Copyright Notice

W3 Lesson 1

DeepLearning.AI

Vectors and Linear Transformations

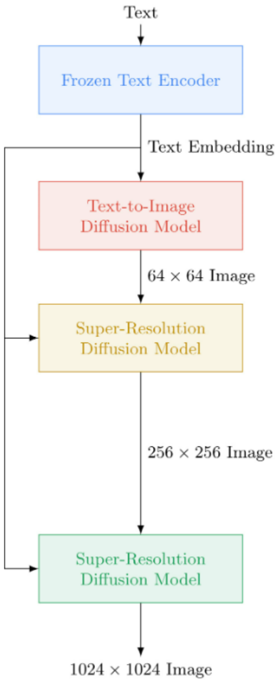
Machine Learning motivation

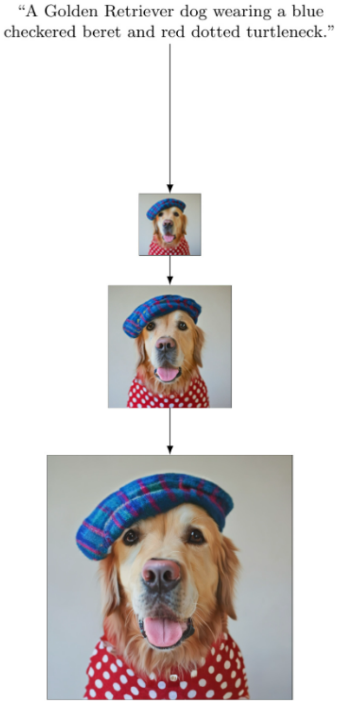
Neural Networks - AI generated images

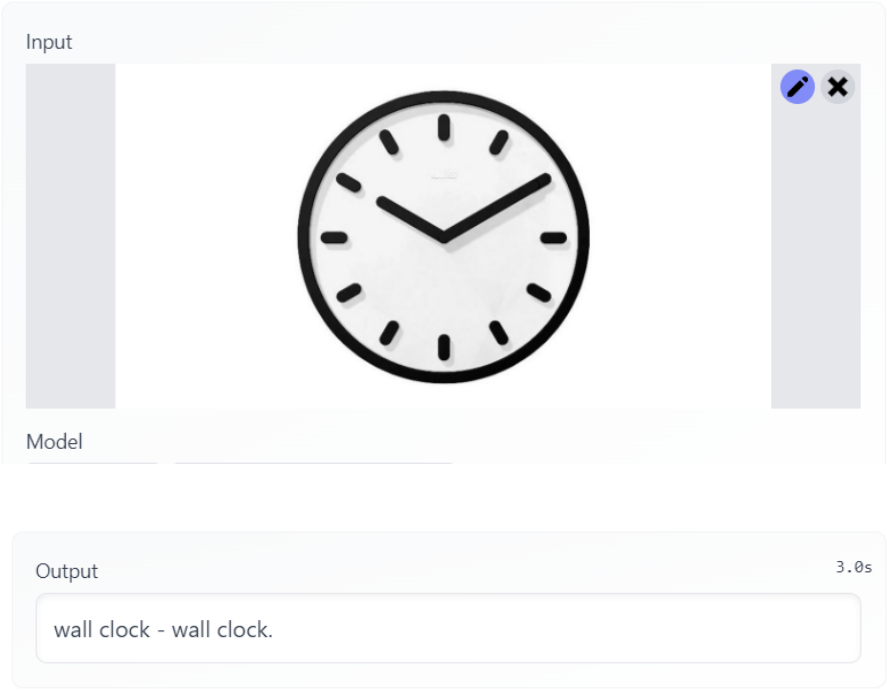
AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation







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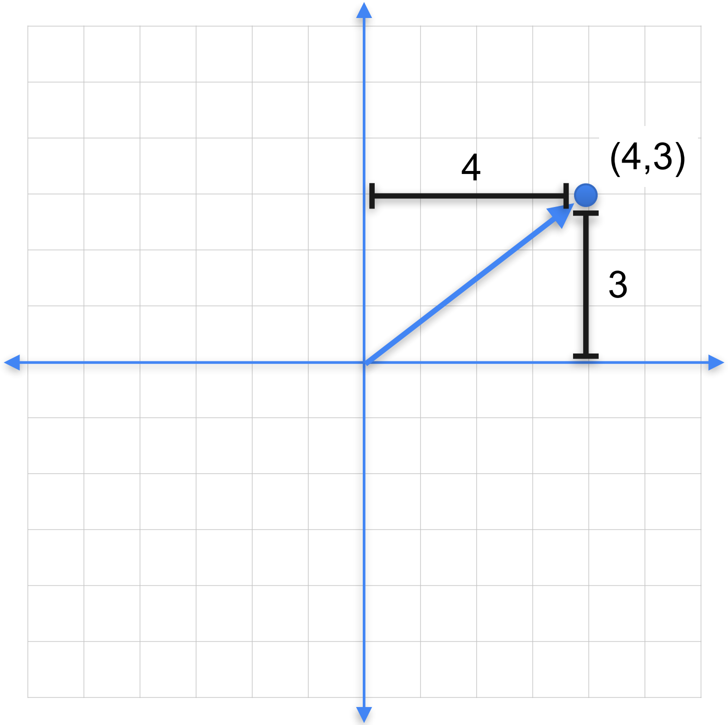
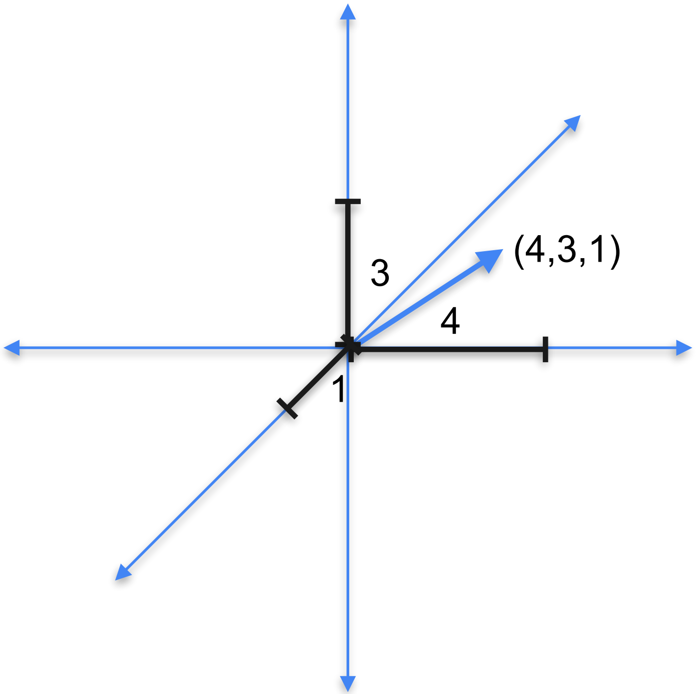
Vectors and Linear Transformations

Vectors and their properties

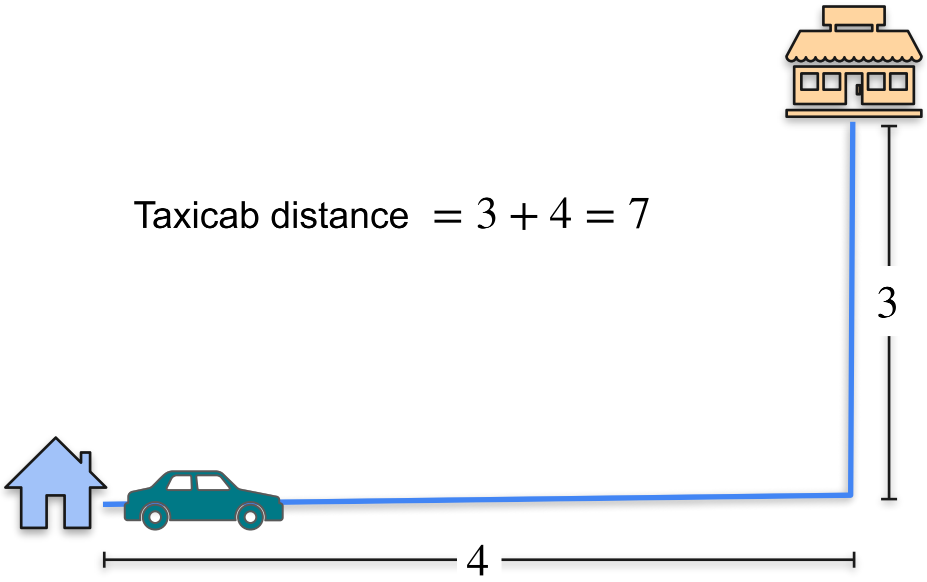
Vectors



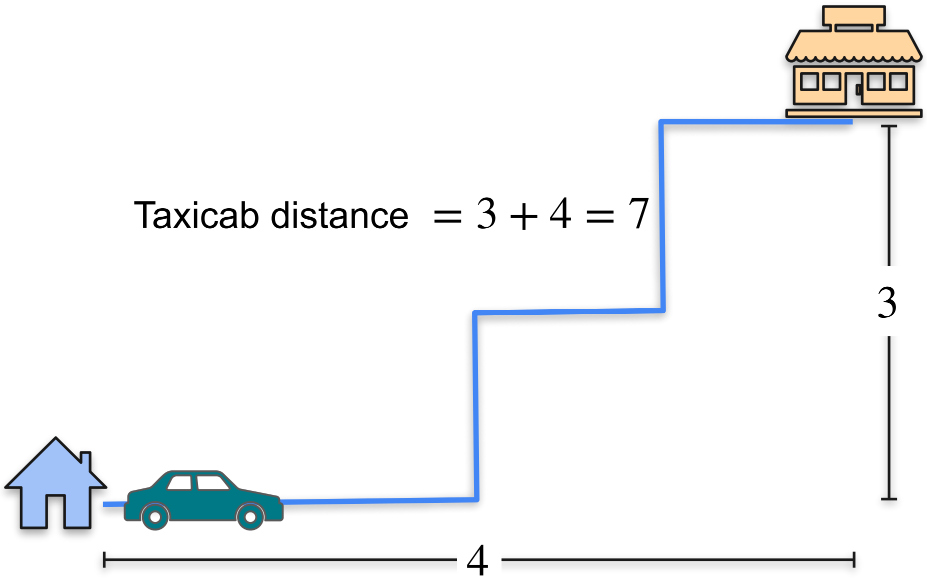


How to get from point A to point B?

Taxicab distance $= 3 + 4 = 7$

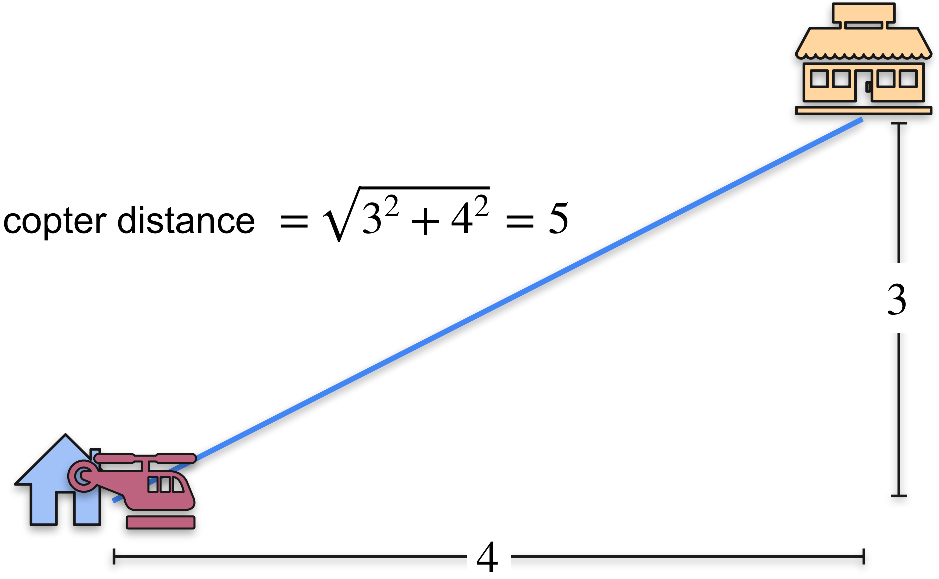


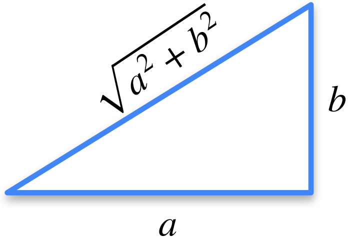
How to get from point A to point B?



How to get from point A to point B?

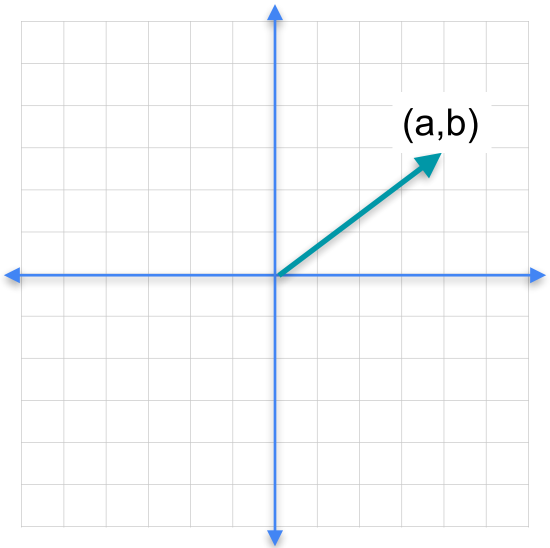
Helicopter distance $= \sqrt{3^2 + 4^2} = 5$



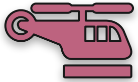


Pythagorean Theorem

Norms

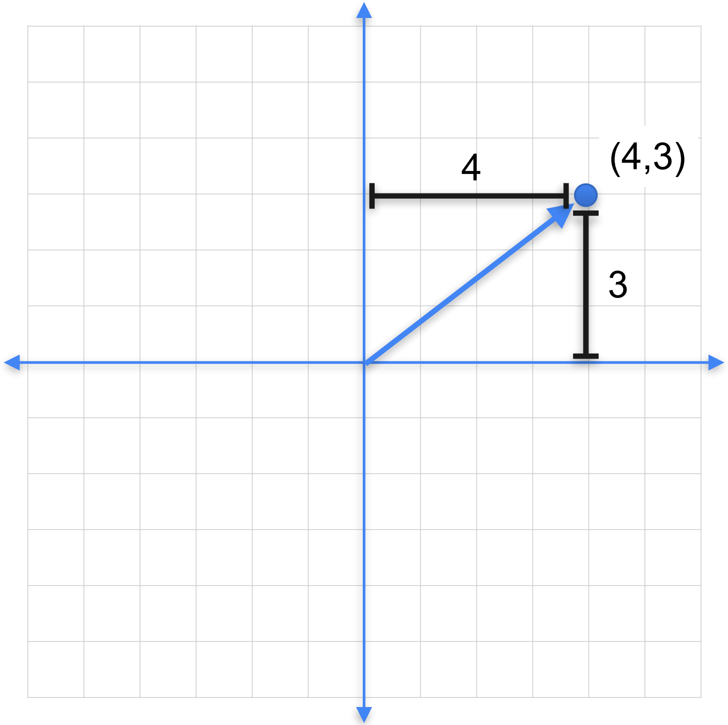


L1-norm $= \|(a,b)\|_1 = |a| + |b|$



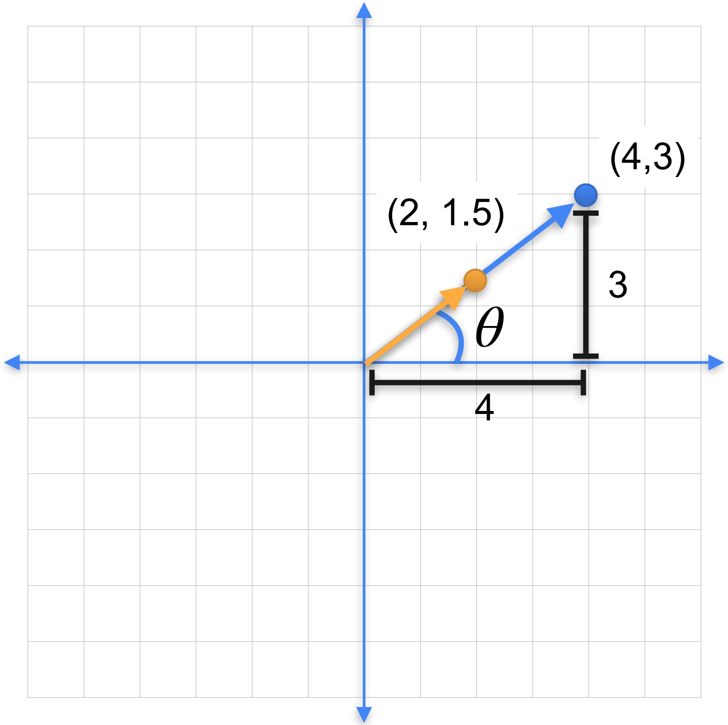
L2-norm $= \|(a,b)\|_2 = \sqrt{a^2 + b^2}$

Norm of a vector



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

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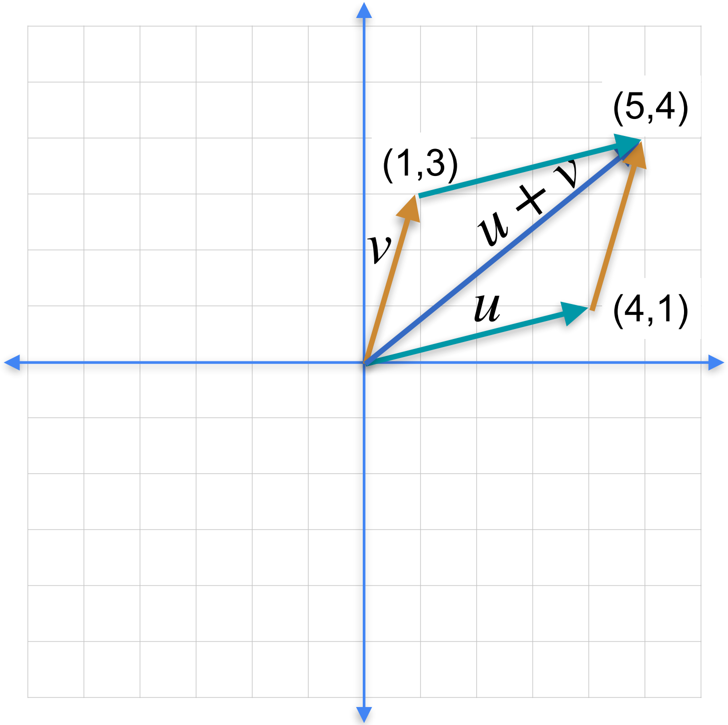
Vectors and Linear Transformations

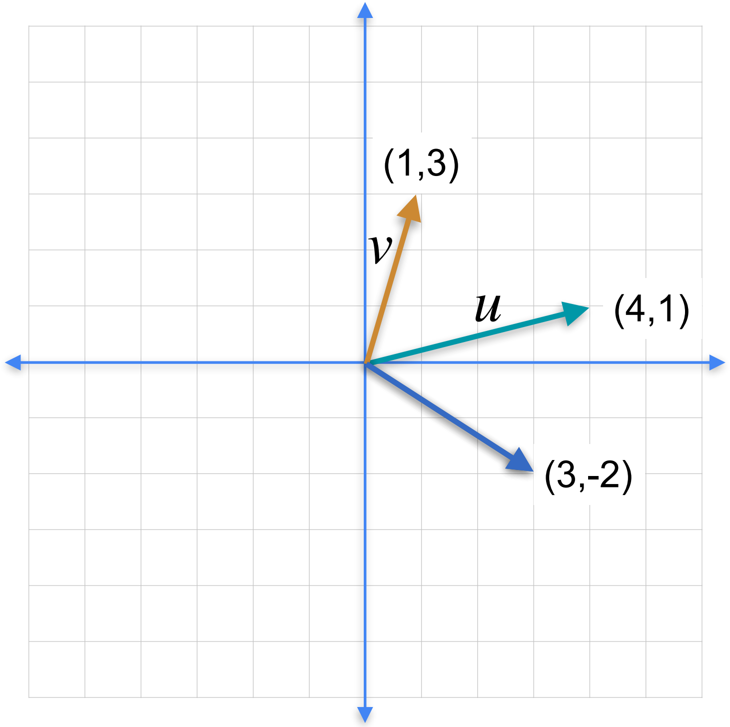
Sum and difference of vectors

Sum of vectors



$$u + v = (4 + 1, 1 + 3) = (5,4)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

General definition: sum and difference

Same number of components

$$x = (x_1 \quad x_2 \quad \cdots \quad x_n) \qquad\qquad y = (y_1 \quad y_2 \quad \cdots \quad y_n)$$

Sum

$$x + y = (x_1 + y_1 \quad x_2 + y_2 \quad \dots \quad x_n + y_n)$$

Sum component by component

Difference

$$x - y = (x_1 - y_1 \quad x_2 - y_2 \quad \dots \quad x_n - y_n)$$

Subtract component by component

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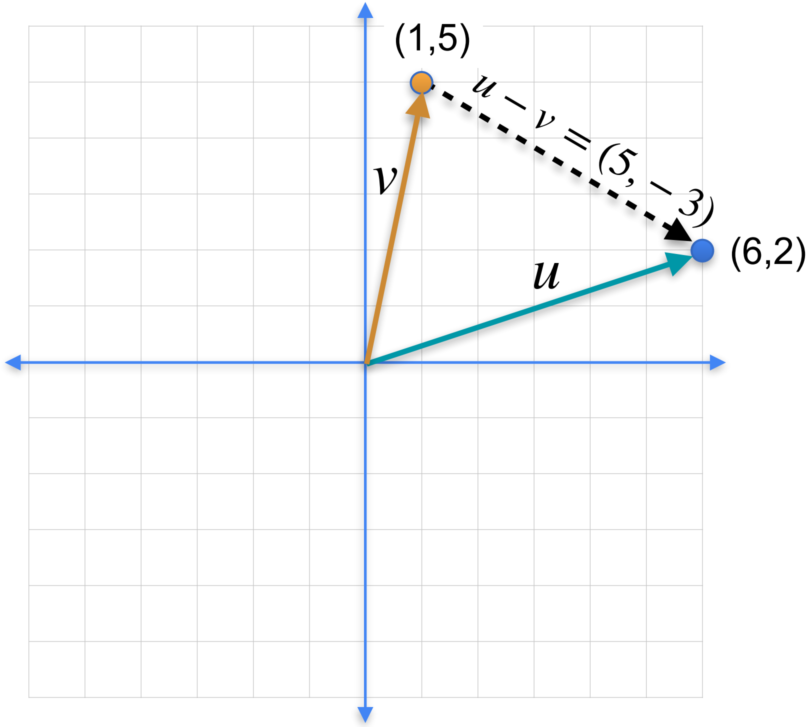
Vectors and Linear Transformations

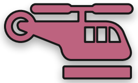
Distance between vectors

Distances



L1-distance

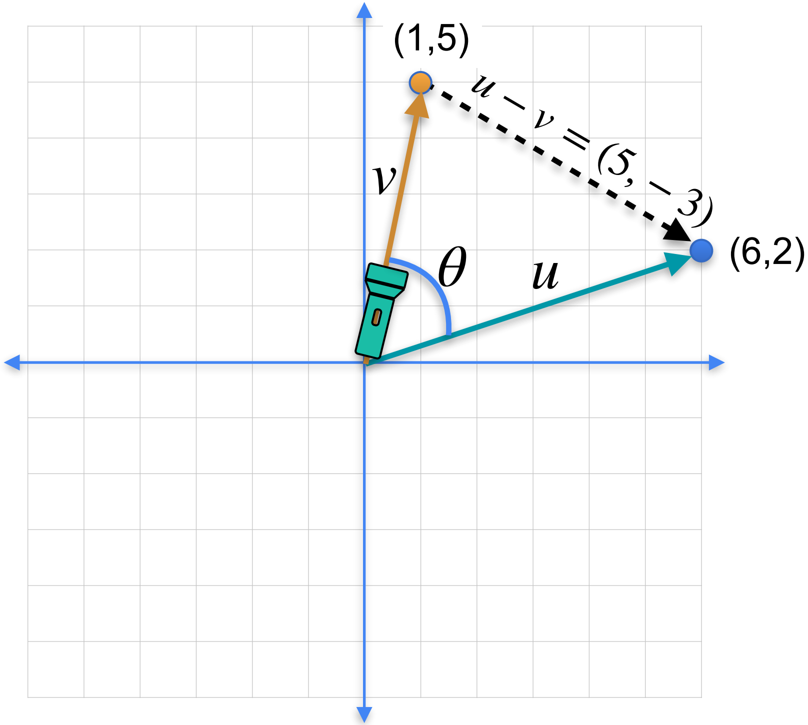
$$\|u - v\|_1 = |5| + |-3| = 8$$



L2-distance

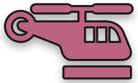
$$\|u - v\|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



L1-distance

$$\|u - v\|_1 = |5| + |-3| = 8$$



L2-distance

$$\|u - v\|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Cosine distance

$$\cos(\theta)$$

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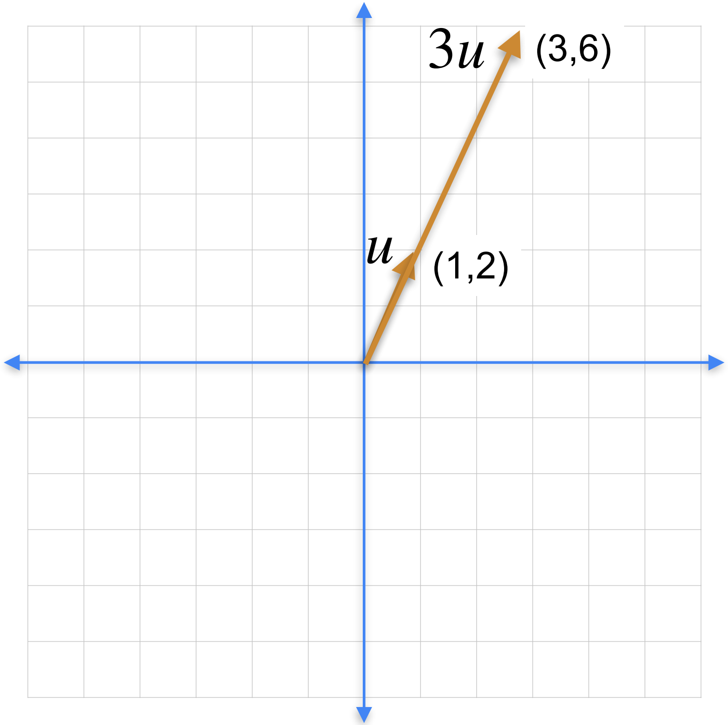
Vectors and Linear Transformations

Multiplying a vector by a scalar

Multiplying a vector by a scalar

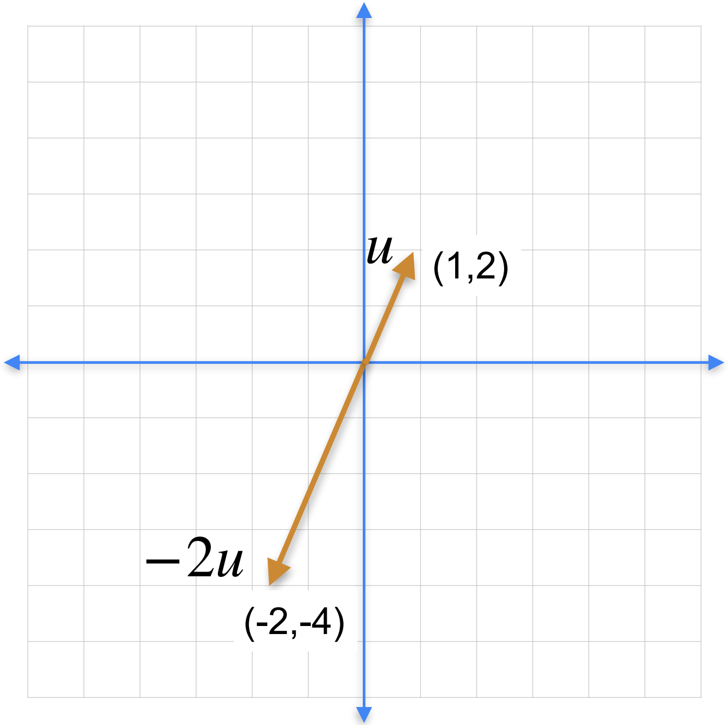


$$u = (1,2)$$

$$\lambda = 3$$

$$\lambda u = (3,6)$$

If the scalar is negative



$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

General definition

Multiplication by a Scalar

$$x = (x_1 \quad x_2 \quad \cdots \quad x_n)$$

$$\lambda x = \left(\lambda x_1 \quad \lambda x_2 \quad \ldots \quad \lambda x_n\right)$$

Video 6: The dot product

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Vectors and Linear Transformations

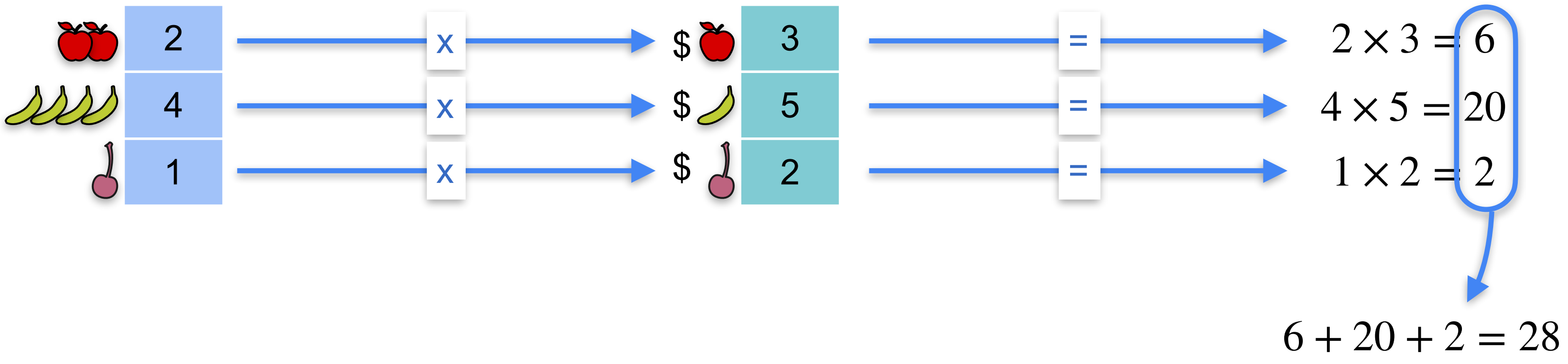
The dot product

A shortcut for linear operations

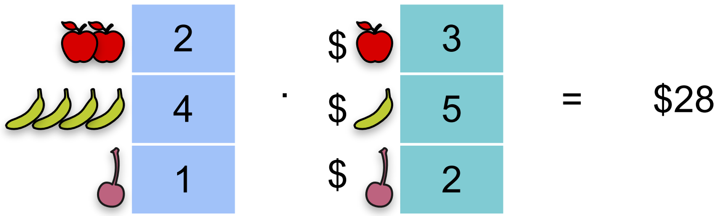
Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

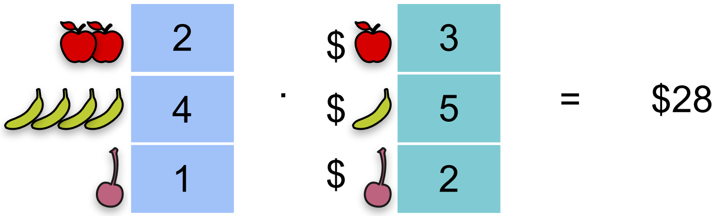
Total price
$28



The dot product

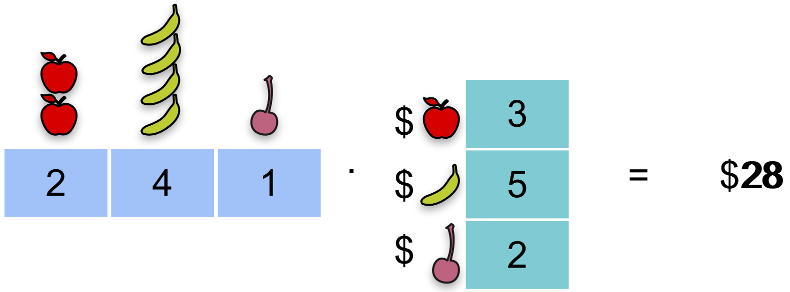


The dot product



$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

The dot product



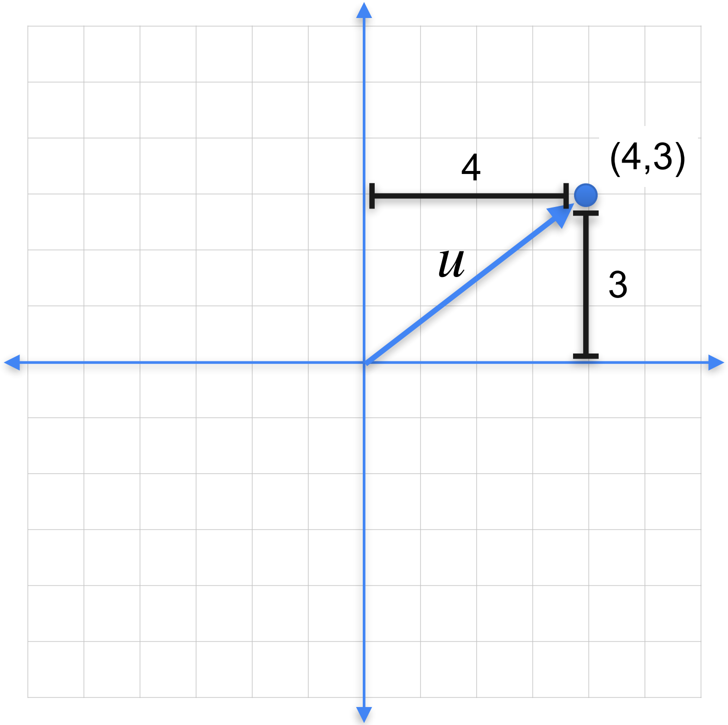
$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

The dot product

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{dot \ \ product(u, u)}$$

$$\|u\|_2 = \sqrt{\langle u, u \rangle}$$

Vector Transpose

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

Vector Transpose

Transpose: convert columns to rows

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28 \qquad \begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28 \qquad 2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

Vector Transpose

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}^T =$$

Vector Transpose

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 1 \end{bmatrix}$$

Vector Transpose

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix}^T =$$

Vector Transpose

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

Matrix Transpose

$$\begin{bmatrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{bmatrix}^T =$$

Transpose the columns!

Matrix Transpose

$$\begin{bmatrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 1 \end{bmatrix}$$

Matrix Transpose

$$\begin{bmatrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 1 \\ 5 & 7 & 3 \end{bmatrix}$$

3×2 $\qquad$ 3×2

Columns $\rightarrow$ Rows

General definition: dot product

Same number of components

$$x = (x_1 \quad x_2 \quad \cdots \quad x_n) \qquad\qquad y = (y_1 \quad y_2 \quad \cdots \quad y_n)$$

$$x \cdot y = (x_1 \times y_1) + (x_2 \times y_2) + \ldots + (x_n \times y_n)$$

$\langle x, y \rangle$ is another notation for the dot product

$$x \cdot y^T = (x_1 \quad x_2 \quad \cdots \quad x_n) \cdot \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

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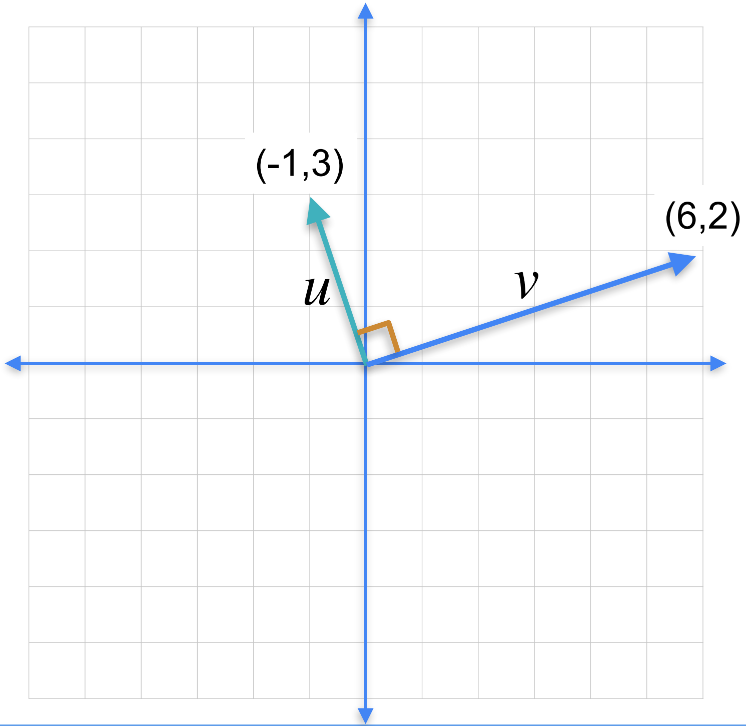
Vectors and Linear Transformations

Geometric dot product

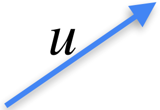
Orthogonal vectors have dot product 0



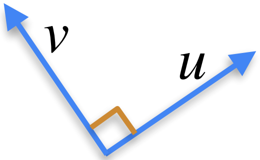
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\langle u, v \rangle = 0$$

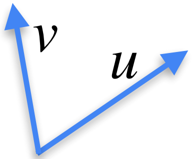
The dot product



$$\langle u, u \rangle = \|u\|^2$$

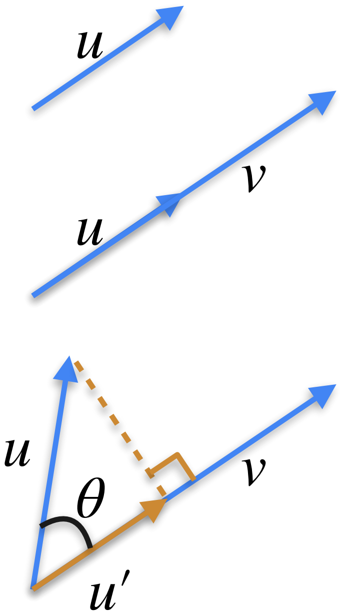


$$\langle u, v \rangle = 0$$



$$\langle u, v \rangle = ?$$

The dot product



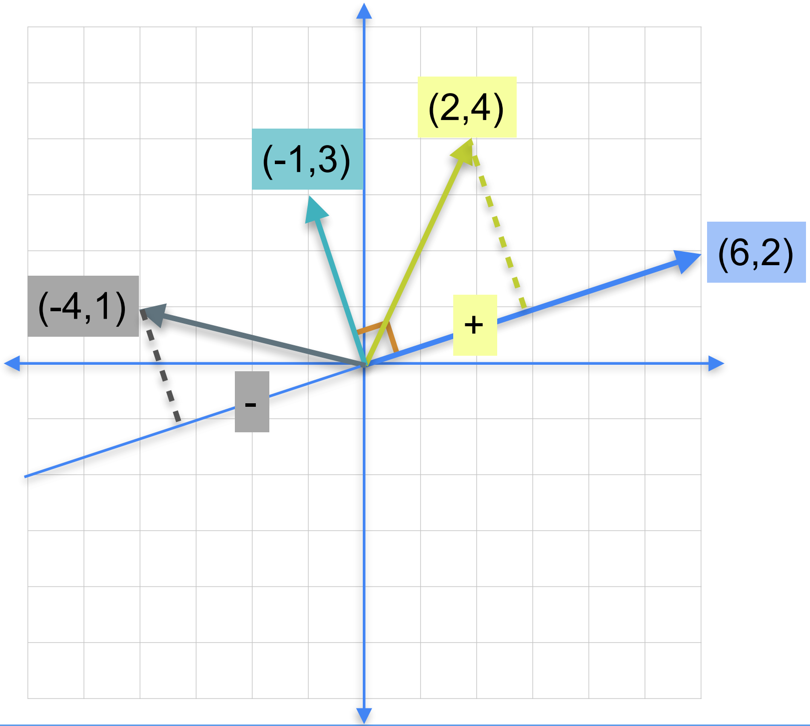
$$\langle u, u \rangle = \|u\|^2 = \|u\| \cdot \|u\|$$

$$\langle u, v \rangle = \|u\| \cdot \|v\|$$

$$\langle u, v \rangle = \|u'\| \cdot \|v\|$$

$$= \|u\| \|v\| \cos(\theta)$$

Geometric dot product

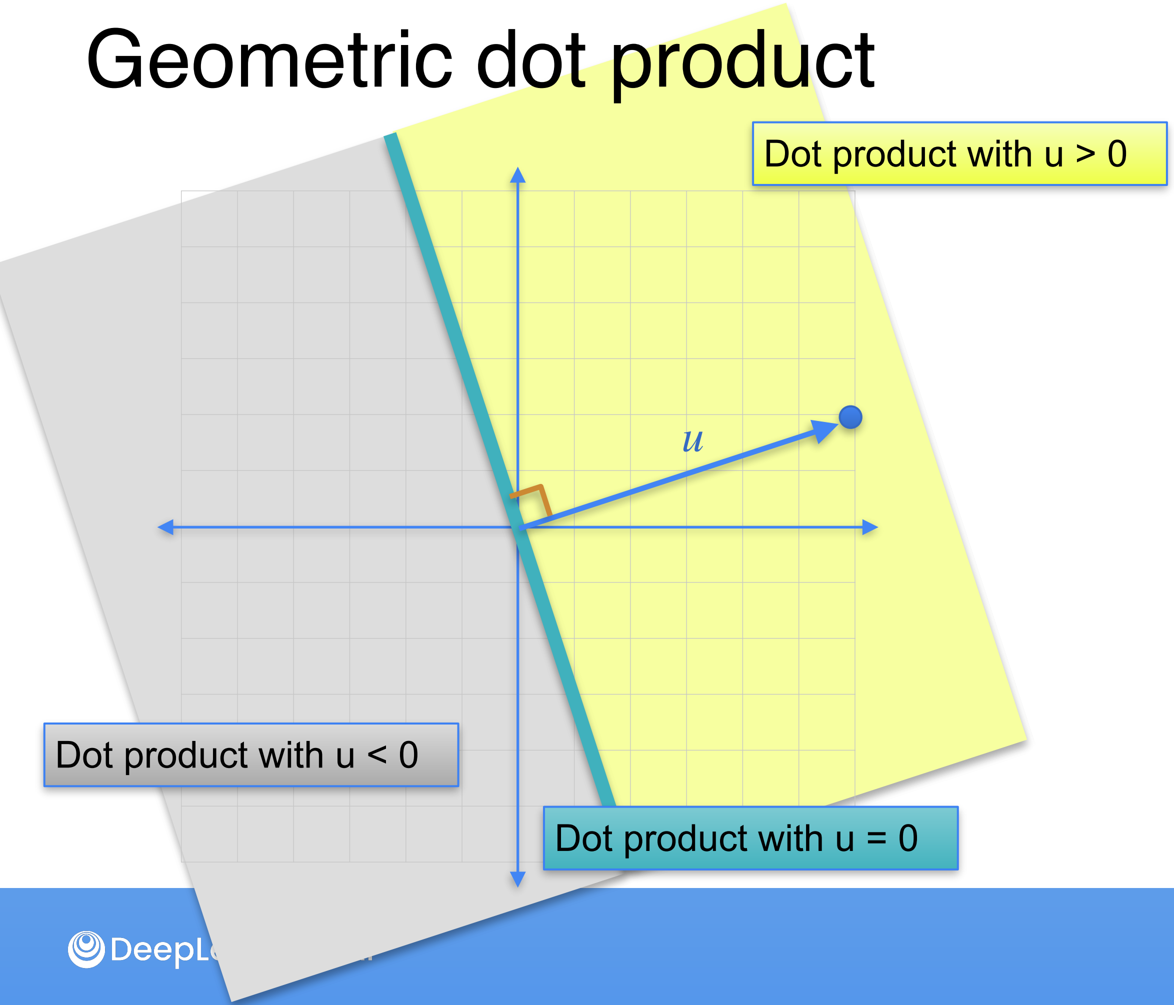


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22 \quad \text{Negative}$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$

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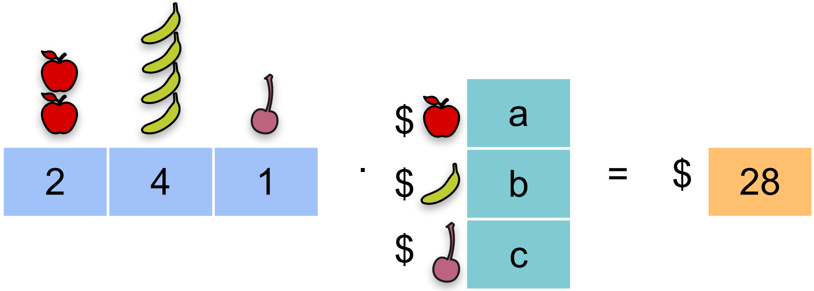
Vectors and Linear Transformations

Multiplying a matrix by a vector

Equations as dot product

$$2a + 4b + c = 28$$



Equations as dot product

$a + b + c = 10$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$\ a \\ \$\ b \\ \$\ c \end{bmatrix} = \$\ 10$$

$a + 2b + c = 15$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$\ a \\ \$\ b \\ \$\ c \end{bmatrix} = \$\ 15$$

$a + b + 2c = 12$

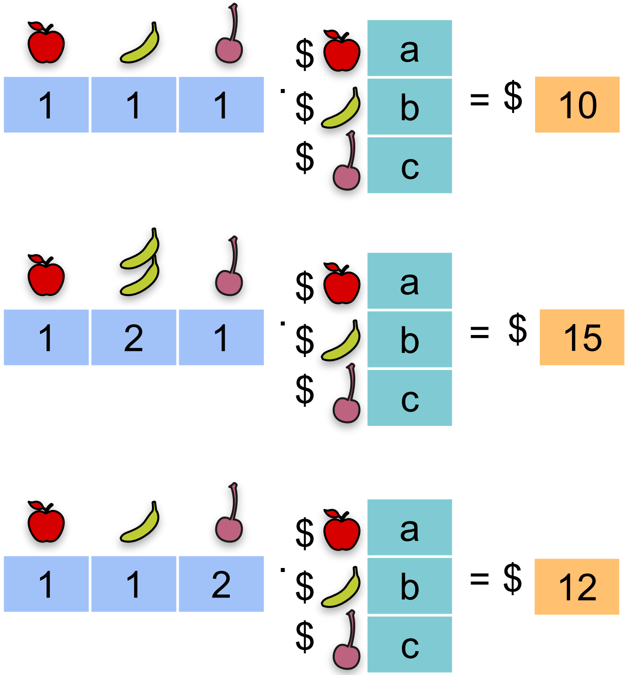
$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \$\ a \\ \$\ b \\ \$\ c \end{bmatrix} = \$\ 12$$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

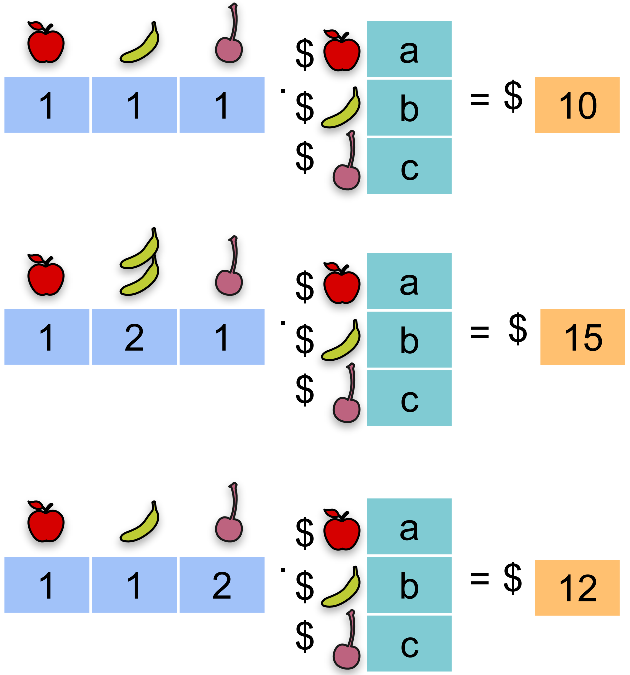


Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$



Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{array}{c} \$ \\ \$ \\ \$ \end{array} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

Equations as dot product

Matrix product

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

$3 \times \boxed{3}$ Length $\boxed{3}$

Matrix can be rectangular

columns = length of vector

Equations as dot product

Matrix product

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \\ 13 \end{bmatrix}$$

$\boxed{4} \times \boxed{3}$ Length $\boxed{3}$ Length $\boxed{4}$

Matrix can be rectangular

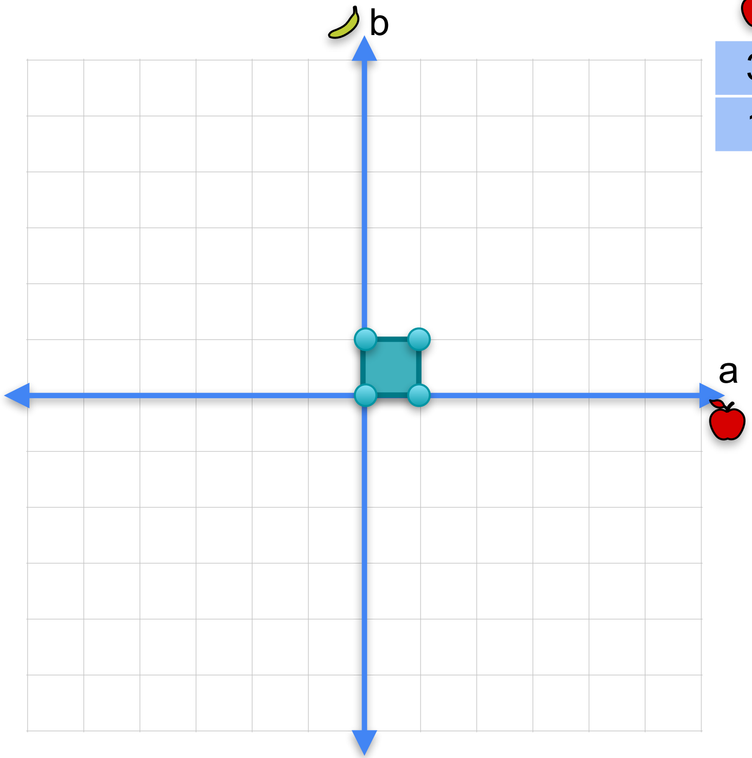
columns = length of vector

W3 Lesson 2

Vectors and Linear Transformations

Matrices as linear transformations

Matrices as linear transformations



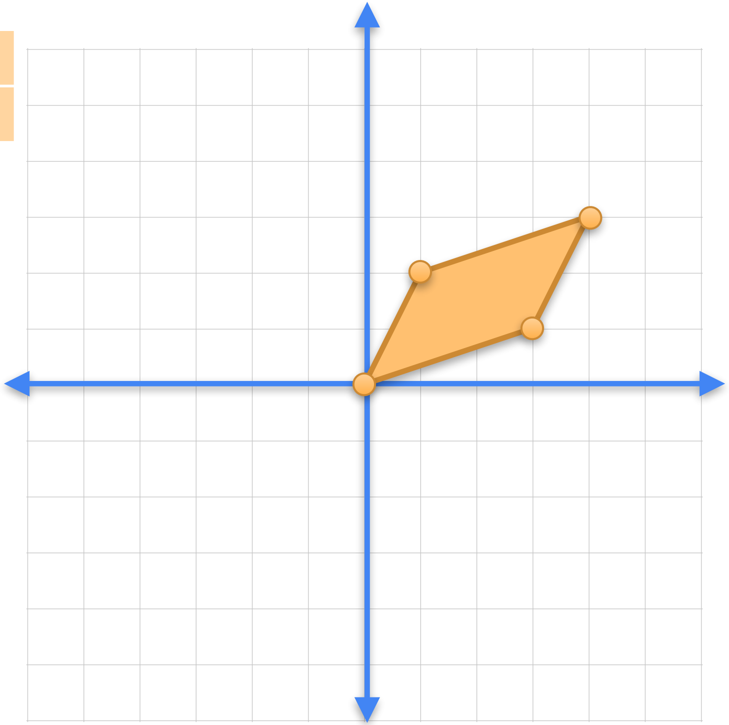
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

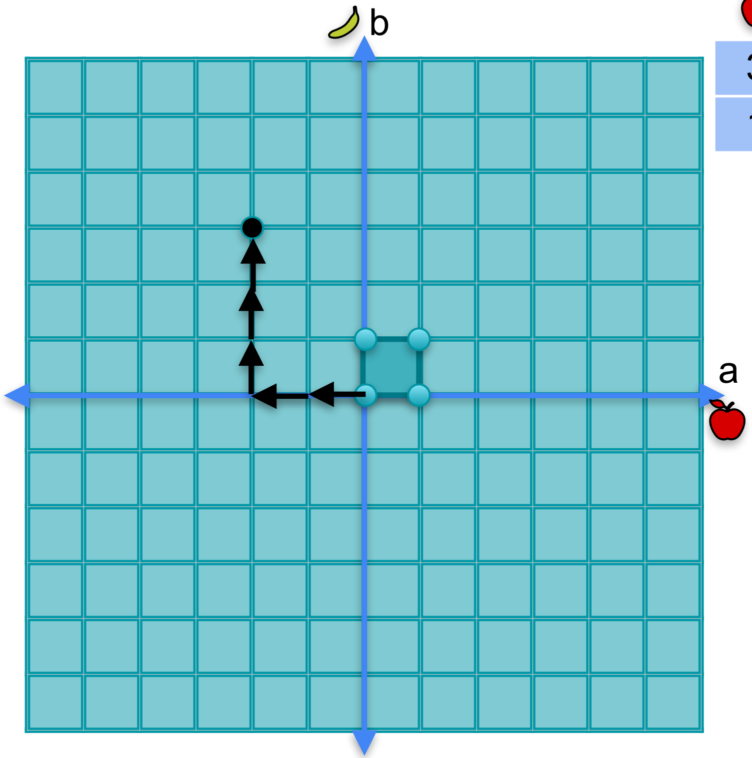
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)

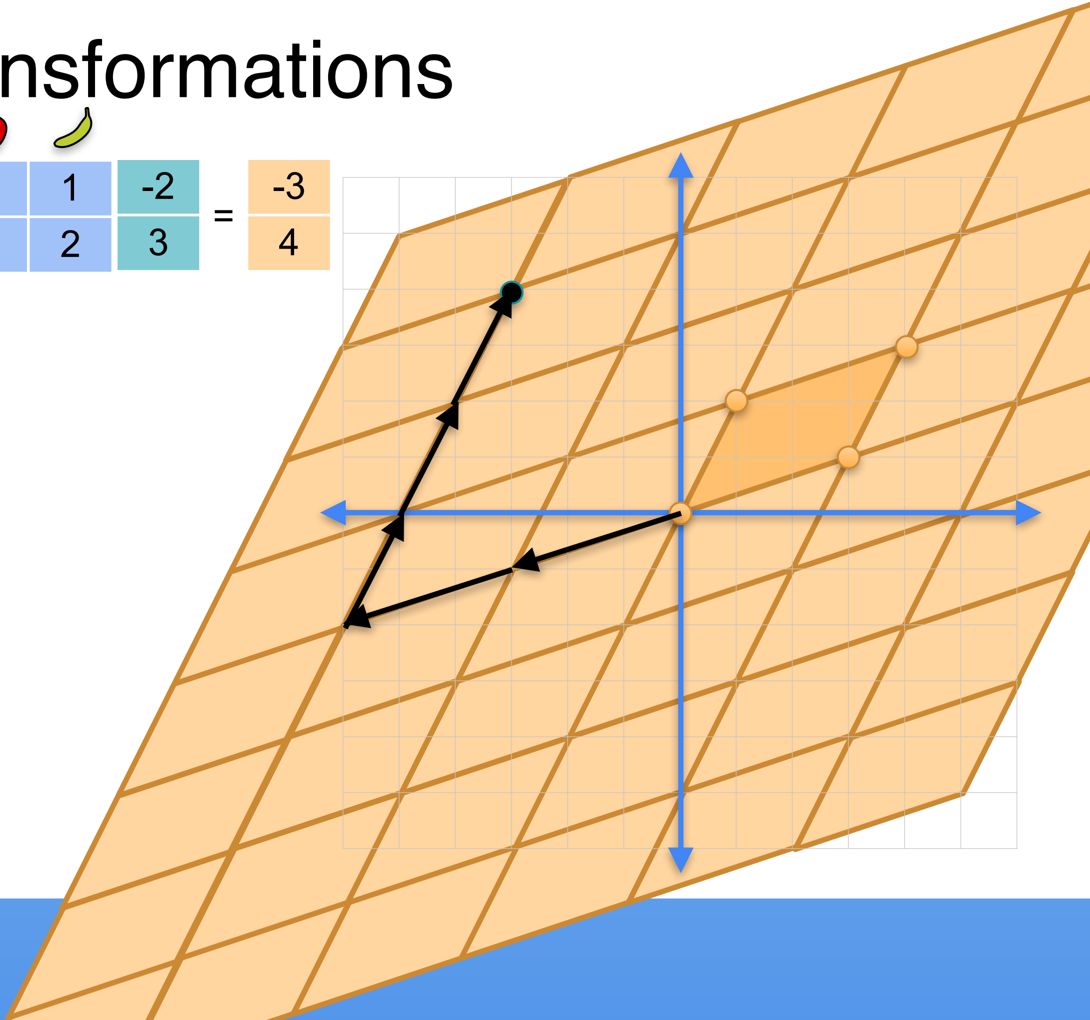


Matrices as linear transformations

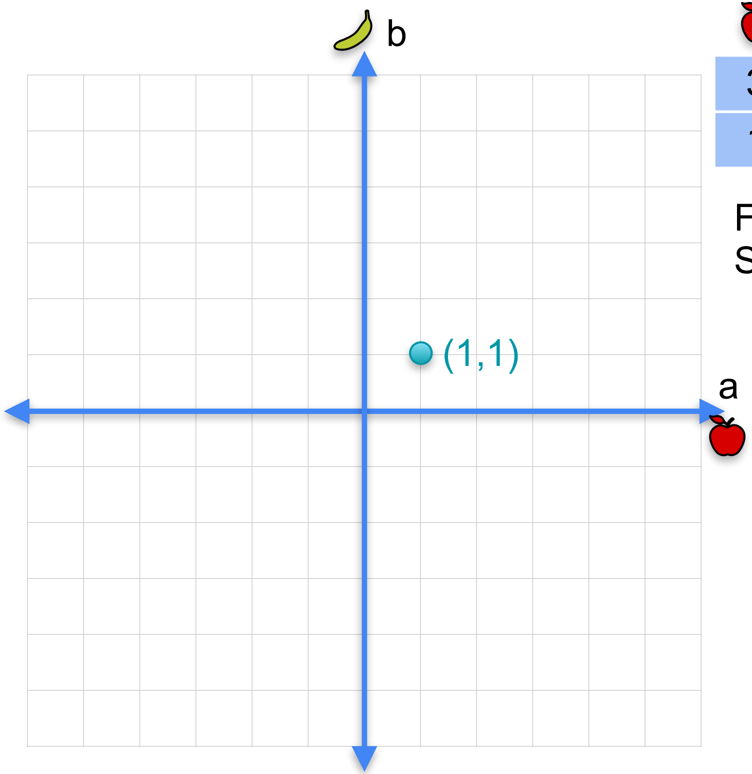


$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



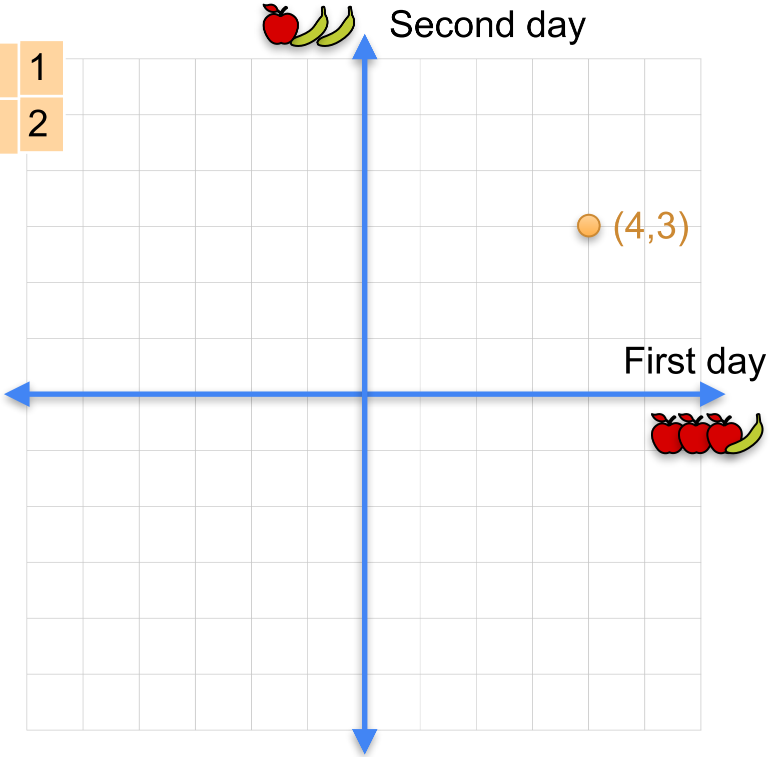
Systems of equations as linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$

(1,1) → (4,3)



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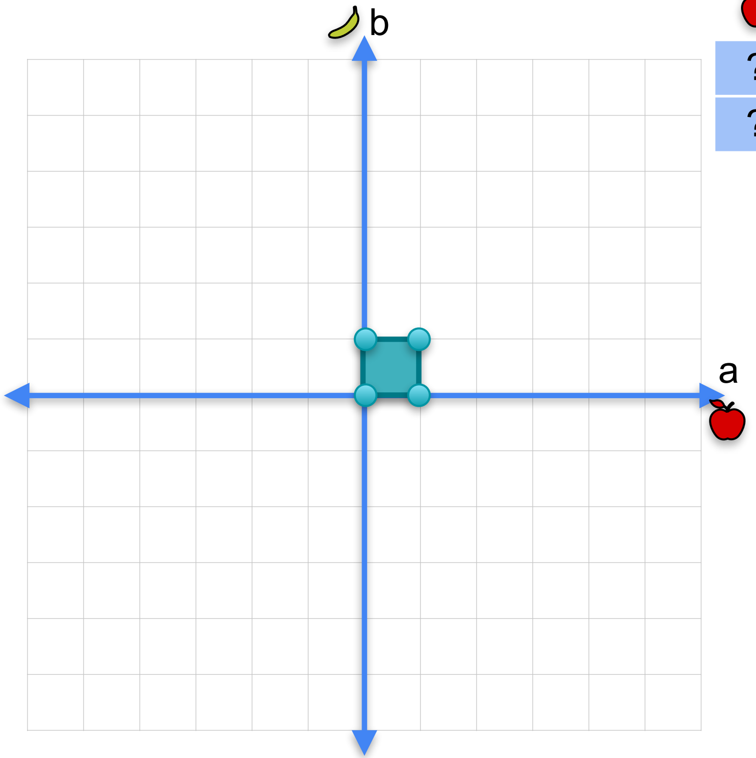
Vectors and Linear Transformations

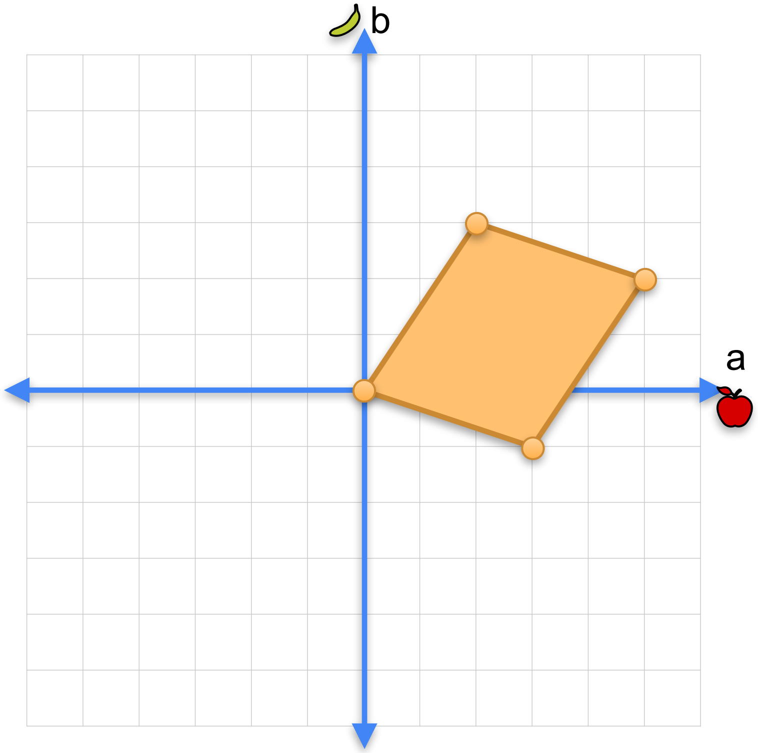
Linear transformations as matrices

Linear transformations as matrices

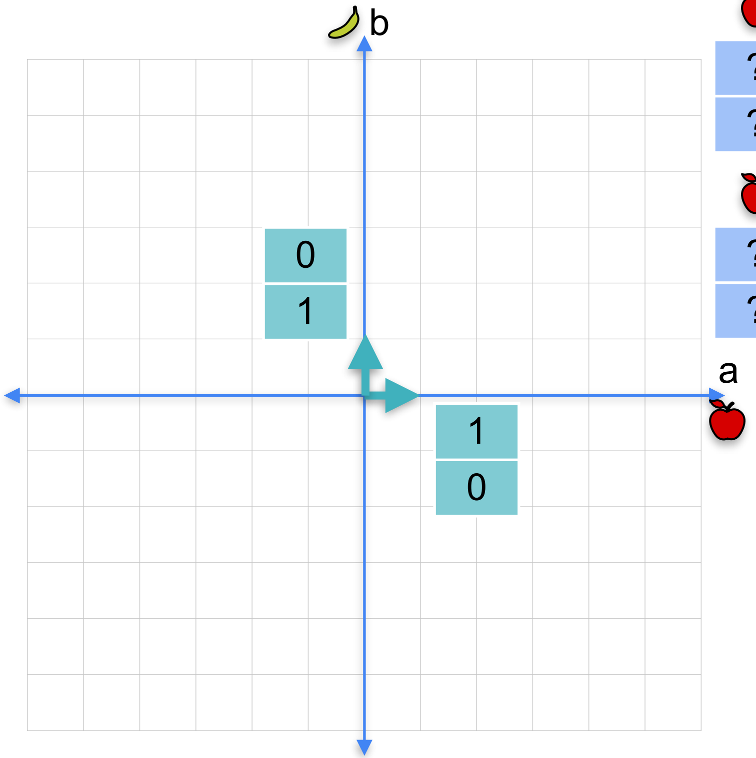


?	?
?	?

(0,0) → (0,0)
(1,0) → (3,-1)
(0,1) → (2,3)
(1,1) → (5,2)



Linear transformations as matrices



$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$

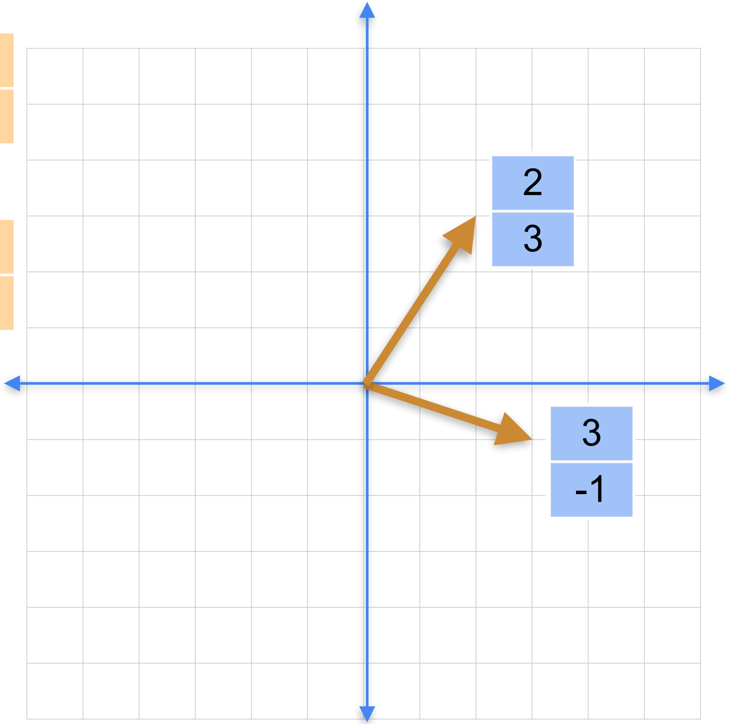
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



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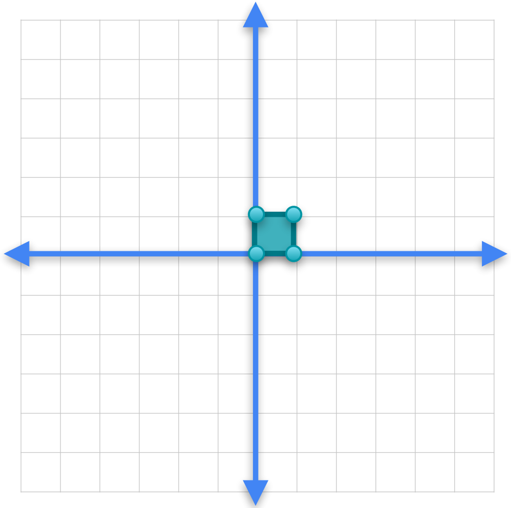
Vectors and Linear Transformations

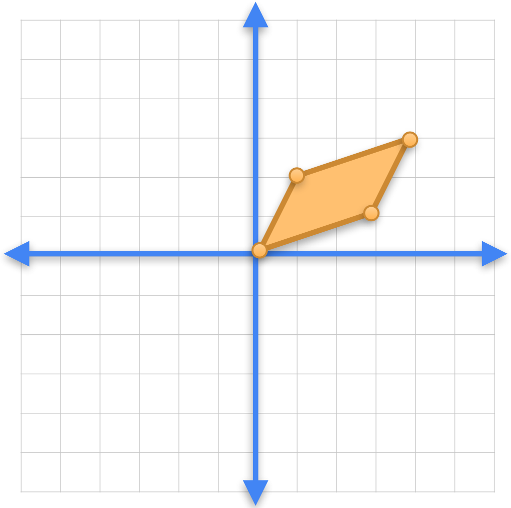
Matrix multiplication

Combining linear transformations

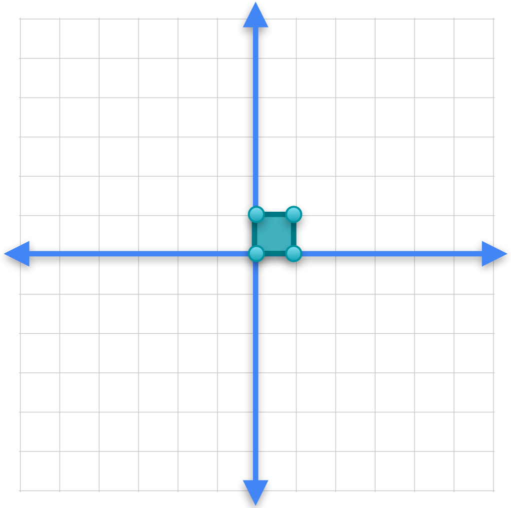


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

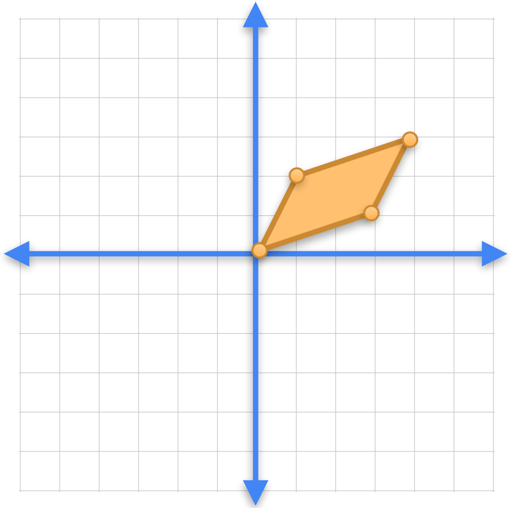


Combining linear transformations

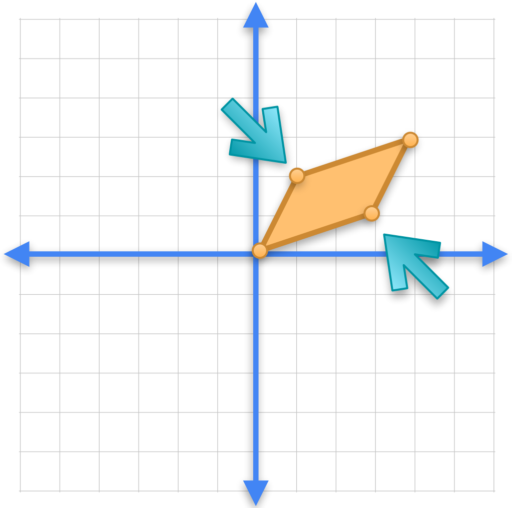


$$\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$$

$$\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$$

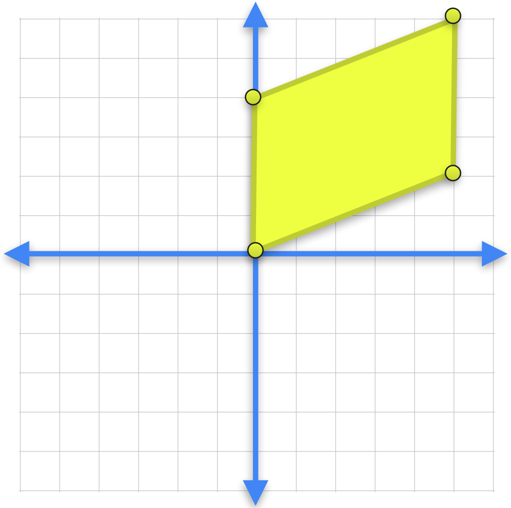


Combining linear transformations

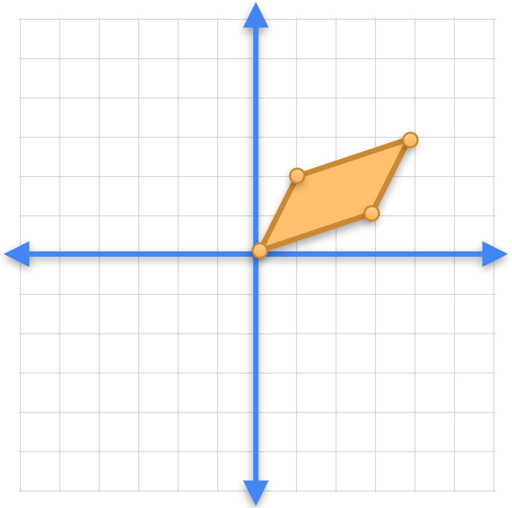


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

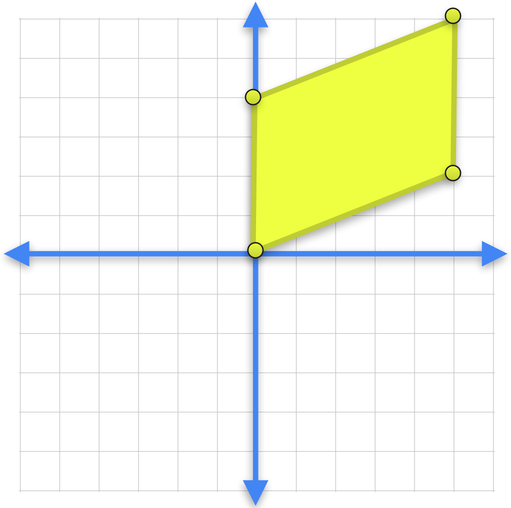


Combining linear transformations

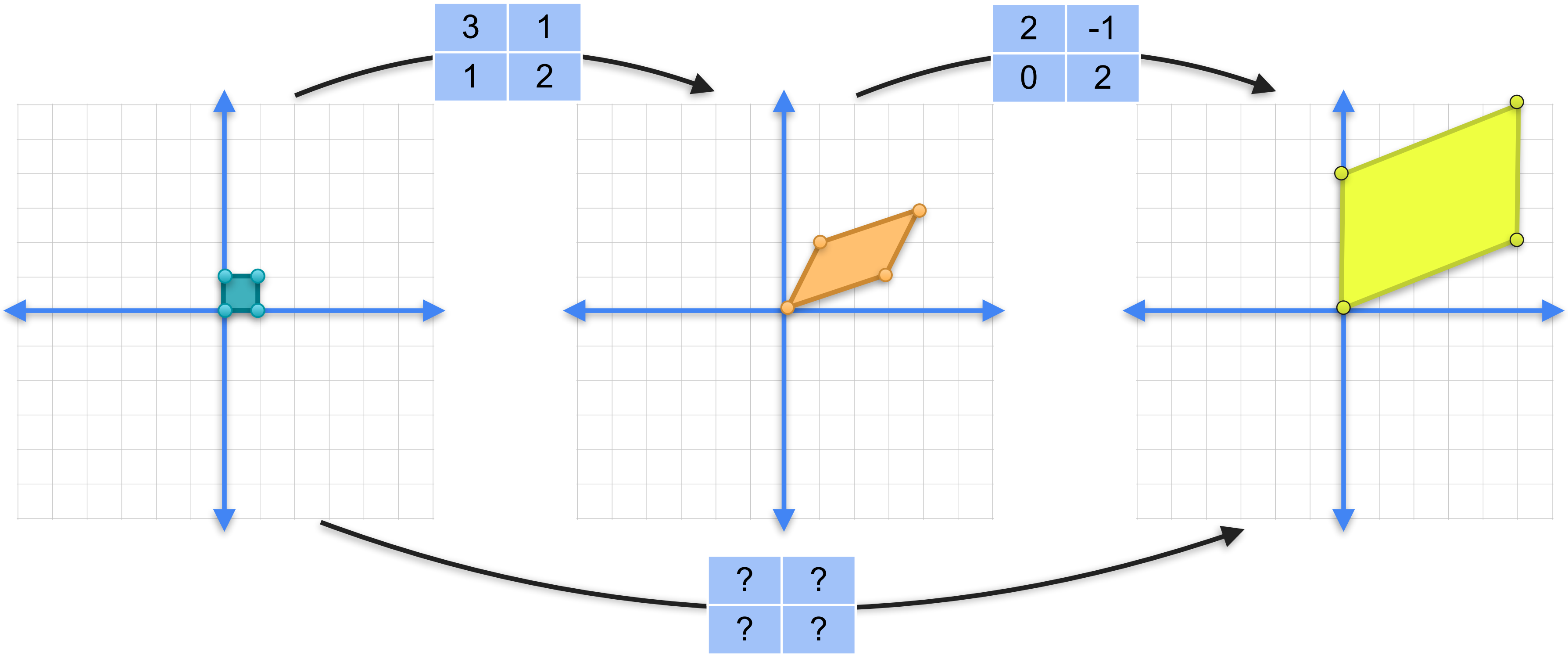


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

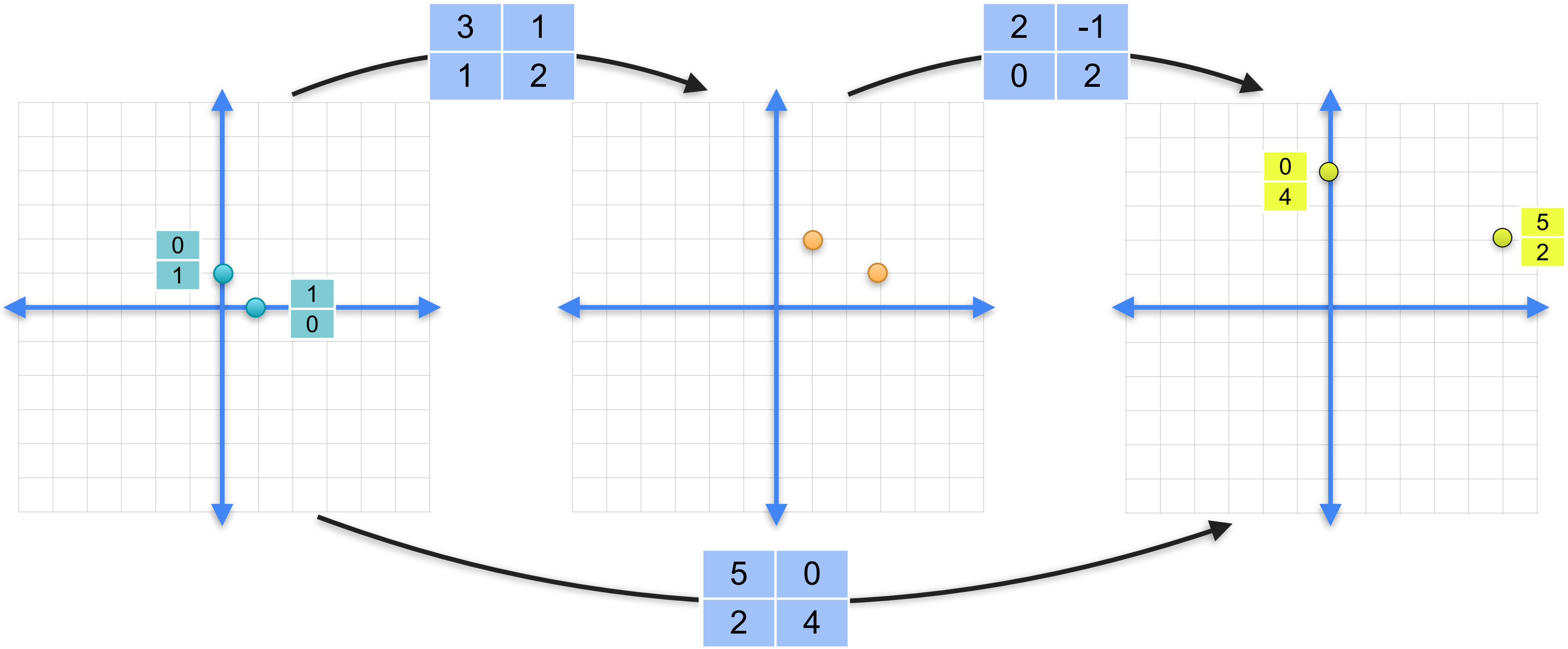
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$



Combining linear transformations



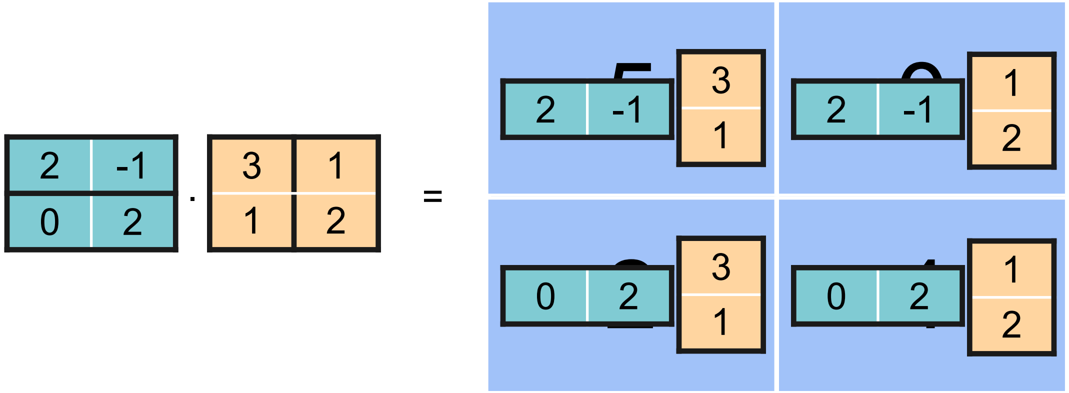
Combining linear transformations



Combining linear transformations

$$\underset{\text{Second}}{\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}} \cdot \underset{\text{First}}{\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Dimension of the matrices



Dimension of the matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

2×2 $\quad$ 2×2 $\quad$ 2×2

Dimension of the matrices

$$\underset{2 \times 3}{\begin{bmatrix} 3 & 1 & 4 \\ 2 & -1 & 2 \end{bmatrix}} \cdot \underset{3 \times 4}{\begin{bmatrix} 3 & 0 & 1 & -2 \\ 1 & 5 & 2 & 0 \\ -2 & 1 & 4 & 0 \end{bmatrix}} = \underset{2 \times 4}{\begin{bmatrix} \; & \; & \; & \; \\ ? & & & \end{bmatrix}}$$

Dimension of the matrices

$$\begin{bmatrix}3&1&4\\2&-1&2\end{bmatrix}\cdot\begin{bmatrix}3&0&1&-2\\1&5&2&0\\-2&1&4&0\end{bmatrix}=\begin{bmatrix}2&9&21&-6\\?&-3&8&-4\end{bmatrix}$$

$$\boxed{2}\times\boxed{3} \qquad \boxed{3}\times\boxed{4} \qquad \boxed{2}\times\boxed{4}$$

- Columns of first matrix must match rows of second (numbers in red match)
- Result takes number of rows from first matrix (numbers in blue match)
- Result takes number of columns from second matrix (numbers in purple match)

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Vectors and Linear Transformations

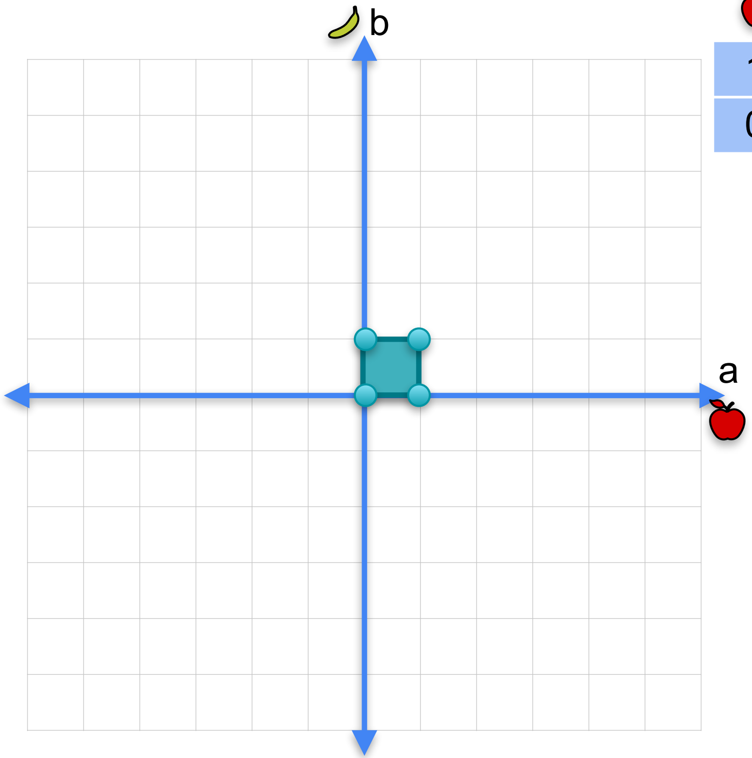
The identity matrix

The identity matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = a$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

The identity matrix



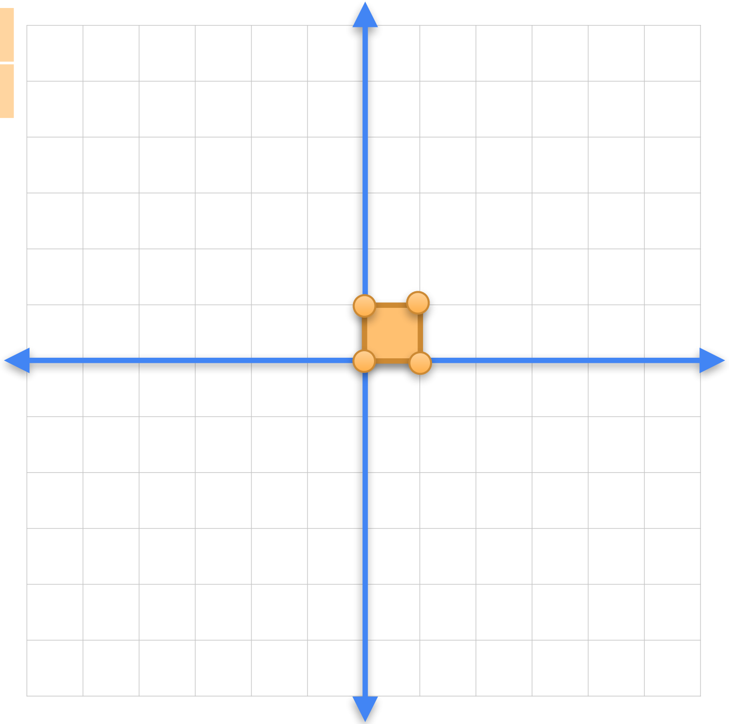
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (1,0)

(0,1) → (0,1)

(1,1) → (1,1)



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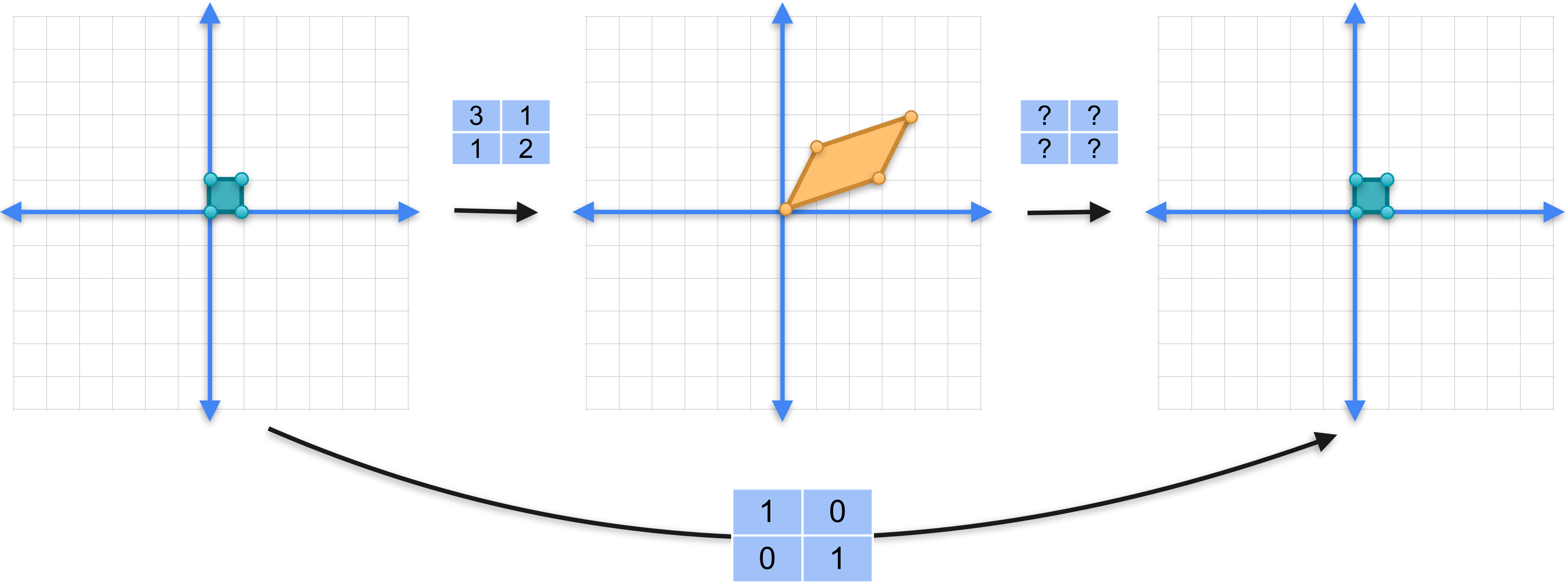
Vectors and Linear Transformations

Matrix inverse

Matrix inverses



Multiplying matrices

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$

How to find an inverse?

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \qquad 3a + 1c = 1 \qquad a = \frac{2}{5}$$

$$\begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \qquad 3b + 1d = 0 \qquad b = -\frac{1}{5}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \qquad 1a + 2c = 0 \qquad c = -\frac{1}{5}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \qquad 1b + 2d = 1 \qquad d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

- **5a + 2c = 1**
- **5b + 2d = 0**
- **a + 2c = 0**
- **b + 2d = 1**

- **a = 1/4**
- **b = -1/4**
- **c = -1/8**
- **d = 5/8**

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a + c = 1$

$2b + 2d = 1$

$2a + 2c = 0$

$b + d = 0$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.

DeepLearning.AI

Vectors and Linear Transformations

Which matrices have an inverse?

Which matrices have inverses?

$5^{-1} = 0.2$ $\qquad$ $8^{-1} = 0.125$ $\qquad$ $0^{-1} = ???$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5

Det = 8

Det = 0

Non-zero determinants

Zero determinant

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Vectors and Linear Transformations

Neural networks and matrices

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point
Win: 2 points
“Win, win the lottery!” : 7points

Rule:
If the number of points of the sentence is bigger than ____, then the email is spam.

Goal: Find the best points and threshold
Lottery: ___ point
Win: ___ point
Threshold: ___ points

Quiz: Natural language processing

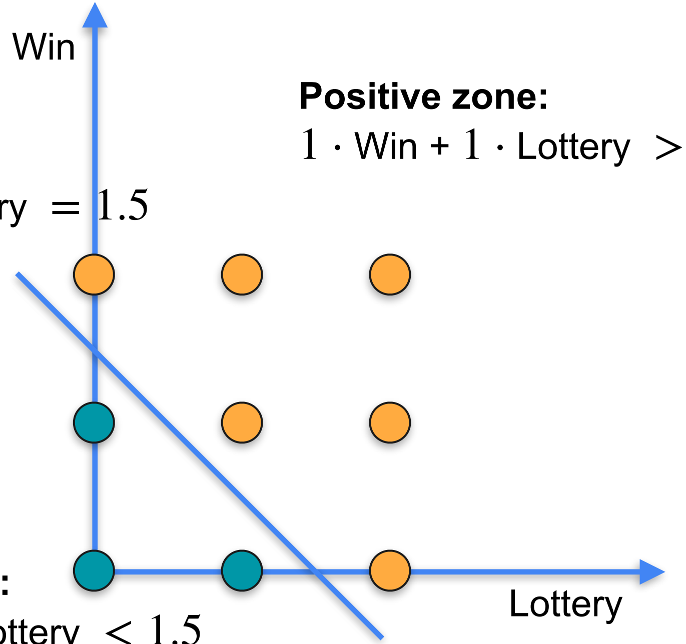
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

Solution:
Lottery: 1 point
Win: 1 point
Threshold: 1.5 points

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Line:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$

Positive zone:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$

Negative zone:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} < 1.5$

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

= 3

Check: > 1.5?

✓

Spam

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

= 1

Check: > 1.5?

✗

Not spam

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Perceptrons

Spam	Word1	Word2	…	WordN
Yes				
Yes				
No				
Yes				
No				
No				
Yes				
Yes				
Yes				

Model
…

=

Prod

Check:

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Threshold

Check

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

Bias

Check: > 0?

Model
1
1
-1.5

Bias

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

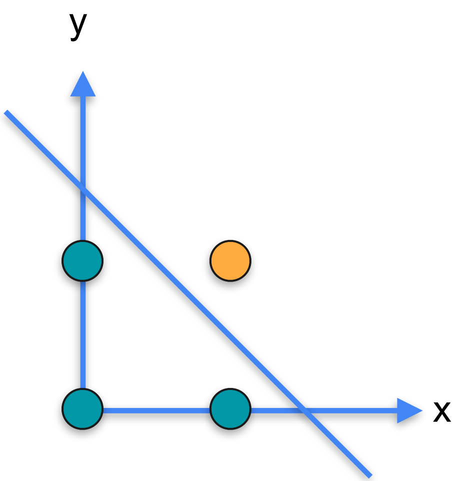
Dot prod
0
1
1
2

Check: >1.5?

Check
No
No
No
Yes

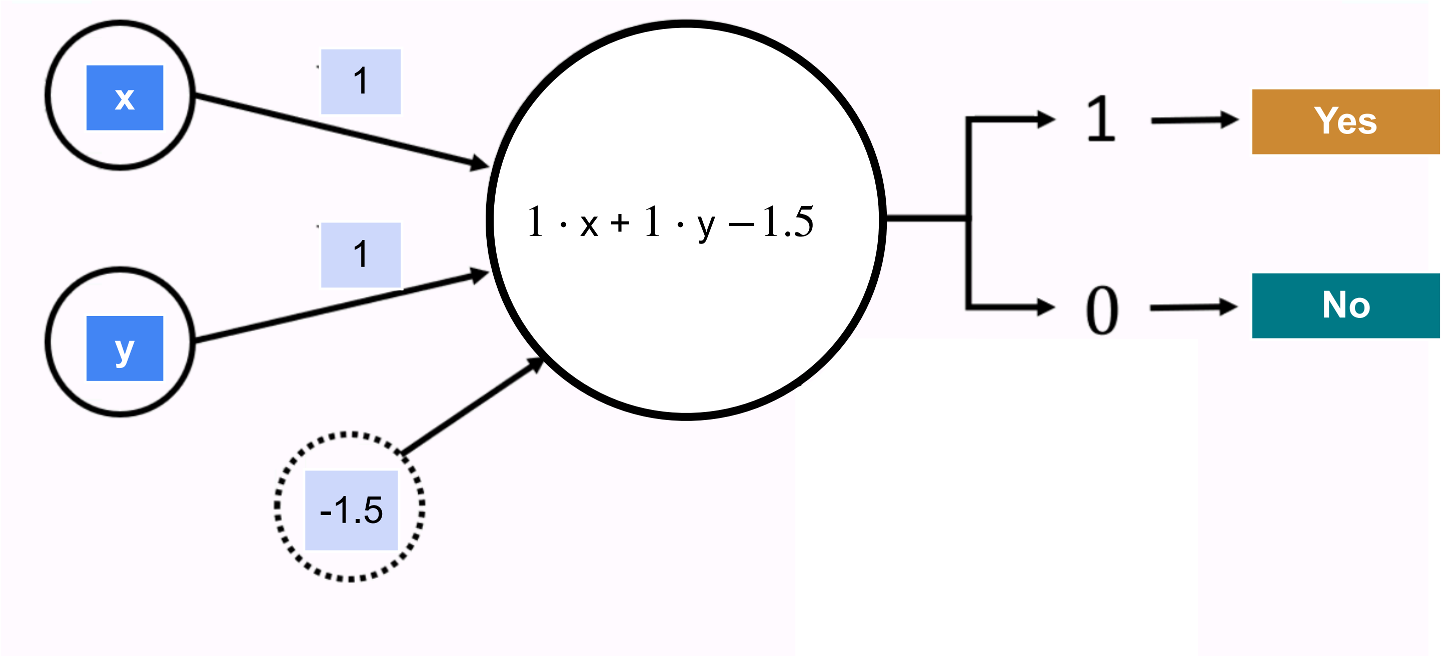
The AND operator

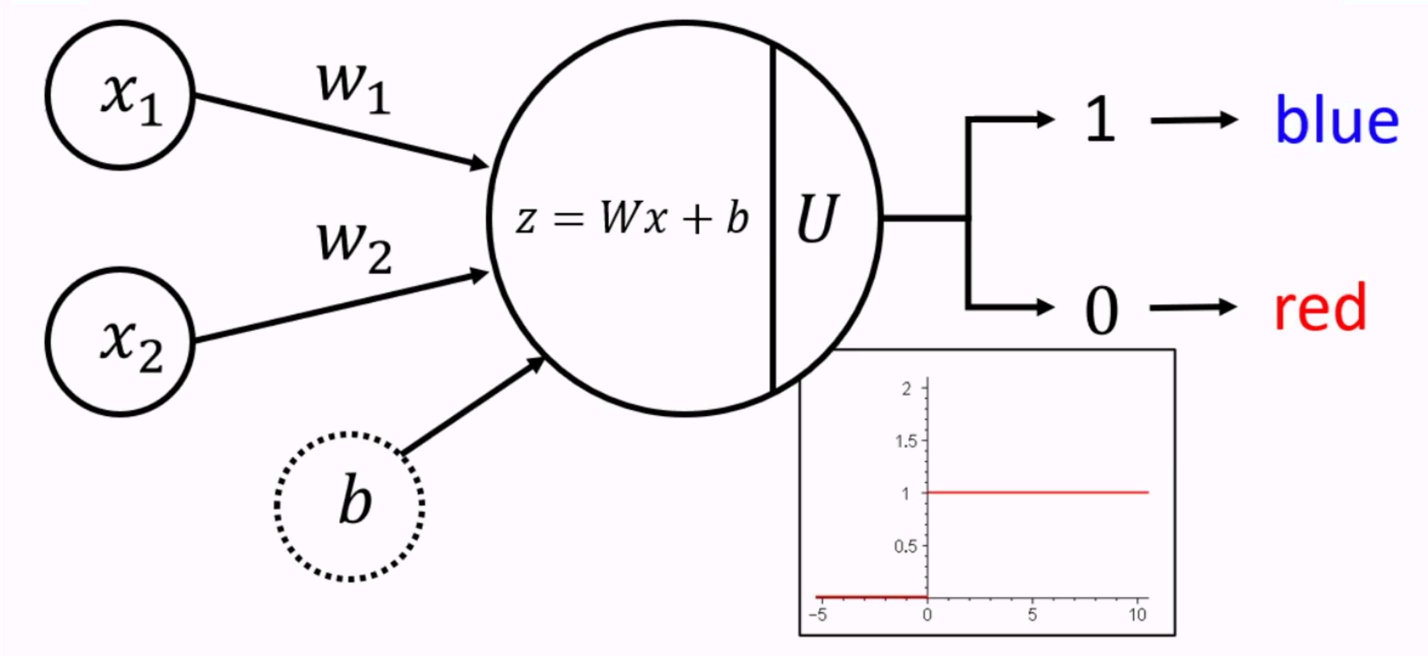
AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1



$$1 \cdot x + 1 \cdot y = 1.5$$

The perceptron





x_1
x_2
w_1
w_2
b
$z = Wx + b$
U
1
blue
0
red
2
1.5
1
0.5
−5
0
5
10

Vectors and Linear Transformations

Conclusion