Fast Empirical Mode Decomposition based on Gaussian Noises

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***Abstract*—Mode-mixing, boundary effects and necessary extrema lacking and etc. are the main problems involved in empirical mode decomposition (EMD). The paper presents an improved empirical mode decomposition based on assisted signals: Gaussian noises. Firstly, the given 1D Gaussian noise and its negative counterpart are added to the original respectively to construct the two s to be decomposed. Secondly, the decomposed IMFs from the two signals are added together to get the IMFs, in which the added noises are canceled out with less mode-mixing and boundary effects. Lastly, the efficiency and performance of the method are given through theoretical analysis and experiments.**

Keywords—Empirical mode decomposition (EMD); 1D Gaussian Noise; Assisted Signal; Intrinsic Mode Functions (IMF)

I. INTRODUCTION

The empirical mode decomposition (EMD) has been developed as a valuable time-frequency-magnitude resolution tool for non-stationary signals [1] and been widely explored [2-17,23-30]. The algorithm is simple and efficient in situations where other methods fail. However, it has some drawbacks, tied with some of the assumptions needed to implement the algorithm, leading to unexpected results, especially in processing. However, there are some problems that should be solve in EMD. The first problem in EMD is the selection of optimal interpolation scheme, which is still an open question now despite of the previous EMD methods as following. The second problem is the boundary effect. In 1D case, to achieve the goal of extension of data, numerous methods, such as the linear prediction, mirror or anti-mirror extension, neural networks, and vector machines, to name a few, have been used. However, due to various rigid stationary and linear assumptions, this extension can hardly deal well with the nonlinear non-stationary data. There are other problems, such as the determination of sifting number, noise variance and mode-mixing, to be solved even if these problems have been addressed in 1D case [5,8-13]. Particularly, there have been no published papers covering the 1D case up till now. In this paper, a signal assisted EMD is explored. The 1D Gaussian noise assisted improved EMD is explored but without the above drawbacks.

II. ASSISTED SIGNAL BASED EMD

A. Traditional EMD

Before the introduction of our method, the traditional EMD is reviewed firstly.

Therefore, traditional EMD for $f(t)$ is defined as follows:

(1) Initialization: $res(t) = f(t)$.
(2) Identify all the minima and maxima of $res(t)$, respectively.
(3) Compute the lower and upper envelopes, $E_{dw}(t)$ and $E_{up}(t)$, through interpolating the minima and maxima using 1D interpolation functions, respectively.
(4) Compute the mean of $E_{dw}(t)$ and $E_{up}(t)$: $M_{en}(t) = \left(E_{dw}(t) + E_{up}(t)\right)/2$.
(5) Subtract $M_{en}(t)$ from $res(t)$: $Mod(t) = res(t) - M_{en}(t)$.
(6) If $Mod(t)$ is not an IMF, $res(t) = Mod(t)$, turn to step (1).
(7) Obtain the IMF: $imf(t) = Mod(t)$.
(8) $res(t) = res(t) - imf(t)$.
(9) Repeat the steps (1)-(8) until the residue $res(t)$ satisfies the stopping condition (in this paper the fixed number of IMFs is given in advance).

After the sifting process of steps (1)-(9), one has the following equation

$$f(t) = \sum_{l=1}^{L} imf_l(t) + res(t) \quad . \qquad (1)$$

B. ASEMD

Here we give two methods: the ASEMD1 is shown in [13], and the ASEMD2 is presented by this paper.

ASEMD1 is defined as follows.

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(1) Add the different random zero-mean white Gaussian noise series $n_j(t)$ ($j=1,2,\cdots,J$) with the same variance to the targeted $f(t)$: $fn_j^{(1)}(t)=n_j(t)+f(t)$.

(2) Decompose the data $fn_j^{(1)}(t)$ using the traditional EMD defined above in part II. A:

$$fn_j^{(1)}(t)=\sum_{l=1}^{L}imf_{n,lj}^{(1)}(t)+res_{n,j}^{(1)}(t).$$

(3) Compute the IMFs and residue:

$$imf_{n,l}^{(1)}(t,J)=\frac{1}{J}\sum_{j=1}^{J}imf_{n,lj}^{(1)}(t),$$
$$res_{n}^{(1)}(t,J)=\frac{1}{J}\sum_{j=1}^{J}res_{n,j}^{(1)}(t).$$

Finally we obtain

$$f_n^{(1)}(t,J)=\sum_{l=1}^{L}imf_{n,l}^{(1)}(t,J)+res_n^{(1)}(t,J) \tag{2}$$

Here the superscript "(1)" denotes the method of ASEMD1.

Taking into account the ASEMD1 approach and the relation $fn_j^{(1)}(t)=n_j(t)+f(t)$, the equation in step (2) of ASEMD2 can also be rewritten as

$$\begin{aligned} fn_j^{(1)}(t)&=\sum_{l=1}^{L}imf_{n,lj}^{(1)}(t)+res_{n,j}^{(1)}(t)\\ &=\sum_{l=1}^{L}\left\{imf_{n,lj}^{(1)}(t)\Big|_{f(t)}+imf_{n,lj}^{(1)}(t)\Big|_{n_j(t)}\right\}\\ &\quad+\left\{res_{n,j}^{(1)}(t)\Big|_{f(t)}+res_{n,j}^{(1)}(t)\Big|_{n_j(t)}\right\} \end{aligned} \tag{3}$$

Where $\bullet\big|_{f(t)}$ denotes the part that comes from $f(t)$, and $\bullet\big|_{n_j(t)}$ denotes the part that comes from $n_j(t)$.

Therefore, the equations of step (3) in ASEMD1 can be rewritten as

$$\begin{aligned} imf_{n,l}^{(1)}(t,J)&=\frac{1}{J}\sum_{j=1}^{J}imf_{n,lj}^{(1)}(t)\\ &=\frac{1}{J}\sum_{j=1}^{J}\left\{imf_{n,lj}^{(1)}(t)\Big|_{f(t)}+imf_{n,lj}^{(1)}(t)\Big|_{n_j(t)}\right\} \end{aligned} \tag{4}$$

$$\begin{aligned} res_{n}^{(1)}(t,J)&=\frac{1}{J}\sum_{j=1}^{J}res_{n,j}^{(1)}(t)\\ &=\frac{1}{J}\sum_{j=1}^{J}\left\{res_{n,j}^{(1)}(t)\Big|_{f(t)}+res_{n,j}^{(1)}(t)\Big|_{n_j(t)}\right\} \end{aligned} \tag{5}$$

Thus, equation (2) reduces to

$$\begin{aligned} f_n^{(1)}(t,J)&=\sum_{l=1}^{L}\left\{\frac{1}{J}\sum_{j=1}^{J}\left\{imf_{n,lj}^{(1)}(t)\Big|_{f(t)}+imf_{n,lj}^{(1)}(t)\Big|_{n_j(t)}\right\}\right\}\\ &\quad+\frac{1}{J}\sum_{j=1}^{J}\left\{res_{n,j}^{(1)}(t)\Big|_{f(t)}+res_{n,j}^{(1)}(t)\Big|_{n_j(t)}\right\}\\ &=\frac{1}{J}\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(1)}(t)\Big|_{f(t)}+res_{n,j}^{(1)}(t)\Big|_{f(t)}\right\}\\ &\quad+\frac{1}{J}\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(1)}(t)\Big|_{n_j(t)}+res_{n,j}^{(1)}(t)\Big|_{n_j(t)}\right\}\\ &=\frac{1}{J}\sum_{j=1}^{J}\left\{f(t)\right\}+\frac{1}{J}\sum_{j=1}^{J}\left\{n_j(t)\right\}\\ &=f(t)+\frac{1}{J}\sum_{j=1}^{J}\left\{n_j(t)\right\} \end{aligned} \tag{6}$$

In principle, $\frac{1}{J}\sum_{j=1}^{J}\left\{n_j(t)\right\}$ should be zero for the reason of zero-mean white Gaussian noises. However, in practice, $\frac{1}{J}\sum_{j=1}^{J}\left\{n_j(t)\right\}$ tends to zero, and with the increasing of J , $\lim_{J\to\infty}\left\{\frac{1}{J}\sum_{j=1}^{J}\left\{n_j(t)\right\}\right\}=0$. Equation (6) shows that the IMFs and residue can not reconstruct the original $f(x,y)$ absolutely using ASEMD1, and the difference between the original $f(t)$ and the reconstructed $f_n^{(1)}(t,J)$ is $\frac{1}{J}\sum_{j=1}^{J}\left\{n_j(t)\right\}$.

ASEMD1 is the direct extension of the 1D case[13]. As 1D case, ASEMD1 utilizes the full advantage of the statistical characteristics of white noise[5] and the filter-like characteristics[14] of EMD to perturb the signal in its true solution neighborhood and to cancel itself out after serving its purpose. With the increasing of J , $f_n^{(1)}(t,J)$, $imf_{n,l}^{(1)}(t,J)$ and $res_n^{(1)}(t,J)$ are more and more close to their true values $f(t)$, $imf(t)$ and $res(t)$, respectively.

Therefore, equation (2) is equal to the following expression

$$f(t)\underset{J\to\infty}{\leftarrow}f_n^{(1)}(t,J)=\sum_{l=1}^{L}imf_{n,l}^{(1)}(t,J)+res_n^{(1)}(t,J) \tag{7}$$

ASEMD2 is defined as follows.

(1) Add the different random zero-mean white Gaussian noise series $n_j(t)$ ($j=1,2,\cdots,J$) with the same variance to the targeted $f(t)$: $fn_j^{(2),+}(t)=n_j(t)+f(t)$.

(2) Decompose the data $fn_j^{(2),+}(t)$ using the traditional EMD defined above in part II. A: $fn_j^{(2),+}(t)=\sum_{l=1}^{L} imf_{n,lj}^{(2),+}(t)+res_{n,j}^{(2),+}(t)$.

(3) Subtract the same random zero-mean white Gaussian noise series $n_j(t)$ ($j=1,2,\cdots,J_1$) given in step (1) from the targeted $f(t)$: $fn_j^{(2),-}(t)=f(t)-n_j(t)$.

(4) Decompose the data $fn_j^{(2),-}(t)$ using the traditional EMD defined above in part II. A: $fn_j^{(2),-}(t)=\sum_{l=1}^{L} imf_{n,lj}^{(2),-}(t)+res_{n,j}^{(2),-}(t)$.

(5) Compute the mean:

$$imf_{n,lj}^{(2)}(t)=\frac{\left(imf_{n,lj}^{(2),+}(t)+imf_{n,lj}^{(2),-}(t)\right)}{2},$$
$$res_{n,j}^{(2)}(t)=\frac{\left(res_{n,j}^{(2),+}(t)+res_{n,j}^{(2),-}(t)\right)}{2}.$$

(6) Compute the IMFs and residue:

$$imf_{n,l}^{(2)}(t,J)=\frac{1}{J}\sum_{j=1}^{J} imf_{n,lj}^{(2)}(t),$$
$$res_{n}^{(2)}(t,J)=\frac{1}{J}\sum_{j=1}^{J} res_{n,j}^{(2)}(t).$$

Finally we obtain

$$f_n^{(2)}(t,J)=\sum_{l=1}^{L} imf_{n,l}^{(2)}(t,J)+res_n^{(2)}(t,J). \quad (8)$$

Here the superscript "(2)" denotes the method of ASEMD2.

Similarly, taking into account the ASEMD2 approach and the relation $fn_j^{(2),+}(t)=n_j(t)+f(t)$, the equation in step (2) of ASEMD2 can also be rewritten as

$$\begin{aligned} fn_j^{(2),+}(t) &= \sum_{l=1}^{L} imf_{n,lj}^{(2),+}(t)+res_{n,j}^{(2),+}(t) \\ &= \sum_{l=1}^{L}\left\{ imf_{n,lj}^{(2),+}(t)\Big|_{f(t)} + imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}\right\} \\ &\quad + \left\{ res_{n,j}^{(2),+}(t)\Big|_{f(t)} + res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}\right\} \end{aligned} \quad (9)$$

In the same manner, we have

$$\begin{aligned} fn_j^{(2),-}(t) &= \sum_{l=1}^{L} imf_{n,lj}^{(2),-}(t)+res_{n,j}^{(2),-}(t) \\ &= \sum_{l=1}^{L}\left\{ imf_{n,lj}^{(2),-}(t)\Big|_{f(t)} + imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}\right\} \\ &\quad + \left\{ res_{n,j}^{(2),-}(t)\Big|_{f(t)} + res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\} \end{aligned} \quad (10)$$

Based on step (5) in ASEMD2, we have

$$\begin{aligned} imf_{n,lj}^{(2)}(t) &= \frac{\left\{ imf_{n,lj}^{(2),+}(t)\Big|_{f(t)} + imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}\right\}}{2} \\ &\quad + \frac{\left\{ imf_{n,lj}^{(2),-}(t)\Big|_{f(t)} + imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2} \end{aligned} \quad (11)$$

$$\begin{aligned} res_{n,j}^{(2)}(t) &= \frac{\left\{ res_{n,j}^{(2),+}(t)\Big|_{f(t)} + res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}\right\}}{2} \\ &\quad + \frac{\left\{ res_{n,j}^{(2),-}(t)\Big|_{f(t)} + res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2} \end{aligned} \quad (12)$$

Based on the above relations, the equations in step (5) of ASEMD2 are rewritten as follows:

$$imf_{n,l}^{(2)}(t,J)=\frac{\sum_{j=1}^{J}\left\{\frac{\left\{ imf_{n,lj}^{(2),+}(t)\Big|_{f(t)} + imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}\right\}+\left\{ imf_{n,lj}^{(2),-}(t)\Big|_{f(t)} + imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2}\right\}}{J} \quad (13)$$

$$res_n^{(2)}(t,J)=\frac{1}{J}\sum_{j=1}^{J}\left\{\frac{\left\{res_{n,j}^{(2),+}(t)\Big|_{f(t)}+res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}\right\}+\left\{res_{n,j}^{(2),-}(t)\Big|_{f(t)}+res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2}\right\} \tag{14}$$

Therefore, equation (8) reduces to

$$\begin{aligned}
f_n^{(2)}(t,J)&=\sum_{l=1}^{L}imf_{n,l}^{(2)}(t,J)+res_n^{(2)}(t,J)\\
&=\sum_{l=1}^{L}\left\{\frac{1}{J}\sum_{j=1}^{J}\left\{\frac{\left\{imf_{n,lj}^{(2),+}(t)\Big|_{f(t)}+imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}\right\}+\left\{imf_{n,lj}^{(2),-}(t)\Big|_{f(t)}+imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2}\right\}\right\}\\
&\quad+\frac{1}{J}\sum_{j=1}^{J}\left\{\frac{\left\{res_{n,j}^{(2),+}(t)\Big|_{f(t)}+res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}\right\}+\left\{res_{n,j}^{(2),-}(t)\Big|_{f(t)}+res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2}\right\}\\
&=\frac{1}{2J}\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(2),+}(t)\Big|_{f(t)}+res_{n,j}^{(2),+}(t)\Big|_{f(t)}\right\}+\frac{1}{2J}\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}+res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}\right\}\\
&\quad+\frac{1}{2J}\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(2),-}(t)\Big|_{f(t)}+res_{n,j}^{(2),-}(t)\Big|_{f(t)}\right\}+\frac{1}{2J}\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}+res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\}
\end{aligned} \tag{15}$$

Since

$$\frac{\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(2),+}(t)\Big|_{f(t)}+res_{n,j}^{(2),+}(t)\Big|_{f(t)}\right\}}{J}=f(t)$$

$$\frac{\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(2),-}(t)\Big|_{f(t)}+res_{n,j}^{(2),-}(t)\Big|_{f(t)}\right\}}{J}=f(t)$$

$$\frac{1}{J}\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}+res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}\right\}=\frac{\sum_{j=1}^{J}\{n_j(t)\}}{J}$$

$$\frac{1}{J}\sum_{j=1}^{J}\left\{\sum_{l=1}^{L}imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}+res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\}=\frac{\sum_{j=1}^{J}\{-n_j(t)\}}{J}$$

Therefore, equation (15) reduces to

$$\begin{aligned}
&f_n^{(2)}(t,J)\\
&=f(t)+\frac{\sum_{j=1}^{J}\{n_j(t)\}}{2J}+\frac{\sum_{j=1}^{J}\{-n_j(t)\}}{2J}\\
&=f(t)
\end{aligned} \tag{16}$$

Therefore, equation (8) is equal to the following expression

$$f(t)\equiv f_n^{(2)}(t,J)=\sum_{l=1}^{L}imf_{n,l}^{(2)}(t,J)+res_n^{(2)}(t,J) \tag{17}$$

Equation (17) shows that the IMFs and residue can reconstruct the original $f(t)$ perfectly using ASEMD2, and there is no difference between the original $f(t)$ and the reconstructed $f_n^{(2)}(t,J)$, which is the superiority of ASEMD2 over ASEMD1. Thus we obtain the following conclusion:

Conclusion 1: ASEMD2 approach can reconstruct the original $f(t)$ perfectly despite of what J is ($J\ge 1, J\in N$); ASEMD1 approach can not reconstruct the original $f(t)$ perfectly unless $J=+\infty$, and the reconstruction error ($\frac{1}{J}\sum_{j=1}^{J}\{n_j(t)\}$) will decrease with the increasing of J.

However, are $imf_{n,l}^{(2)}(t,J)$ and $res_n^{(2)}(t,J)$ equal to $imf_l(t)$ and $res(t)$ in ASEMD2, respectively? From equations (11) and (12), we have

$$imf_{n,lj}^{(2)}(t)=\frac{\left\{imf_{n,lj}^{(2),+}(t)\Big|_{f(t)}+imf_{n,lj}^{(2),-}(t)\Big|_{f(t)}\right\}}{2}+\frac{\left\{imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}+imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2} \quad (18)$$

$$res_{n,j}^{(2)}(t)=\frac{\left\{res_{n,j}^{(2),+}(t)\Big|_{f(t)}+res_{n,j}^{(2),-}(t)\Big|_{f(t)}\right\}}{2}+\frac{\left\{res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}+res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2} \quad (19)$$

From the filter-like characteristics[14] of EMD, only the frequency is taken into account but it has nothing to do with the negative and positive magnitudes. Therefore

$$imf_{n,lj}^{(2),+}(t)\Big|_{f(t)}=imf_{n,lj}^{(2),-}(t)\Big|_{f(t)}=imf_{n,lj}^{(2)}(t)\Big|_{f(t)} \quad (20)$$

$$res_{n,j}^{(2),+}(t)\Big|_{f(t)}=res_{n,j}^{(2),-}(t)\Big|_{f(t)}=res_{n,j}^{(2)}(t)\Big|_{f(t)} \quad (21)$$

Therefore we have

$$imf_{n,lj}^{(2)}(t)=imf_{n,lj}(t)\Big|_{f(t)}+\frac{\left\{imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}+imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2} \quad (22)$$

$$res_{n,j}^{(2)}(t)=res_{n,j}(t)\Big|_{f(t)}+\frac{\left\{res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}+res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2} \quad (23)$$

If $imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}=-imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}$, then $imf_{lj}(t)=imf_{n,lj}^{(2)}(t)$.

If $res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}=-res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}$,then $res(t)=res_{n,j}^{(2)}(t)$.

However, in practice it is impossible that $imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}\equiv -imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}$ and $res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}\equiv -res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}$ hold true. Therefore, the step (6) in ASEMD2 is employed to make sure that the effect of noises tends to zero as much as possible:

$$imf_{n,l}^{(2)}(t)=imf_{n,l}(t)\Big|_{f(t)}+\sum_{j=1}^{J}\frac{\left\{imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}+imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2J} \quad (24)$$

$$res_{n,j}^{(2)}(t)=res_{n,j}(t)\Big|_{f(t)}+\sum_{j=1}^{J}\frac{\left\{res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}+res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}\right\}}{2J} \quad (25)$$

Since the most parts of $imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}$ ($res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}$) and $imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}$ ($res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}$) are canceled out

$$(imf_{n,lj}^{(2),+}(t)\Big|_{n_j(t)}\approx -imf_{n,lj}^{(2),-}(t)\Big|_{n_j(t)}$$
$$res_{n,j}^{(2),+}(t)\Big|_{n_j(t)}\approx -res_{n,j}^{(2),-}(t)\Big|_{n_j(t)}),$$

the residual parts of them can be attenuated greatly with the increasing of J . However, in ASEMD1, there are the two relations:

$$imf_{n,l}^{(1)}(t)=imf_{n,l}(t)\Big|_{f(t)}+\sum_{j=1}^{J}\frac{\left\{imf_{n,lj}^{(1)}(t)\Big|_{n_j(t)}\right\}}{J} \quad (26)$$

$$res_{n,j}^{(1)}(t)=res_{n,j}(t)\Big|_{f(t)}+\sum_{j=1}^{J}\frac{\left\{res_{n,j}^{(1)}(t)\Big|_{n_j(t)}\right\}}{J} \quad (27)$$

Thus the most parts of $imf_{n,lj}^{(1)}(t)\Big|_{n_j(t)}$ ($res_{n,j}^{(1)}(t)\Big|_{n_j(t)}$) can only be attenuated using J . Therefore, the influences of J in ASEMD1 and ASEMD2 are not the same. Simulations also show that the number J needn't to be very high in ASEMD2. The later section will address the influence of the number J .

Note that in steps (2) and (4) of ASEMD2 the numbers of IMFs are both L, which avoids the problem that the mean can not be computed because of the lacking of the according IMFs. Thus the number of IMFs should be given in advance. In general, five IMFs and one residue are sufficient for the most usages in processing. In this paper, we will decompose the into five IMFs and one residue.

Different with ASEMD2, the number J should be very high in ASEMD1. The variance $\sigma_t^{(1)}(J)$ of ASEMD1 in every IMF or residue has the following relation

$$\sigma_t^{(1)}(J) \propto \frac{\sigma_t}{\sqrt{J}} \tag{28}$$

Instead, the variance $\sigma_t^{(2)}(J)$ of ASEMD2 in every IMF or residue has the following relation

$$\sigma_t^{(2)}(J) \propto \frac{\sigma_t}{(2J)^{\alpha}} \quad (\alpha >> 1/2) \tag{29}$$

where σ_t is the variance of the given 1D random zero-mean white Gaussian noise series,

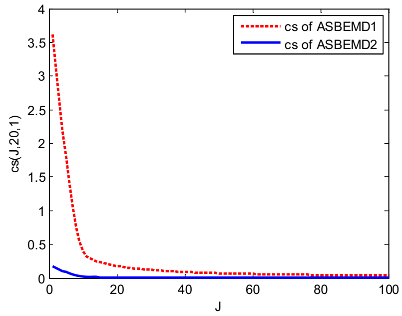
$$\sigma_t^{(s)}(J) = \mathrm{var}\left(imf_n^{(s)}(t,J) - imf(t) \right) \ (s = 1,2),$$

here " var " is the variance operator.

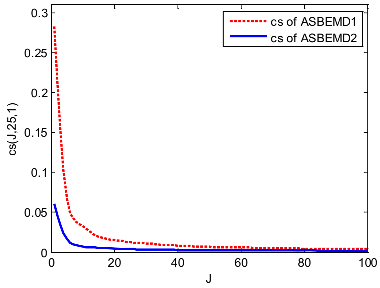
To decrease the variance $\sigma_t^{(s)}(J)$ ($s = 1,2$), J must increase. Note the time-cost of ASEMD1 is much greater than that of ASEMD2 for the same results. If $J \to \infty$, $\sigma_{x,y}^{(s)}(J) \to 0$, we will obtain the true values of IMFs and residue. The IMFs and residue will converge to their true values with the increasing of J. Thus the convergence speed (*cs*) is employed as the performance valuation of the two methods.

$$cs^{(s)}(J,\sigma_t,l) = \frac{\left\| imf_{n,l}^{(s)}(t,J) - imf_{n,l}^{(s)}(t,\infty) \right\|_{L^2(R)}}{\left\| imf_{n,l}^{(s)}(t,\infty) \right\|_{L^2(R)}} \quad (s = 1,2) \tag{30}$$

If $cs^{(s)}(J,\sigma_t,l) = 0$, it denotes that the IMFs and residue converge to their true values. If $cs^{(s)}(J,\sigma_t,l)$ is high, it denotes that the IMFs and residue are far from their true values, and vice versa. Since the true component $imf_{n,l}^{(s)}(t,\infty)$ is not accessible in reality, here we make $imf_{n,l}^{(s)}(t,100\sigma_t^2)$ be the true component instead of $imf_{n,l}^{(s)}(t,\infty)$ approximately.



(a) Convergence speeds for $\sigma_t = 40$ of IMF_1



(b) Convergence speeds for $\sigma_t = 15$ of IMF_2

Fig. 1 The convergence speed comparison of ASEMD1 and ASEMD2

It clearly shows that the convergence speed of ASEMD2 is much faster than that of ASEMD1 in Fig.1. The above theoretical analysis also shows the same conclusion. For simpleness, here only two IMFs' results are given in Fig.1.

Thus we have the following conclusion:

Conclusion 2: ASEMD2 approach can cancel most noise parts in IMFs and residue for one time's noise, but ASEMD1 fails. ASEMD2 needs much lower J than ASEMD1 for the same IMF and residue.

However, for the same J, the time-cost (tc) of ASEMD1 is only about half of that of ASEMD2. The time-cost of them are shown in equations (31) and (32).

$$tc^{(1)}(J,L) \propto J \cdot L \tag{31}$$
$$tc^{(2)}(J,L) \propto 2J \cdot L \tag{32}$$

where L is the max decomposition level of EMD. Still, since ASEMD1 needs much higher J than ASEMD2, thus the time-cost of ASEMD1 is much higher than that of ASEMD2 for the same or similar decomposed results.

III. CONCLUSION

Although EMD has been proven to be an adaptive and valuable nonlinear non-stationary tool in signal processing, there are still a lot of questions that are open up till now. One signal assisted empirical mode decomposition (ASEMD) is developed in this paper, in which the zero-mean Gaussian white noise is employed as the assisted signal. The reconstruction error and time-cost are discussed. Theoretical analysis for reconstruction error and convergence speed is discussed in great details, and ASEMD2 can reconstruct the perfectly and ASEMD1 fails.

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