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# Math for Machine Learning

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## Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix

# W2 Lesson 1



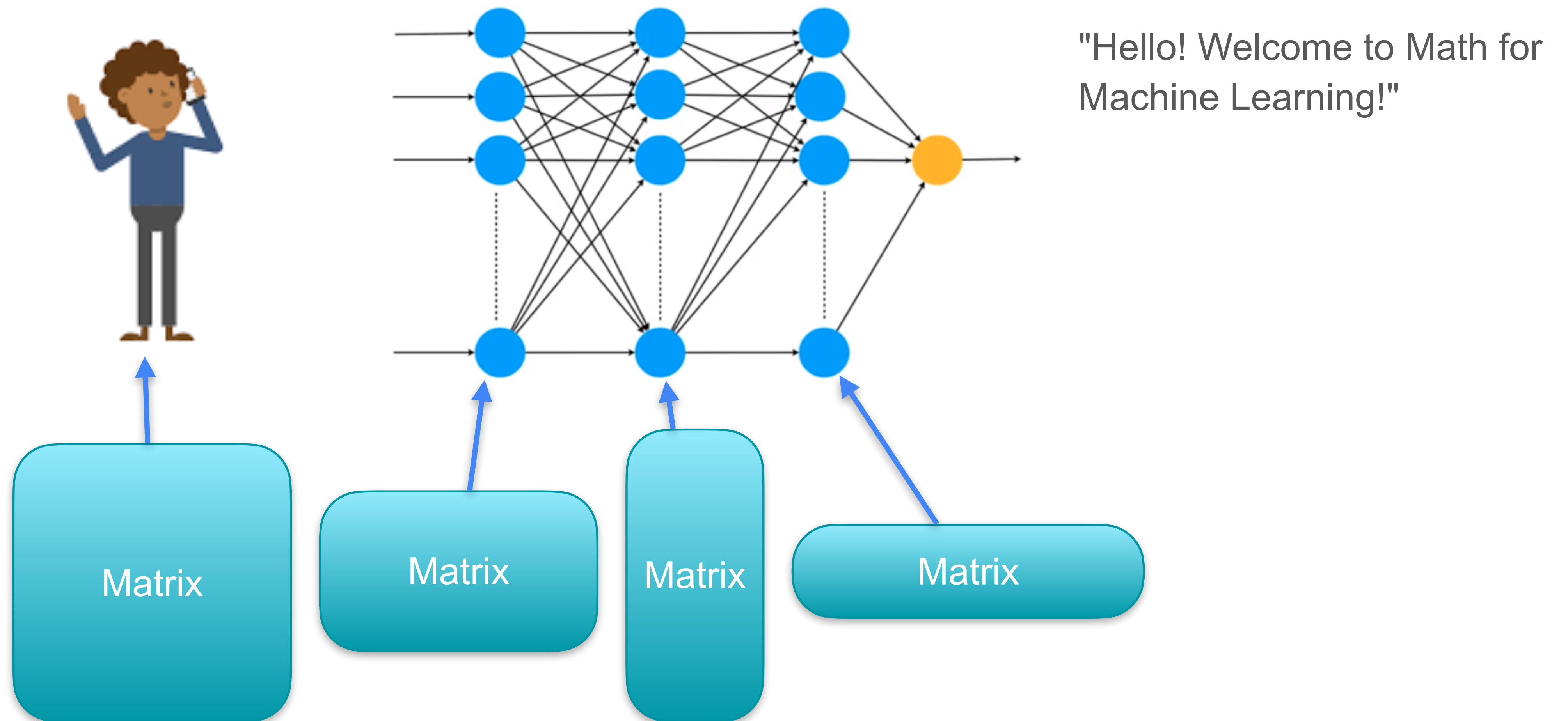
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# Solving System of Linear Equations

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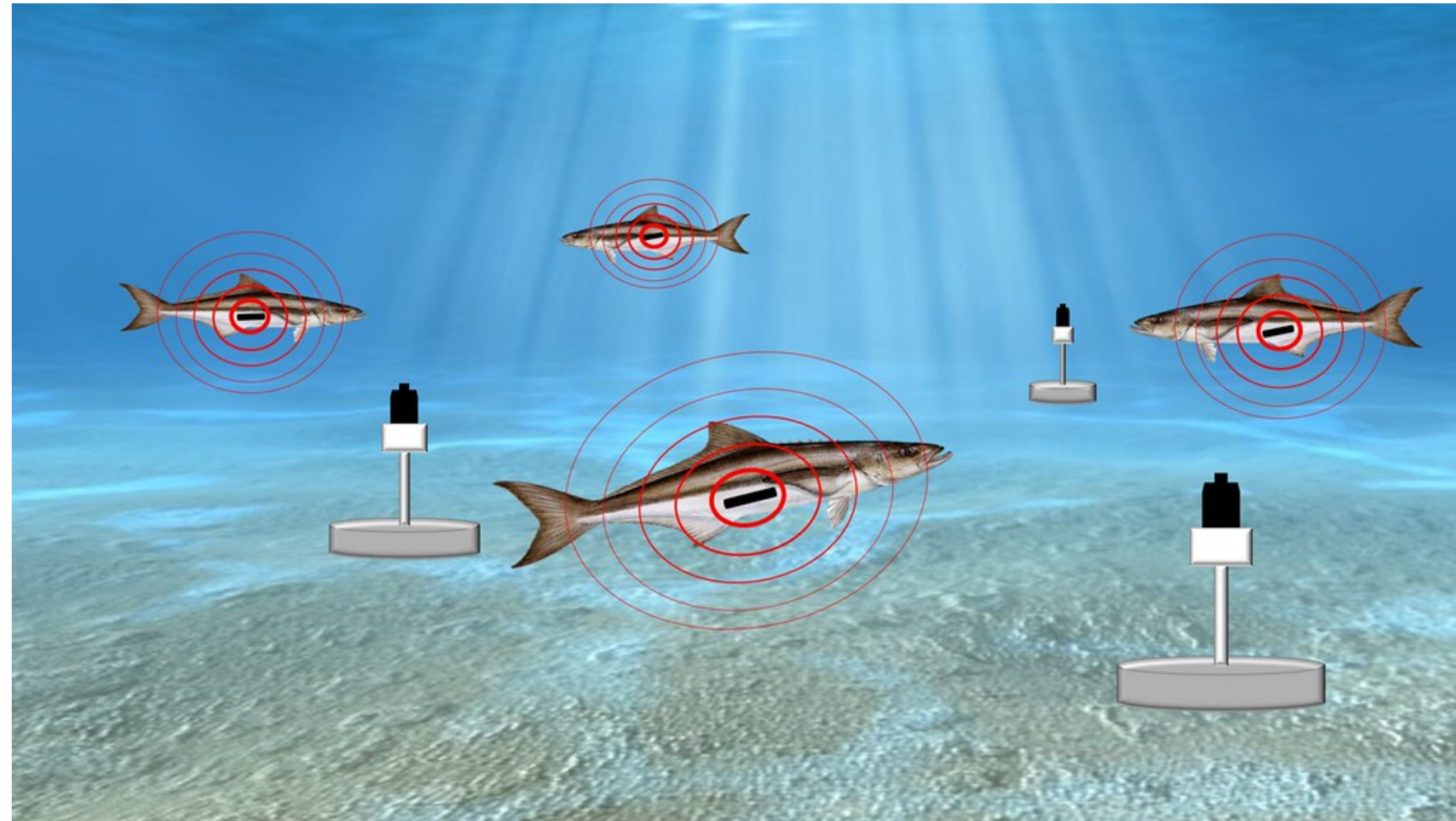
**Machine learning motivation**

# Neural networks - Matrix operations





# Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.



# Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.



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# Solving System of Linear Equations



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**Solving non-singular system  
of linear equations**



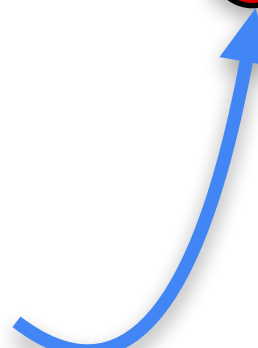






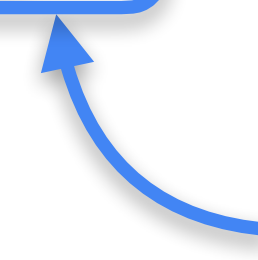
# Solving systems of equations

## System

- $a + b = 10$   
 

- $a + 2b = 12$   
 

 +  = \$10  
\$8  \$2 

 +  +  = \$12  
 \$2

# Solving systems of equations

**System**

- $a + b = 10$



- $a + 2b = 12$



**Some process**

**Manipulating equations**

**Swapping equations**

**Adding equations**

**Multiplying equations by a constant**

**Solved system**

- $a = 8$



- $b = 2$



# Solving systems of equations

## System

- $a + b = 10$



- $a + 2b = 12$



Eliminate 'a' from this equation

## Solved system

- $a = 8$



- $b = 2$



# Manipulating equations

## Multiplying by a constant

$$\begin{array}{r} a + b = 10 \\ \times \qquad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$\begin{array}{r} a + b = 10 \\ + \quad 2a + 3b = 22 \\ \hline 3a + 4b = 32 \end{array}$$



# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$

Eliminate 'a'  
from this equation

## Divide by coefficient of a

- $a + 0.2b = 3.4$
- $a - 0.75b = 1.5$

## Subtract equation 1 from equation 2

$$a - 0.75b = 1.5$$

$$\begin{array}{r} a - 0.75b = 1.5 \\ - (a + 0.2b = 3.4) \\ \hline \end{array}$$

$$0a - 0.95b = -1.9$$

$$-0.95b = -1.9$$

$$b = 2$$

## Solved system

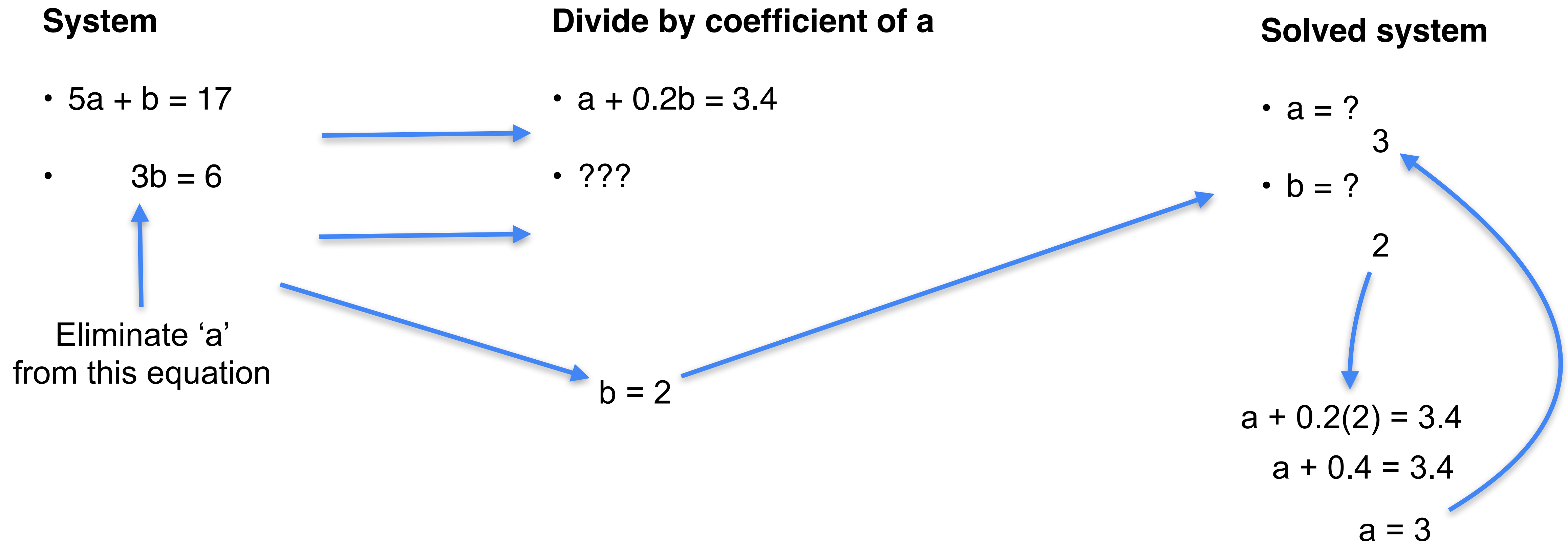
- $a = ?$  3
- $b = ?$  2

$$a + 0.2(2) = 3.4$$

$$a + 0.4 = 3.4$$

$$a = 3$$

# What if one of the coefficients of a is zero?



# Quiz

- Solve the following system of equations

## System

- $2a + 5b = 46$
- $8a + b = 32$

# Solution

- Solve the following system of equations

## System

- $2a + 5b = 46$
- $8a + b = 32$

## Solution

- $a = 3$
- $b = 8$





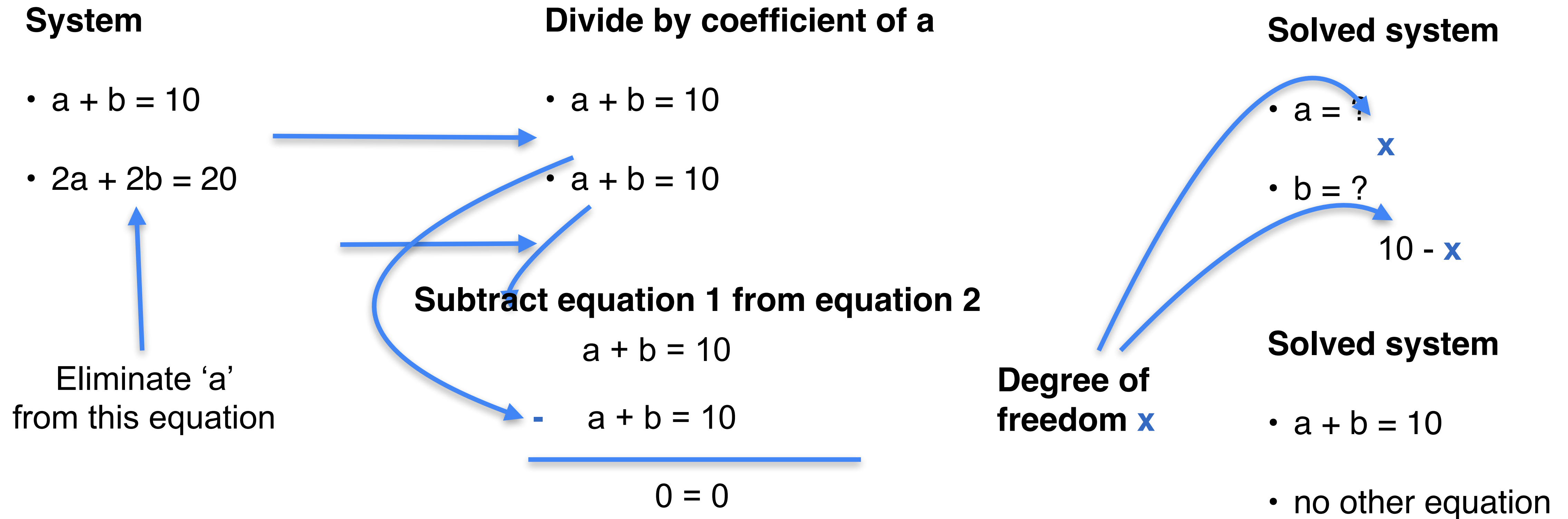
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# Solving System of Linear Equations

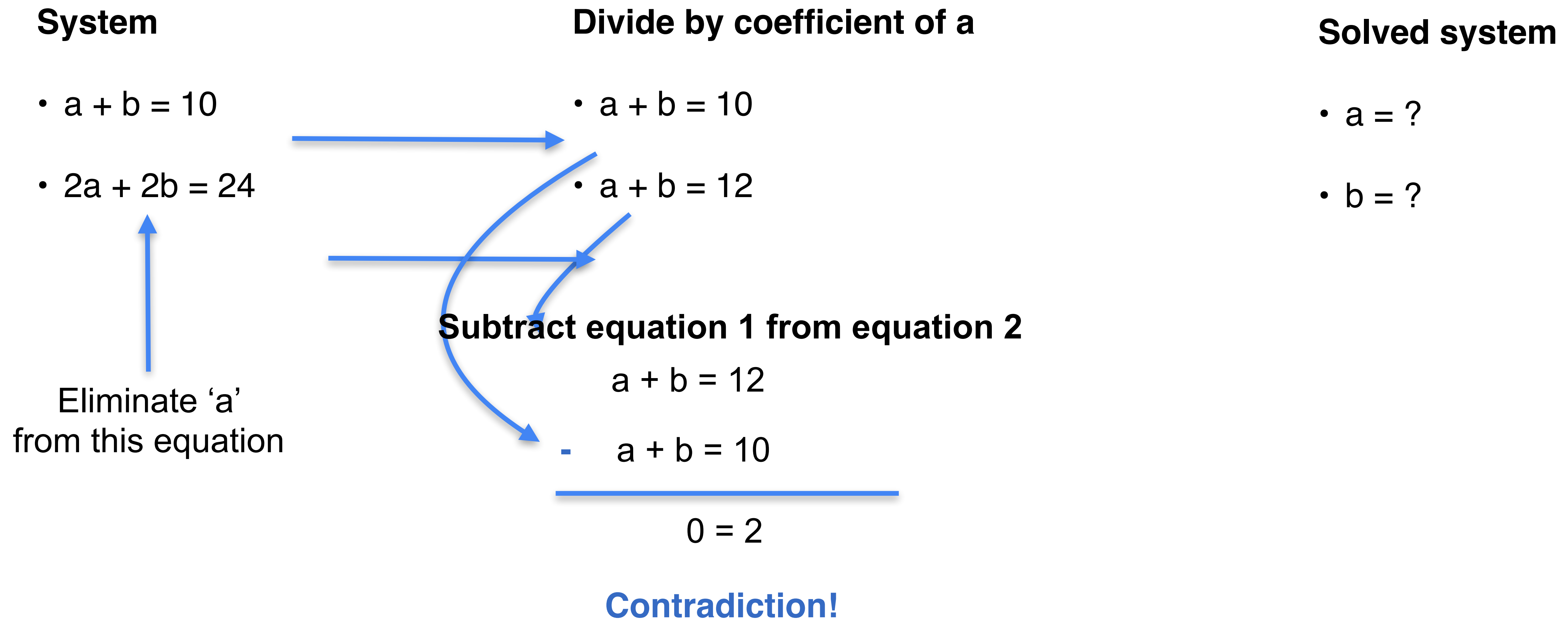
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**Solving singular system of  
linear equations**

# What if the system is singular (redundant)?



# What if the system is singular (contradictory)?



# Quiz

- Solve the following system of equations

## System

- $5a + b = 11$
- $10a + 2b = 22$



# Solution

- Solve the following system of equations

## System

- $5a + b = 11$
- $10a + 2b = 22$

**Solution:** If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



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# Solving System of Linear Equations

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**Solving system of equations  
with more variables**

# Elimination method

## System



- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

Leave 'a' by  
itself

Divide each row  
by the  
coefficient of 'a'



- $a + b + 2c = 12$

- $a - b - \frac{1}{3}c = 1$

- $a - \frac{b}{2} + 3c = 12$

Use the first  
equation to  
remove 'a' from  
the others

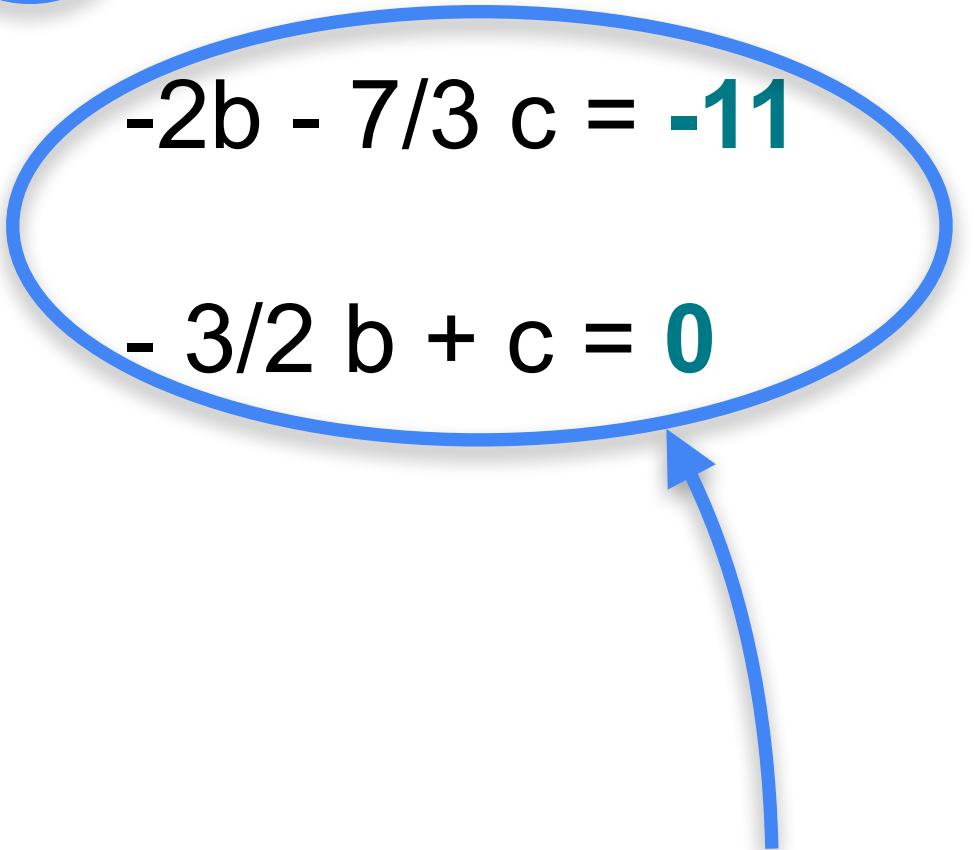


- $a + b + 2c = 12$

- $-2b - \frac{7}{3}c = -11$

- $-\frac{3}{2}b + c = 0$

Isolated 'a'



Solve this new  
system of 2  
equations

# Elimination method

## System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $-11/6 c = -11/2$

Isolated 'b'

$c = 3$



# Elimination method

## System

- $a + b + 2c = 12$  
- $b + \frac{7}{6}c = \frac{11}{2}$  
- $c = 3$

$a + 2 + 6 = 12$   
 $a = 4$

$b + \frac{7}{2} = \frac{11}{2}$   
 $b = 2$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

The solution is  
 $a = 4$   
 $b = 2$   
 $c = 3$



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# Solving System of Linear Equations

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## **Matrix row reduction**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

Row echelon form

## Diagonal matrix

1	0
0	1

Reduced row echelon form

# Systems of equations to matrices

## Original system

- $a + b = 10$
- $2a + 2b = 20$

## Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

## Original matrix

1	1
2	2

## Upper diagonal matrix

1	1
0	0

Row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 11$
- $10a + 2b = 22$

## Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

## Original matrix

5	1
10	2

## Upper diagonal matrix

1	0.2
0	0

Row echelon form

# Systems of equations to matrices

## Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

## Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

## Original matrix

0	0
0	0

## Upper diagonal matrix

0	0
0	0

Row echelon form



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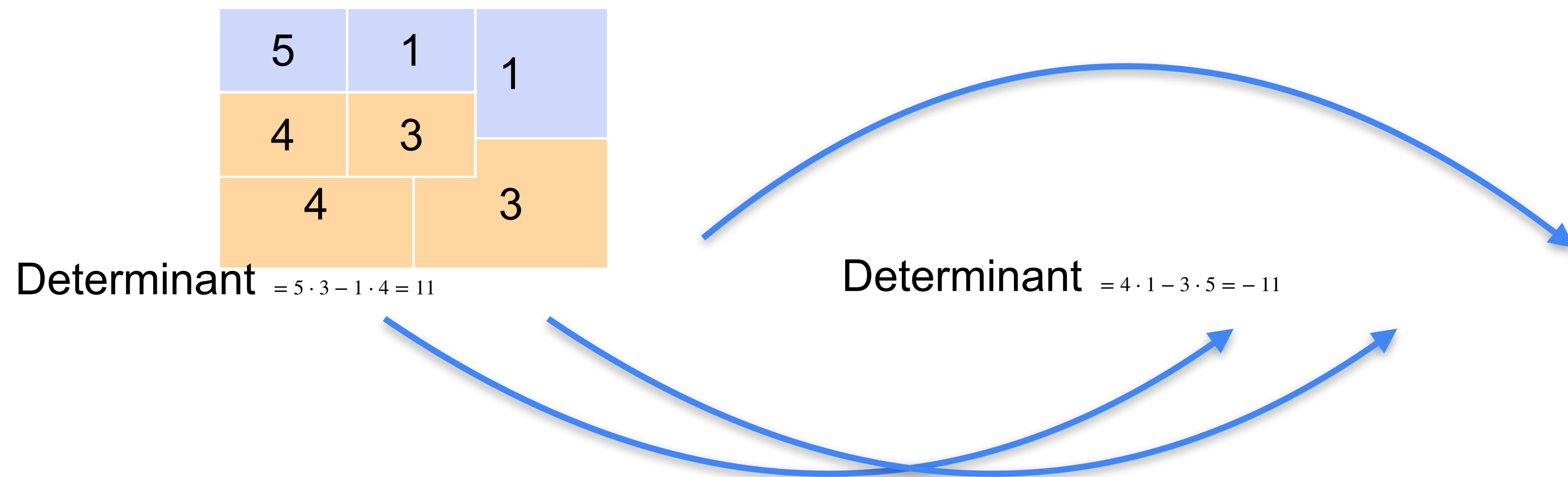
# Solving System of Linear Equations

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**Row operations that preserve  
singularity**



# Switching rows



# Multiplying a row by a (non-zero) scalar

5	1	1
4	3	
4		-3

Determinant  $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

$\times 10 =$

50	10
----	----

Determinant  $= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$

$= 10 \cdot 11$

# Adding a row to another row



A 3x3 matrix diagram. The top row consists of three blue squares containing the numbers 5, 1, and 1. The middle row consists of two orange squares containing 4 and 3, followed by a blue square containing 1. The bottom row consists of two orange squares containing 4 and 3.

Determinant  $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

+

---



A 2x2 matrix diagram with two teal squares containing the numbers 9 and 4 in the top row, and 4 and 3 in the bottom row.

Determinant  $= 9 \cdot 3 - 4 \cdot 4$

$= 11$

# W2 Lesson 2



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# Solving System of Linear Equations

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## **Rank of a matrix**

# Compressing Images - Reducing rank

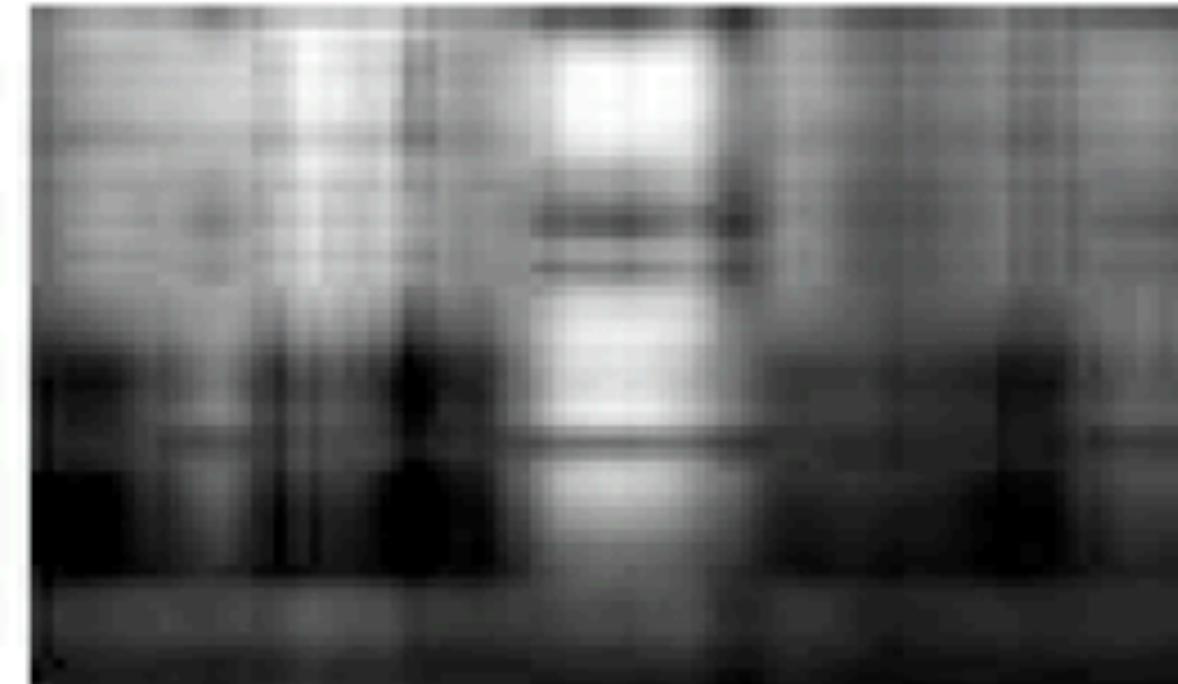
Original (Rank 200)



Rank 1



Rank 2



Rank 5



Rank 15





Rank 50



# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

## System 2


 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

Rank = 1

## System 3

 The dog  
 The dog

Two sentences

Zero pieces of information

Rank = 0



# Systems of equations

System 1

$a + b = 0$   
   
 $a + 2b = 0$   
 

	
1	1
1	2





Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$   
   
 $2a + 2b = 0$   
 

	
1	1
2	2

Rank = 1



Two equations

One piece of information

Rank = 1

System 3

$0a + 0b = 0$   
 $0a + 0b = 0$

	
0	0
0	0

Rank = 0

Two equations

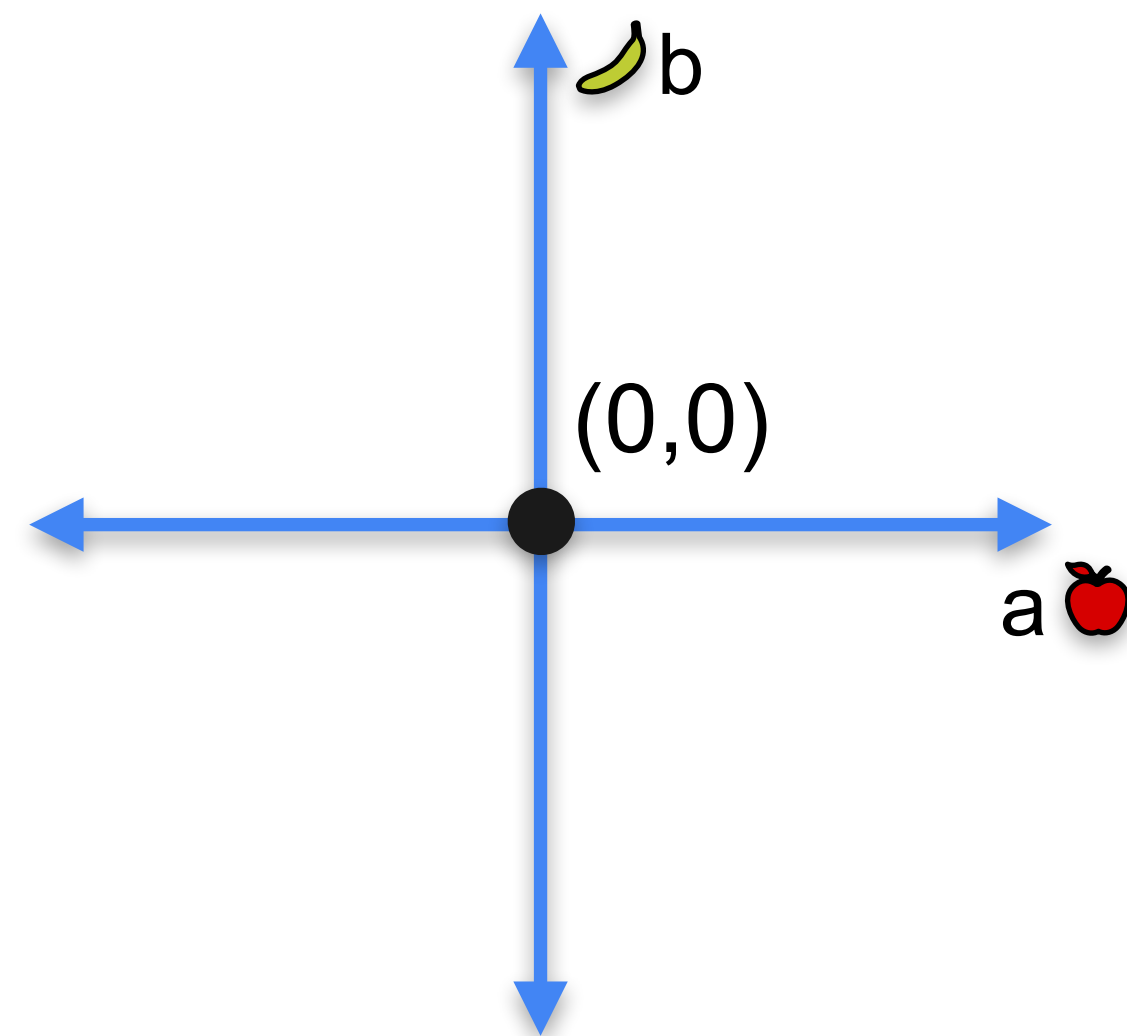
Zero pieces of information



Rank = 0

# Rank and solutions to the system

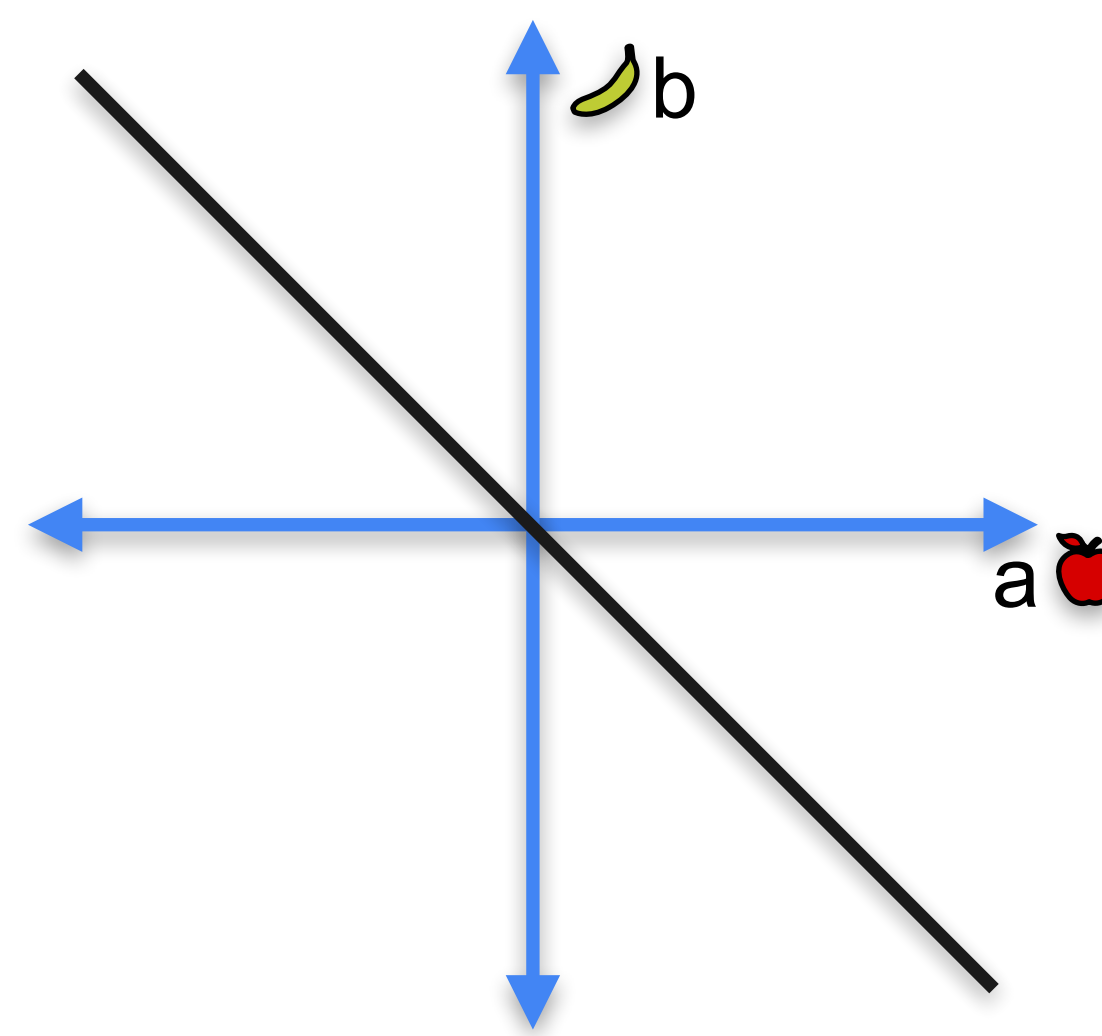
	
1	1
<sup>1</sup> Rank = 2	2

Dimension of solution space = 0



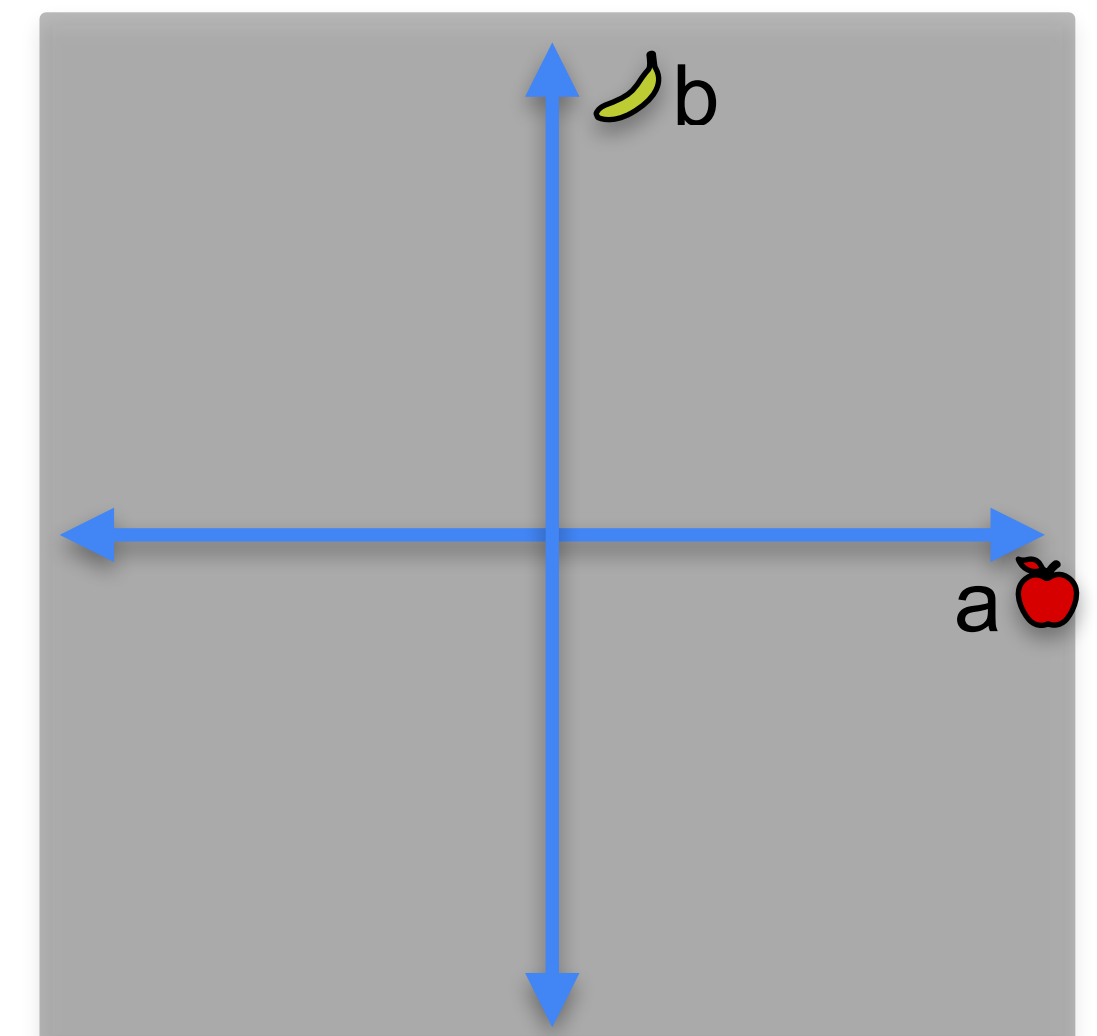
	
1	1
<sup>2</sup> Rank = 1	2

Dimension of solution space = 1



	
0	0
<sup>0</sup> Rank = 0	0

Dimension of solution space = 2



# Rank of a matrix

	
1	1
<sup>1</sup> Rank = 2	2

Dimension of solution space = 0

	
1	1
<sup>2</sup> Rank = 1	2

Dimension of solution space = 1

	
0	0
<sup>0</sup> Rank = 0	0



Dimension of solution space = 2

$$\text{Rank} = 2 - (\text{Dimension of solution space})$$



# Rank and singularity

	
1	1
<sup>1</sup> Rank = 2	2

Non-singular

	
1	1
<sup>2</sup> Rank = 1	2

Singular

	
0	0
<sup>0</sup> Rank = 0	0

Singular

# Quiz: Rank of a matrix

Determine the rank of the following two matrices

**Matrix 1**

5	1
-1	3

**Matrix 2**

2	-1
-6	3

# Solutions: Rank of a matrix

Determine the rank of the following two matrices

**Matrix 1:** Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

**Matrix 2:** Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3



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# Solving System of Linear Equations

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**Rank of a matrix:  
General case**



# Rank for matrices

## System 1

$a + b + c = 0$  ✓  
 $a + 2b + c = 0$  ✓  
 $a + b + 2c = 0$  ✓

3 Equations  
3 Pieces of information

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$a + b + c = 0$  ✓  
 $a + b + 2c = 0$  ✗  
 $a + b + 3c = 0$  ✓

3 Equations  
2 Pieces of information

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$a + b + c = 0$  ✓  
 $2a + 2b + 2c = 0$  ✗  
 $3a + 3b + 3c = 0$  ✗

3 Equations  
1 Piece of information

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$0a + 0b + 0c = 0$  ✗  
 $0a + 0b + 0c = 0$  ✗  
 $0a + 0b + 0c = 0$  ✗

3 Equations  
0 Pieces of information

## Rank 0

0	0	0
0	0	0
0	0	0

# Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



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# Solving System of Linear Equations

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**Row echelon form**

# Row echelon form of a matrix

Original matrix

5	1
4	-3
5	1
10	2
0	0
0	0

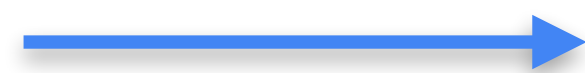
Row echelon form

1	0.2
0	1
1	1
0	0
0	0
0	0

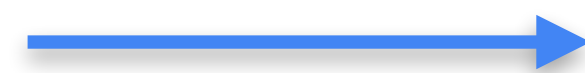
# Row echelon form

Original matrix

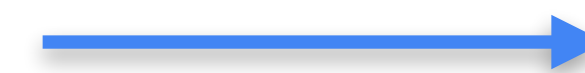
5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Row echelon form

1	0.2
0	1

Divide each row by  
the leftmost coefficient

-	1	-0.75
	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by  
the leftmost coefficient

1	0.2
1	0.2

Row echelon form

1	0.2
0	0

Divide the second row by  
the leftmost non-zero coefficient

1	0.2
?	?

-	1	0.2
	1	0.2
<hr/>		
	0	0

# Row echelon form for singular matrices

## Row echelon form

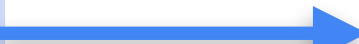
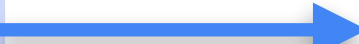
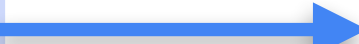
Original matrix



Divide each row by  
the leftmost coefficient



# Row echelon form, singularity, and rank

Non-singular matrix	<table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>-3</td></tr></table> 	5	1	4	-3	<table><tr><td>1</td><td>0.2</td></tr><tr><td>0</td><td>1</td></tr></table>	1	0.2	0	1	<b>Rank 2</b> 2 ones in the diagonal
5	1										
4	-3										
1	0.2										
0	1										
Singular matrix	<table><tr><td>5</td><td>1</td></tr><tr><td>10</td><td>2</td></tr></table> 	5	1	10	2	<table><tr><td>1</td><td>0.2</td></tr><tr><td>0</td><td>0</td></tr></table>	1	0.2	0	0	<b>Rank 1</b> 1 one in the diagonal
5	1										
10	2										
1	0.2										
0	0										
Singular matrix	<table><tr><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td></tr></table> 	0	0	0	0	<table><tr><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td></tr></table>	0	0	0	0	<b>Rank 0</b> 0 ones in the diagonal
0	0										
0	0										
0	0										
0	0										



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# Solving System of Linear Equations

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**Row echelon form:  
General case**

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

## Matrix

1	1	2
3	-3	-1
2	-1	6



## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

## Row echelon form matrix

1	1	2
0	-6	7
0	0	6

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

**Rank 5**

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

**Rank 3**

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

# Row echelon form

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

$\div 3$

$\div 1$

$\div (-4)$

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

## Note:

- In general, pivots different than 1 are allowed
- For this class, pivots are 1. This makes no mathematical difference.

# Another example

**Matrix**

1	1	1
1	2	1
1	1	2

**Row echelon form**

1	1	1
0	1	0
0	0	1

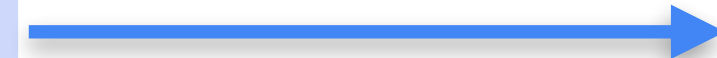
Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

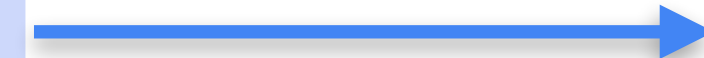
1	1	1
1	1	2
1	1	3

Subtract the first row  
from the second and  
the third ones



1	1	1
0	0	1
0	0	2

Subtract twice the  
second row from the  
third one



**Row echelon form**

1	1	1
0	0	1
0	0	0

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the first  
row from the second  
row



1	1	1
0	0	0
3	3	3

Subtract three times  
the first row from the  
third row



**Row echelon form**

1	1	1
0	0	0
0	0	0



# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Rank = 2**

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Rank = 1**

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Rank = 0**

## Row echelon forms

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**



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# Solving System of Linear Equations

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## **Reduced row echelon form**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

Row echelon form

## Diagonal matrix

1	0
0	1

Reduced row echelon form

# Reduced row echelon form

Row echelon form

1	0.2
0	1

0	1
---	---

$\times$  **0.2**

---

0	0.2
---	-----

Reduced row echelon form

1	0.2
0	1

1	0.2
0	0.2

-

---

1	0
---	---

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Rank 5

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by the  
value of the pivot

Reduced row  
echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a  
pivot to 0

# Reduced row echelon form

**Row echelon form**

1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one

1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one

1	0	0
0	1	4
0	0	1

Subtract 4 times the third row from the second one

**Reduced row echelon form**

1	0	0
0	1	0
0	0	1



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# Solving System of Linear Equations

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## **The Gaussian Elimination Algorithm**



# Augmented matrix

$$\begin{array}{rcl} 2a - b + c & = & 0 \\ 2a + 2b + 4c & = & 0 \\ 4a + b & = & 0 \end{array}$$



**Augmented matrix**

2	-1	1
2	2	4
4	1	0

**Proceed with the elimination method**

# Augmented matrix

$$2a - b + c = 1$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$



$R_1$	2	-1	1		1
$R_2$	2	2	4		-2
$R_3$	4	1	0		-1



# Pivoting

$$2a - b + c = 1$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$

 $R_1$ 

2	-1	1	1
---	----	---	---

 $R_2$ 

2	2	4	-2
---	---	---	----

 $R_3$ 

4	1	0	-1
---	---	---	----

$$R_1 \leftarrow \frac{1}{2}R_1$$

$$R_1 \leftarrow \frac{1}{2}$$

 $=$ 

1	-1/2	1/2	1/2
---	------	-----	-----

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$

$R_1$	1	-1/2	1/2	1/2
$R_2$	2	2	4	-2
$R_3$	4	1	0	-1

$$R_2 \leftarrow R_2 - 2R_1$$

	2	2	4	-2
-2	1	-1/2	1/2	1/2
<hr/>				
$R_2 \leftarrow$	0	3	3	-3

$$R_3 \leftarrow R_3 - 4R_1$$

	4	1	0	-1
-4	1	-1/2	1/2	1/2
<hr/>				
$R_3 \leftarrow$	0	3	-2	2

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$
$$3b + 3c = -3$$

$$3b - 2c = 2$$



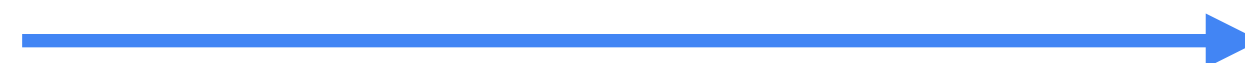
$R_1$	1	-1/2	1/2	1/2
$R_2$	0	3	3	-3
$R_3$	0	3	-2	2

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$3b + 3c = -3$$

$$3b - 2c = 2$$


 $R_1$ 

1	-1/2	1/2	1/2
---	------	-----	-----

 $R_2$ 

0	3	3	-3
---	---	---	----

 $R_3$ 

0	3	-2	2
---	---	----	---

$$R_2 \leftarrow \frac{1}{3}R_2$$

$$R_2 \leftarrow \frac{1}{3}$$

0	3	3	-3
---	---	---	----

 $=$ 

0	1	1	-1
---	---	---	----

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$3b - 2c = 2$$

 $R_1$ 

1	-1/2	1/2	1/2
---	------	-----	-----

 $R_2$ 

0	1	1	-1
---	---	---	----

 $R_3$ 

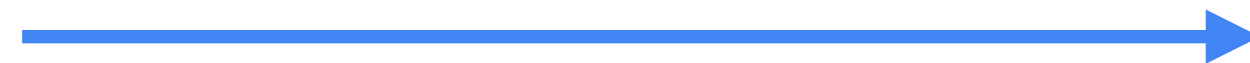
0	3	-2	2
---	---	----	---

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$3b - 2c = 2$$



$R_1$	1	-1/2	1/2		1/2
$R_2$	0	1	1		-1
$R_3$	0	3	-2		2

$$R_3 \leftarrow R_3 - 3R_2$$

	0	3	-2		2
-3	0	1	1		-1
<hr/>					
$R_3 \leftarrow$	0	0	-5		5



# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$-5c = 5$$

 $R_1$ 

1	-1/2	1/2		1/2
---	------	-----	--	-----

 $R_2$ 

0	1	1		-1
---	---	---	--	----

 $R_3$ 

0	0	-5		5
---	---	----	--	---

$$R_3 \leftarrow -\frac{1}{5}R_3$$

$$R_3 \leftarrow -\frac{1}{5}$$

0	0	-5		5
---	---	----	--	---

$$=$$

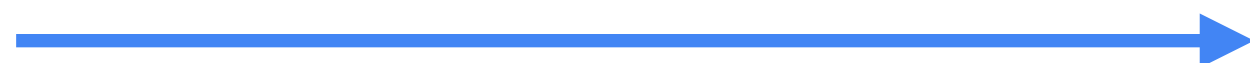
0	0	1		-1
---	---	---	--	----

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 $R_1$ 

1	-1/2	1/2	1/2
---	------	-----	-----

 $R_2$ 

0	1	1	-1
---	---	---	----

 $R_3$ 

0	0	1	-1
---	---	---	----

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$



$R_1$

$R_2$

$R_3$

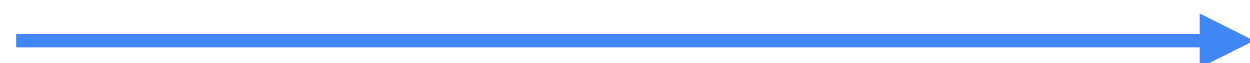
1	-1/2	1/2	1/2
0	1	1	-1
0	0	1	-1

# Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 $R_1$ 

1	-1/2	1/2	1/2
0	1	1	-1
0	0	1	-1

 $R_2$  $R_3$

# Back substitution

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

$R_1$	1	-1/2	1/2		1/2
$R_2$	0	1	1		-1
$R_3$	0	0	1		-1

$$R_2 \leftarrow R_2 - R_3$$

	0	1	1		-1
-	0	0	1		-1

$$R_2 \leftarrow$$

0	1	0		0
---	---	---	--	---

$$R_1 \leftarrow R_1 - \frac{1}{2}R_3$$

$$-\frac{1}{2}$$

$$R_1 \leftarrow$$


0	0	1		-1
1	-1/2	1/2		1/2
1	-1/2	0		1

# Back substitution

$$a - \frac{1}{2}b = 1$$

$$b = 0 \quad -1$$

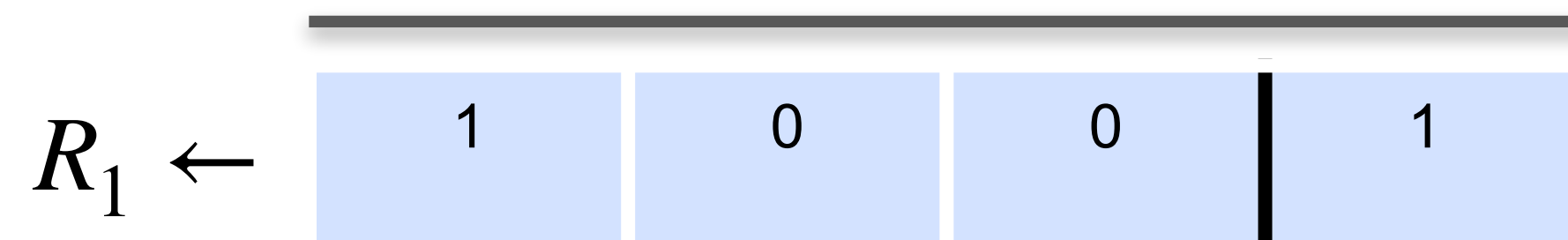
$$c = -1$$



$R_1$	1	-1/2	0		1
$R_2$	0	1	0		0
$R_3$	0	0	1		-1

$$R_1 \leftarrow R_1 + \frac{1}{2}R_2$$

$$+\frac{1}{2}$$



$R_1 \leftarrow$	1	0	0		1
------------------	---	---	---	--	---

# The result

$a = 1$   
 $b = 0$   
 $c = -1$



$R_1$

1	0	0	1
---	---	---	---

$R_2$

0	1	0	0
---	---	---	---

$R_3$

0	0	1	-1
---	---	---	----

Solution to the system

Identity matrix

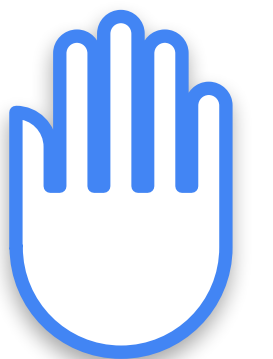
# What if the system is singular?

1	2	-1	5
2	4	5	1
3	6	4	6

After row reduction...



1	2	-1	5
0	0	-7	9
0	0	0	0



There is no need to worry!



# Checking if it has infinitely many or no solutions

1	2	-1	5
2	4	5	1
3	6	4	6

After row reduction...



Look at the column of constants

1	2	-1	5
0	0	-7	9
0	0	0	0



$$0a + 0b + 0c = 0$$

Infinitely many  
solutions

# Checking if it has infinitely many or no solutions

1	2	-1	5
2	4	5	1
3	6	4	10

After row reduction...



Look at the column of constants

1	2	-1	5
0	0	-7	9
0	0	0	0
0	0	0	4

$$0a + 0b + 0c = 4$$

The system has  
no solutions

# Checking if it has infinitely many or no solutions

- Row full of zeroes in row echelon form
- Constant in that row is zero
- **Infinitely many solutions**

1	2	-1	5
0	0	-7	9
0	0	0	0

- Row full of zeroes in row echelon form
- Constant in that row is not zero
- **No solutions**

1	2	-1	5
0	0	-7	9
0	0	0	4

# Gaussian Elimination - Summary

1. Create the augmented matrix
2. Get the matrix into reduced row echelon form
3. Complete back substitution
4. Stop if you encounter a row of 0s



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# Solving System of Linear Equations

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## Conclusion