Exercise Sheet 2

Linear Algebra and PCA

Deadline: 23.09.2022 08:00

Exercise 2.1 - Linear Algebra (0.5+1+2 points)

a) Compute the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

b) Given a matrix A for which an inverse exists, find the relationship between the eigenvalues of A and A^{-1}.

c) Show that if λ is an eigenvalue of AB, then it is also an eigenvalue of BA where $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times n}$.

Exercise 2.2 - Matrix Calculus (0.5+1+1+1 points)

In this lecture we will often compute the derivatives of multivariate functions and matrix valued functions. Let $f : \mathbb{R}^n \to \mathbb{R}$; $w, x, c \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times n}$. Prove that the following rules hold.

a) $f(x) = w^T x$, then $\nabla_x f(x) = w$

b) $f(x) = x^T A x$, then $\nabla_x f(x) = Ax + A^T x$

c) $f(x) = \|Bx\|_2^2$, then $\nabla_x f(x) = 2B^T Bx$

d) $f(x) = \|Bx - c\|_2^2$, then $\nabla_x f(x) = 2B^T(Bx - c)$

Exercise 2.3 - PCA (3 points)

See assignment2.ipynb

Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

- Please submit the assignment as a **team of two to three** students.

- Write the Microsoft Teams user name, student id and the name of each member of your team on your submission.

- Hand in zip file containing a `single` PDF with your solutions and the completed ipython notebook. Do not include any data or cache files (e.g. __pycache__).

- Important: please name the submitted zip folder and files inside using the format: **Name1_id1_Name2_id2**.

- Your assignment solution must be uploaded by only **one** of your team members to the 'Assignments' tab of the tutorial team (in **Microsoft Teams**). Please remember to press the **Hand In** button after uploading your work.

- If you have any trouble with the submission, contact your tutor **before** the deadline.