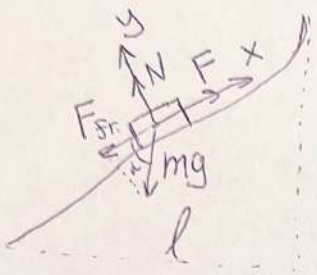
Homework Assignment 3

Problem 1.

$\mu = K$



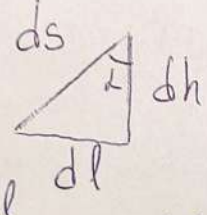
$Ox: ma_x = F - F_{fr} - \cancel{A}mg\cos\alpha$

$a_x = 0 \Rightarrow F = +\mu mg\cos(90-\alpha) + mg\cos\alpha \Leftrightarrow$

$\Leftrightarrow F = +\mu mg\sin\alpha + mg\cos\alpha$

$W_f = \int \vec{F} d\vec{s} = \int_0^{S_0} +\mu mg\sin\alpha ds +$

$+ \int_0^{S_0} mg\cos\alpha ds \ (=)$



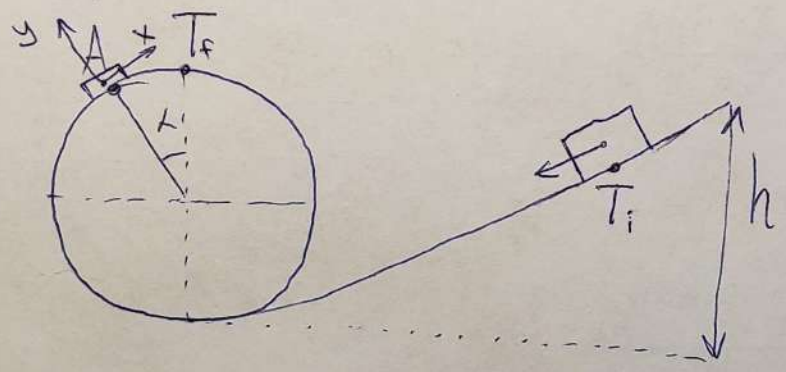
$\sin\alpha ds = dl$

$\cos\alpha ds = dh$

$(=) \int_0^l \mu mg dl + \int_0^h mg dh = mg(\mu l + h)$

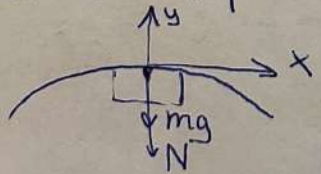
Answer: $W_f = mg(\mu l + h)$

Problem 2.



$E_{mec}^{T_i} = U_{T_i} = mgh$

Let's consider the moment of time when a cart is located at the top of the dead loop (point T_f):



$Oy: -ma = -mg - N^{\to 0}$

$\frac{mv_f^2}{R} = mg \Leftrightarrow v_f = \sqrt{gR}$

$E_{mec}^{T_f} = E_{mec}^{T_i} \Leftrightarrow \frac{mv_f^2}{2} + 2mgR = mgh \Leftrightarrow \frac{mgR}{2} + 2mgR =$

$= mgh \Leftrightarrow \frac{gR}{2} + 2gR = gh \Leftrightarrow R = \frac{2}{5}h$

Let's write all forces acting on the cart at the point A:

$Ox: -mg\sin\alpha = ma_t$

$Oy: -mg\cos\alpha - N = ma_c$

$$-mg\cos\alpha - N = -\frac{mv_A^2}{R} \Leftrightarrow N = \frac{mv_A^2}{R} - mg\cos\alpha$$

$$E_{mec}^A = \frac{mv_A^2}{2} + mg(R + R\cos\alpha) = mgh = E_{mec}^{T_i}$$

$$v_A^2 = 2gh - 2g(R + R\cos\alpha)$$

$$N = \frac{m(2gh - 2gR(1+\cos\alpha))}{R} - mg\cos\alpha =$$

$$= m(5g - 2g - 2g\cos\alpha) - mg\cos\alpha = mg(3 - 3\cos\alpha)$$

Answer: $N = 3mg(1-\cos\alpha)$

Problem 3.

(a) $E_s = \frac{kx^2}{2}$

$$E_s = U_f \Leftrightarrow \frac{kx^2}{2} = mg(x+H) \Leftrightarrow k = \frac{2mg(x+H)}{x^2} =$$

$$= \frac{2 \cdot 80 \cdot 9{,}81(0{,}5+70)}{0{,}5 \cdot 0{,}5} = \underline{442627{,}2 \ \frac{kg}{s^2}}$$

(b) $$\frac{kx^2}{2} = \frac{mv_0^2}{2} \Leftrightarrow v_0 = \sqrt{\frac{kx^2}{m}} = \sqrt{\frac{442627{,}2 \cdot 0{,}5 \cdot 0{,}5}{80}} =$$

$\approx 37{,}19 \ m/s \approx \underline{133{,}9 \ km/h}$

Problem 4.

$$P = \frac{dW}{dt} = \frac{d}{dt} Fx\cos\varphi = F\cos\varphi v$$

$$F = \frac{P}{\cos\varphi v} = \frac{90 \cdot 10^6}{\frac{90}{3{,}6}} = 3{,}6 \cdot \frac{90 \cdot 10^6}{90} = 5{,}4 \ MN$$

Answer: $F = 5{,}4 \ MN$