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CS D

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Assignment 1
Soft Computing

(1) Projection of fuzzy relation:
we can project a fuzzy relation $R \subseteq A \times B$ with respect to A or B as in the following manner.

$$\mu_{R_A}(x) = \max \mu_R(x, y) : \text{projection to A}$$
$$\mu_{R_B}(y) = \max \mu_R(x, y) : \text{projection to B}$$

Ex: $\underline{R} = \begin{array}{c} x_1 \\ x_2 \end{array} \begin{bmatrix} 0.8 & 0.1 \\ 0.0 & 0.8 \end{bmatrix}$

$\underline{R}$ on x $\quad \pi_x(x_1) = 0.8$
$\pi_x(x_2) = 0.8$

$\underline{R}$ on y $\quad \pi_y(y_1) = 0.8$
$\pi_y(y_2) = 0.8$

(2) Cylindrical Extension:
If a fuzzy set or relation R is defined in spec axis, this relation can be extended to $A \times B \times C$ and we obtain a new set written as as $C(R)$

$$\mu_{C(R)}(a, b, c) = \mu_R(a, b)$$
$$a \in A,\ b \in B,\ c \in C$$

$$M_C(R_A) = \begin{array}{c} a_1 \\ a_2 \end{array} \begin{bmatrix} 1.0 & 1.0 \\ 0.8 & 0.8 \end{bmatrix}$$

$\mu_C R_A(a_1, b_1) = \mu_{RA}(a_1) = 1.0$
$\mu_C R_A(a_1, b_2) = \mu_{RA}(a_1) = 1.0$
$\mu_C R_A(a_2, b_1) = \mu_{RA}(a_2) = 0.8$

Reflexivity :

Relation R is reflexive if every element in set is associated with itself i.e. $\mu_R(x, x) = 1, \forall x \in X$

example: $X = \{1, 2, 3\}$

$$\tilde{R} = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0.8 & 0.1 \\ 0.4 & 1 & 0.3 \\ 0.3 & 0.6 & 1 \end{bmatrix} \end{array}$$

as, we can see $\mu_R(1,1) = \mu_R(2,2) = \mu_R(3,3) = 1$

$\therefore \mu_R(x, x) = 1$, so it is reflexive

Anti-reflexivity

R is an anti-reflexive if $\forall x \in X\ (x, x) \notin \underline{R}$ i.e. $\mu_R(x, x) = 0$

$$R = \begin{array}{c} 1 \\ 2 \end{array} \begin{bmatrix} 0 & 0.2 \\ 0.4 & 0 \end{bmatrix}$$

$\because \mu_R(1,1) = \mu_R(2,2) = 0$ so its anti-reflexive

Symmetricity

If element x is related to element y, then element y must be related to x.

i.e. $\mu_R(x, y) = \mu_R(y, x), \forall x, y \in X$

eg:
$$\tilde{R} = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.0 & 0.8 & 0.1 \\ 0.8 & 1 & 0.3 \\ 0.1 & 0.3 & 0.8 \end{bmatrix} \end{array}$$

$\because \mu_R(1,2) = \mu_R(2,1) = 0.8$

$\mu_R(1,3) = \mu_R(3,1) = 0.1$

similarly $\mu_R(2,3) = \mu_R(3,2) = 0.3$

$\therefore$ It is symmetric

Anti-symmetric :

if $\mu_R(x,y) > 0$, then $\mu_R(y,x) = 0$

$x, y \in X,\ x \neq y$

eg:

$$\tilde{R} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 0.7 \\ 2 & 0.2 & 0 & 0 \\ 3 & 0 & 0.2 & 0 \end{array}$$

$\mu_R(1,2) \neq \mu_R(2,1) = 0, 0.2$

same goes for $\mu_R(1,3) \neq \mu_R(3,1) = 0.7, 0$

$\therefore$ It is anti-symmetric

as $R(1,3)$ $R(2,1)$ $R(3,2)$ are non zero.

& $R(3,1)$ $R(1,2)$ $R(2,3)$ are zero

Transitivity

If x is related to y & y is related to z then x must be related to z

let $\lambda_1 = \mu_R(x_i, x_j)$, $\lambda_2 = \mu_R(x_j, x_k)$

$\lambda = \mu_R(x_i, x_k)$

$R^2 = R \cdot R$

R is transitive if $\mu_R^2(x,y) < \mu_R(x,y)$

Anti - Symmetary / Partial Order Relation:
If R is similarity relation then its complement is anti-symmetry.

$$\mu_{\bar{R}}(x,y) = 1 - \mu_R(x,y)$$
$$\mu_R'(x,y) \geq \min(\max(\mu_R(x,y), \mu_R(y,z))$$

eg: $\bar{R} = \begin{bmatrix} 1 & 0.1 & 0.7 \\ 0.1 & 1 & 0.7 \\ 0.7 & 0.7 & 1 \end{bmatrix}$

$\therefore \mu_R = 1 - \mu_R(x,y)$

$\Rightarrow \begin{bmatrix} 0 & 0.9 & 0.3 \\ 0.9 & 0 & 0.3 \\ 0.3 & 0.3 & 0 \end{bmatrix}$

Weak - Similarity
If relation R is reflexive & symmetric but not transitive then it is called weak similarity

Order Relation
- Relation R is order relation if it is transitive
- Relation R is pre-order if it is reflexive & transitive.
- Relation R is half-order relation if it is reflexive & weak anti-symmetric relation.

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order relation $R \subset X \times X$ is transitive relation

$$\mu_R(x,z) \cdot = \max(\min(\mu_R(x,y), \mu_R(y,z)))$$

eg:

$$\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & 1 & 0.7 & 0.8 \\ x_2 & 0 & 0.1 & 0.3 \\ x_3 & 0 & 0.7 & 1 \end{array}$$

<u>Half-Order</u>

$$\mu_R(x,x) = 1 \quad \forall x \in X$$

& weakly anti-symmetric

$\mu_R(x,y) > 0$ & $\mu_R(y,x) > 0$

then $x = y$

$$R = \begin{bmatrix} 1 & 0.8 & 0.2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$