alice

May 1, 2020

[2]:
```
from sympy import *
```

[3]:
```
l, m0, k, g, t, psi0 = symbols('l m_0 k g t psi0', nonnegative=True)
u = symbols('u', real=True)
```

[4]:
```
x1 = Function('x_1')(t)
x2 = Function('x_2')(t)
psi1 = Function('psi_1')(t)
psi2 = Function('psi_2')(t)
```

[5]:
```
x1s, x2s, psi1s, psi2s = symbols('x1 x2 psi1 psi2')
```

[6]:
```
deq1 = Eq(x1.diff(t), -x2 * g - k * x1**2 / x2**2 + l * u)
deq1
```

[6]:
$$\frac{d}{dt}\operatorname{x}_{1}(t) = -g\operatorname{x}_{2}(t) - \frac{k\operatorname{x}_{1}^{2}(t)}{\operatorname{x}_{2}^{2}(t)} + lu$$

[7]:
```
deq2 = Eq(x2.diff(t), -u)
deq2
```

[7]:
$$\frac{d}{dt}\operatorname{x}_{2}(t) = -u$$

[9]:
```
deq3 = Eq(psi1.diff(t), psi0 / x2 + 2 * k * psi1 * x1 / x2**2)
deq3
```

[9]:
$$\frac{d}{dt}\psi_{1}(t) = \frac{2k\psi_{1}(t)\operatorname{x}_{1}(t)}{\operatorname{x}_{2}^{2}(t)} + \frac{\psi_{0}}{\operatorname{x}_{2}(t)}$$

[10]:
```
deq4 = Eq(psi2.diff(t), psi1 * g - psi0 * x1 / x2**2 - 2 * k * psi1 * x1**2 /␣
 ↪x2**3)
deq4
```

[10]:
$$\frac{d}{dt}\psi_{2}(t) = g\psi_{1}(t) - \frac{2k\psi_{1}(t)\operatorname{x}_{1}^{2}(t)}{\operatorname{x}_{2}^{3}(t)} - \frac{\psi_{0}\operatorname{x}_{1}(t)}{\operatorname{x}_{2}^{2}(t)}$$

[11]:
```
F = psi1 * l - psi2
F
```

[11]:

$l\psi_1(t) - \psi_2(t)$

[12]:
```
dF = F.diff(t).subs({
    x1.diff(t): deq1.rhs,
    x2.diff(t): deq2.rhs,
    psi1.diff(t): deq3.rhs,
    psi2.diff(t): deq4.rhs
})
dF = simplify(dF)
dF
```

[12]:

$$\frac{-g\psi_1(t)\,\mathrm{x}_2{}^3(t) + 2k\psi_1(t)\,\mathrm{x}_1{}^2(t) + l\left(2k\psi_1(t)\,\mathrm{x}_1(t) + \psi_0\,\mathrm{x}_2(t)\right)\mathrm{x}_2(t) + \psi_0\,\mathrm{x}_1(t)\,\mathrm{x}_2(t)}{\mathrm{x}_2{}^3(t)}$$

[13]:
```
s = latex(dF.subs({x1: x1s, x2: x2s, psi1: psi1s, psi2: psi2s}))
print(s)
```

```
\frac{- g \psi_{1} x_{2}^{3} + 2 k \psi_{1} x_{1}^{2} + l x_{2} \left(2 k
\psi_{1} x_{1} + \psi_{0} x_{2}\right) + \psi_{0} x_{1} x_{2}}{x_{2}^{3}}
```

[14]:
```
d2F = simplify(dF.diff(t).subs({
    x1.diff(t): deq1.rhs,
    x2.diff(t): deq2.rhs,
    psi1.diff(t): deq3.rhs,
    psi2.diff(t): deq4.rhs
}))
d2F
```

[14]:

$$\frac{-2gkl\psi_1(t)\,\mathrm{x}_2{}^3(t) - 6gk\psi_1(t)\,\mathrm{x}_1(t)\,\mathrm{x}_2{}^2(t) - 2g\psi_0\,\mathrm{x}_2{}^3(t) + 2k^2l\psi_1(t)\,\mathrm{x}_1{}^2(t) + 2kl^2u\psi_1(t)\,\mathrm{x}_2{}^2(t) + 2kl\psi_0\,\mathrm{x}_1(t)\,\mathrm{x}_2(t)}{\mathrm{x}_2{}^4(t)}$$

[15]:
```
sol = solve(d2F, u)
u_spec_mode = simplify(sol[0])
u_spec_mode
```

[15]:

$$\frac{gkl\psi_1(t)\,\mathrm{x}_2{}^3(t) + 3gk\psi_1(t)\,\mathrm{x}_1(t)\,\mathrm{x}_2{}^2(t) + g\psi_0\,\mathrm{x}_2{}^3(t) - k^2l\psi_1(t)\,\mathrm{x}_1{}^2(t) - kl\psi_0\,\mathrm{x}_1(t)\,\mathrm{x}_2(t) - \frac{k\psi_0\,\mathrm{x}_1{}^2(t)}{2}}{kl^2\psi_1(t)\,\mathrm{x}_2{}^2(t) + 4kl\psi_1(t)\,\mathrm{x}_1(t)\,\mathrm{x}_2(t) + 3k\psi_1(t)\,\mathrm{x}_1{}^2(t) + l\psi_0\,\mathrm{x}_2{}^2(t) + \psi_0\,\mathrm{x}_1(t)\,\mathrm{x}_2(t)}$$

[16]:
```
s = latex(u_spec_mode.subs({x1: x1s, x2: x2s, psi1: psi1s, psi2: psi2s}))
print(s)
```

```
\frac{g k l \psi_{1} x_{2}^{3} + 3 g k \psi_{1} x_{1} x_{2}^{2} + g \psi_{0}
x_{2}^{3} - k^{2} l \psi_{1} x_{1}^{2} - k l \psi_{0} x_{1} x_{2} - \frac{k
\psi_{0} x_{1}^{2}}{2}}{k l^{2} \psi_{1} x_{2}^{2} + 4 k l \psi_{1} x_{1} x_{2}
+ 3 k \psi_{1} x_{1}^{2} + l \psi_{0} x_{2}^{2} + \psi_{0} x_{1} x_{2}}
```