

Computational Practicum. Variant 1

```
class ImprovedEuler(Solution):
    def __init__(self, x0, y0, h, X):
        super().__init__(x0, y0, h, X)

    def improved_euler_method(self, X, Y):
        return (Y + 0.5 * self.h * (self.f_prime(X, Y) + self.f_prime(X +
self.h, self.euler_method(X, Y))))

    def euler_method(self, X, Y):
        return (Y + self.h * (self.f_prime(X, Y)))

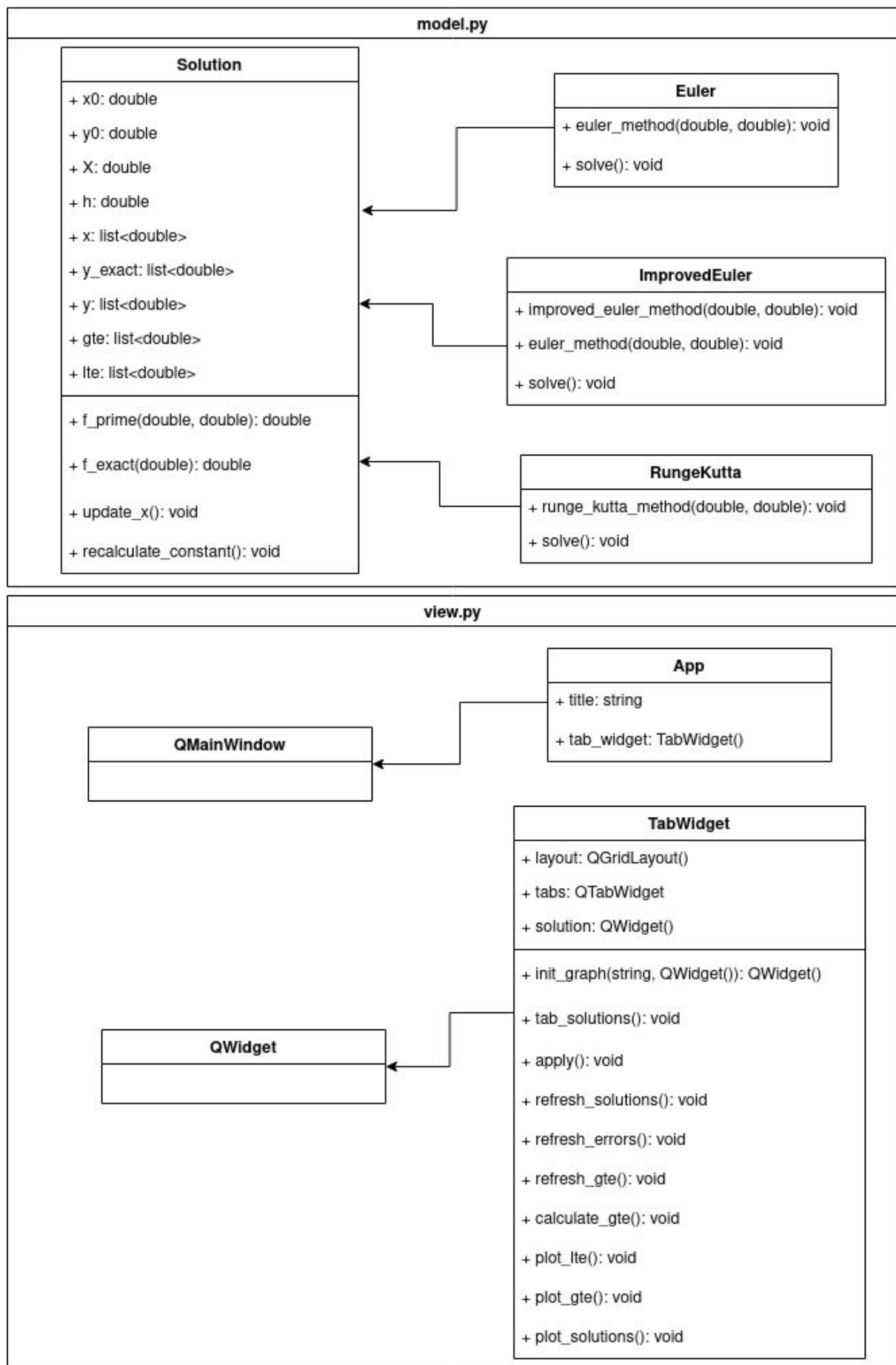
    def solve(self):
        while (self.x[-1] < self.X - self.h/2):
            y_improved = self.improved_euler_method(self.x[-1],
self.y[-1])
            self.y.append(y_improved)
            self.update_x()
            exact = self.f_exact(self.x[-1])
            self.y_exact.append(exact)
            self.gte.append(abs(exact - self.y[-1]))
            self.lte.append(abs(exact -
self.improved_euler_method(self.x[-2], self.f_exact(self.x[-2]))))

class RungeKutta(Solution):
    def __init__(self, x0, y0, h, X):
        super().__init__(x0, y0, h, X)

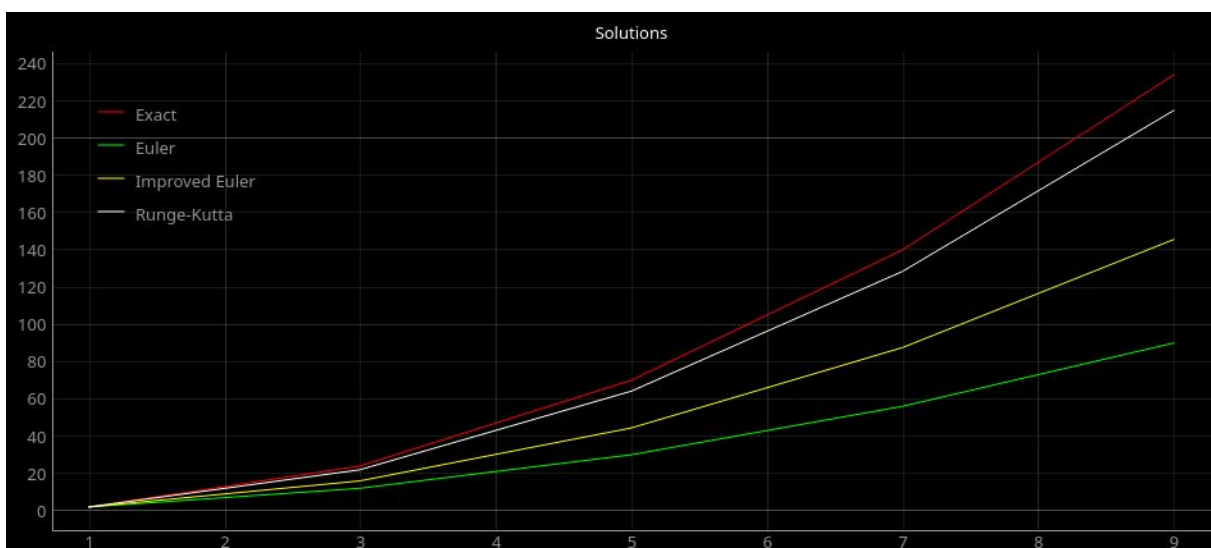
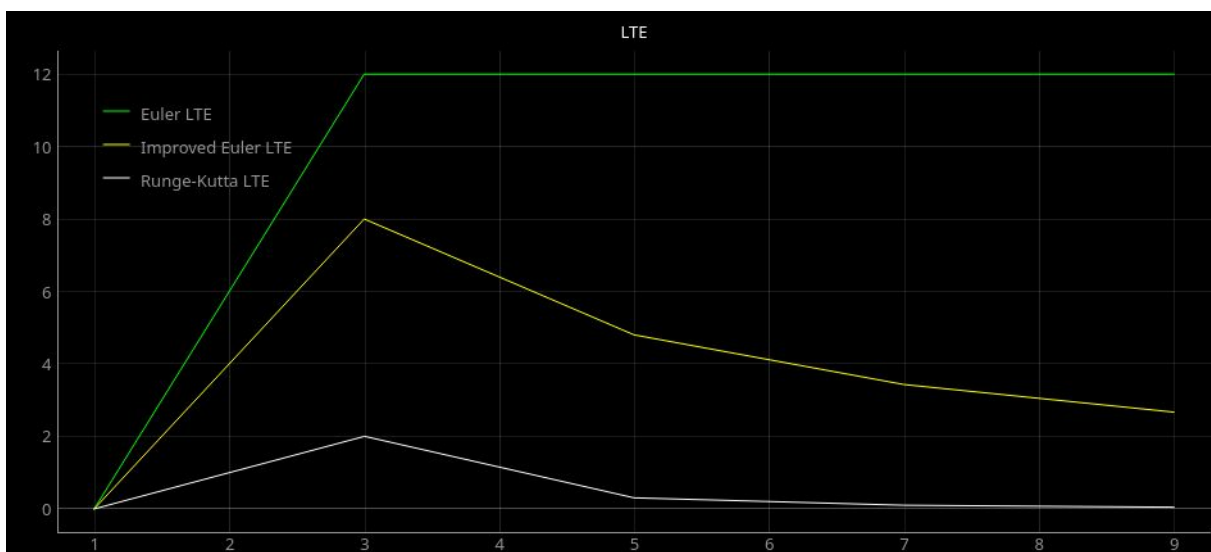
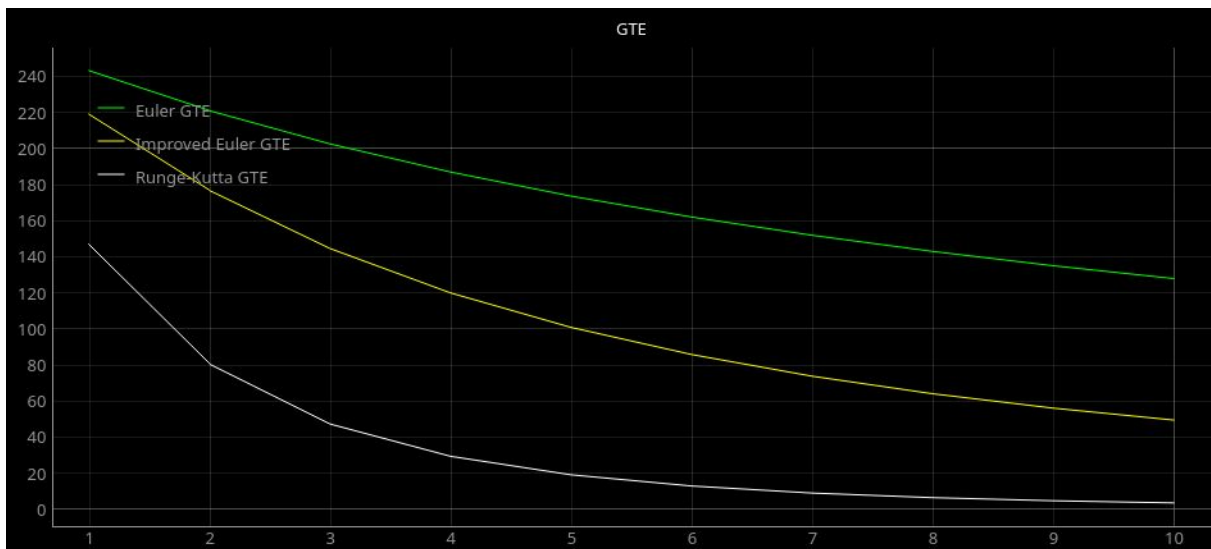
    def runge_kutta_method(self, X, Y):
        k1 = self.f_prime(X, Y)
        k2 = self.f_prime(X + 0.5 * self.h, Y + 0.5 * self.h * k1)
        k3 = self.f_prime(X + 0.5 * self.h, Y + 0.5 * self.h * k2)
        k4 = self.f_prime(X + self.h, Y + self.h * k3)
        return (Y + self.h * (k1 + 2 * k2 + 2 * k3 + k4) / 6)

    def solve(self):
        while (self.x[-1] < self.X - self.h/2):
            y_runge = self.runge_kutta_method(self.x[-1], self.y[-1])
            self.y.append(y_runge)
            self.update_x()
            exact = self.f_exact(self.x[-1])
            self.y_exact.append(exact)
            self.gte.append(abs(exact - self.y[-1]))
            self.lte.append(abs(exact -
self.runge_kutta_method(self.x[-2], self.f_exact(self.x[-2]))))
```

UML DIAGRAM



Graphs



x0 = 1.0
y0 = 2.0
X = 10.0
h = 1.0

Euler Method

	X	Y (exact)	LTE	Y (approximated)	GTE
0	1	2.0	0.0	2.0	0.0
1	3	24.0	12.0	12.0	12.0
2	5	70.0	12.0	30.0	40.0
3	7	140.0	12.0	56.0	84.0
4	9	234.0	12.0	90.0	144.0

Improved Euler Method

	X	Y (exact)	Y (approximated)	LTE	GTE
0	1	2.0	2.0	0.0	0.0
1	3	24.0	16.0	8.0	8.0
2	5	70.0	44.4	4.799999999999999 97	25.6
3	7	140.0	87.56571428571428 8	3.4285714285714 45	52.434285714285 72
4	9	234.0	145.607437641723 35	2.666666666666666 57	88.392562358276 65

Runge-Kutta Method

	X	Y (exact)	Y (approximated)	LTE	GTE
0	1	2.0	2.0	0.0	0.0
1	3	24.0	22.0	2.0	2.0
2	5	70.0	64.16666666666666 66	0.3000000000000011 37	5.833333333333333 43
3	7	140.0	128.47883597883 595	0.095238095238102	11.521164021164 054
4	9	234.0	214.91640946502 054	0.0416666666666685 614	19.083590534979 464