Untitled1

December 19, 2019

[1]:
```
from sympy import *
from sympy.plotting import plot_parametric
```

[2]:
```
A = Matrix([[-12, 8],
            [-18, 12]])
A
```

[2]:
$$\begin{bmatrix} -12 & 8 \\ -18 & 12 \end{bmatrix}$$

[3]:
```
B = Matrix([[-5],
            [-3]])
B
```

[3]:
$$\begin{bmatrix} -5 \\ -3 \end{bmatrix}$$

[6]:
```
x0 = Matrix([[-7],
             [-9]])
x0
```

[6]:
$$\begin{bmatrix} -7 \\ -9 \end{bmatrix}$$

[5]:
```
x1 = zeros(2, 1)
x1
```

[5]:
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[7]:
```
eigs = A.eigenvects()
pprint(eigs)
```

```
       2/3
0, 2,
        1
```

[8]:
```
t = symbols('t', real=True)
```

[20]:
```
Phi = Matrix([[2, 2 * t],
              [3, 3 * t + 1/4]])
Phi
```

[20]:
$$\begin{bmatrix} 2 & 2t \\ 3 & 3t + 0.25 \end{bmatrix}$$

[21]:
```
expA = Phi @ Phi.subs({t: 0}).inv()
expA
```

[21]:
$$\begin{bmatrix} 1 - 12.0t & 8.0t \\ -18.0t & 12.0t + 1.0 \end{bmatrix}$$

[87]:
```
expA = Matrix([[1 - 12*t, 8*t],
               [-18 * t, 12*t + 1]])
expA
```

[87]:
$$\begin{bmatrix} 1 - 12t & 8t \\ -18t & 12t + 1 \end{bmatrix}$$

[88]:
```
l1, l2 = symbols('l1 l2', real=True)
l = Matrix([[l1],
            [l2]])
l
```

[88]:
$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$$

[89]:
```
tau, t1 = symbols('tau t1', real=True)
t0 = 0
```

[90]:
```
H = -expA.subs({t: - tau}) @ B
H
```

[90]:
$$\begin{bmatrix} 36\tau + 5 \\ 54\tau + 3 \end{bmatrix}$$

[91]:
```
Q = simplify(integrate(H @ H.T, (tau, t0, t1)))
Q
```

[91]:
$$\begin{bmatrix} t_1 \left(432t_1^2 + 180t_1 + 25\right) & 3t_1 \left(216t_1^2 + 63t_1 + 5\right) \\ 3t_1 \left(216t_1^2 + 63t_1 + 5\right) & 9t_1 \left(108t_1^2 + 18t_1 + 1\right) \end{bmatrix}$$

[92]:
```
constr = l.dot(x0) - 1
constr
```

[92]:
$$-7l_1 - 9l_2 - 1$$

[93]:
```
lam = symbols('lambda', real=True)
```

[94]:
```
L = simplify(l.dot(Q @ l) + lam * constr)
L
```

[94]:

$$l_1 t_1 \left(l_1 \left(432t_1^2 + 180t_1 + 25\right) + 3l_2 \left(216t_1^2 + 63t_1 + 5\right)\right) + 3l_2 t_1 \left(l_1 \left(216t_1^2 + 63t_1 + 5\right) + 3l_2 \left(108t_1^2 + 18t_1 + 1\right)\right) - \lambda \left(7l_1 + 9l_2 + 1\right)$$

[98]:
```
L_kate = t1 * (5*l1 + 3*l2)**2 + 108 * t1 ** 3 * (2*l1 + 3*l2)**2 + t1**2 *␣
↪(5*l1 + 3*l2) * (36*l1+54*l2) + lam * constr
L_kate
```

[98]:

$$\lambda \left(-7l_1 - 9l_2 - 1\right) + 108t_1^3 \left(2l_1 + 3l_2\right)^2 + t_1^2 \left(5l_1 + 3l_2\right)\left(36l_1 + 54l_2\right) + t_1 \left(5l_1 + 3l_2\right)^2$$

[106]:
```
sol = solve([L.diff(l1), L.diff(l2), L.diff(lam)], (l1, l2, lam))
pprint(simplify(sol))
```

```
               2                           2                               3
     - 108 t    + 63 t  + 8         216 t    - 99 t  - 40            -243 t
l :                        , l :                         ,  :
             2                           2                         2
    4 27 t    - 36 t  + 16       12 27 t   - 36 t  + 16      54 t    - 72 t




+ 32
```

[107]:
```
l1_opt, l2_opt = sol[l1], sol[l2]
l_opt = Matrix([[l1_opt],
                [l2_opt]])
l_opt
```

[107]:

$$\begin{bmatrix} \frac{-108t_1^2+63t_1+8}{4\left(27t_1^2-36t_1+16\right)} \\ \frac{216t_1^2-99t_1-40}{12\left(27t_1^2-36t_1+16\right)} \end{bmatrix}$$

[108]:
```
t1_cond = simplify(l.dot(Q @ l).subs({l1: l1_opt, l2:l2_opt}))
t1_cond
```

[108]:

$$\frac{243t_1^3}{4\left(27t_1^2 - 36t_1 + 16\right)}$$

[109]:
```
sol = solve(t1_cond - 1, t1)
sol
```

[109]: [4/9]

[110]:
```
t1_opt = sol[0]
t1_opt
```

[110]: $\frac{4}{9}$

[113]:
```
u = (H.T @ l_opt / sqrt(l_opt.dot(Q @ l_opt))).subs(t1, t1_opt)
u
```

[113]: $\left[\frac{3}{2} - \frac{81\tau}{8}\right]$

[117]:
```
expA.subs(t, t1_opt - t0) @ x0 + integrate(expA.subs(t, t1_opt - tau) @ B @ u,␣
↪(tau, t0, t1_opt))
```

[117]: $\begin{bmatrix}0\\0\end{bmatrix}$

[ ]: