1 Relate Ω_j and ω_j in terms of h

$$\frac{\rho_i + \rho_k + \rho_\Lambda}{\rho_c} = \frac{\rho_i + \rho k + \rho \Lambda}{\frac{3H^2}{8\pi G}} = \Omega_i + \Omega_k + \Omega_\Lambda$$

$$\frac{\rho_i + \rho_k + \rho_\Lambda}{\frac{3(100h)^2}{8\pi G}} = \Omega_i + \Omega_k + \Omega_\Lambda$$

$$(\omega_i + \omega_k + \omega_\Lambda)/h^2 = \Omega_i + \Omega_k + \Omega_\Lambda$$

$$\omega_i + \omega_k + \omega_\Lambda = h^2(\Omega_i + \Omega_k + \Omega_\Lambda)$$

2 Write the Friedmann Equation in terms of ω_j and h

From the Friedmann equation we have

$$\frac{8\pi G}{3}(\rho_i + \rho_k + \rho_\Lambda) = H^2$$

$$\rho_i + \rho_k + \rho_\Lambda = \frac{3H^2}{8\pi G}$$

Dividing both sides of the second expression by ρ_c^{100} gives us

$$\omega_i + \omega_k + \omega_\Lambda = \rho_c/\rho_c^{100} = h^2$$

3 Find the value of ρ_c^{100} in energy units

From the back of K&T The critical density is quoted as $8.0992h^2 \times 10^{-47}\mathrm{GeV}^4$. To get ρ_c^{100} we let h go to 1 here, to get $\rho_c^{100} = 8.0992 \times 10^{-47}\mathrm{GeV}^4$

4 Matlab code attached

5 Find ω_r using accounting for neutrinos

K&T gives the value of $8.09 \times 10^{-34}\mathrm{g\,cm}^{-3}$ or $8.09 \times 10^{-31}\mathrm{Kg\,m}^{-3}$. Now to convert this to energy units, multiply by c and divide by $(\hbar c)^{-3}$ to get $3.487 \times 10^{-15}\mathrm{eV}^4$ or $3.487 \times 10^{-51}\mathrm{GeV}^4$. This yields $\rho_r = 3.487 \times 10^{-51}\mathrm{GeV}^4\, a^{-4}$, and after dividing by ρ_c^{100} gives

$$\omega_r = 4.305 \times 10^{-5}\, a^{-4}$$

6 Find when matter and radiation densities are equal

$$\rho_m(a_{eq}) = \rho_r(a_{eq})$$
$$.958 \times 10^{-47}\, a_{eq}^{-3} = 3.487 \times 10^{-51}\, a_{eq}^{-4}$$
$$a_{eq} = 3.487 \times 10^{-51} / .958 \times 10^{-47} = 3.64 \times 10^{-4}$$

7 Plots

included on the next pages

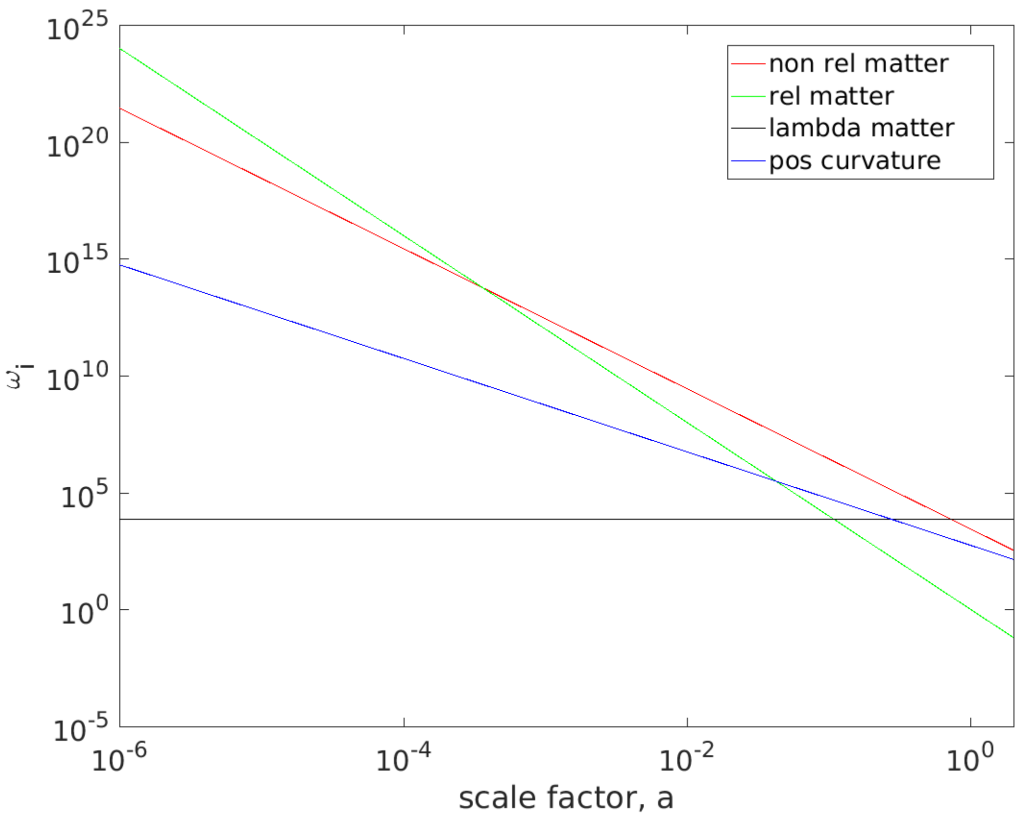

Figure 1: using a curvature density with positive initial condition

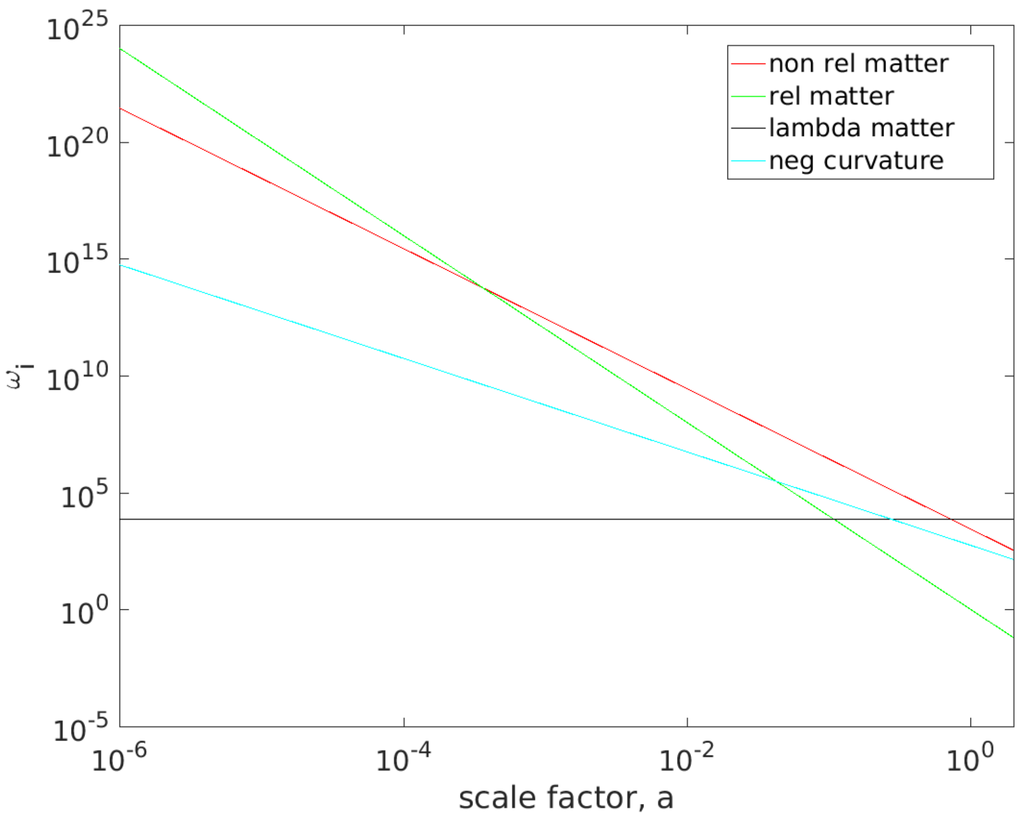

Figure 2: using a curvature density with negative initial condition

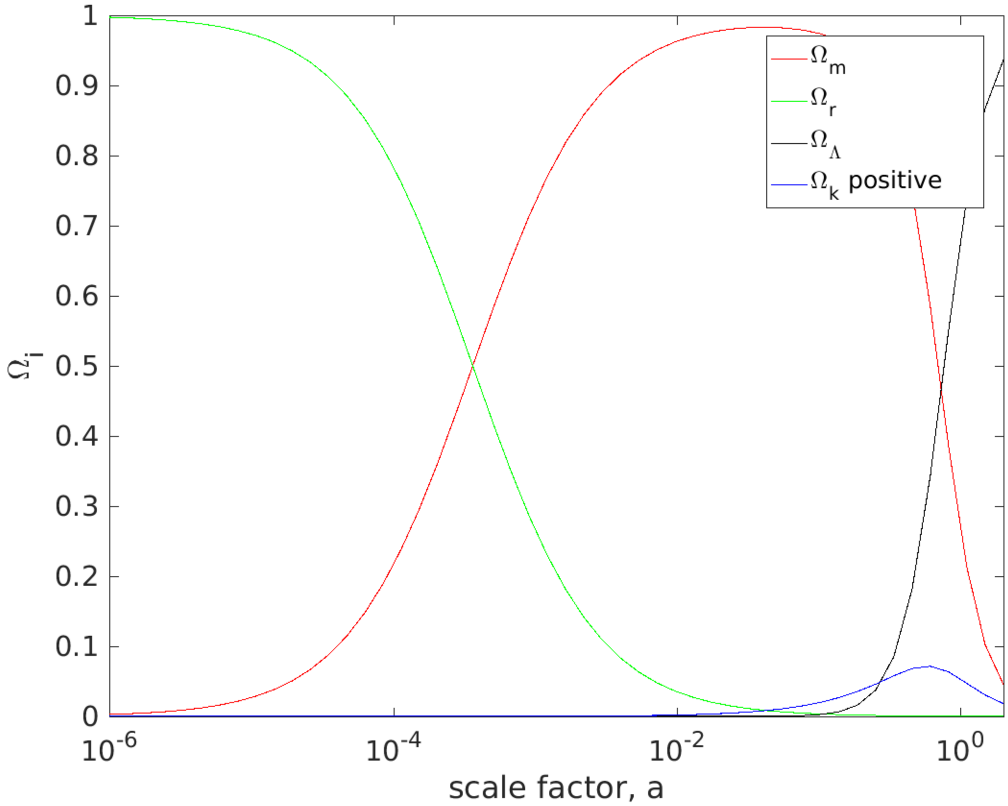

Figure 3: using a curvature density with positive initial condition

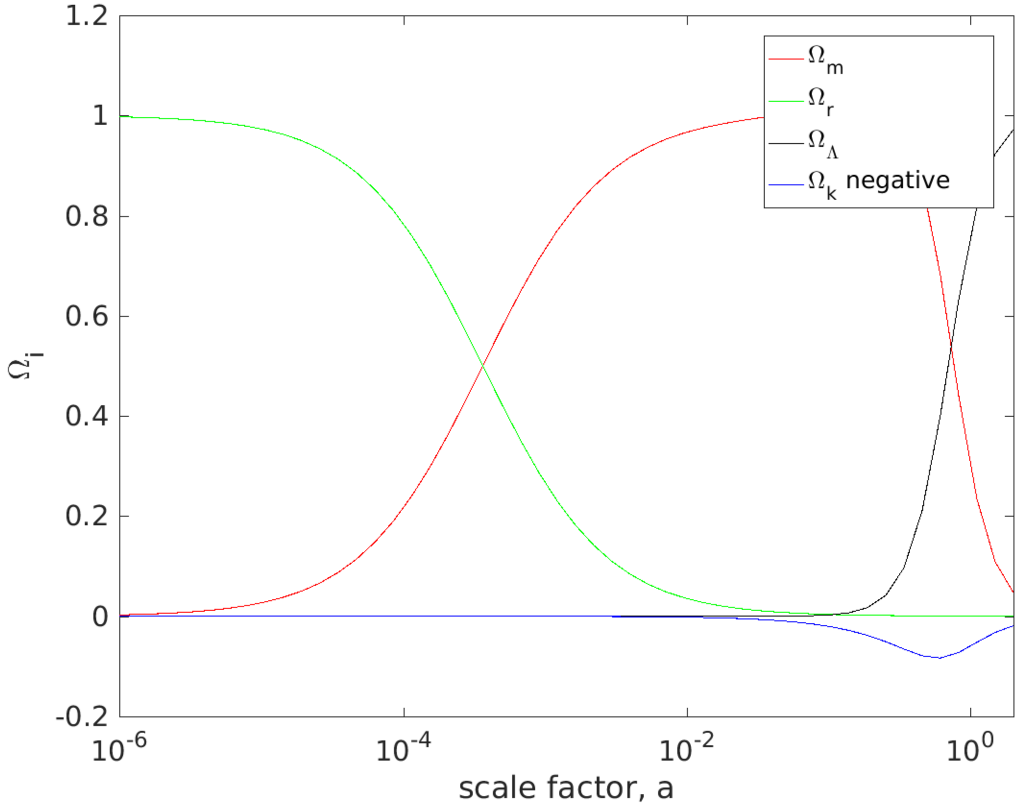

Figure 4: using a curvature density with negative initial condition