With only Work done by Gravity

$$\Sigma W = W_g = \Delta K$$

$$W_g = \int_A^B F_g ds$$

$$W_g = \int_A^B -mgds$$

$$W_g = -mgs_B + mgs_A$$

$$W_g = mgs_A - mgs_B$$

$$mgs_A - mgs_B = \Delta K$$

$$mgs_A - mgs_B = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$mgs_A + \frac{1}{2}mv_A^2 = mgs_B + \frac{1}{2}mv_B^2$$

Define $U_g = mgs$

$$U_{g,A} + K_A = U_{g,B} + K_B$$

With only Work done by Spring

$$\Sigma W = W_e = \Delta K$$

$$W_e = \int_A^B F_e dx$$

$$W_e = \int_A^B -kx dx$$

$$W_e = -\frac{1}{2}kx_B^2 + \frac{1}{2}kx_A^2$$

$$W_e = \frac{1}{2}kx_A^2 - \frac{1}{2}kx_B^2$$

$$\frac{1}{2}kx_A^2 - \frac{1}{2}kx_B^2 = \Delta K$$

$$\frac{1}{2}kx_A^2 - \frac{1}{2}kx_B^2 = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$\frac{1}{2}kx_A^2 + \frac{1}{2}mv_A^2 = \frac{1}{2}kx_B^2 + \frac{1}{2}mv_B^2$$

Define $U_e = \frac{1}{2}kx^2$

$$U_{e,A} + K_A = U_{e,B} + K_B$$

With arbitrary conservative force F_C

$$\Sigma W = W_C = \Delta K$$

$$W_C = \int_A^B F_C ds$$

By definition of Conservative Force, There exists a function $f(s)$ where

$$W_C = f(s_B) - f(s_A)$$

Define $W_C = -\Delta U_C$

$$f(s_B) - f(s_A) = -\Delta U_C$$

$$f(s_B) - f(s_A) = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$-\Delta U_C = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$-(U_{C,B} - U_{C,A}) = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$-U_{C,B} + U_{C,A} = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$U_{C,A} + \frac{1}{2}mv_A^2 = U_{C,B} + \frac{1}{2}mv_B^2$$