Definitions

$$v(t) = \frac{ds}{dt} = s'(t)$$

$$a(t) = \frac{dv}{dt} = s''(t)$$

Let

$$s(t) = At^2 + Bt$$

$$s'(t) = 2At + B = v(t)$$

$$s''(t) = 2A = a(t)$$

$$v_i = v(0) = B$$

$$a = a(t) = 2A$$

Thus

$$v(t) = v_f = v_i + at$$

$$s(t) = s = v_i t + \frac{1}{2}at^2$$

Proof of

$$v_f^2 = v_i^2 + 2as$$

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{-v_i \pm \sqrt{v_i^2 + 2as}}{a}$$

$$\frac{v_f - v_i}{a} = \frac{-v_i \pm \sqrt{v_i^2 + 2as}}{a}$$

$$v_f = \pm\sqrt{v_i^2 + 2as}$$

$$v_f^2 = v_i^2 + 2as$$