Proof of Work-Energy Theorem

$$\Sigma W_{AB} = \int_A^B \Sigma \vec{F} \cdot d\vec{s}$$

$$\Sigma W_{AB} = \int_A^B (\Sigma \vec{F}_x \hat{i} + \Sigma \vec{F}_y \hat{j} + \Sigma \vec{F}_z \hat{k}) \cdot (d\vec{s}_x \hat{i} + d\vec{s}_y \hat{j} + d\vec{s}_z \hat{k})$$

$$\Sigma W_{AB} = \int_A^B \Sigma \vec{F}_x d\vec{s}_x + \Sigma \vec{F}_y d\vec{s}_y + \Sigma \vec{F}_z d\vec{s}_z$$

$$\Sigma W_{AB} = \int_A^B \Sigma \vec{F}_x d\vec{s}_x + \int_A^B \Sigma \vec{F}_y d\vec{s}_y + \int_A^B \Sigma \vec{F}_z d\vec{s}_z$$

$$\Sigma \vec{F}_x = m\vec{a}_x$$

$$\vec{a}_x = \frac{d\vec{v}_x}{dt}$$

$$d\vec{s}_x = \vec{v}_x dt$$

$$\Sigma \vec{F}_x d\vec{s}_x = m\vec{a}_x \vec{v}_x dt$$

$$\Sigma \vec{F}_x d\vec{s}_x = m\frac{d\vec{v}_x}{dt} \vec{v}_x dt$$

$$\Sigma \vec{F}_x d\vec{s}_x = m\vec{v}_x d\vec{v}_x$$

$$\Sigma W_{AB} = \int_A^B m\vec{v}_x d\vec{v}_x + \int_A^B m\vec{v}_y d\vec{v}_y + \int_A^B m\vec{v}_z d\vec{v}_z$$

$$\Sigma W_{AB} = \frac{1}{2} m\vec{v}_x^2 \Big|_A^B + \frac{1}{2} m\vec{v}_y^2 \Big|_A^B + \frac{1}{2} m\vec{v}_z^2 \Big|_A^B$$

$$K_x = \frac{1}{2} m\vec{v}_x^2$$

$$\Sigma W_{AB} = \Delta K_x + \Delta K_y + \Delta K_z$$

$$\Sigma W_{AB} = \Delta K$$