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# High-Gain Observers in Nonlinear Feedback Control

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**Abstract:** The theory of high-gain observers has been developed for about twenty years. This paper is a brief introduction to high-gain observers in nonlinear feedback control, with emphasis on the peaking phenomenon and the role of control saturation in dealing with it. The paper surveys recent results on the nonlinear separation principle, conditional servo compensators, extended high-gain observers, performance in the presence of measurement noise, sampled-data control, and experimental testbeds.

**Keywords:** High-Gain Observers, Nonlinear control, Nonlinear Observers

## 1. INTRODUCTION

High-gain observers have evolved over the past two decades as an important tool for the design of output feedback control of nonlinear systems. They were used earlier by Doyle and Stein [24] in the design of robust observers for linear systems. The early work on high-gain observers in nonlinear systems appeared in the late 1980's [25], [81]. Soon afterwards the technique was developed independently by two schools of researchers: a French school lead by Gauthier, Hammouri, and others; c.f. [18], [19], [23], [30], [31], [33], [96], and a U.S. school lead by Khalil at Michigan State University. The work of Gauthier and others in the French school focused on deriving global results under global Lipschitz conditions. The work of Khalil and coworkers took a different turn after it was demonstrated by Esfandiari and Khalil [26] that, in the lack of global Lipschitz conditions, high-gain observers could destabilize the closed-loop system as the observer gain is driven sufficiently high. The destabilization was explained using the *peaking phenomenon*, which is a term used in asymptotic high-gain design to describe the fact that driving the gain sufficiently high produces an impulsive like-behavior. The destabilizing effect of the interaction of peaking with nonlinearity had already been exposed by Sussmann and Kokotović[92] in high-gain state feedback. The presence of peaking in linear high-gain observers was observed earlier [69], [75], but its destabilizing effect in nonlinear feedback control was observed for the first time in [26]. More importantly, [26] proposed a seemingly simple solution for the problem. It suggested that the control should be designed as a globally bounded function of the state estimates so that it saturates during the peaking period. Because the high-gain observer is designed to be much faster than the closed-loop dynamics under state feedback, the peaking period is very short relative to the time scale of the plant variables, which remain very close to their initial values. This separation of time scales was used in [26] to prove that the output feedback controller stabilizes the closed-loop system for sufficiently high observer gain. About seven years later, Atassi and Khalil [14] proved another fundamental property of the saturated control. They showed that the trajectories of the state variables under output feedback come arbitrarily close the ones under state feedback, as the observer gain becomes high enough. This property shows that the output feedback controller recovers the performance of the state feedback controller, not just its stability properties.

Shortly after the appearance of [26], Teel and Praly used its ideas to prove the first nonlinear separation principle and develop a set of tools for semiglobal stabilization of nonlinear systems [93], [94]. Their work drew attention to [26], and soon afterwards many leading nonlinear control researchers starting using high-gain observers; c.f. [11], [20], [32], [34], [36], [37], [38], [39], [40], [56], [57] [58], [60], [61], [63], [78], [80], [83], [87], [88], [90], [97]. These papers have studied a wide range of nonlinear control problems, including stabilization, regulation, tracking, and adaptive control. Most of them achieve semiglobal or regional results, which is the price of saturating the control. In the last few year Praly and other have shown that, by using dynamic observer gain, global results can be achieved without global Lipschitz conditions; c.f. [12], [52], [54], [76], [77].

At Michigan State University, Khalil and coworkers have developed the high-gain observer theory in a number of directions, covering stabilization [26], [50], sliding mode control [72], [73], regulation (or servomechanisms) [41], [42], [45], [46], [64], [65], [85], [86], [89], adaptive control [8], [9], [43], [53], [84], separation principle [5], [14], [15], [16], logic-based switching [27], [28], robustness to unmodeled dynamics [7], [49], [62], sampled data control [21], [22], [48], effect of measurement noise [5], [95], disturbance estimation [29], and connection with Extended Kalman Filter [1].

This paper is intended as a tutorial/survey paper on the use of high-gain observers in nonlinear control. The survey focuses on the work at Michigan State University over the last ten years; earlier work was surveyed in [44]. In Section 2, we use a second-order example to illustrate the main ideas. In Section 3, we review some

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recent results on the nonlinear separation principle. In Section 4, we review the tool of conditional servocompensators. In Section 5, we review some recent results on extended high-gain observers. In Sections 6 and 7, we discuss the effect of measurement noise, robustness to unmodeled dynamics, and sampled-data control. Finally, in Section 8 we report experimental results.

## 2. MOTIVATING EXAMPLE

Consider the second-order nonlinear system

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & \phi(x,u,w,d) \\ y & = & x_1 + v \end{array} \tag{1}$$

where $x = [x_1, x_2]^T$ is the state vector, $u$ is the control input, $y$ is the measured output, $d$ is a vector of disturbance inputs, $w$ is a vector of known exogenous signals, and $v$ is the measurement noise. The function $\phi$ is locally Lipschitz in $(x, u)$ and continuous in $(d, w)$. We assume that $d(t)$, $v(t)$ and $w(t)$ are bounded measurable functions of time. Suppose the state feedback control $u = \gamma(x, w)$ stabilizes the origin $x = 0$ of the closed-loop system

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & \phi(x,\gamma(x,w),w,d) \end{array} \tag{2}$$

uniformly in $(w, d)$, where $\gamma(x, w)$ is locally Lipschitz in $x$ and continuous in $w$. To implement this feedback control using only measurements of the output $y$, we use the observer

$$\begin{array}{rcl} \dot{\hat{x}}_1 & = & \hat{x}_2 + h_1(y-\hat{x}_1) \\ \dot{\hat{x}}_2 & = & \hat{\phi}(\hat{x},u,w) + h_2(y-\hat{x}_1) \end{array} \tag{3}$$

where $\hat{\phi}(x, u, w)$ is a nominal model of $\phi(x, u, w, d)$. The estimation error

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \end{bmatrix}$$

satisfies the equation

$$\begin{array}{rcl} \dot{\tilde{x}}_1 & = & -h_1\tilde{x}_1 + \tilde{x}_2 + h_1 v \\ \dot{\tilde{x}}_2 & = & -h_2\tilde{x}_1 + \delta(x,\tilde{x},w,d) + h_2 v \end{array} \tag{4}$$

where

$$\delta(x,\tilde{x},w,d) = \phi(x,\gamma(\hat{x},w),w,d) - \hat{\phi}(\hat{x},\gamma(\hat{x},w),w)$$

In the absence of $\delta$ and $v$, asymptotic error convergence is achieved when the matrix

$$\begin{bmatrix} -h_1 & 1 \\ -h_2 & 0 \end{bmatrix}$$

is Hurwitz, which is the case for any positive constants $h_1$ and $h_2$. In the presence of $\delta$, we design $H$ with the additional goal of rejecting the effect of $\delta$ on $\tilde{x}$. This is ideally achieved if the transfer function

$$G_o(s) = \frac{1}{s^2 + h_1 s + h_2}\begin{bmatrix} 1 \\ s + h_1 \end{bmatrix}$$

from $\delta$ to $\tilde{x}$ is identically zero. While this is not possible, we can make $\sup_{\omega\in R}\|G_o(j\omega)\|$ arbitrarily small by choosing $h_2 \gg h_1 \gg 1$. In particular, taking

$$h_1 = \frac{\alpha_1}{\varepsilon}, \quad h_2 = \frac{\alpha_2}{\varepsilon^2} \tag{5}$$

for some positive constants $\alpha_1$, $\alpha_2$, and $\varepsilon$, with $\varepsilon \ll 1$, it can be shown that

$$G_o(s) = \frac{\varepsilon}{(\varepsilon s)^2 + \alpha_1 \varepsilon s + \alpha_2}\begin{bmatrix} \varepsilon \\ \varepsilon s + \alpha_1 \end{bmatrix}$$

Hence, $\lim_{\varepsilon\to 0} G_o(s) = 0$. The disturbance rejection property of the high-gain-observer can be also seen in the time domain by using the scaled estimation errors

$$\eta_1 = \frac{\tilde{x}_1}{\varepsilon}, \quad \eta_2 = \tilde{x}_2 \tag{6}$$

which satisfy the singularly perturbed equation

$$\begin{array}{rcl} \varepsilon\dot{\eta}_1 & = & -\alpha_1\eta_1 + \eta_2 - (\alpha_1/\varepsilon)v \\ \varepsilon\dot{\eta}_2 & = & -\alpha_2\eta_1 + \varepsilon\delta(x,\tilde{x},w,d) - (\alpha_2/\varepsilon)v \end{array} \tag{7}$$

This equation shows that reducing $\varepsilon$ diminishes the effect of $\delta$. However, it shows also a tradeoff between the steady state errors due to the model uncertainty $\delta$ and the measurement noise $v$. It is not hard to see that $\|\eta\|$ and, consequently, $\|x - \hat{x}\|$ satisfy an inequality of the form

$$\|x(t) - \hat{x}(t)\| \le c_1\varepsilon + c_2\frac{\mu}{\varepsilon}, \qquad \forall\, t \ge T \tag{8}$$

for some positive constants $c_1$, $c_2$, and $T$, where $\mu = \sup_{t\ge 0}|v(t)|$. This ultimate bound, sketched in Figure 1, shows that the presence of measurement noise puts a lower bound on the choice of $\varepsilon$. For higher values of $\varepsilon$ we can reduce the steady-state error by reducing $\varepsilon$, but $\varepsilon$ should not be reduced lower than $c_a\sqrt{\mu}$ because the steady-state error will increase significantly beyond this point. Another tradeoff we face in the presence of measurement noise is the one between the steady-state error and the speed of state recovery. Equation (7) shows that, for small $\varepsilon$, $\eta$ will be much faster than $x$. Fast convergence of $\eta$ plays an important role in recovering the performance of the state feedback controller, as we shall illustrate shortly. The presence of measurement noise prevents us from making the observer as fast as we wish. This tradeoff can be handled by using two values of $\varepsilon$, a small value during the transient period to achieve the desired fast convergence, followed by a relatively larger value when the error is small enough. Two approaches that use switching and nonlinear gain are discussed in Section 6.

The transient response of the high-gain observer suffers from the *peaking phenomenon*. The initial condition $\eta_1(0)$ could be $O(1/\varepsilon)$ when $x_1(0) \neq \hat{x}_1(0)$. Consequently, the transient response of (7) could contain a term of the form $(1/\varepsilon)e^{-at/\varepsilon}$ for some $a > 0$. While this exponential mode decays rapidly, it exhibits an impulsive-like behavior where the transient peaks to $O(1/\varepsilon)$ values before it decays rapidly towards zero. In fact, the function $(1/\varepsilon)e^{-at/\varepsilon}$ approaches an impulse function as $\varepsilon$ tends to zero. In addition to inducing unacceptable transient response, the peaking phenomenon could destabilize the closed-loop nonlinear system; see [47, Section 14.6]. A key idea to overcome the peaking phenomenon is to design the control law $\gamma(\hat{x}, w)$ and the nominal function $\hat{\phi}(\hat{x}, u, w)$ to be globally bounded in $\hat{x}$, that is, bounded for all $\hat{x}$ when $w$ is bounded. This property can be always achieved by saturating $u$ and/or $\hat{x}$ outside compact sets of interest. The global boundedness of $\gamma$ and $\hat{\phi}$ in $\hat{x}$ provides a buffer that protects the plant from peaking because during the peaking period the control $\gamma(\hat{x}, w)$ saturates. Since the peaking period shrinks to zero as $\varepsilon$ tends to zero, for sufficiently small $\varepsilon$ the peaking period is so small that the state of the plant $x$ remains close to its initial value. After the peaking period, the estimation error becomes of the order $O(\varepsilon + \mu/\varepsilon)$ and the feedback control $\gamma(\hat{x}, w)$ becomes $O(\varepsilon + \mu/\varepsilon)$ close to $\gamma(x, w)$. Consequently, the trajectories of the closed-loop system under the output feedback controller asymptotically approach an $O(\sqrt{\mu})$ neighborhood of its trajectories under the state feedback controller as $\varepsilon$ tends to $c_a\sqrt{\mu}$. This leads to recovery of the performance achieved under state feedback.

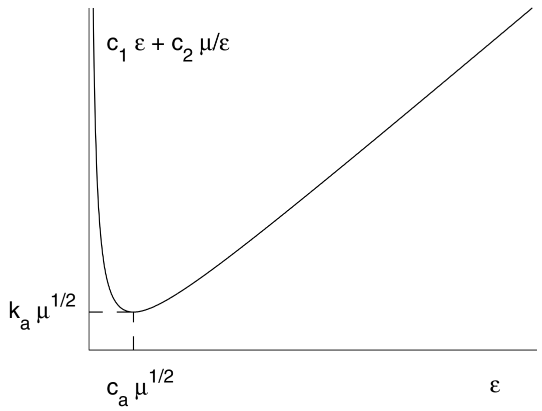


Fig. 1 A sketch of $c_1\varepsilon + c_2\mu/\varepsilon$; $c_a = \sqrt{c_2/c_1}$, $k_a = 2\sqrt{c_1 c_2}$.

The analysis of the closed-loop system under output feedback proceeds as follows. The system is represented in the singularly perturbed form

$$\left.\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & \phi(x, \gamma(\hat{x}, w), w, d) \end{array}\right\} \quad (9)$$

$$\left.\begin{array}{rcl} \varepsilon\dot{\eta}_1 & = & -\alpha_1\eta_1 + \eta_2 - (\alpha_1/\varepsilon)v \\ \varepsilon\dot{\eta}_2 & = & -\alpha_2\eta_1 + \varepsilon\delta(x, \tilde{x}, w, d) - (\alpha_2/\varepsilon)v \end{array}\right\} \quad (10)$$

where $\hat{x}_1 = x_1 - \varepsilon\eta_1$ and $\hat{x}_2 = x_2 - \eta_2$. The slow equation (9) coincides with the closed-loop system under state feedback (2) when $\eta = 0$. The homogeneous part of the fast equation (10) is $\varepsilon\dot{\eta} = \begin{bmatrix} -\alpha_1 & 1 \\ -\alpha_2 & 0 \end{bmatrix} \eta \stackrel{\text{def}}{=} A_0\eta$. Let $V(x)$ be a Lyapunov function for the slow subsystem, which is guaranteed to exist for any stabilizing state feedback control, and $W(\eta) = \eta^T P_0 \eta$ be a Lyapunov function for the fast subsystem, where $P_0$ is the solution of the Lyapunov equation $P_0 A_0 + A_0^T P_0^T = -I$. Define the sets $\Omega_c$ and $\Sigma$ by $\Omega_c = \{V(x) \leq c\}$ and $\Sigma = \{W(\eta) \leq (\sigma_1\varepsilon + \sigma_2\mu/\varepsilon)^2\}$, where $c > 0$ is chosen such that $\Omega_c$ is in the interior of the region of attraction of (2). The analysis is divided in two steps. In the first step we show that for appropriately chosen $\sigma_1$ and $\sigma_2$ and for sufficiently small $\mu$, there is $\varepsilon_1^* > c_a\sqrt{\mu}$ such that for each $c_a\sqrt{\mu} < \varepsilon < \varepsilon_1^*$ the origin of the closed-loop system is asymptotically stable and the set $\Omega_c \times \Sigma$ is a positively invariant subset of the region of attraction. The proof makes use of the fact that in $\Omega_c \times \Sigma$, $\eta$ is $O(\varepsilon + \mu/\varepsilon)$. In the second step we show that for any bounded $\hat{x}(0)$ and any $x(0) \in \Omega_b$, where $0 < b < c$, there exists $\varepsilon_2^* > c_a\sqrt{\mu}$ such that for each $c_a\sqrt{\mu} < \varepsilon < \varepsilon_2^*$ the trajectory enters the set $\Omega_c \times \Sigma$ in finite time. The proof makes use of the fact that $\Omega_b$ is in the interior of $\Omega_c$ and $\gamma(\hat{x}, w)$ is globally bounded. There exists $T_1 > 0$, independent of $\varepsilon$, such that any trajectory starting in $\Omega_b$ remains in $\Omega_c$ for all $t \in [0, T_1]$. Using the fact that $\eta$ decays faster than an exponential mode of the form $(1/\varepsilon)e^{-at/\varepsilon}$, we can show that the trajectory enters the set $\Omega_c \times \Sigma$ within a time interval $[0, T(\varepsilon)]$ where $\lim_{\varepsilon \to 0} T(\varepsilon) = 0$. Thus, by choosing $\varepsilon$ small enough we can ensure that $T(\varepsilon) < T_1$. Figure 2 illustrates the idea of the proof.

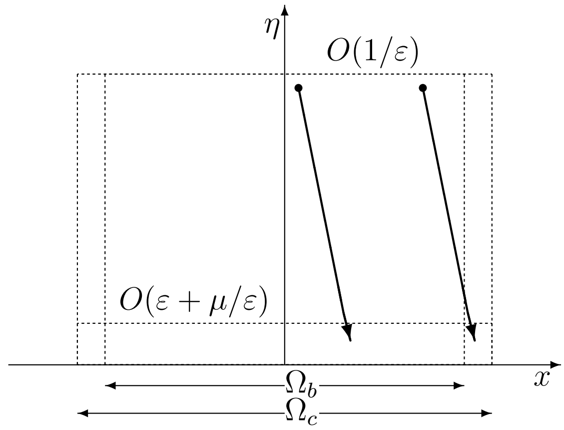


Fig. 2 Illustration of fast convergence to the set $\Omega_c \times \Sigma$.

The second-order observer (3) estimates $x_1$ and its derivative $\dot{x}_1 = x_2$. If an estimate of the second derivative $\ddot{x}_1 = \dot{x}_2$ was available, we would have had an estimate of the function $\phi$, which could play an important role in the control design. An observer that estimates the function $\phi$ is called an *extended high-gain observer*. To illustrate its idea, consider the system

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & b(x, w, d) + a(x, w, d)u \\ y & = & x_1 + v \end{array} \quad (11)$$

which is a special case of (1) with $\phi = b + au$. Let $\hat{a}(x, w)$ be a nominal model of $a(x, w, d)$ and assume that both $a$ and $\hat{a}$ are different from zero over the domain of interest. Let

$$\sigma = b(x, w, d) + [a(x, w, d) - \hat{a}(x, w)]u$$

and extend the dimension of (11) by adding $\sigma$ as a state variable, to obtain

$$\begin{array}{rcl}\dot{x}_1 &=& x_2 \\ \dot{x}_2 &=& \sigma + \hat{a}(x,w)u \\ \dot{\sigma} &=& \varphi(x,w,d,u,\dot{u}) \\ y &=& x_1 + v\end{array} \tag{12}$$

where $\varphi = \dot{\sigma}$. Assume that $\varphi$ is bounded over the domain of interest. A high-gain observer for (12) can be taken as

$$\begin{array}{rcl}\dot{\hat{x}}_1 &=& \hat{x}_2 + (\alpha_1/\varepsilon)(y - \hat{x}_1) \\ \dot{\hat{x}}_2 &=& \hat{\sigma} + \hat{a}(\hat{x},w)u + (\alpha_2/\varepsilon^2)(y - \hat{x}_1) \\ \dot{\hat{\sigma}} &=& (\alpha_2/\varepsilon^3)(y - \hat{x}_1)\end{array} \tag{13}$$

By considering $\sigma$ as an additional measurement of the system, we can design a feedback controller that depends on $x_1$, $x_2$ and $\sigma$; then implement the output feedback controller by using the estimates $\hat{x}_1$, $\hat{x}_2$, and $\hat{\sigma}$ in lieu of $x_1$, $x_2$, and $\sigma$, respectively. For example, if the state feedback controller is taken as

$$u = \frac{1}{\hat{a}}[-\sigma - k_1 x_1 - k_2 x_2]$$

then

$$u = \frac{1}{\hat{a}}[-b - (a - \hat{a})u - k_1 x_1 - k_2 x_2]$$

By simple manipulation we arrive at

$$u = \frac{1}{a}[-b - k_1 x_1 - k_2 x_2]$$

which yields the closed-loop system

$$\begin{array}{rcl}\dot{x}_1 &=& x_2 \\ \dot{x}_2 &=& -k_1 x_1 - k_2 x_2\end{array} \tag{14}$$

The system (14) is independent of the functions $a$ and $b$ and its performance can recovered by the output feedback controller for sufficiently small $\mu$ and $\varepsilon$. The extended high-gain observer is discussed further in Section 5.

## 3. SEPARATION PRINCIPLE

The combination of globally bounded state feedback control and high-gain observers allows for a separation approach where the state feedback controller is designed first to meet the design objectives, then the high-gain observer is designed, fast enough, to recover the performance achieved under state feedback. This separation approach is used in most of the papers that utilize high-gain observers. The first separation theorem for high-gain observer was proved by Teel and Praly [93], where it is shown that global stabilizability by state feedback and uniform observability imply semiglobal stabilizability by output feedback. A more comprehensive separation theorem was proved by Atassi and Khalil [14]. They consider a class of multi-input-multi-output nonlinear systems of the form

$$\begin{array}{rcl}\dot{x} &=& Ax + B\phi(x,z,u) \\ \dot{z} &=& \psi(x,z,u) \\ y &=& Cx \\ \zeta &=& q(x,z)\end{array} \tag{15}$$

where $u$ is the control input, $y$ and $\zeta$ are measured outputs, and $x$ and $z$ constitute the state vector. The triple $(A, B, C)$ comprises block diagonal matrices that represent $p$ channels of integrators with $r_i$ integrators per channel. In a normal form representation, the vector $z$ would be the state of the internal (zero) dynamics, but it is not limited to this interpretation. For example, in an electromechanical system it could represent the state of the actuator. The variable $\zeta$ represents additional measured signals that will be fed back to the controller but will not drive the high-gain observer. For example, in an electromechanical system the vector $y$ would typically comprise positions of various components whose velocities, and may be accelerations, are to be estimated, while $\zeta$ would consist of armature currents of electric motors.

The goal of [14] is to design a feedback controller to stabilize the origin of the closed-loop system using only the measured outputs $y$ and $\zeta$. A two-step approach is followed. First a partial state feedback controller that uses measurements of $x$ and $\zeta$ is designed to asymptotically stabilize the origin. Then a high-gain observer is used to estimate $x$ from $y$. The state feedback controller is allowed to be a dynamic system of the form

$$\begin{array}{rcl}\dot{\vartheta} &=& \Gamma(\vartheta,x,\zeta) \\ u &=& \gamma(\vartheta,x,\zeta)\end{array} \tag{16}$$

where $\gamma$ and $\Gamma$ are globally bounded functions of $x$. In the output feedback controller, $x$ is replaced by its estimate $\hat{x}$, provided by the high-gain observer

$$\dot{\hat{x}} = A\hat{x} + B\phi_0(\hat{x},\zeta,u) + H(y - C\hat{x}) \tag{17}$$

The observer gain $H$ is a block diagonal matrix whose $i$th diagonal block is chosen as

$$H_i = \left[\alpha_1^i/\varepsilon,\ \alpha_2^i/\varepsilon^2,\ \ldots,\ \alpha_{r_i}^i/\varepsilon^{r_i}\right]^T \tag{18}$$

where the positive constants $\alpha_j^i$ are chosen such that the polynomial

$$s^{r_i} + \alpha_1^i s^{r_i - 1} + \cdots + \alpha_{r_i-1}^i s^1 + \alpha_{r_i}^i$$

is Hurwitz, for each $i = 1, \ldots, p$, $\varepsilon$ is a small positive constant, and $\phi_0(x,\zeta,u)$ is a nominal model of $\phi(x,z,u)$. It is shown in [14] that the output feedback controller recovers the performance of the state feedback controller for sufficiently small $\varepsilon$. The performance recovery manifests itself in three points. First, the origin $(x = 0,\ z = 0,\ \vartheta = 0,\ \hat{x} = 0)$ of the closed-loop system under output feedback is asymptotically stable. Second, the output feedback controller recovers the region of attraction of the state feedback controller in the sense that if $\mathcal{R}$ is the region of attraction under state feedback, then for any compact set $\mathcal{S}$ in the interior of $\mathcal{R}$ and any compact set $\mathcal{Q} \subset R^r$, where $r = r_1 + \cdots + r_p$, the set $\mathcal{S} \times \mathcal{Q}$ is included in the region of attraction under output feedback control. Third, the trajectory of $(x, z, \vartheta)$ under output feedback approaches the trajectory under state feedback as $\varepsilon \to 0$.

Atassi and Khalil extended the results of [14] in two directions. In [16] they present the separation principle for the more general case where the system is time varying and the state feedback controller renders a certain compact set positively invariant and asymptotically attractive. This more general result allows the separation principle to be applied to finite-time convergence to a set, ultimate boundedness and regulation. The results of [16] are proved in Attasi's Ph.D. dissertation [13]. In [15], they show that separation results similar to those of [14] can be obtained for other high-gain-observer designs such as the ones presented in [24], [30], [74], [79, Section 4.4.1], [79, Section 4.4.2], and [81], provided the state feedback controller is globally bounded in $x$.

Recently, Ahrens and Khalil [4], [5] extended the results of [16] to the case when the output measurement is corrupted by noise. They consider a special case of (15) where $u$ and $y$ are scalars and $(A, B, C)$ represents a channel of $r$ integrators. They replace the output equation $y = Cx$ by $y = Cx + v$, where $v(t)$ is a bounded measurable function of time with $|v(t)| \le \mu$. They prove that if $\varepsilon$ is restricted to an interval of the form $\left(c_a\mu^{1/r}, \varepsilon_a\right]$, then for sufficiently small $\mu$ the performance of the state feedback controller can be recovered by the output feedback controller by choosing $\varepsilon$ small enough.

A different approach to the separation principle was introduced by Maggiore and Passino [60]. Unlike the work of Khalil and coworkers, they do not saturate the state estimates or the control. Instead, they use a dynamic projection algorithm to project the state estimates within a predetermined set. They prove a separation theorem for a class of nonlinear systems which are not necessarily uniformly observable.

## 4. CONDITIONAL SERVOCOMPENSATORS

High-gain observers have played an important role in advancing the nonlinear regulation theory for systems that can be transformed into the normal form. They allow the design to be performed in the error coordinates, where the regulation error and its derivatives are used as state variables. The state transformation to the error coordinates usually depends on unknown disturbances; so it cannot be used to implement the controller even if the states were measured in the original coordinates. However, in output feedback control a high-gain observer is used to estimate the error derivatives from the measured error, thus allowing us to design the state feedback controller in the error coordinates. This techniques has been used effectively by Khalil and Isidori, among others; see [35], [36], [42], [45], [46], [64], [65], [66], [82], [83]. More recently, Khalil and coworkers used high-gain observers to introduce the conditional servocompensator, which achieves zero steady-state regulation error without degrading the transient response. The technique was originally introduced in a sliding mode control framework in [85], [86] and then extended to a more general setup in [67], [68], [89]. The paper [86] considers the nonlinear system

$$\begin{array}{rcl} \dot{z} & = & \phi(z, e, \nu, w, \theta) \\ \dot{e}_i & = & e_{i+1}, 1 \le i \le \rho - 1 \\ \dot{e}_\rho & = & b(z, e, \nu, w, \theta) + a(z, e, \nu, w, \theta)u \end{array} \tag{19}$$

where $e_1$ is the measured tracking error, $\theta$ is a vector of unknown constants, $\nu(t)$ is a bounded exogenous signal with $\lim_{t\to\infty} \nu(t) = 0$, $w(t)$ is a bounded exogenous signal that is generated by the exosystem $\dot{w} = S_0 w$, where $S_0$ has simple eigenvalues on the imaginary axis, and $a(\cdot) \ge a_0 > 0$. The function $\phi(\cdot)$ satisfies $\phi(0, 0, 0, w, \theta) = 0$ and the equation $\dot{z} = \phi$ satisfies certain stability requirements, which imply that the system is minimum phase. The control task is to design an output feedback controller that stabilizes the zero-error manifold $\{z = 0, e = 0\}$ and ensures that, for a compact set of initial conditions, all trajectories converge to the zero-error manifold. The steady-state control that maintains the trajectories on the zero-error manifold is given by

$$\chi(w, \theta) = -\frac{b(0, 0, 0, w, \theta)}{a(0, 0, 0, w, \theta)}$$

and is assumed to be generated by the internal model

$$\frac{\partial \tau(w, \theta)}{\partial w} S_0 w = S\tau(w, \theta), \qquad \chi(w, \theta) = \Gamma\tau(w, \theta)$$

where the pair $(S, \Gamma)$ is observable and $S$ has simple eigenvalues on the imaginary axis. Assume for the present that the state $e$ is available for feedback. Define $\zeta^T = [e_1\ e_2\ \cdots\ e_{\rho-1}]$ and $K_2 = [k_1\ k_2\ \cdots\ k_{\rho-1}]$, where $K_2$ is chosen such that the polynomial

$$\lambda^{\rho-1} + k_{\rho-1}\lambda^{\rho-2} + \cdots + k_2\lambda + k_1$$

is Hurwitz. The conditional servocompensator is defined by

$$\dot{\sigma} = (S - JK_1)\sigma + \mu J \mathrm{sat}(s/\mu) \tag{20}$$

where the pair $(S, J)$ is controllable, $K_1$ is designed such that $(S - JK_1)$ is Hurwitz, $\mu$ is a small positive constant, and $s$ is defined by

$$s = K_1\sigma + K_2\zeta + e_\rho \tag{21}$$

The control is given by

$$u = -k\,\mathrm{sat}(s/\mu) \tag{22}$$

where $k > 0$ is chosen large enough to ensure that $s\dot{s}$ is negative outside the set $\{|s| \le \mu\}$, which ensures that $s$ reaches the set $\{|s| \le \mu\}$ in finite time. Inside this set, the control (22) and the conditional servo compensator (20) reduce to

$$u = -\frac{k}{\mu}s, \qquad \dot{\sigma} = S\sigma + K_2\zeta + e_\rho$$

It is shown in [86] that these equations ensure convergence to the zero-error manifold. The key feature of the conditional servocompensator is that its state $\sigma$ will be of order $O(\mu)$ all the time, provided it is $O(\mu)$ at the initial time. This causes the trajectories of closed-loop system under the control (22) to be $O(\mu)$ close to the trajectories of the closed-loop system under a sliding mode controller that does not include a servocompensator. This property is proved in [86] for the state feedback controller as well as the output feedback controller in which a high-gain observer is used to estimate $e$.

The paper [85] dealt with a special case where the conditional servocompensator is simply a conditional integrator. The results of [85], [86] were extended in [67], [89] to more general stabilizing control strategies. Both papers start from a fairly general state feedback controller that stabilizes the system in the absence of nonvanishing disturbances. By using Lyapunov design, a new stabilizing controller is derived in saturated high-gain feedback form that resembles (22) and a conditional servocompensator is introduced by mimicking the sliding mode control case. The advantage of this Lyapunov redesign extension is illustrated in [68] where it is applied to regulation of a linear system under constrained control.

## 5. PERFORMANCE RECOVERY USING EXTENDED HIGH-GAIN OBSERVERS

The use of observers to estimate perturbations or disturbance inputs is wide spread in the control literature. The main idea is to view the disturbance inputs as additional state variables, augment them with the plant, and design the observer for the augmented system. In many mechanical and electromechanical applications, e.g. [55], [59], a slowly varying disturbance torque is approximated by a constant that is modeled as the solution of $\dot{\tau} = 0$. Augmenting this equation with the plant, an observer is designed to estimate the torque. In most of these studies the observer is designed using linear observer theory without rigorously analyzing the impact of the slowly varying torque, but good simulation or experimental results are reported.

For nonlinear systems in the normal form with matched disturbance, the design of a disturbance observer amounts to estimating an extra derivative of the output because viewing the disturbance input as a state variable adds one more integrator to a chain of integrators. This task can be achieved by a high-gain observer, as we have seen in Section 2. Because the observer's dynamics are fast, we do not need to restrict the disturbance to be slowly varying. Such observer is called "extended" because its dimension will be $n + 1$ for a system of relative degree $n$.

In [29], Freidovich and Khalil consider a class of minimum phase, feedback linearizable systems of the form

$$\begin{aligned} \dot{x} &= Ax + B\left[b(x, z, w) + a(x, z, w)\, u\right] \\ \dot{z} &= f_0(x, z, w) \\ y &= Cx \end{aligned} \tag{23}$$

where $x \in R^n$ and $z \in R^m$ are the state variables, $u \in R$ is the control input, $y \in R$ is the measured output, $w \in R^\ell$ is a disturbance input, the triple $(A, B, C)$ represents a chain of $n$ integrators, and $a(\cdot) \geq a_0 > 0$. The goal is to design an output feedback controller to asymptotically regulate the output $y(t)$ to zero, for all initial states in a given compact set, while meeting ceratin requirements on the transient response.

The extended high-gain observer is taken as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B\left[\hat{\sigma} + \hat{b}(\hat{x}) + \hat{a}(\hat{x})u\right] + H(y - C\hat{x}) \\ \dot{\hat{\sigma}} &= (\alpha_{n+1}/\varepsilon^{n+1})(y - C\hat{x}) \end{aligned} \tag{24}$$

where $\hat{a}(\cdot) \geq a_0 > 0$ and $\hat{b}(\cdot)$ are globally bounded functions that model $a(\cdot)$ and $b(\cdot)$, respectively,

$$H = \left[\alpha_1/\varepsilon, \ \ldots, \ \alpha_n/\varepsilon^n\right]^T$$

$\alpha_1, \ldots, \alpha_n, \alpha_{n+1}$ are chosen such that the polynomial

$$s^{n+1} + \alpha_1 s^n + \cdots + \alpha_{n+1}$$

is Hurwitz, and $\varepsilon > 0$ is a small parameter. The control is taken as

$$u = M \operatorname{sat}\left(\frac{-\hat{\sigma} - \hat{b}(\hat{x}) + \phi(\hat{x})}{M\hat{a}(\hat{x})}\right) \tag{25}$$

where $\phi$ stabilizes the origin of

$$\dot{x} = Ax + B\phi(x) \tag{26}$$

and meets given transient response specifications. One choice is $\phi(x) = -Kx$, where $(A - BK)$ is Hurwitz. The saturation level $M$ is chosen such that the control saturates only outside a compact set of interest. Let

$$k_a = \max\left|\frac{a(x, z, w) - \hat{a}(x)}{\hat{a}(x)}\right|$$

$$G(s) = \frac{\alpha_{n+1}}{s^{n+1} + \alpha_1 s^n + \cdots + \alpha_{n+1}}$$

where the maximization is taken over a compact set of interest. It is shown in [29] that if $k_a < 1/\sup_\omega |G(j\omega)|$, then, for sufficiently small $\varepsilon$, the trajectories under output feedback control are bounded and $x(t)$ satisfies

- $\|x(t) - x^\star(t)\| \to 0$ as $\varepsilon \to 0$, uniformly in $t$, for $t \geq 0$, where $x^\star(t)$ is the solution of (26) with $x(0) = x^\star(0)$, and
- $\|x(t)\|$ is uniformly ultimately bounded by $\delta(\varepsilon)$, where $\delta(\varepsilon) \to 0$ as $\varepsilon \to 0$

The extended high-gain observer is used in [71] to derive a stabilizing feedback controller for a nonminimum phase system. The technique builds on earlier work by Isidori [37], where the function $b(\cdot)$ in (23) is used as the output for an auxiliary system that includes the unstable internal dynamics. A dynamic controller for the auxiliary system is implemented by replacing $b$ by its estimate $\hat{\sigma}$, generated by the extended high-gain observer. The output feedback controller is shown to recover the performance of the state feedback controller in which $\hat{\sigma}$ is replaced by $b + (a - \hat{a})u$. An application to the translational oscillator with rotating actuator (TORA) system in [70] demonstrates the effectiveness of the control strategy.

In [27], [28], the extended high-gain observer is used to develop a Lyapunov-based switching control strategy. The paper [27] considers a minimum-phase nonlinear system with large parametric uncertainty. The system can be represented globally in the normal form and the goal is to find an output feedback controller to ensure that the output tracks a bounded reference signal. The set of uncertain parameters is partitioned into smaller subsets, a robust controller is designed for each subset, and the controller is switched if, after a dwell time, the derivative of a Lyapunov function does not satisfy a certain inequality. The extended high-gain observer is used to estimate the derivatives of the output as well as the derivative of the Lyapunov function. The paper [28] introduces a switching strategy that uses on-line information to decide on the controller to switch to, instead of using a pre-sorted list as in [27]. It also deals with a more general class of uncertain nonlinear systems. In particular, it requires neither the sign of the high-frequency gain to be known nor the system to be minimum-phase.

# 6. SWITCHING AND NONLINEAR-GAIN OBSERVERS

The performance of high-gain observers in the presence of measurement noise has been studied in [2], [4], [5], [17], [95]. In [95], the effect of measurement noise is studied when the observer is used to estimate the derivatives of a signal in real time. The measurement noise is modeled as a bounded measurable signal and the ultimate bound on the estimation error is calculated as function of the observer parameter $\varepsilon$ and the bound on the noise $\mu$; see Fig. 1. The analysis of [95] is carried further in [4], [5] where the stability properties of the closed-loop system are studied. The effect of measurement noise on the closeness of trajectories under output feedback to the ones under state feedback is studied. The papers illustrate the tradeoff when selecting the observer gain between state reconstruction speed and robustness to model uncertainty, on the one hand, and amplification of measurement noise on the other. Based on this tradeoff, it is proposed to use a high-gain observer that switches between two gain values. This scheme is able to quickly recover the system states during large estimation error and reduce the effect of noise when the error is small.

In [17] an alternative idea to change the observer gain is used. The high-gain observer has a nonlinear gain, which is chosen to have a higher gain when the error $|y - \hat{x}_1|$ is larger than a threshold and a lower gain otherwise. The nonlinear-gain approach bypasses the complications associated with switching, with little degradation in performance.

# 7. IMPLEMENTATION ISSUES

To move high-gain observers towards practical implementation, three issues have to be addressed: measurement noise, unmodeled fast dynamics, and digital implementation. In the previous section we dealt with measurement noise; in the current section we discuss the other two issues.

Since high-gain observers extend the bandwidth of the controller as the observer gain is increased, it is important to study robustness to unmodeled fast (high-frequency) dynamics. In a series of papers [7], [62], [49], Khalil and coworkers studied the robustness of a high-gain-observer-based stabilizing controller to unmodeled actuator and sensor dynamics. It is shown in [49] that the controller is robust provided the actuator and sensor dynamics are sufficiently fast relative to the dynamics of the closed-loop system under state feedback, but not necessarily faster than the observer dynamics.

Sampled-data control of high-gain observers was studied in [3], [6], [22], with a related result in [48]. The paper [22] studies digital implementation of high-gain-observer-based output feedback controllers. By a separation approach, a continuous-time state feedback controller is designed, discretized, and applied using estimates provided by discrete-time implementation of a continuous-time high-gain observer. The sampling frequency is chosen of the order of the observer bandwidth. It is shown in [22] that the output feedback controller stabilizes the origin of the closed-loop system for sufficiently high sampling frequency and recovers the performance under continuous-time state feedback control.

The paper [3] develops a multirate sampled-data output feedback control for a class of nonlinear systems using high-gain observers where the measurement sampling rate is higher than the control update rate. It is shown that if the sampled-data state feedback controller globally stabilizes the origin, then the multirate output feedback controller will achieve semiglobal practical stabilization. This multirate scheme allows for more computationally intensive controllers because the control sampling period will not shrink as we increase the observer gain. The development of a multirate scheme was motivated by an application to a smart-actuated control system[6].

In [48], the continuous-time high-gain observer is replaced by a discrete-time observer, designed using a discrete-time model of a chain of integrators. Performance recovery results, similar to those of [22], are proved. This design gives a simpler observer procedure compared to [22], but the observer of [22] could, in general, be designed to yield better performance.

# 8. EXPERIMENTAL TESTBEDS

To demonstrate the practicality of high-gain observers in nonlinear feedback control, several experiments were conducted as the technique was applied to mechanical and electromechanical systems. In [22], sampled-data control is applied to the pendubot. The pendubot is also used in [89] to test regulation using conditional integrators. In [6], multirate sampled-data control is applied to a smart-material actuated system where a discrete-time implementation of a high-gain observer is combined with a hysteresis inversion controller. In these three experiments the high-gain observer estimates velocity from displacement measurement. In [10], it is used in nonlinear control of an induction motor, where the observer estimates shaft velocity and acceleration from the shaft position. In [51], [91] the high-gain observer is used in sensorless control of induction motors, where shaft velocity is estimated from stator current measurement. Finally, in [29], an extended high-gain observer uses measurement of the shaft position of a DC motor to estimate its velocity as well as a torque due to nonlinear friction.

## 9. CONCLUSIONS

Despite the progress in high-gain-observer theory over the past twenty years, there are still challenging problems and open avenues for research. The work reported here on conditional servocompensators and extended high-gain observers is still in an early stage. We believe that there is a lot to learn about how to use these tools to improve the performance of nonlinear controllers. We are also just scratching the surface regarding how to modify the design of high-gain observers to cope better with the effect of measurement noise.

## REFERENCES


[1] A.H. Ahrens and H.K. Khalil. Closed-loop behavior of a class of nonlinear systems under ekf-based control. *IEEE Trans. Automat. Contr.*, 52:536–540, 2007.

[2] J.H. Ahrens and H.K. Khalil. Output feedback control using high-gain observers in the presence of measurement noise. In *Proc. American Control Conf.*, pages 4114–4119, Boston, MA, 2004.

[3] J.H. Ahrens and H.K. Khalil. Multirate sampled-data output feedback using high-gain observers. In *Proc. IEEE Conf. on Decision and Control*, pages 4885–4890, San Diego, CA, 2006.

[4] J.H. Ahrens and H.K. Khalil. High-gain observers in the presence of measurement noise: A switched-gain approach. In *The 17th IFAC World Congress*, pages 7606–7611, Seoul, Korea, 2008.

[5] J.H. Ahrens and H.K. Khalil. High-gain observers in the presence of measurement noise: A switched-gain approach. *Automatica*, 2009.

[6] J.H. Ahrens, X. Tan, and H.K. Khalil. Multirate sampled-data output feedback control of smart material actuated systems. In *Proc. American Control Conf.*, pages 4327–4332, New York, 2007.

[7] R.W. Aldhaheri and H.K. Khalil. Effect of unmodeled actuator dynamics on output feedback stabilization of nonlinear systems. *Automatica*, 32(9):1323–1327, 1996.

[8] B. Aloliwi and H.K. Khalil. Adaptive output feedback regulation of a class of nonlinear systems: convergence and robustness. *IEEE Trans. Automat. Contr.*, 42:1714–1716, 1997.

[9] B. Aloliwi and H.K. Khalil. Robust adaptive output feedback control of nonlinear systems without persistence of excitation. *Automatica*, 33:2025–2032, 1997.

[10] B. Aloliwi, H.K. Khalil, and E.G. Strangas. Robust speed control of induction motors. In *Proc. American Control Conf.*, Albuquerque, NM, 1997. WP16:4.

[11] J. Alvarez-Ramirez and R. Fermat. Robust PI stabilization of a class of chemical reactors. *Syst. Contr. Lett.*, 38:219–225, 1999.

[12] V. Andrieu, L. Praly, and A. Astolfi. High gain observers with updated gain and homogeneous correction terms. *Automatica*, 44, 2008. To appear.

[13] A.N. Atassi. *A separation principle for the control of a class of nonlinear systems*. PhD thesis, Michigan State University, East Lansing, 1999.

[14] A.N. Atassi and H.K. Khalil. A separation principle for the stabilization of a class of nonlinear systems. *IEEE Trans. Automat. Contr.*, 44:1672–1687, 1999.

[15] A.N. Atassi and H.K. Khalil. Separation results for the stabilization of nonlinear systems using different high-gain observer designs. *Syst. Contr. Lett.*, 39:183–191, 2000.

[16] A.N. Atassi and H.K. Khalil. A separation principle for the control of a class of nonlinear systems. *IEEE Trans. Automat. Contr.*, 46, 2001.

[17] A.A. Ball and H.K. Khalil. High-gain observers in the presence of measurement noise: A nonlinear gain approach. In *Proc. IEEE Conf. on Decision and Control*, Cancun, 2008.

[18] G. Besançon. High-gain observation with disturbance attenuation and application to robust fault detection. *Automatica*, 39:1095–1102, 2003.

[19] K. Busawon, M. Farza, and H. Hammouri. Observer design for a special class of nonlinear systems. *Int. J. Contr.*, 71:405–418, 1998.

[20] C.I. Byrnes, F. Celani, and A. Isidori. Omega-limit sets of a class of nonlinear systems that are semiglobally practically stabilized. *Int. J. Robust and Nonlinear Control*, 15::315–333, 2005.

[21] A. Dabroom and H.K. Khalil. Discrete-time implementation of high-gain observers for numerical differentiation. *Int. J. Contr.*, 72:1523–1537, 1999.

[22] A.M. Dabroom and H.K. Khalil. Output feedback sampled-data control of nonlinear systems using high-gain observers. *IEEE Trans. Automat. Contr.*, 46:1712–1725, 2001.

[23] F. Deza, E. Busvelle, J.P. Gauthier, and J. Rakotopara. High gain estimation for nonlinear systems. *Syst. Contr. Lett.*, 18:295–299, 1992.

[24] J.C. Doyle and G. Stein. Robustness with observers. *IEEE Trans. Automat. Contr.*, AC-24(4):607–611, August 1979.

[25] F. Esfandiari and H.K. Khalil. Observer-based design of uncertain systems: recovering state feedback robustness under matching conditions. In *Proc. Allerton Conf.*, pages 97–106, Monticello, IL, September 1987.

[26] F. Esfandiari and H.K. Khalil. Output feedback stabilization of fully linearizable systems. *Int. J. Contr.*, 56:1007–1037, 1992.

[27] L.B. Freidovich and H.K. Khalil. Logic-based switching for the robust control of minimum-phase nonlinear systems. *Syst. Contr. Lett.*, 54:713–727, 2005.

[28] L.B. Freidovich and H.K. Khalil. Lyapunov-based switching control of nonlinear systems using high-gain observers. *Automatica*, 43:150–157, 2007.

[29] L.B. Freidovich and H.K. Khalil. Performance recovery of feedback-linearization-based designs. *IEEE Trans. Automat. Contr.*, 2009.

[30] J.P. Gauthier, H. Hammouri, and S. Othman. A simple observer for nonlinear systems application to bioreactors. *IEEE Trans. Automat. Contr.*, 37(6):875–880, 1992.

[31] J.P. Gauthier and I.A.K. Kupka. Observability and observers for nonlinear systems. *SIAM J. Contr. Optim.*, 32:975–994, 1994.

[32] J. Gonzalez-Trejo, J. Alvarez Ramirez, and G. Fernandez. Robust control with uncertainty estimation for feedback linearizable systems: application to control of distillation columns. *J. Process Control*, 9:221–231, 1999.

[33] H. Hammouri, B. Targui, and F. Armanet. High gain observer based on a triangular structure. *Int. J. Robust and Nonlinear Control*, 12:497–518, 2002.

[34] L. Imsland, R. Findeisen, E. Bullinger, F. Allgower, and B.A. Foss. A note on stability, robustness and performance of output feedback nonlinear model predictive control. *J. Process Control*, 13:633–644, 2003.

[35] A. Isidori. *Nonlinear Control Systems*. Springer-Verlag, New York, 3rd edition, 1995.

[36] A. Isidori. A remark on the problem of semiglobal nonlinear output regulation. *IEEE Trans. Automat. Contr.*, 42(12):1734–1738, 1997.

[37] A. Isidori. A tool for semiglobal stabilization of uncertain non-minimum-phase nonlinear systems via output feedback. *IEEE Trans. Automat. Contr.*, 45:1817–1827, 2000.

[38] M. Janković. Adaptive output feedback control of nonlinear feedback linearizable systems. *Int. J. Adaptive Control and Signal Processing*, 10:1–18, 1996.

[39] M. Janković. Adaptive nonlinear output feedback tracking with a partial high-gain observer and backstepping. *IEEE Trans. Automat. Contr.*, 42(1):106–113, 1997.

[40] Z-P. Jiang, D.J. Hill, and Y. Guo. Stabilization and tracking via output feedback for the nonlinear benchmark system. *Automatica*, 34:907–915, 1998.

[41] H.S. Kay and H.K. Khalil. Universal integral controllers with non-linear integral gains. *Int. J. Contr.*, 77:1521–1531, 2004.

[42] H.K. Khalil. Robust servomechanism output feedback controllers for a class of feedback linearizable systems. *Automatica*, 30(10):1587–1599, 1994.

[43] H.K. Khalil. Adaptive output feedback control of nonlinear systems represented by input-output models. *IEEE Trans. Automat. Contr.*, 41:177–188, 1996.

[44] H.K. Khalil. High-gain observers in nonlinear feedback control. In H. Nijmeijer and T.I. Fossen, editors, *New Directions in Nonlinear Observer Design*, volume 244 of *Lecture Notes in Control and Information Sciences*, pages 249–268. Springer, London, 1999.

[45] H.K. Khalil. On the design of robust servomechanisms for minimum phase nonlinear systems. *Int. J. Robust and Nonlinear Control*, 10:339–361, 2000.

[46] H.K. Khalil. Universal integral controllers for minimum phase nonlinear systems. *IEEE Trans. Automat. Contr.*, 45:490–494, 2000.

[47] H.K. Khalil. *Nonlinear Systems*. Prentice Hall, Upper Saddle River, New Jersey, 3rd edition, 2002.

[48] H.K. Khalil. Performance recovery under output feedback sampled-data stabilization of a class of nonlinear systems. *IEEE Trans. Automat. Contr.*, 49:2173–2184, 2004.

[49] H.K. Khalil. A note on the robustness of high-gain-observer-based controllers to unmodeled actuator and sensor dynamics. *Automatica*, 41:1821–1824, 2005.

[50] H.K. Khalil and F. Esfandiari. Semiglobal stabilization of a class of nonlinear systems using output feedback. *IEEE Trans. Automat. Contr.*, 38(9):1412–1415, 1993.

[51] H.K. Khalil, E.G. Strangas, and S. Jurkovic. Speed observer and reduced nonlinear model for sensorless control of induction motors. *IEEE Trans. Contr. Syst. Tech.*, 2009. To appear.

[52] P. Krishnamurthy and F. Khorrami. Dynamic high-gain scaling: state and output feedback with application to systems with iss appended dynamics driven by all states. *IEEE Trans. Automat. Contr.*, 49:2219–2239, 2004.

[53] K.W. Lee and H.K. Khalil. Adaptive output feedback control of robot manipulators using high-gain observers. *Int. J. Contr.*, 67(6):869–886, 1997.

[54] H. Lei and W. Lin. Universal adaptive control of nonlinear systems with unknown growth rate by output feedback. *Automatica*, 42:1783–1789, 2006.

[55] W. Leonhard. *Control of Electrical Drives*. Springer, 2nd edition, 1996.

[56] W. Lin and C. Qian. Semiglobal robust stabilization of nonlinear systems by partial state and output feedback. In *Proc. IEEE Conf. on Decision and Control*, pages 3105–3110, Tampa, FL, December 1998.

[57] Z. Lin and A. Saberi. Robust semi-global stabilization of minimum-phase input-output linearizable systems via partial state and output feedback. *IEEE Trans. Automat. Contr.*, 40(6):1029–1041, 1995.

[58] Z. Lin and A. Saberi. Low-and-high gain design technique for linear systems subject to input saturation - A direct method. *Int. J. Robust and Nonlinear Control*, 7:1071–1101, 1997.

[59] R.D. Lorenz. Advances in electric drive control. In *International Conference on Electric Machines and Drives*, pages 9–16, 1999.

[60] M. Maggiore and K. Passino. A separation principle for non-UCO systems. *IEEE Trans. Automat. Contr.*, 48:1122–1133, 2003.

[61] M. Maggiore and K. Passino. Output feedback tracking: a separation principle approach. *IEEE Trans. Automat. Contr.*, 50:111–117, 2005.

[62] M.S. Mahmoud and H.K. Khalil. Robustness of high-gain observer-based nonlinear controllers to unmodeled actuators and sensors. *Automatica*, 38:361–369, 2002.

[63] M.S. Mahmoud and M. Zribi. Output feedback stabilization of systems with nonlinear actuators. *Int. J. Contr.*, 73:187–196, 2000.

[64] N.A. Mahmoud and H.K. Khalil. Asymptotic regulation of minimum phase nonlinear systems using output feedback. *IEEE Trans. Automat. Contr.*, 41(10):1402–1412, 1996.

[65] N.A. Mahmoud and H.K. Khalil. Robust control for a nonlinear servomechanism problem. *Int. J. Contr.*, 66(6):779–802, 1997.

[66] L. Marconi, A. Isidori, and A. Serrani. Non-resonance conditions for uniform observability in the problem of nonlinear output regulation. *Syst. Contr. Lett.*, 53:281–298, 2004.

[67] A. Memon and H.K. Khalil. Lyapunov redesign approach to output regulation of nonlinear systems using conditional servocompensators. In *Proc. American Control Conf.*, pages 395–400, Seattle, 2008.

[68] A. Memon and H.K. Khalil. Output regulation of linear systems subject to input constraints. In *Proc. IEEE Conf. on Decision and Control*, Cancun, 2008.

[69] T. Mita. On zeros and responses of linear regulators and linear observers. *IEEE Trans. Automat. Contr.*, AC-22(3):423–428, 1977.

[70] S. Nazrulla and H.K. Khalil. A novel nonlinear output feedback control applied to the tora benchmark system. In *Proc. IEEE Conf. on Decision and Control*, Cancun, 2008.

[71] S. Nazrulla and H.K. Khalil. Robust stabilization of non-minimum phase nonlinear systems using extended high gain observers. In *Proc. American Control Conf.*, pages 1734–1739, Seattle, 2008.

[72] S. Oh and H.K. Khalil. Output feedback tracking for nonlinear systems using variable structure control. In *Preprints of the 1995 IFAC Symposium on Nonlinear Control System Design*, Tahoe City, CA, 1995.

[73] S. Oh and H.K. Khalil. Nonlinear output feedback tracking using high-gain observer and variable structure control. *Automatica*, 33:1845–1856, 1997.

[74] I.R. Petersen and C.V. Holot. High-gain observers applied to problems in disturbance attenuation, H-infinity optimization and the stabilization of uncertain linear systems. In *Proc. American Control Conf.*, pages 2490–2496, Atlanta, GA, June 1988.

[75] V.N. Polotskii. On the maximal errors of an asymptotic state identifier. *Automation and Remote Control*, 11:1116–1121, 1979.

[76] L. Praly. Asymptotic stabilization via output feedback for lower triangular systems with output dependent incremental rate. *IEEE Trans. Automat. Contr.*, 48:1103–1108, 2003.

[77] L. Praly and Z.P. Jiang. Linear output feedback with dynamic high gain for nonlinear systems. *Syst. Contr. Lett.*, 53:107 116, 2004.

[78] C. Qian. Semi-global stabilization of a class of uncertain nonlinear systems by linear output feedback. *IEEE Trans. Circuits Syst.-II: Express Briefs*, 52, 2005.

[79] A. Saberi, B.M. Chen, and P. Sannuti. *Loop Transfer Recovery: Analysis and Design*. Springer-Verlag, New York, 1993.

[80] A. Saberi, Z. Lin, and A.R. Teel. Control of linear systems with saturating actuators. *IEEE Trans. Automat. Contr.*, 41:368–378, 1996.

[81] A. Saberi and P. Sannuti. Observer design for loop transfer recovery and for uncertain dynamical systems. *IEEE Trans. Automat. Contr.*, 35(8):878–897, 1990.

[82] A. Serrani, A. Isidori, and L. Marconi. Semiglobal robust output regulation of minimum-phase nonlinear systems. *Int. J. Robust and Nonlinear Control*, 10:379–396, 2000.

[83] A. Serrani, A. Isidori, and L. Marconi. Semiglobal nonlinear output regulation with adaptive internal model. *IEEE Trans. Automat. Contr.*, 46:1178–1194, 2001.

[84] S. Seshagiri and H.K. Khalil. Output feedback control of nonlinear systems using RBF neural networks. *IEEE Trans. Neural Networks*, 11:69–79, 2000.

[85] S. Seshagiri and H.K. Khalil. Robust output feedback regulation of minimum-phase nonlinear systems using conditional integrators. *Automatica*, 41:43–54, 2005.

[86] S. Seshagiri and H.K. Khalil. Robust output regulation of minimum phase nonlinear systems using conditional servocompensators. *Int. J. Robust and Nonlinear Control*, 15:83–102, 2005.

[87] H. Shim, Y.I. Son, and J.H. Seo. Semi-global observer for multi-output nonlinear systems. *Syst. Contr. Lett.*, 42:233–244, 2000.

[88] H. Shim and A.R. Teel. Asymptotic controllability and observability imply semiglobal practical asymptotic stabilizability by sampled-data output feedback. *Automatica*, 39:441–454, 2003.

[89] A. Singh and H.K. Khalil. Regulation of nonlinear systems using conditional integrators. *Int. J. Robust and Nonlinear Control*, 15:339–362, 2005.

[90] S.N. Singh, R. Zhang, P. Chandler, and S. Banda. Decentralized nonlinear robust control of UAVs in close formation. *Int. J. Robust and Nonlinear Control*, 13:1057–1078, 2003.

[91] E.G. Strangas, H.K. Khalil, B. Aloliwi, L. Laubinger, and J. Miller. Robust tracking controllers for induction motors without rotor position sensor: analysis and experimental results. *IEEE Trans. Energy conversion*, 14:1448–1458, 1999.

[92] H.J. Sussmann and P.V. Kokotović. Peaking and stabilization. In *Proc. IEEE Conf. on Decision and Control*, pages 1379–1384, Tampa, FL, December 1989.

[93] A. Teel and L. Praly. Global stabilizability and observability imply semi-global stabilizability by output feedback. *Syst. Contr. Lett.*, 22:313–325, 1994.

[94] A. Teel and L. Praly. Tools for semiglobal stabilization by partial state and output feedback. *SIAM J. Contr. Optim.*, 33:1443–1488, 1995.

[95] L.K. Vasiljevic and H.K. Khalil. Error bounds in differentiation of noisy signals by high-gain observers. *Syst. Contr. Lett.*, 2009. To appear.

[96] F. Viel, E. Busvelle, and J.P. Gauthier. Stability of polymerization reactors using I/O linearization and a high-gain observer. *Automatica*, 31:971–984, 1995.

[97] X. Ye. Semiglobal output feedback control of uncertain nonlinear systems with unknown high frequency gain sign. *IEEE Trans. Automat. Contr.*, 45:2402–2405, 2000.