time_l2

December 23, 2019

[1]:
```
from sympy import *
from sympy.plotting import plot_parametric
```

[2]:
```
A = Matrix([[12, 9],
            [-16, -12]])
A
```

[2]:
$$\begin{bmatrix} 12 & 9 \\ -16 & -12 \end{bmatrix}$$

[5]:
```
B = Matrix([[2],
            [-1]])
B
```

[5]:
$$\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

[6]:
```
x0 = Matrix([[3],
             [-4]])
x0
```

[6]:
$$\begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

[7]:
```
x1 = zeros(2, 1)
x1
```

[7]:
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[8]:
```
eigs = A.eigenvects()
pprint(eigs)
```

```
       -3/4
0, 2,
        1
```

[9]:
```
v = eigs[0][2][0]
u = A.pinv() @ v
```

```
u
```

[9]:
$$\begin{bmatrix} -\frac{1}{25} \\ -\frac{3}{100} \end{bmatrix}$$

[25]:
```
t, tau, t1 = symbols('t tau t1', real=True, nonnegative=True)
t0 = 0
```

[19]:
```
Phi = v.col_insert(1, t * v + u)
Phi
```

[19]:
$$\begin{bmatrix} -\frac{3}{4} & -\frac{3t}{4} - \frac{1}{25} \\ 1 & t - \frac{3}{100} \end{bmatrix}$$

[20]:
```
expA = Phi @ Phi.subs(t, 0).pinv()
expA
```

[20]:
$$\begin{bmatrix} 12t + 1 & 9t \\ -16t & 1 - 12t \end{bmatrix}$$

[23]:
```
H = -expA.subs(t, t0 - tau) @ B
H
```

[23]:
$$\begin{bmatrix} 15\tau - 2 \\ 1 - 20\tau \end{bmatrix}$$

[24]:
```
l1, l2 = symbols('l1 l2', real=True)
l = Matrix([[l1],
            [l2]])
l
```

[24]:
$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$$

$$\langle l, x \rangle = 1, \quad l2 = \frac{3}{4} l1 - \frac{1}{4}$$

[44]:
```
simplify(integrate((H.T @ l) ** 2, (tau, t0, t1))[0])
```

[44]:
$$\frac{t_1 \left(12 l_1^2 - 12 l_1 l_2 + 3 l_2^2 + t_1^2 \left(225 l_1^2 - 600 l_1 l_2 + 400 l_2^2\right) + 15 t_1 \left(-6 l_1^2 + 11 l_1 l_2 - 4 l_2^2\right)\right)}{3}$$

[29]:
```
L = integrate((H.T @ l) ** 2, (tau, t0, t1)).subs(l2, 3 * l1 / 4 - Rational(1,␣
↪4))[0]
L
```

[29]:

$$t_1^3\left(75l_1^2 - 200l_1\left(\frac{3l_1}{4} - \frac{1}{4}\right) + \frac{400\left(\frac{3l_1}{4} - \frac{1}{4}\right)^2}{3}\right) + t_1^2\left(-30l_1^2 + 55l_1\left(\frac{3l_1}{4} - \frac{1}{4}\right) - 20\left(\frac{3l_1}{4} - \frac{1}{4}\right)^2\right) + t_1\left(4l_1^2 - 4l_1\left(\frac{3l_1}{4} - \frac{1}{4}\right) + \left(\frac{3l_1}{4} - \frac{1}{4}\right)^2\right)$$

[30]:
```
sol = solve(L.diff(l1), l1)
sol
```

[30]:
```
[2*t1 - 1/5]
```

[31]:
```
l1_opt = sol[0]
l1_opt
```

[31]: $2t_1 - \frac{1}{5}$

[36]:
```
l2_opt = 3 * l1_opt / 4 - Rational(1, 4)
l2_opt
```

[36]: $\frac{3t_1}{2} - \frac{2}{5}$

[37]:
```
l_opt = Matrix([[l1_opt],
                [l2_opt]])
l_opt
```

[37]: $\begin{bmatrix} 2t_1 - \frac{1}{5} \\ \frac{3t_1}{2} - \frac{2}{5} \end{bmatrix}$

[38]:
```
expr = simplify(integrate((H.T @ l_opt) ** 2, (tau, t0, t1))[0])
expr
```

[38]: $\frac{25t_1^3}{12}$

[41]:
```
sol = solve(expr - 1, t1)
sol[0]
```

[41]: $\frac{\sqrt[3]{60}}{5}$

[42]:
```
t1_opt = sol[0]
```

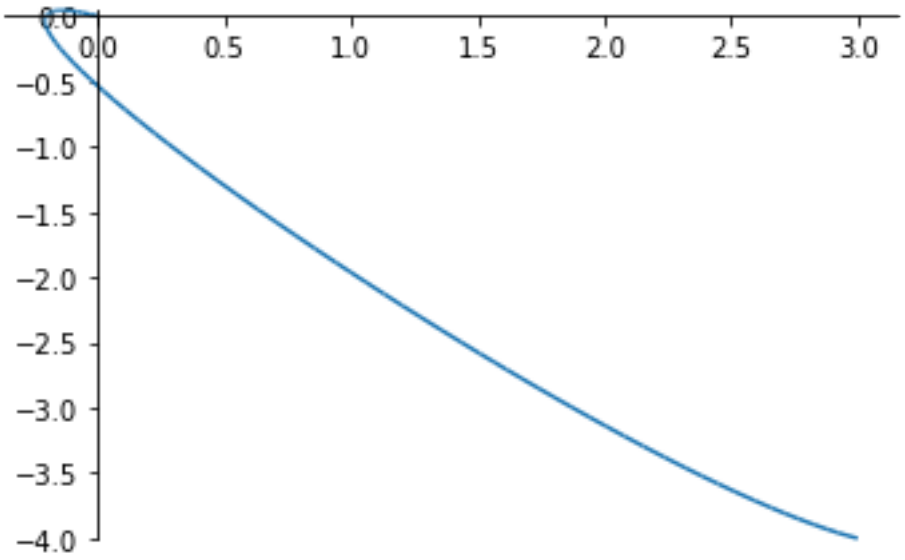
[49]:
```
u_opt = simplify(((H.T @ l_opt)[0] / integrate(((H.T @ l_opt)[0]) ** 2, (tau,␣
↪t0, t1_opt))).subs(t1, t1_opt))
u_opt
```

[49]: $5\tau - \frac{\sqrt[3]{60}}{2}$

[52]:
```
x_opt = simplify(integrate(expA.subs(t, t - tau) @ B * u_opt, (tau, t1_opt, t)))
x_opt
```

[52]:
$$\begin{bmatrix} \frac{25t^3}{2} - \frac{15\sqrt[3]{60}t^2}{4} + 5t^2 - \sqrt[3]{60}t + 3 \\ -\frac{50t^3}{3} - \frac{5t^2}{2} + 5\sqrt[3]{60}t^2 + \frac{\sqrt[3]{60}t}{2} - 4 \end{bmatrix}$$

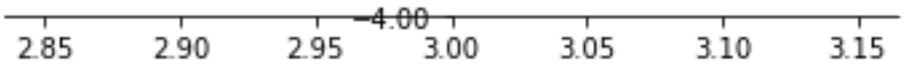
[55]:
```
plot_parametric(x_opt[0], x_opt[1], (t, t0, t1_opt))
```



[55]:
```
<sympy.plotting.plot.Plot at 0x7f23268d6cd0>
```

, , x_0

[59]:
```
plot_parametric((expA @ x0)[0], (expA @ x0)[1], (t, t0, t1_opt))
```



[59]: <sympy.plotting.plot.Plot at 0x7f231117cd50>