2018 FALL
COMPUTATIONAL STATISTIC
FINAL

Sequential Importance Sampling

For Object Tracking Problem: Following a Moving Target

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1. Sequential Importance Sampling (SIS)
   - Sequential Monte–Carlo
   - Weight Degeneracy & Rejuvenation
   - SIS for Hidden Markov Models

2. Following a Moving Target
   - Resample–Move Algorithm
   - Implementation

3. Discussion

1. Sequential Importance Sampling (SIS)

useful for time stochastic process

SIS

Sequential Importance Sampling

▶ Sequential Monte–Carlo Method (SMC)

When

Target density f가 High–Dimension일 때

$$f(X_1, \dots, X_t)$$

How

Split into a sequence of simpler steps & Update the previous one

$$f(X_1, \dots, X_t) = f(X_1) \cdot f(X_2|X_1) \cdot \cdots \cdot f(X_t|X_1, \dots, X_t)$$

➡ Time Stochastic Process를 포함한

사물의 실시간 궤도 추정, 고분자 화합물의 확산 등에 관한 연구에 이용

SIS
Sequential Importance Sampling

IS

Sample from ISF g (envelope)

Calculate Importance weight $w = f/g$

IS Estimator

$$\hat{\mu}_{IS} = \sum_{i=1}^{n} w(X_i) \cdot h(X_i) / \sum_{i=1}^{n} w(X_i)$$

Sequential IS

Target $f(x_{1:t}) = f(x_1)f(x_2|x_1) \cdot \cdots \cdot f(x_t|x_{1:t-1})$

Envelope $g(x_{1:t}) = g(x_1)g(x_2|x_1) \cdot \cdots \cdot g(x_t|x_{1:t-1})$

Importance Weight $w(x_{1:t}) = w(x_1)w(x_2|x_1) \cdot \cdots \cdot w(x_t|x_{1:t-1})$

$$w_{1:t} = w_{t-1} * u_t, \qquad u_t = \frac{f(x_t|x_{1:t-1})}{g(x_t|x_{1:t-1})}$$

SIS

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Sequential IS

t = 1 Sample X_1 from ISF g_1 (envelope)

Calculate Importance weight $w_1 = f_1/g_1 = u_1$

t > 1 Sample $X_t | x_{1:t-1} \sim g_t(X_t | x_{1:t\text{-}1})$

Append x_t to $x_{1:t\text{-}1}$

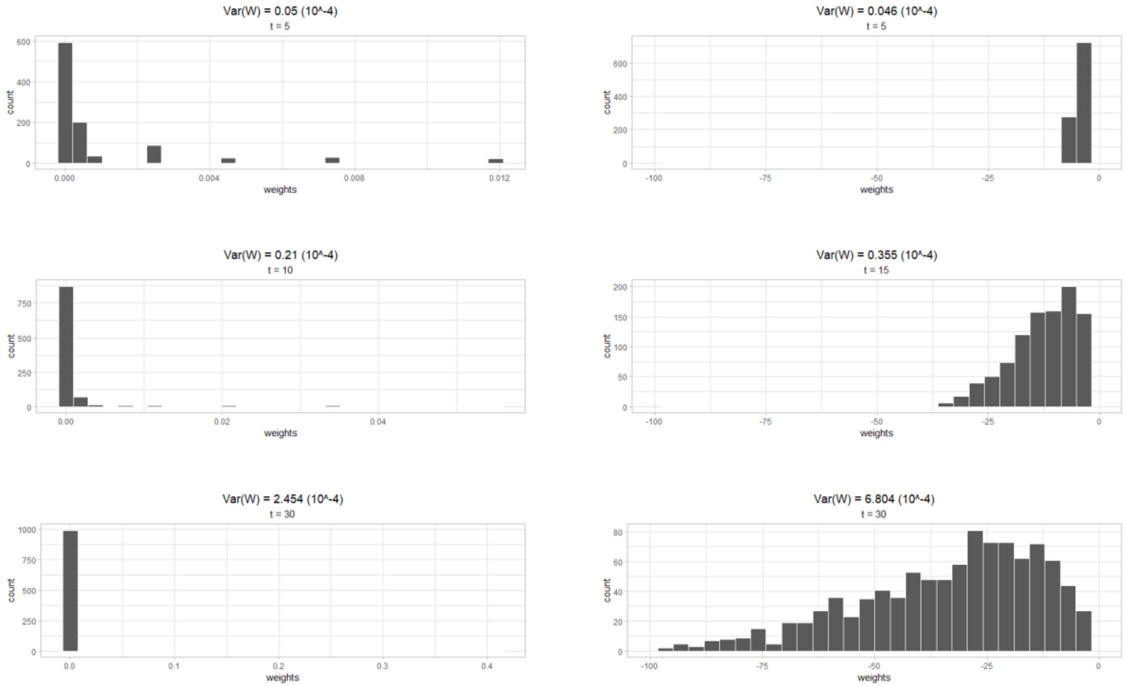
Calculate $u_t = f_t(x_t | x_{1:t\text{-}1}) / g_t(x_t | x_{1:t-1})$

Update weight $w_t = w_{t\text{-}1} \cdot u_t$

SIS
Sequential Importance Sampling

▶ Weight Degeneracy & Rejuvenation

- t가 증가함에 따라 가중치가 계속 누적됨
- n개 중 특정 sequence에 일반적이지 않은 sample이 포함될 경우 가중치 약화 (회복 불가)
- 일부 sequence들에만 가중치가 집중될 가능성이 있음



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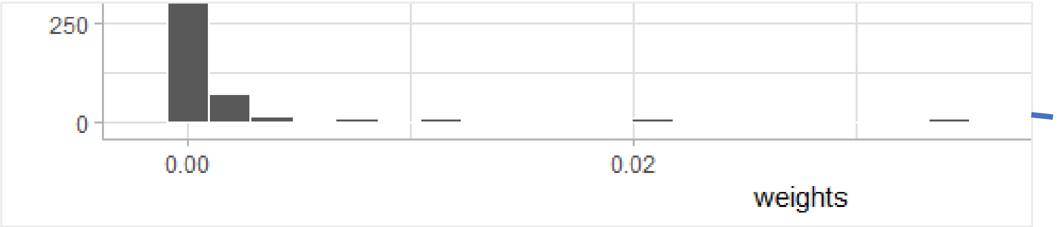
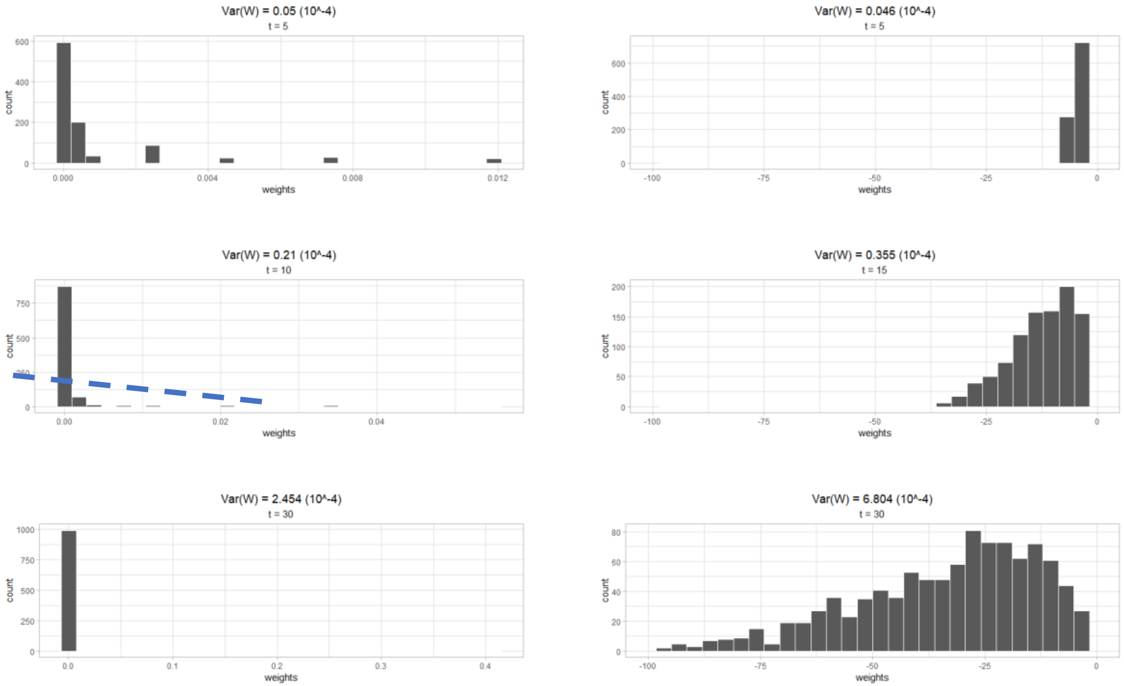




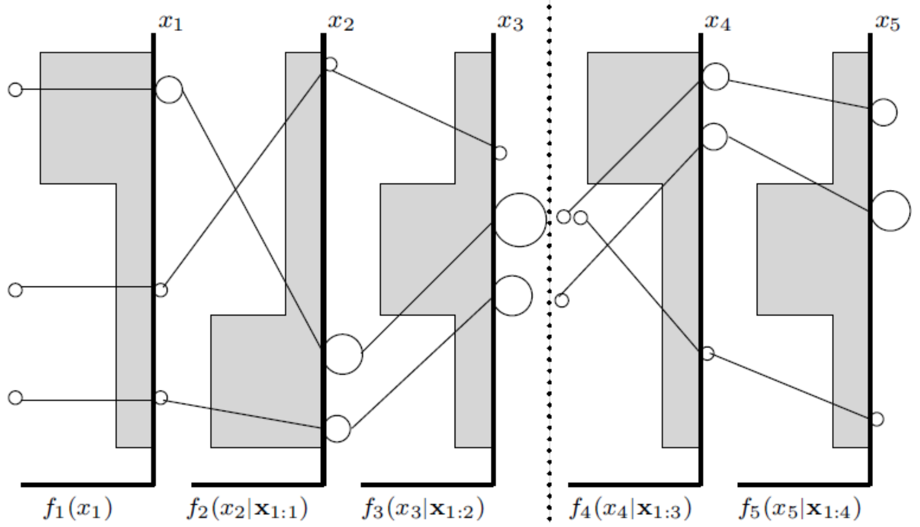
SISR

Sequential Importance Sampling with Resampling

▶ SIS with Resampling

- 특정 시점에 해당 시점까지의 weight을 이용해 Resampling
- Resampling한 sequence들로 sample을 재구성
- Weight을 1/n으로 동일하게 초기화함

$$\widehat{N}(g, f) = \frac{1}{\sum_{i=1}^{n} w(x^{(i)})^2}$$



▶ Particle Filters: Bootstrap Filter

- 모든 시점에 Resampling step을 수행하자!
- t 시점의 weight는 해당 시점의 Resampling step에만 사용됨

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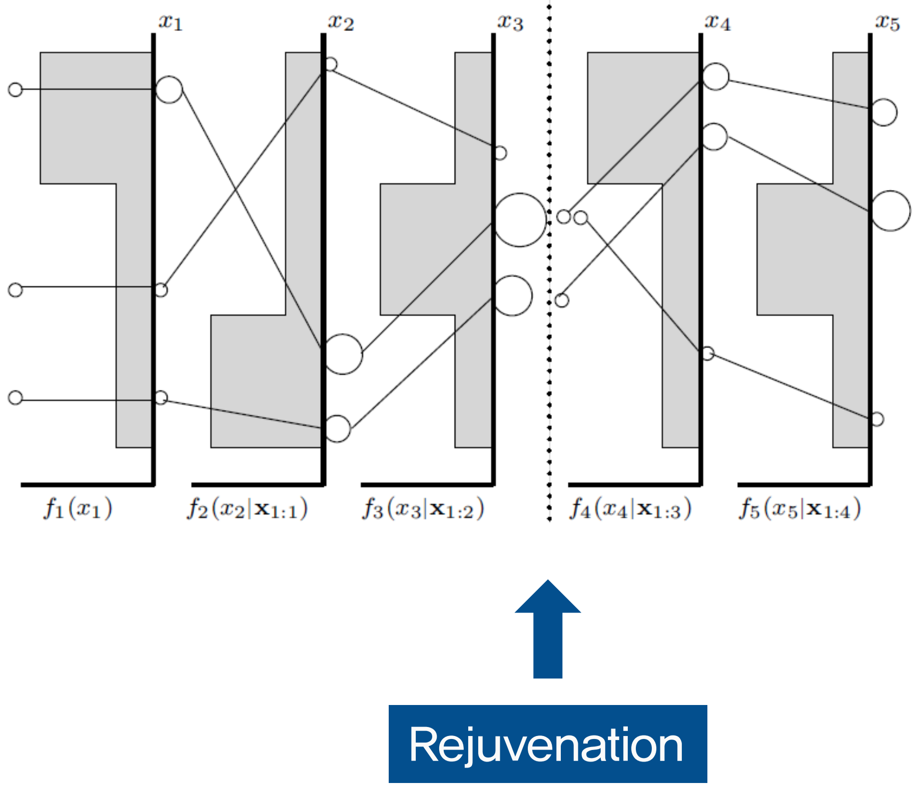

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▶ SIS for Hidden Markov Models

- 관측이 불가능한 Markov Sequence $X_0, X_1, \dots, X_t$
- 관측 가능한 $Y_0, Y_1, \dots, Y_t$

$$f(X_t|x_1, \dots, x_{t-1}) = f(X_t|x_{t-1})$$

Markov

$$Y_t \sim f_y(y_t|x_t) \quad \& \quad X_t \sim f_x(x_t|x_{t-1})$$

Target

Posterior density

$$f_t(x_{1:t}|y_{1:t}) = f_t(x_{1:t-1}|y_{1:t-1}) \cdot f_x(x_t|x_{t-1}) f_y(y_t|x_t)$$

$$\frac{f_t(x_{1:t}|y_{1:t})}{f_t(x_{1:t-1}|y_{1:t-1}) f_x(x_t|x_{t-1})} = f_y(y_t|x_t) = u_t$$

➡ Importance Weight ∝ Likelihood

2. Object Tracking Problem: Following a Moving Target

– Monte Carlo inference for dynamic Bayesian models (W. R. Gilks and Berzuini, 2001)

Object Tracking Problem

Following a Moving Target – W. R. Gilks and Berzuini (2001)

Bearings–only Tracking

– 움직이는 선박의 실시간 궤도를 추정
– t 시점의 선박의 x축, y축 위치는 관측 불가능
– 관측지점인 원점으로부터 각도 + noise 관측 가능

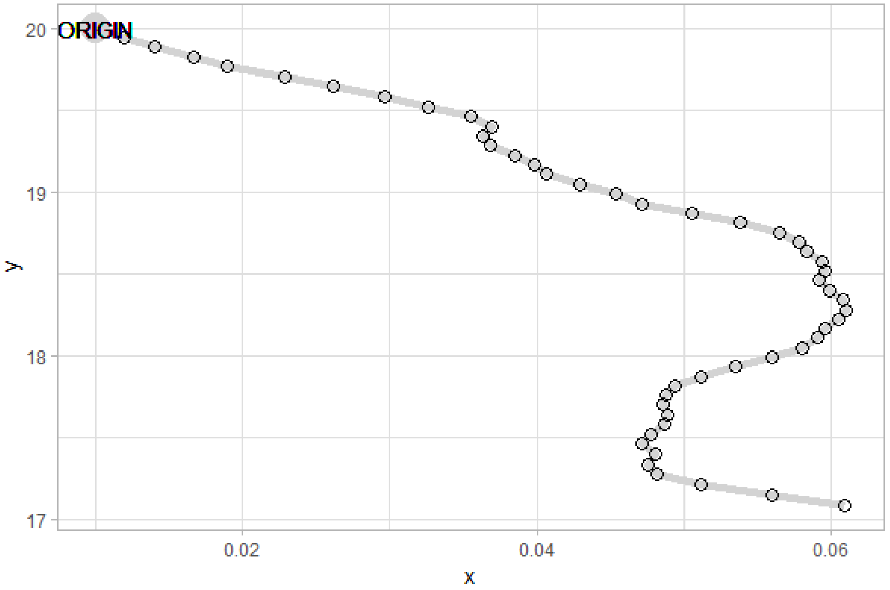
$$\dot{x}_t \sim N(\dot{x}_{t-1}, \tau^{-1})$$

$$\dot{y}_t \sim N(\dot{y}_{t-1}, \tau^{-1})$$

$$x_t = x_{t-1} + \dot{x}_{t-1}$$

$$y_t = y_{t-1} + \dot{y}_{t-1}$$

$$z_t = \tan^{-1}(y_t / x_t) + N(0, \eta^2)$$



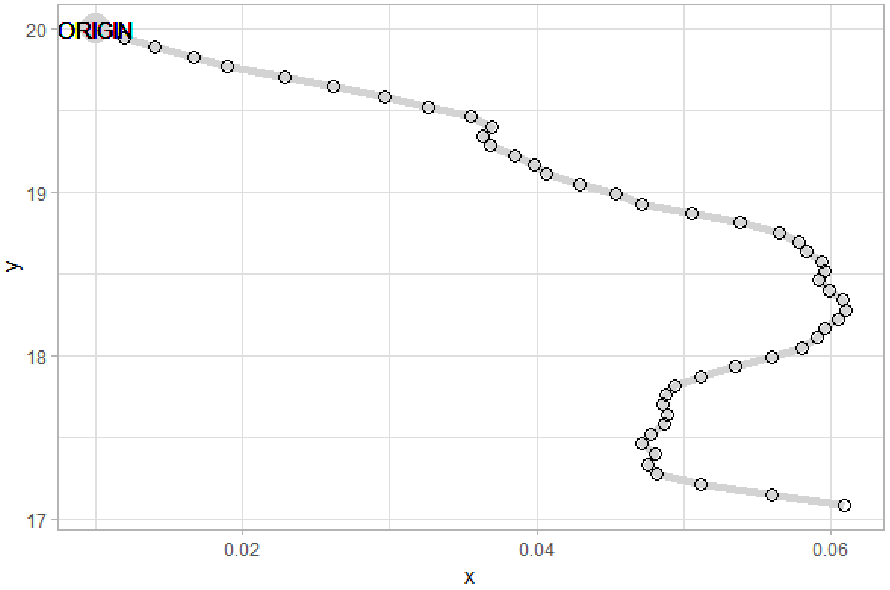
$$\theta_t = (\tau^{-1}, x_1, y_1, \dot{x}_1, \dot{y}_1, \dots, \dot{x}_t, \dot{y}_t)$$

Set of Parameters of Interest at time t

Object Tracking Problem

Following a Moving Target – W. R. Gilks and Berzuini (2001)

Bearings–only Tracking



Target Posterior distribution

$\propto$ Prior · Likelihood

$$\pi_t(\theta_t) = p(\theta\,|z_{1:t}) \quad \propto \quad p(\theta_t) \cdot \prod_{i=1}^{t} p(z_i|\theta_i)$$

$$p(\theta_t) \propto \tau \cdot exp\{-0.5\tau \sum (\dot{x}_t - \dot{x}_{t-1})^2 - 0.5\tau \sum (\dot{y}_t - \dot{y}_{t-1})^2\}$$

Object Tracking Problem

Following a Moving Target – W. R. Gilks and Berzuini (2001)

▶ Resample–Move Algorithm: MCMC Sampling과 Sequential ISR의 결합

Initialization

T = 1 sample size n의 독립적인 sample set $S_1 = (\tau, x_1, y_1, \dot{x}_1, \dot{y}_1)$을 구성한다.

Rejuvenation

n개의 sample particle (i)에 대한 Importance Weights

$w_t^{(i)} = p(z_t^{(i)} | \theta_{t-1}^{(i)})$을 계산한다.

T > 1 〈 Resample Step 〉

weigh를 사용하여 n개의 particle들을 Resampling하여 Sample set을 다시 구성한다.

〈 Move Step 〉

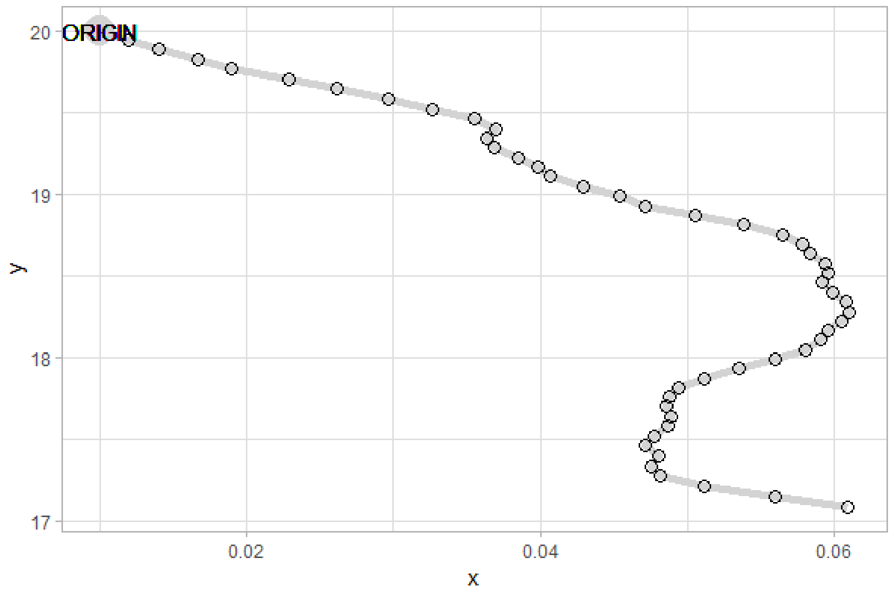
재구성된 sample set에 대하여 unknown parameter τ를

조건부 sampling을 이용하여 update한다.

Object Tracking Problem

Following a Moving Target – W. R. Gilks and Berzuini (2001)

▶ Generated Real Data

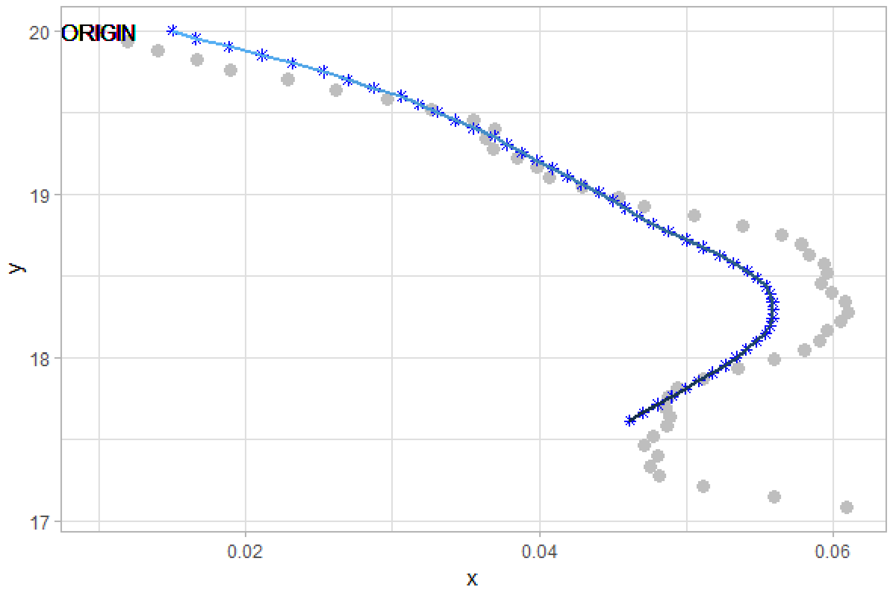


x_1	y_1	$\dot{x}_1$	$\dot{y}_1$	τ^{-1}	η
0.01	20	0.002	-0.06	0.000001	0.005

Object Tracking Problem

Following a Moving Target – W. R. Gilks and Berzuini (2001)

- Predicted by Resample–Move (n = 1000)



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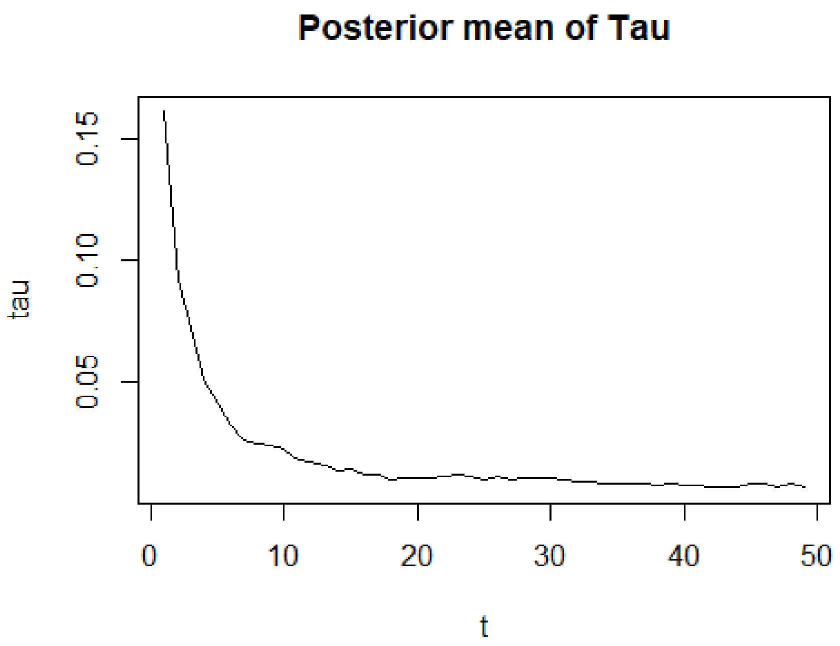

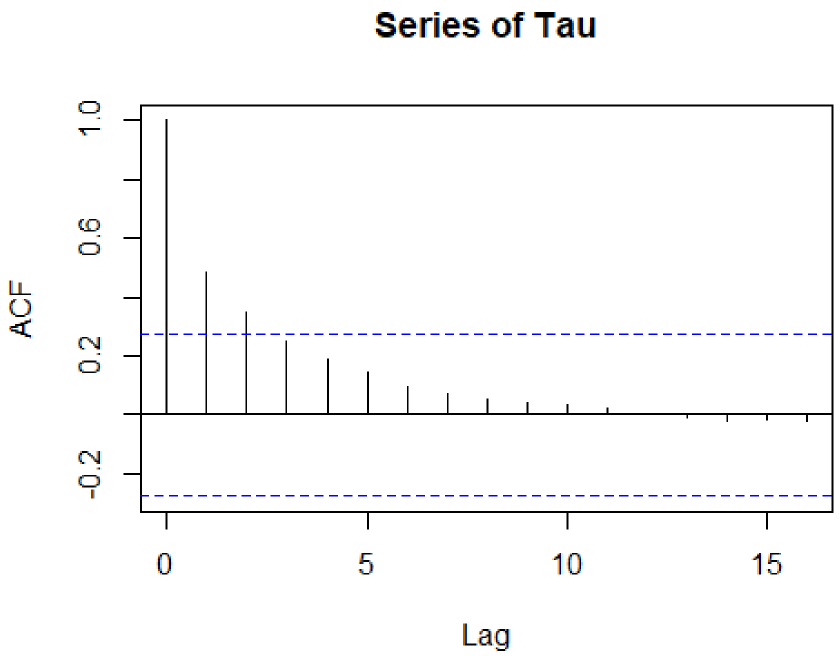

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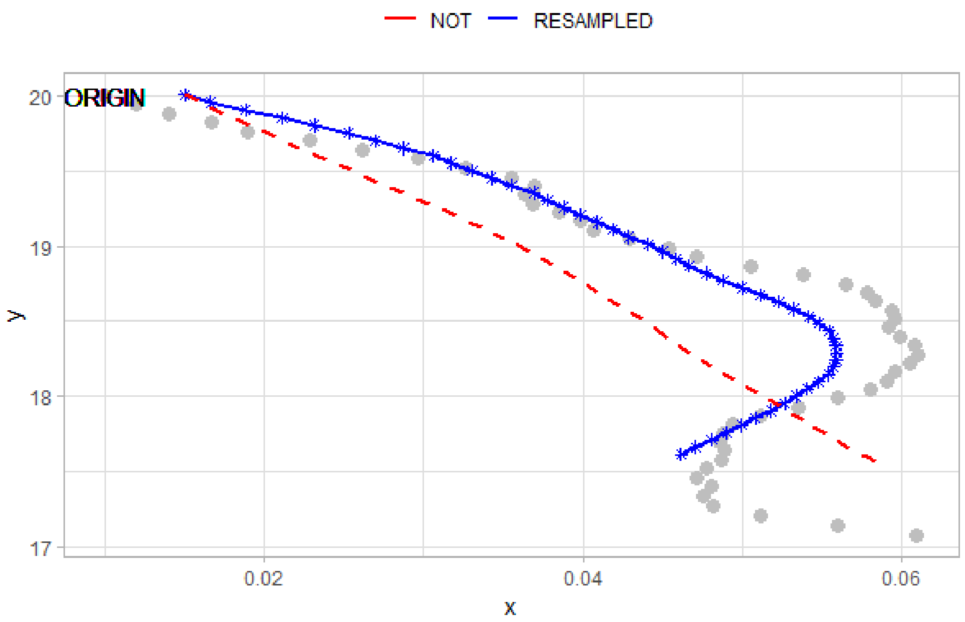
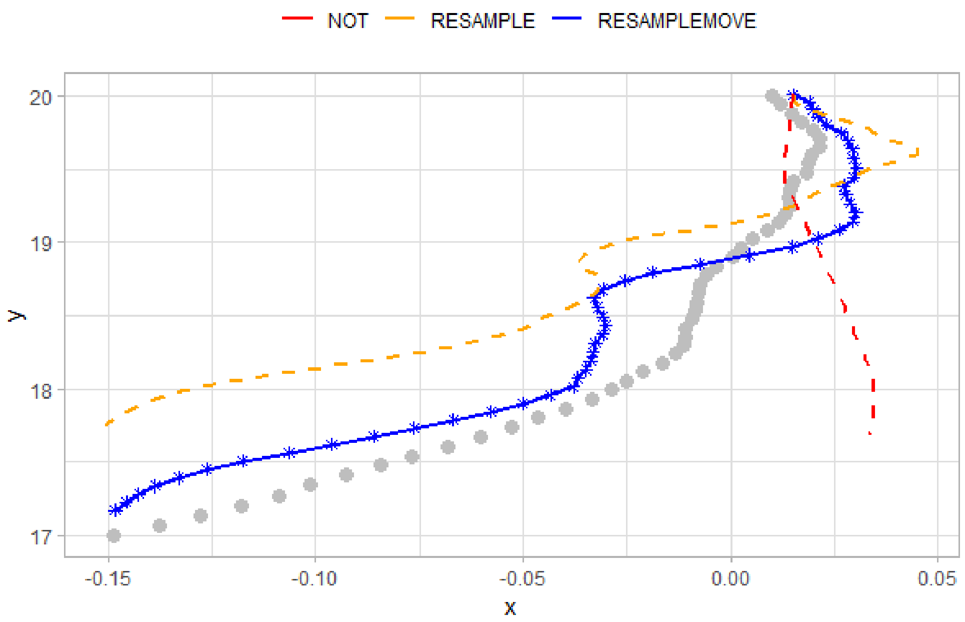




Resample–Move & SIS

Resample–Move & Resample & SIS

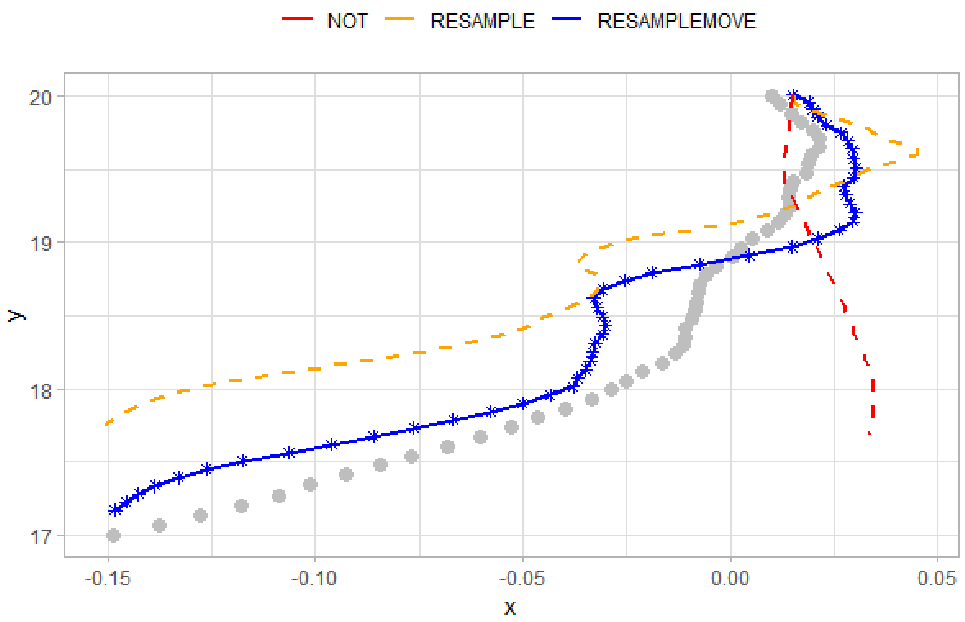
Object Tracking Problem

Following a Moving Target – W. R. Gilks and Berzuini (2001)

t = 50 x와 y 위치의 사후 평균과 표준오차

X50	RS–M	RS	NOT
mean	–0.1492	–0.1592	0.0321
sd	0.0392	0.0421	2.0220

Y50	RS–M	RS	NOT
mean	17.1925	17.7886	17.6322
sd	1.4475	1.8205	1.9644



Resample–Move & Resample & SIS

3. Discussion

Summary

- Real–Time Sequential Forecasting

  ▶ Financial & Medical Time Series, Control Engineering, Speech Recognition 등

- Resample–Move Algorithm은 다양한 MC Simulation 및 Bayesian Inference & Prediction에 하나의 framework을 제공함
- 특히 누적된 정보를 이용하여 Unknown hyperparameter를 Update할 필요가 있는 경우와 High–Dimensional (Evolving) Target Distribution에 효과적으로 이용 가능
- MCMC의 계산 부담을 줄일 수 있음

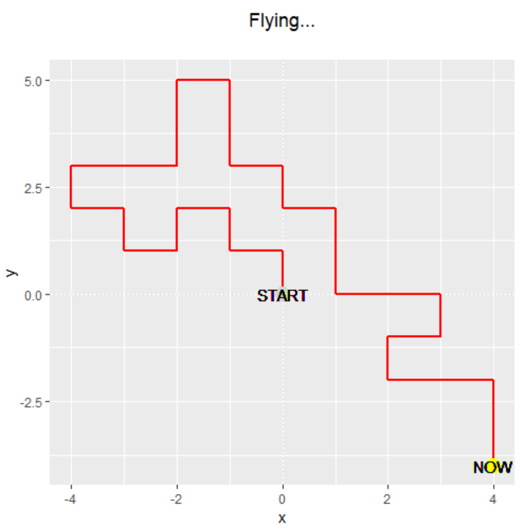
- 추가적인 고려사항

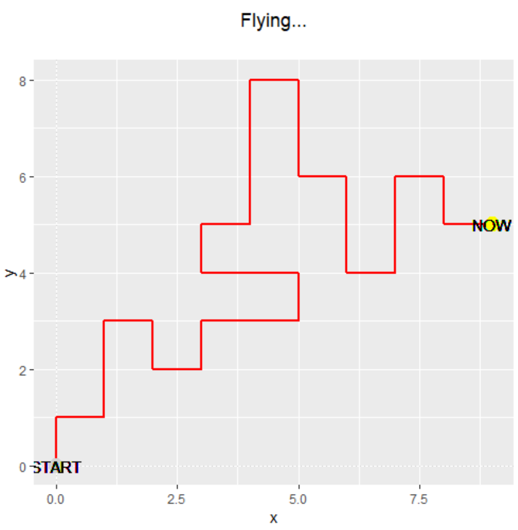
  ▶ the number of Rejuvenation Step, Which Parameters to move

THANK YOU

Self–Avoiding Walk (SAW)
in Infinite Lattice

Untrapped (example) t = 30





- 가능한 경우의 수가 매우 많음
- 고분자 화합물에 관한 연구 등에서 Simulation의 중요성이 큼

Trapped