Saarland University, Department of Computer Science
Neural Network Assignment 2

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Exercise 2.1

2.1.a

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$(4-\lambda)(3-\lambda) - 2 = 0$$

$$\lambda^2 - 7\lambda + 10 = 0$$

$$\lambda(\lambda - 2) - 5(\lambda - 2) = 0$$

$$\lambda_1 = 5; \lambda_2 = 2$$

$$\text{ Case for } \lambda_1 = 5;$$

$$4x_1 + 2x_2 = \lambda x_1 \Rightarrow 4x_1 + 2x_2 = 5x_1$$

$$x_1 + 3x_2 = \lambda x_2 \Rightarrow x_1 + 3x_2 = 5x_2$$

$$\Rightarrow -x_1 + 2x_2 = 0 \Rightarrow x_1 = 2x_1$$

$$x_1 - 2x_2 = 0$$

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\text{ Case for } \lambda_2 = 2'$$

$$4x_1 + 2x_2 = 2x_1$$

$$x_1 + 3x_2 = 2x_2$$

$$2x_1 + 2x_2 = 0$$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

$$V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

2.1.b

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$$

$$\begin{aligned}
&|A^{-1} - \lambda I| = 0 \\
&|\begin{bmatrix} \frac{3}{10} & -\frac{1}{5} \\ \frac{-1}{10} & -\frac{2}{5} \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}| = 0 \\
&\left(\frac{3}{10} - \lambda\right)\left(\frac{2}{5} - \lambda\right) - \left(\frac{-1}{5}x - \frac{1}{10}\right) = 0 \\
&\frac{3}{25} - \frac{3}{10}\lambda - \frac{2}{5}\lambda + \lambda^2 - \frac{1}{50} = 0 \\
&\frac{1}{10} - \frac{7}{10}\lambda + \lambda^2 = 0 \qquad (1) \\
&\lambda^2 - \frac{7}{10}\lambda + \frac{1}{10} = 0 \\
&\lambda^2 - \frac{1}{5}\lambda - \frac{1}{2}\left(\lambda - \frac{1}{5}\right) = 0 \\
&\lambda\left(\lambda - \frac{1}{5}\right) - \frac{1}{2}\left(\lambda - \frac{1}{5}\right) = 0 \\
&\lambda_1 = \frac{1}{2}, \quad \lambda_2 = \frac{1}{5}
\end{aligned}$$

The relationship between eigenvalues of the inverse matrix; A^{-1} are equal to the inverse of the eigenvalues of the original matrix A: Eigenvalues of A; 5 and 2 and A^{-1}; $\frac{1}{2}$ and $\frac{1}{5}$

2.1.c

WILL CHECK LATER ON HOW TO GO ABOUT IT, FEEL FREE TO DO IF YOU KNOW TOO

Exercise 2.2

2.2 a

(a) Using first-derivative formulae

$$\begin{aligned}
&f'(x) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \\
&f(x) = \omega^\top x; f(x+h) = \omega^\top(x+h) \\
&f'(x) = \lim_{h \to 0} \frac{\omega^\top(x+h) - \omega^\top x}{h} \\
&F'(x) = \lim_{h \to 0} \frac{w^\top x + w \cdot h - w^\top x}{h} \\
&= \lim_{h \to 0} \frac{w \cdot h}{h} \\
&\text{As } h \to 0; \quad f'(x) = w
\end{aligned}$$

2.2 b

$$\begin{aligned}
&f(x) = x^\top Ax; f(x+h) = (x+h)^\top A(x+h)\\
&f'(x) = \lim_{h\to 0} \frac{\left(x^\top + h\right)^\top A(x+h) - x^\top Ax}{h}\\
&f'(x) = \lim_{h\to 0} \frac{x^\top Ax + x^\top Ah + h^\top Ax + h^\top Ah - x^\top Ax}{h}\\
&f'(x) = \lim_{h\to 0} \frac{x^\top Ah + h^\top Ax + h^\top Ah}{h}\\
&f'(x) = \lim_{h\to 0} x^\top A + Ax + h^\top A\\
&F'(x) = Ax + A^\top x, \text{ as } h \to 0
\end{aligned} \tag{2}$$

2.2 c

$$\begin{aligned}
f(x) &= (Bx)^2, f(x+h) = (B(x+h))^2\\
f'(x) &= \lim_{h\to 0} \frac{B(x+h)\cdot B(x+h) - (Bx)^2}{h}\\
f'(x) &= \lim_{h\to 0} \frac{(Bx)^2 + 2B^\top Bxh + (Bh)^2 - (Bx)^2}{h}\\
f'(x) &= \lim_{h\to 0} \frac{2B^\top Bxh + (Bh)^2}{h}\\
f'(x) &= \lim_{h\to 0} 2B^\top Bx + B^2 \cdot h\\
f'(x) &= 2B^\top Bxh \to 0
\end{aligned}$$

2.2 d

will include over the weekend, I AM EXHAUSTED !!! :)