Andrea Tilli, Marcello Montanari

A low-noise estimator of angular speed and acceleration from shaft encoder measurements

Abstract— **This paper deals with the differentiation of discrete-time, quantized signal provided by an encoder. In particular, speed and acceleration estimation with large bandwidth is required for feedback control in automation and robotics applications. The main problem in the differentiation of the encoder measurements is to combine the large bandwidth and the strong filtering of the quantization noise. The proposed speed-acceleration estimator relies on typical filtered differentiator nonlinearly combined with a state-variable filter. The former produces a strongly filtered estimation, while the latter is characterized by a large bandwidth, in order to track fast signal transients. In this way, the resulting estimator has a variable bandwidth depending on the harmonic content of the encoder signals. Two versions of the estimator are presented and compared. The first one gives only a speed estimation, while the second provides also an acceleration signal. Simulation and experimental tests verify the performance of the proposed solutions. A 5k c/r encoder and a high resolution sine/cosine encoder are considered.**

Keywords— **acceleration estimation, encoder measurements, nonlinear observers, real differentiators, velocity estimation.**

1. Introduction

In the field of robotics and automation, the main objective of motion control is to guarantee accurate tracking of position or speed trajectories characterized by large harmonic content. In order to achieve this purpose, a cascade structure for the controller is usually adopted. This architecture is constituted by "nested" control loops: the outer position and speed control loops and the inner "effort" control loop [1] [2]. The outer loops mainly deal with the mechanical part of the plant, while the inner loop controls the actuator producing the effort which "moves" the system (generally an electric motor which produces torque). It is well known that the action yielded by these controllers can be split in two main parts: the feedforward one and the feedback one. The former is based on the "inversion" of the nominal dynamical model of the plant and mainly determines a fast response to reference variations. The latter is based on measurements and should make the tracking performance robust with respect to all the non-idealities, as parameter uncertainties and unexpected loads.

Since the model of the controlled system and the knowledge of external loads are often quite rough, the tracking performances are essentially related to the feedback control characteristics. In order to obtain feedback controllers with wide bandwidth (i.e. high and fast rejection of disturbances) it is essential to have accurate and large bandwidth sensors. For position/speed controller, good measurements of position and velocity are required. Moreover, if an acceleration measurement is also available a significant improvement in the rejection of sudden loads can be achieved [3]. Nevertheless, in practical implementation only the position is measured by means of an optical encoder. Rarely the velocity is measured using tachogenerator while direct measurement of the acceleration is still a research topic [4]. Hence, the speed and, in case, the acceleration should be estimated in some ways using the position measurement.

Pure derivative operators link position, velocity and acceleration. The simplest numerical method for differentiating a signal is the backward difference. This is the most common approach used to obtain the velocity from the position measurement and, in principle, it can be used to derive also the acceleration. By the way, this solution produces very noisy estimations owing to the combination of the following reasons: A) The position obtained by an optical encoder is a discrete-time, quantized signal; hence a so-called quantization noise is superimposed to the real value. This is a broadband noise whose amplitude is proportional to the encoder resolution. B) Backward difference operator has a noise-amplifying characteristic, which is the larger the more the sampling time is small.

The noisiness of these estimations has different effects on the control scheme. 1) The acceleration is practically useless owing to the very low Signal-to-Noise (S/N) ratio, unless very high-resolution encoders are adopted (e.g. sine-cosine interpolating encoders). 2) With large bandwidth controllers, the high frequency noise determines an unjustified large harmonic content of the position/speed controller output (typically the torque command). This can yield mechanical vibration and, even with absence of vibrations, it leads to a reduction of the energy efficiency in the electric motor. Hence it is necessary to develop estimation schemes which filter out the unavoidable noise present in the speed measurements, without impairing the control bandwidth.

In the literature, different solutions for the differentiation of signals with noise attenuation have been proposed. The aim of the methods is to realize filters that approximate the ideal differentiator in a certain range of frequencies. Assuming that the position can be approximated with a low-degree polynomial, in [5] a differentiator based on the Newton predictor has been proposed. Differentiators based on FIR or IIR filters have been presented in [6]; these approaches are normally called predictive postfiltering [7]. A different approach for the speed estimation relies on state observers' theory. Assuming the knowledge of the plant

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model, estimators based on Kalman filter [8] [9], Luenberger observers and nonlinear observers [10] [11] [12] have been presented. Other solutions based on the sliding mode observers are presented in [13] [14].

The objective of this paper is to present a new speed and acceleration estimator from the encoder position measurement which is characterized by low noise and large bandwidth. The proposed scheme relies on typical filtered differentiators nonlinearly combined with a non-model-based position/speed observer.

The basic idea is the following. The purpose is to minimize the effect of the quantization noise, maintaining a large bandwidth. Hence a very narrow low-pass filtering ("slow filtering") of the estimations is performed when the speed, velocity and acceleration signals have a narrow bandwidth, while a wider-band filter ("fast filtering") is applied when the signals have large bandwidth. The smooth switching between the two kinds of filtering is based on the position estimation error. If this error is inside an interval equal to the encoder resolution the slow filtering is applied, otherwise the fast filtering is enabled. Thanks to this solution, in all the conditions with low harmonic signal content (e.g. constant speed or acceleration) a very large noise rejection will be obtained. Obviously, when the signal harmonic content is larger the same absolute noise attenuation cannot be achieved, since the fast filtering has to be used to guarantee a good tracking of the real signals. However, the performance in terms of S/N ratio will be still good, owing to the increasing of the signal harmonic component. It is worth to note that the proposed solution is suitable for applications where the mechanical characteristics of the plant are uncertain; in fact, no plant model is required.

About the coupling between the proposed differentiator and speed controller the following remark is relevant. The feedback regulator should be designed taking into account the variable-structure nature of the estimator, but this topic is not extensively covered in this work

The paper is organized as follows. First, in section 2, the speed estimator based on the above-exposed concepts is described. Then, in section 3, the previous scheme is extended to the case of simultaneous estimation of speed and acceleration. In section 4 simulation and experimental results are presented for both speed and speed-acceleration estimators, when a common 5k pulses/revolution encoder is adopted. Some additional simulation results show the performance of the speed-acceleration estimator, in the case of a high-resolution sinusoidal encoder (8M pulses/revolution). Finally, in section 5 some conclusions are obtained.

2. Speed estimator

The basic scheme of the proposed estimator is reported in fig.1. In accordance with the Introduction, the presented solution is based on a standard filtered differentiator nonlinearly connected with a state-variable filter (i.e. an observer with unknown inputs). The former element is characterized by a very narrow low-pass bandwidth and it gives a "slow" estimation of the velocity and high noise rejection.

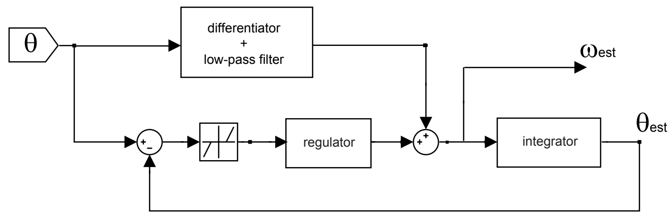

Fig. 1. Structure of the speed/acceleration estimator.

The latter element, instead, is characterized by a very large bandwidth, which allows to track fast-varying speed signals ("fast" estimation). The smooth switching between the two estimators is realized by means of the dead-zone block inserted in the feedback loop of the state-variable filter. When the velocity has a narrow bandwidth the "slow" estimator will track the speed and the resulting position error will be inside the dead-zone, hence the "fast" estimator will be disabled. On the other hand, when the harmonic content of the speed is larger than the "slow" estimator bandwidth, the position error will grow exiting from the dead-band, hence the fast estimator will start to operate in order to guarantee a good speed tracking. The dead-zone amplitude is crucial since it determines the switching between the fast and the slow filtering. It is reasonable to impose a dead band equal to $[-\Delta_\theta, \Delta_\theta]$, where Δ_θ is the angular resolution of the adopted encoder. In fact, owing to the quantization of the position measurement, this is the minimum significant error.

In fig.2 a more detailed scheme of the proposed discrete-time speed estimator is reported. The "slow" part, $G_s(z)$, is the discretization, by means of the Tustin method, of a simple continuous-time ideal differentiator with a first order filtering. In particular its transfer function is:

$$G_s(z) = \left.\frac{s}{\tau s + 1}\right|_{s=\frac{2}{T}\frac{z-1}{z+1}} = \frac{z-1}{(\tau + T/2)z - (\tau - T/2)}$$

where T is the sampling time and τ is the filter time-constant. It is worth observing that the ideal Tustin differentiator has a pole equal to -1, yielding to a ringing response. This pole disappears when the discretization of the filtered version is considered. The "fast" part of the speed estimator is implemented by means of a first order state-variable filter, whose feedback regulator is just a gain K. This gain determines directly the bandwidth of this branch. The discrete-time integrator used in the state-variable filter is obtained with the backward derivative method to avoid algebraic loops, while the integrator, used to correctly reconstruct the position from the "slow" speed estimation, has to be based on the Tustin approximation. Nevertheless, both of the integrators can be realized with a unique state variable, with an additional feed-forward action related to the Tustin integrator of the "slow" branch only.

It is remarkable to note that a pole in 1 is present in the fast feedback loop. Hence a position error inside the dead-band seems achievable in condition of null speed only. Actually, owing to the feed-forward action produced by the "slow" estimation branch, an error inside the dead-band

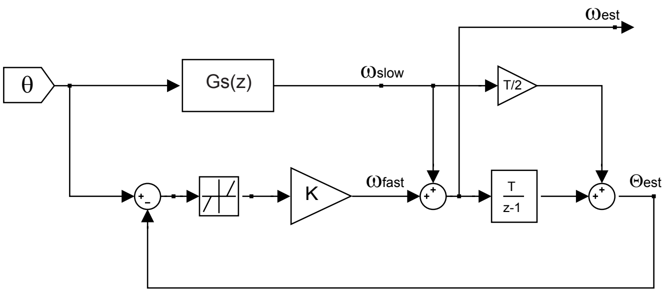

Fig. 2. Speed estimator.

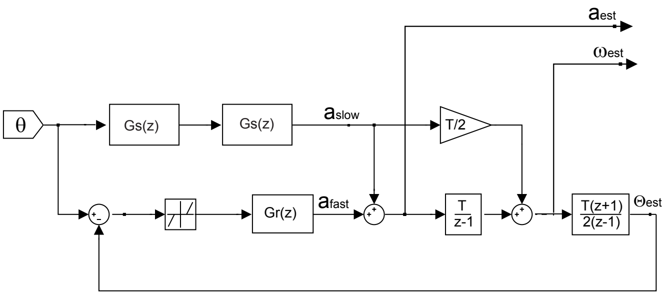

Fig. 3. Speed and acceleration estimator.

can be obtained also with any constant speed. In these conditions, after an initial transient, only the "slow" estimation branch will operate and a very large rejection of the quantization noise will be obtained in the estimated speed.

3. Speed and acceleration estimator

The structure of the speed/acceleration estimator is presented in fig.3 and it is a generalization of the one presented above. In this solution, in order to obtain the acceleration estimation, the number of differentiations in the "slow" estimator is increased by one and, in the same way, the order of the "fast" state-variable filter is set to two. In particular, the double differentiator with the low-pass filter and the double integrator for the "slow" part are still discretized with the Tustin approximation. The double integrator for the "fast" filtering is the discrete-time model of $\frac{1}{s^2}$ obtained using the exponential matrix method. This discretization method is based on the assumption of constant input (i.e. the acceleration) during a sample period. The obtained integrator chain does not introduce algebraic loops and it is the cascade of a forward-Euler integrator and a Tustin integrator:

$$G(z) = \frac{T}{z-1}\frac{T(z+1)}{2(z-1)}$$

The regulator $G_r(z)$ is a lead network,

$$G_r(z) = \frac{k(z-z_0)}{z-z_p}$$

that, e.g., can be designed in the w-plane domain assigning the bandwidth and the phase margin. The other design parameter for the estimator is the time constant of the slow filter.

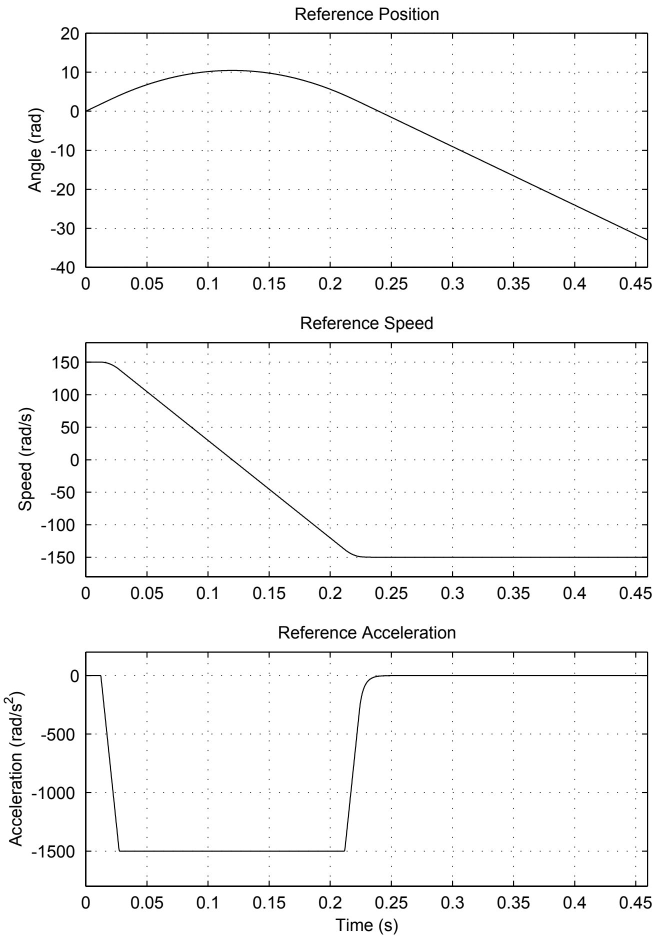

Fig. 4. Position, velocity and acceleration profiles.

The advantage of this scheme w.r.t. the speed estimator relies on the availability of the estimated acceleration and on the increased filtering performance. In fact, thanks to the structure of the "slow" filter and to the double integration in the feedback loop, a position estimation error inside the dead-zone is obtained also with constant acceleration; in this way, an estimated speed with attenuated noise is achieved in a wider range of conditions and also a good estimation of constant acceleration is performed.

The discrete-time schemes presented for speed and speed-acceleration reconstruction are "almost" linear. By the way the dead-zone block plays a key-role in realizing estimators with variable bandwidth, in fact it gives variable-structure properties to the systems. In this paper, the stability properties have been tested by simulation; a detailed theoretical analysis of the convergence of the estimation for both the solutions is omitted.

4. Simulation and experimental results

Some simulation and experimental results are shown to test the performance of the speed and speed/acceleration estimators. In fig.4 the position, speed and acceleration profiles used in the simulations and in the experiments are

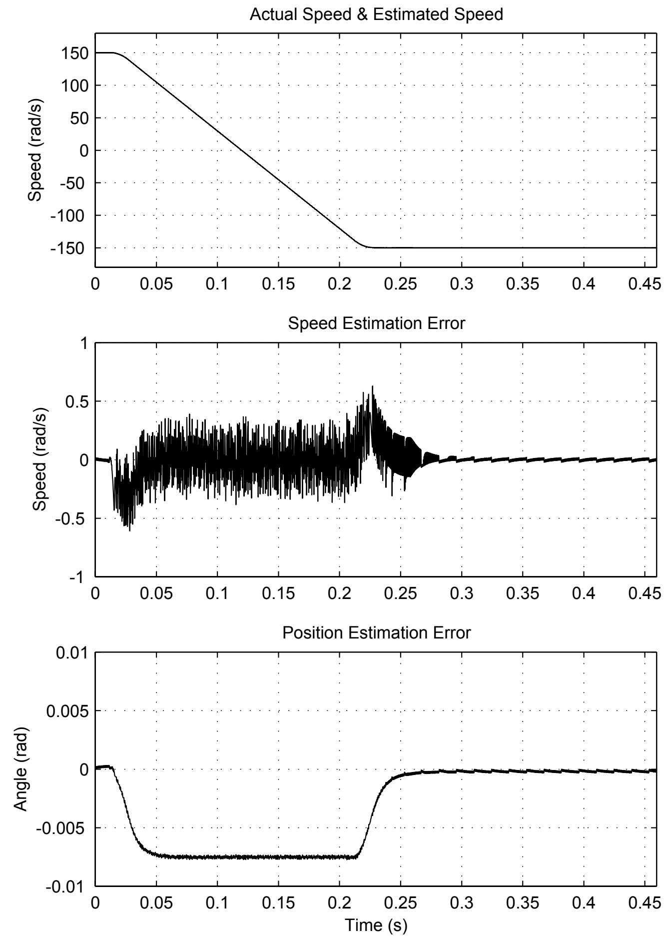

Fig. 5. Speed estimator: simulation results.

shown. The adopted profile is similar to the ones normally used in motion control applications. A speed transient from $+150\ rad/s$ to $-150\ rad/s$ is considered. The maximum acceleration and jerk are respectively 1500 rad/s^2 and $10^5\ rad/s^3$.

In the simulation tests, the encoder output is obtained by means of the quantization of the position profile shown in fig.4. Differently, in the experiments the profiles shown in the same figure are the references for the speed control.

A 5000 c/r incremental encoder with a $200\mu s$ sample period is used both in the simulations and in the experiments. The angular resolution Δ_θ is equal to $\frac{2\pi}{5000\cdot 4} = 3.14 \cdot 10^{-4} rad$, where the term "4" is inserted since the adopted encoder is two-phases type. It is worth to note that, using a pure backward differentiation to reconstruct speed and acceleration, the quantization noise amplitude would be respectively $\Delta_\omega = \frac{\Delta_\theta}{T} = 1.57 rad/s$ and $\Delta_a = \frac{\Delta_\theta}{T^2} = 7854 rad/s^2$.

Accordingly with the previous sections, the design of the proposed estimators is based on the following requirements: A) a main time-constant for the "slow" branch equal to 8 ms in order to obtain strong noise filtering; B) a bandwidth of the "fast" branch equal to 1667 rad/s, which is the maximum complying with the sampling time. The resulting parameters for the estimator of section 2 are $\tau = 8\ ms$

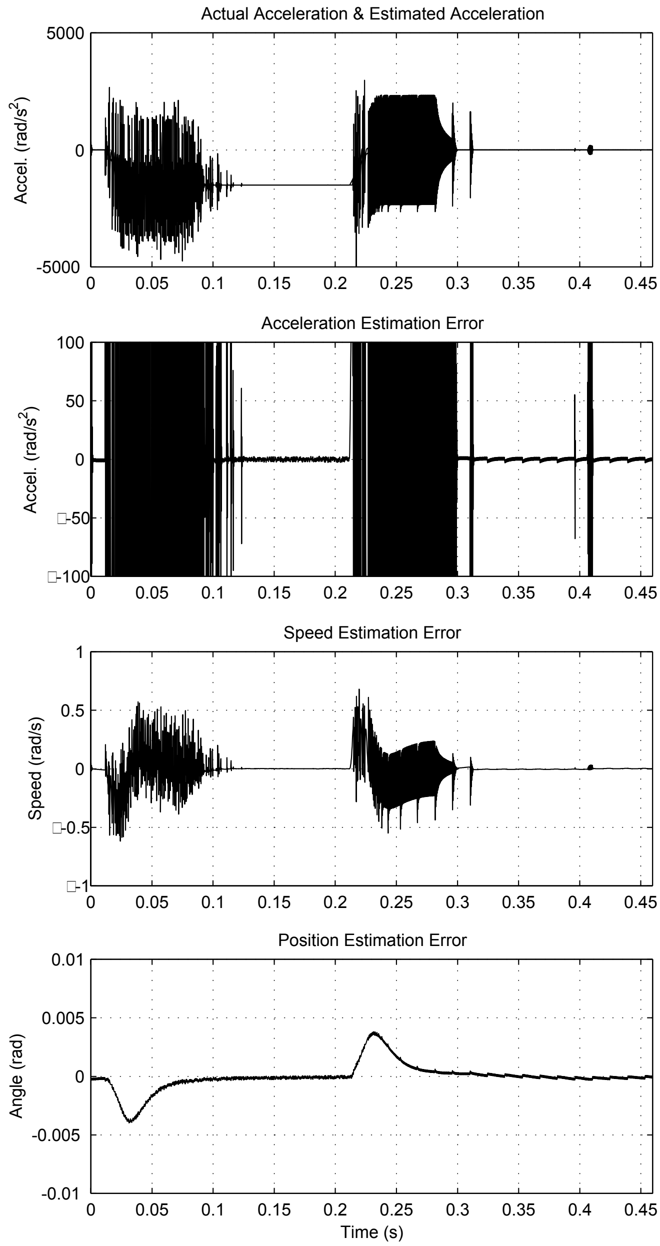

Fig. 6. Speed and acceleration estimator with a 5000 c/r encoder: simulation results.

and $K = 1667\ rad/s$, while the parameters for the solution presented in section 3 are $\tau = 8\ ms$, $k = 8.116 \cdot 10^6$, $z_0 = 0.94$, $z_p = 0.042$. In particular, for the latter case a phase margin of 60 degrees has been imposed for the "fast" branch.

Fig.5 shows the performance of the speed estimator. As expected, with constant speed a large noise attenuation is achieved, since only the "slow" part is operating. During transients, good speed estimation is still obtained thanks to the high bandwidth of the state-space filter, but the absolute noise rejection is lower. In the interval $0.22 - 0.27\ s.$, even if the real speed is constant, the position estimation error is greater than the dead-zone amplitude, owing to the dynamic delay of the "slow" estimator.

In fig.6 the speed and acceleration estimator is consid-

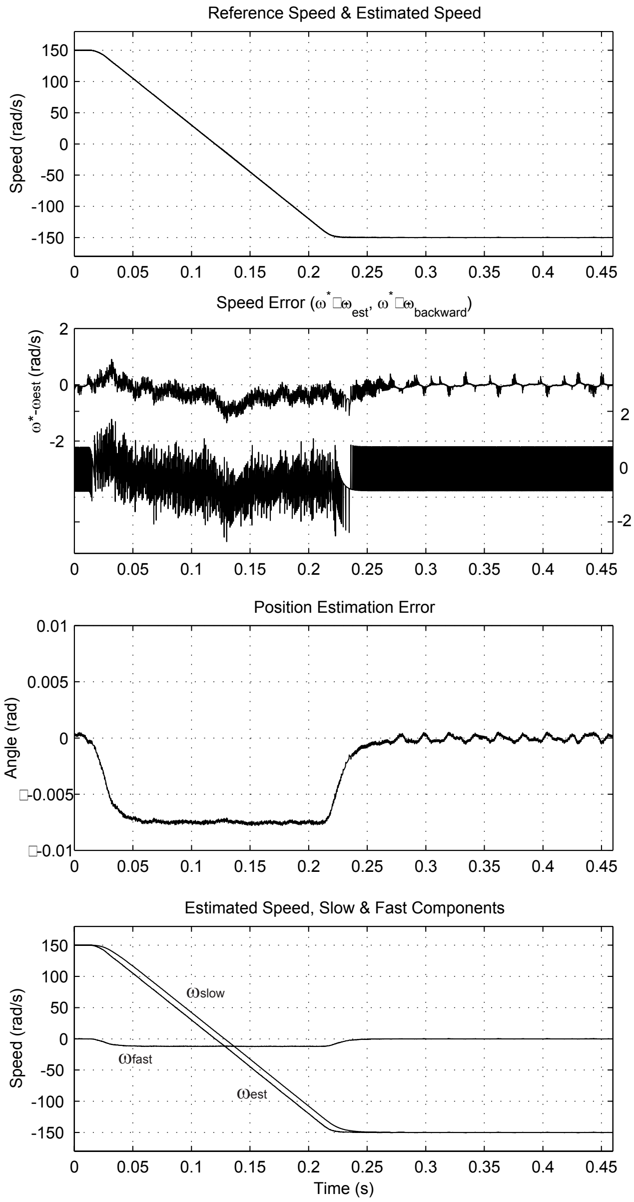

Fig. 7. Speed estimator: experimental results.

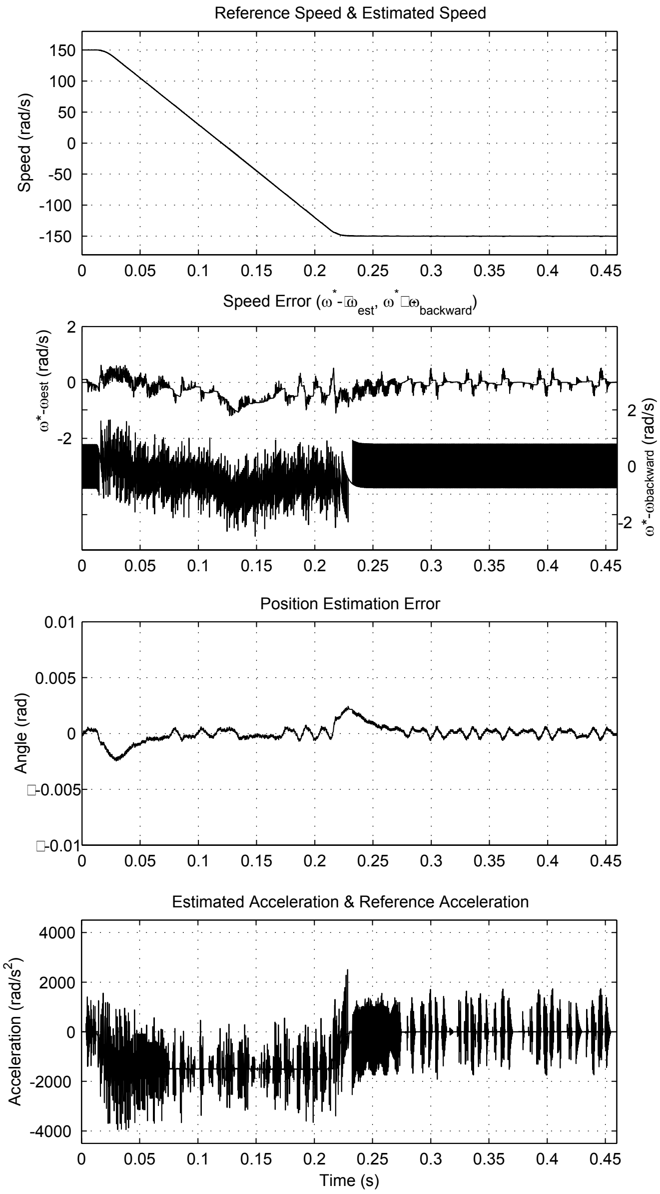

Fig. 8. Speed and acceleration estimator: experimental results.

ered. As expected, with constant acceleration a relevant noise rejection on both the estimated velocity and acceleration is achieved. On the other hand, during acceleration transients a significant noise is superimposed on the estimated acceleration; this is an unavoidable effect of the encoder quantization combined with the large estimator bandwidth. Nevertheless, the speed estimation is excellent. In particular, with respect to the speed estimator, a substantial noise reduction is achieved also during speed ramps.

Experimental tests have been performed using a 1.1 kW, 1410 r.p.m. induction motor controlled by a standard indirect-field oriented control. The speed estimated with the proposed schemes is used as feedback in the speed control loop and in the feed-forward terms. Particular attention has been paid to the tuning of the speed feedback controller in order to guarantee stability for both estimator bandwidths. The tracking performance of the speed controller are maximum when the "fast" estimator is enabled, while a degradation is present when only the "slow" estimator is active. By the way, during transients the "fast" component is usually enabled, owing to the harmonic content of the speed signal.

In figs.7 and 8 the experimental performances of the speed and speed-acceleration estimators are reported respectively. The results well confirm the simulation proofs in terms of quantization noise reduction during steady-state and transient operations. In particular, the speed-acceleration estimator shows better performance during constant acceleration conditions, as can be noted from the speed error and position error profiles. As comparison point, also the backward-difference speed estimate is shown. In particular, in the second picture of figs.7 and 8 the error between the reference speed and the estimated

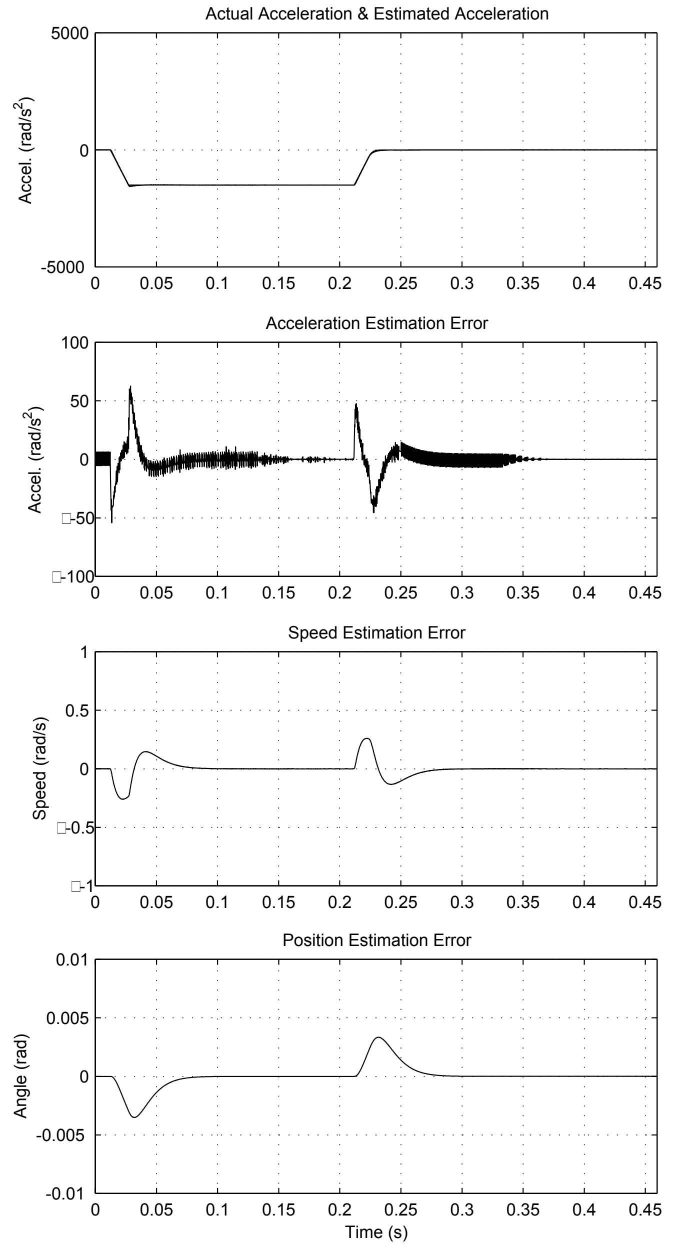

Fig. 9. Speed and acceleration estimator with a sine/cosine encoder: simulation results.

speed with the two different methods is reported to better evaluate the noise rejection in different conditions. Note that, when the reference speed is constant, some small velocity oscillations are present, due to unmodeled mechanical resonances. This phenomenon determines some noise spikes in the estimated speed, because the "fast" state-variable filter is sometimes activated to track the above oscillations; moreover, the speed-acceleration estimator gives noise peaks in the estimated acceleration. Finally, in the fourth picture of fig.7, the "slow" and the "fast" components of the estimated speed are separately reported. It can be noted as, during transient, the "fast" part of the estimation is essential to guarantee accurate speed reconstruction.

A final set of simulations is performed to test the capability of the scheme proposed in section 3 to obtain a satisfactory estimation of the acceleration using a high resolution encoder, such as a sine/cosine type. An encoder with $8M$ c/r resolution is assumed. In fig.9 a very good acceleration estimation is shown; only a negligible error is present during acceleration transients and a low S/N ratio is achieved.

5. Conclusions

Based on an innovative scheme, speed and speed-acceleration estimators have been proposed and tested by means of simulation and experimental results. Good performance are generally obtained. Both of the solutions perform good speed estimation using encoders with standard resolution. The speed-acceleration estimator shows better noise filtering capability with respect to the speed estimator. In addition, it has been shown that with high resolution encoders also a low noise acceleration estimation can be obtained.

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