu|$ and $|\dot{\mu}|$ are arbitrarily small, at least after a finite time interval. Consequently, $\dot{\mu}$ does not *ultimately* affect the stability of A_o in (18) and ζ or $\tilde{\xi}$ can be made arbitrarily small, *modulo* exponentially decaying term.

In order to obtain a norm bound for the time derivative of μ (15) we calculate $\dot{\mu}$ by the expression:

$$\dot{\mu}(t) = -\frac{\mu^2}{\bar{\mu}} \left[\frac{\partial \psi_\mu}{\partial \omega} \dot{\omega} + \frac{\partial \psi_\mu}{\partial t} \right] . \tag{20}$$

Note that, $\dot{\mu}$ is a piecewise continuous time signal which can be upperbounded by

$$|\dot{\mu}(t)| \leq \frac{\left| \frac{\partial \psi_\mu}{\partial \omega} \right|}{1 + \psi_\mu} \mu |\dot{\omega}| + \frac{\left| \frac{\partial \psi_\mu}{\partial t} \right|}{1 + \psi_\mu} \mu . \tag{21}$$

Now, assume that the control strategy and the norm observer are such that the following inequalities hold:

$$|\nu| \leq \psi_\nu(\omega, t) + \pi_3 , \tag{22}$$

$$|\dot{\omega}| \leq \psi_\omega(\omega, t) + \pi_1 , \tag{23}$$

respectively, for some non-negative functions ψ_ω and ψ_ω and vanishing terms π_3, π_1 depending on initial conditions. Hence, one has that:

$$\mu |\nu| \leq \frac{\psi_\nu}{1 + \psi_\mu} \bar{\mu} + \mu \pi_3 , \tag{24}$$

and

$$\mu |\dot{\omega}| \leq \frac{\psi_\omega}{1 + \psi_\mu} \bar{\mu} + \mu \pi_1 . \tag{25}$$

Now, choose the domination function ψ_μ in (15) so that the following property holds with ψ_v in (22) and ψ_ω in (23):
(P0) $\psi_v\,,\ \psi_\omega \le c_{\mu 0}(1+\psi_\mu)$, $\forall t \in [0,t_M)$, where $c_{\mu 0} \ge 0$ is a *known* constant.
If ψ_μ satisfies (P0) then, from (24) and (25), $\mu|v|$ and $\mu|\dot{\omega}|$ can be bounded by

$$\mu|v| \le \mathscr{O}(\bar{\mu}) + \mu\pi_3\,. \tag{26}$$
$$\mu|\dot{\omega}| \le \mathscr{O}(\bar{\mu}) + \mu\pi_1\,. \tag{27}$$

Moreover, our strategy is to design $\psi_\mu(\omega,t)$ such that the following property holds:
(P1) $\left|\frac{\partial\psi_\mu}{\partial\omega}\right|\,,\ \left|\frac{\partial\psi_\mu}{\partial t}\right| \le c_{\mu 1}(1+\psi_\mu)$, $\forall t \in [0,t_M)$, where $c_{\mu 1} \ge 0$ is a *known* constant.
This property is trivially satisfied by polynomial ψ_μ with positive coefficients.

Now, with ψ_μ satisfying (P1), one has that:

$$|\dot{\mu}(t)| \le c_{\mu 1}\mu|\dot{\omega}| + c_{\mu 1}\mu\,. \tag{28}$$

Therefore, from (28), (26) and (27) the following holds:

$$|\dot{\mu}(t)|\,,\ \mu|v| \le \mathscr{O}(\bar{\mu}) + \mu\pi_4\,, \tag{29}$$

where $\pi_4 := c_{\mu 1}\pi_1 + \pi_3$.

Finally, if ψ_μ is designed so that (P0)–(P1) hold and *finite escape is avoided*[1], then from (29) one can verify that there exists a finite $t_\mu \in [0,t_M)$ such that:

$$|\dot{\mu}(t)|\,,\ \mu|v| \le \mathscr{O}(\bar{\mu})\,, \quad \forall t \in [t_\mu,t_M)\,. \tag{30}$$

V. Control

A simple PD controller is used as control approach in order to minimize the error $e(t)$ over time between a given joint position reference vector and the actual joint position. The control variable $v(t)$ is determined by:

$$e(t) = q_{ref} - q \tag{31}$$
$$\dot{e}(t) = \dot{q}_{ref} - \dot{q}_{est} \tag{32}$$
$$x(t) = K_p e(t) + K_d \frac{de(t)}{dt} \tag{33}$$
$$v(t) = \ddot{q}_{ref} + x(t) \tag{34}$$
$$\tag{35}$$

Given a control variable, the control output signal is expressed as

$$u(t) = M_{est}(q)v + C_{est}(q,\dot{q})\dot{q} + g_{est}(q) \tag{36}$$

The matrices $M_{est}(q)$, $C_{est}(q,\dot{q})$ and g_{est} are $M(q)$, $C(q,\dot{q})$ and g with a given measure error according to table I.

VI. Simulation

The parameters used for the robot plant are listed in tables III, I and II. The reference joint position vector q_{ref} is given by $[0.02cos(2\pi/0.55)t \quad 0.9sin(2\pi/1.5)t+1 \quad sin(2\pi/0.45)t+1.8]'$ an approximation of those presented in [**?**]. It's derivatives were obtained using a 'dirty' derivative formula

$$\hat{q} = \left\{ \frac{s}{\varepsilon s+1} \right\} \tag{37}$$

and then used as joint position, velocity and acceleration references in the controller transfer function

The Proportional-Derivative control has parameters defined in table I

[1]This can be guaranteed if an additional technical Property is satisfied, see [**?**] for details. Here, we omitted this property just to simplify the paper presentation.

TABLE I

CONTROL PARAMETERS TABLE

Parameter	Value	Units
K_p	$10I_{3x3}$	
K_d	$10I_{3x3}$	
Mass vector	+1% error	Kg
C_2	+5% error in $C(q,\dot{q})$ and $M(q)$	m
C_3	-5% error in $C(q,\dot{q})$ and $M(q)$	m
I_2	-5% error	$Kg-m^2$

TABLE II

HGO PARAMETERS TABLE

Parameter	Value	Units
μ	10^{-3}	
l_1	1	
l_2	2	

VII. CONCLUSIONS

In this work, we considered the control of a wind energy conversion system (WECS) to extract its maximum power, by applying extremum seeking control (ESC) in an outer control loop to perform the maximum power point tracking (MPPT) and a nonlinear robust controller in an inner control loop. The key idea of the method is to maximize an auxiliary output which is an estimate of the aerodynamic torque, with a real-time control to handle cut-in wind speed to rated wind speed. A sliding mode control with smooth control (SSC) effort was implemented in the inner loop, resulting in a *chattering* free control law. The complete stability analysis, including the ESC employed in the outer loop, was provided. Numerical simulations illustrated the performance of the proposed scheme.

Future possible topics of research are: the evaluation of the generator's active power feedback or the usage of accelerometers in order to avoid the need of time differentiation of the rotor speed; and consider more general class of nonlinear plants representing the WECS, since the SSC can also be applied for linear plants with arbitrary relative degree.

APPENDIX

$$\begin{aligned}
C(1,1) =&0\\
C(1,2) =&-\dot{q_2}(L_2m_3+m_2(C_2+L_2))sin(q_2)-C_3m_3(\dot{q_2}+\dot{q_3})sin(q_2+q_3)\\
C(1,3) =&-C_3m_3sin(q_2+q_3)(\dot{q_2}+\dot{q_3})\\
C(2,1) =&0\\
C(2,2) =&-C_3L_2m_3\dot{q_3}sin(q3)\\
C(2,3) =&-C_3L_2m_3sin(q_3)(\dot{q_2}+\dot{q_3})\\
C(3,1) =&0\\
C(3,2) =&C_3L_2m_3\dot{q_2}sin(q_3)\\
C(3,3) =&0
\end{aligned} \tag{38}$$

$$\begin{aligned}
D(1,1) &= m_1+m_2+m_3\,,\\
D(1,2) &= D(2,1) = (c_3\cos(q_2+q_3)+l_2\cos(q_2))+\\
&\quad m_2(c_2\cos(q_2)+l_2\cos(q_2))\,,\\
D(1,3) &= D(3,1) = c_3m_3\cos(q_2+q_3)\,,\\
D(2,2) &= I_{2z}+I_{3z}+c_2^2m_2+c_3^2m_3+\\
&\quad l_2^2(m_2+m_3)+2c_2l_2m_2+2c_3l_2m_3\cos(q_3)\,,\\
D(2,3) &= D(3,2)m_3c_3^2+l_2m_3\cos(q_3)c_3+I_{3z}\,,\\
D(3,3) &= m_3c_3^2+I_{3z}\,.
\end{aligned}$$

$$
\begin{aligned}
g(1,1) &= -g(m_1+m_2+m_3) \\
g(2,1) &= -C_3 g m_3 cos(q_2+q_3) - g(m_2(C_2+L_2)+L_2 m_3)cos(q_2) \\
g(3,1) &= -C_3 g m_3 cos(q_2+q_3)
\end{aligned} \tag{39}
$$

TABLE III

PLANT PARAMETERS TABLE

Parameter	Value	Units
m_1	21.29	Kg
m_2	8.57	Kg
m_3	2.33	Kg
I_2	0.435	$Kg-m^2$
I_3	0.062	$Kg-m^2$
d_0	0.5	m
L_2	0.425	m
L_3	0.527	m
C_2	-0.339	m
C_3	0.320	m
g	9.81	m/s^2